

Entropy Production during Reheating at Late Times and Neutrino Decoupling

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Abstract

Recent theories have proposed a variety of massive particles, like the moduli, whose abundance or decay endangers standard cosmological results. To dilute them, thermal inflation has been proposed, with its own massive scalar flaton field which goes on decaying in the era of MeV-scale temperatures. In this paper, the effect of late-time entropy production on neutrino decoupling during such reheating is investigated by including a term, arising from the rate of entropy production due to scalar decay, in the Boltzmann equation for the neutrino number density. The effect on the decoupling temperature of massless neutrinos is studied. It is found that a lower bound to the scalar decay rate constant can be set at 10^{-22} GeV.

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1 Introduction

Various massive fields like the gravitino, the Polonyi, the moduli, and the dilaton figure in supersymmetric and string theory models[1]. The corresponding particles are long-lived. These fields pose quite serious cosmological problems. If they decay during baryogenesis or nucleosynthesis(BBN), the baryon to photon ratio may be greatly diluted in the first case and nuclear abundances distorted in the second. If they are stable, there is a problem of over-abundance. People have studied the problem in detail and proposed various ranges of masses and decay rates to minimise damage to standard cosmology[2, 3, 4].

An interesting proposal is that of thermal inflation[5, 6]. Quite apart from primordial inflation[7], a scalar field, called the flaton, is used to generate inflation at late times so that the potentially dangerous fields mentioned above may be diluted away. Typically[8], the inflation starts as the thermal energy density falls below the flaton potential at the origin at a temperature of about 10^7 Gev, and stops when the flaton vev reaches $10^{12} - 10^{14}$ Gev at a temperature \sim the flaton mass, which may be taken to be $\leq 10^3$ Gev. Such a flaton field will go on decaying into the Mev-scale temperature era, and the parameters in thermal inflation must be such that the entropy generated due to flaton decay may not affect nucleosynthesis.

Recently, the effect of the decay of a massive particle, like the flaton, on nucleosynthesis has been subjected to detailed investigation[9, 10, 11]. As the parameters and decay schemes of such particles are yet to be generally agreed upon, it is useful to address the general problem of the effect of late-time entropy production, during reheating[12, 13, 14], in the context of BBN. In the present work, neutrino decoupling is studied in the presence of such late-time entropy production, neutrino decoupling temperature being one of the key parameters determining BB nucleosynthesis. Bernstein's method of "pseudo chemical potentials"[15] is used to introduce the entropy production rate directly into the Boltzmann equation for the neutrino number density. The decoupling temperature is estimated in the presence of this entropy generation.

The paper is organised as follows. Section 1 is this Introduction. In Section 2, the late-time development of a universe, once dominated by a scalar field, is set out with some modification of the usual formalism. In Section 3, the method of "pseudo chemical potentials" is applied to modify the Boltzmann equation for the neutrino number density and incorporate into it the late-time entropy production rate directly. In Section 4, the effect of late-time entropy production on neutrino decoupling is studied, using the results of the previous sections. In Section 5, conclusions are stated.

2 Reheating at Late Times

In this section, a two-component universe is considered, made up of the scalar field energy density ρ_ϕ and a radiation energy density ρ_R . It is assumed [16, 8], further, that either the scalar field parameters have not supported resonance, or, in any case, the times considered are late enough to be beyond a possible preheating [17] interlude.

Taking the scalar decay rate constant to be Γ [13], the equation for the evolution of the scalar energy density is

$$\frac{\partial}{\partial t}\rho_\phi + 3H\rho_\phi = -\Gamma\rho_\phi. \quad (1)$$

Defining $\Phi = a^3\rho_\phi$ and $R = a^4\rho_\phi$, where a is the scale factor, (1) becomes

$$\dot{\Phi} = -\Gamma\Phi. \quad (2)$$

From

$$\frac{\partial}{\partial t}[a^3(\rho_\phi + \rho_R)] + p_R\frac{\partial}{\partial t}a^3 = 0,$$

one obtains

$$\dot{R} = a\Gamma\Phi. \quad (3)$$

The Friedmann equation is

$$H^2 = \frac{K^2}{a^4}(\Phi a + R), \text{ with } K^2 = \frac{8\pi}{3M_{Pl}^2}. \quad (4)$$

2.1 Incomplete Φ -domination

In this subsection, it is assumed that $R \ll \Phi a$, so that the square and higher powers of $R/(\Phi a)$ can still be neglected at these times. (4) gives

$$\dot{H} = -\frac{3}{2}H^2\left(1 + \frac{R}{3(\Phi a + R)}\right).$$

With the $R \ll \Phi a$ assumption, one can write

$$\dot{H} \approx -\frac{3}{2}H^2\left(1 + \frac{R}{3\Phi a}\right). \quad (5)$$

For full matter domination, at yet earlier times, R is neglected compared to Φa , and

$$\dot{H} \approx -\frac{3}{2}H^2,$$

leading to $t = \frac{2}{3H}$. In the era of incomplete Φ -domination, the correction term on the RHS of (5) is not neglected, and may be evaluated to the approximation

$$R - R_I = \frac{dR}{da}(a - a_I), \quad (6)$$

where t_I refers to some initial epoch such that $a \gg a_I$. Also, if it is supposed that the scalar decay produces sufficiently copious radiation, $R_I \ll R$. As a correction term is being dealt with, these approximations should not cause much deviation from the actual evolution.

Then, for t sufficiently later than t_I , but within the regime under consideration, one may write, in the correction term on the RHS of (5),

$$R \approx \frac{\Gamma}{H} \Phi a, \quad (7)$$

using (3) and (6). (5) becomes

$$\dot{H} = -\frac{3}{2} H^2 \left(1 + \frac{\Gamma}{3H}\right). \quad (8)$$

Now, evolution will be described by the new variable

$$x = \frac{\Gamma}{H}, \quad (9)$$

instead of the time t . (4) can be written as

$$H^2 = \frac{K^2 \Phi}{a^3} \left(1 + \frac{R}{\Phi a}\right) \quad (10)$$

$$= \frac{K^2 \Phi}{a^3} (1 + x), \quad (11)$$

using (7) in the correction term on the RHS of (10).

Then, (2) leads to the evolution equation for Φ

$$\frac{d\Phi}{dx} = -\frac{2}{3+x} \Phi, \quad (12)$$

with the solution

$$\Phi = \Phi_I \frac{(1 + \frac{1}{3}x_I)^2}{(1 + \frac{1}{3}x)^2}, \quad (13)$$

in the approximation $x \ll 1$. To obtain R , (13) is substituted in (3), a being calculated from (11), using (9). The result is

$$\frac{dR}{dx} = \frac{2}{3} \left(\frac{K}{\Gamma}\right)^{\frac{2}{3}} x^{\frac{2}{3}} \Phi_I^{\frac{4}{3}} \frac{(1 + \frac{1}{3}x_I)^{\frac{8}{3}} (1+x)^{\frac{1}{3}}}{(1 + \frac{1}{3}x)^{\frac{11}{3}}}. \quad (14)$$

Integration will give R .

To compare with earlier work, at early times, one goes to the approximation $x, x_I \ll 1$. (14) becomes

$$\frac{dR}{dx} = \frac{2}{3} \left(\frac{K}{\Gamma}\right)^{\frac{2}{3}} \Phi_I^{\frac{4}{3}} x^{\frac{2}{3}}.$$

On integration, taking $R_I \ll R$,

$$\begin{aligned} R &= \frac{2}{5} \left(\frac{K}{\Gamma}\right)^{\frac{2}{3}} \Phi_I^{\frac{4}{3}} (x^{\frac{5}{3}} - x_I^{\frac{5}{3}}) \\ &= \frac{2}{5} \frac{\Gamma}{K} \Phi_I^{\frac{1}{2}} (a^{\frac{5}{2}} - a_I^{\frac{5}{2}}), \end{aligned}$$

which agrees with [16].

2.2 Incomplete Radiation domination

Now, one may consider even later times when $\Phi a \ll R$, such that $\Phi a/R$ cannot be neglected, but its higher powers can. The Friedmann equation reads

$$H^2 = \frac{K^2}{a^4} R \left(1 + \frac{\Phi a}{R}\right), \quad (15)$$

which leads to

$$\dot{H} = -2H^2 \left[1 - \frac{\Phi a}{4(\Phi a + R)}\right]. \quad (16)$$

If, well into this epoch, the correction term on the RHS of (16) is neglected, the full radiation domination relations are found:

$$\begin{aligned} H &= \frac{1}{2t}, \text{ and} \\ a &= At^{\frac{1}{2}}, \end{aligned} \quad (17)$$

A being a constant.

(2) has, as solution, a falling exponential in t , viz. $\Phi \sim e^{-\Gamma t}$. The new evolution variable $x = \Gamma/H$ is now sought to be introduced in place of t . To do this, instead of taking the falling exponential in t directly, suitable approximations to the correction terms on the RHS of (15) and (16), are first worked out. Let t_0 be a sufficiently late epoch, when $\Phi = \Phi_0 \approx 0$. Then, for use only in the correction terms, one takes

$$\Phi - \Phi_0 = \tilde{\Phi}\left(\frac{1}{t}\right) - \tilde{\Phi}\left(\frac{1}{t_0}\right) = \frac{d\tilde{\Phi}}{d\frac{1}{t}}\Big|_{t_0} \left(\frac{1}{t} - \frac{1}{t_0}\right).$$

Neglecting $\Phi_0, 1/t_0$ compared to $\Phi, 1/t$, respectively, an approximation

$$\Phi \approx \frac{B}{t}, \quad (18)$$

will be used only in the correction terms, i.e. in the correction terms, the falling exponential will be approximated by a rectangular hyperbola. B is a constant. Using this approximation in the correction terms, the final form of Φ will be found below to be a falling exponential in $(1/2)x$, with a pre-exponential correction.

Next, a similar approximation is considered for R . It ought to be mentioned that R refers to the total radiation present, and not only to that produced by decay. However, the change in R is due to ϕ decay and consequent entropy production. In the absence of this decay, $\dot{R} = 0$.

Using (17) and (18) in (3), and, integrating, one obtains, for use only in the correction terms,

$$R - R_E \approx 2AB\Gamma \left(t^{\frac{1}{2}} - t_E^{\frac{1}{2}}\right).$$

If t_E is sufficiently early compared to t , though within the regime under consideration, and there is sufficiently copious radiation production since t_E , it is sufficient to take

$$R \approx 2AB\Gamma t^{\frac{1}{2}}$$

in the correction terms. (17) and (18) are now used to give, in the correction terms,

$$\begin{aligned}\frac{R}{\Phi a} &\approx \frac{\Gamma}{H}, \text{ once again, as in (7),} \\ &= x.\end{aligned}\tag{19}$$

The approximation is now $x \gg 1$. (15) and (16) become

$$\begin{aligned}H^2 &= \frac{K^2}{a^4} R \left(1 + \frac{1}{x}\right), \\ \text{and, } \dot{H} &= -2H^2 \frac{x + \frac{3}{4}}{x + 1}.\end{aligned}\tag{20}$$

This last equation and (2) give

$$\Phi = \Phi_E \left(\frac{4x_E + 3}{4x + 3}\right)^{\frac{1}{8}} e^{-\frac{1}{2}(x-x_E)}\tag{21}$$

One can see that this solution is dominated by the falling exponential. The pre-exponential factor arises because radiation domination is not yet complete, and $(1/2)x$ is not quite Γt . For sufficiently late times t, t_E , one has $x, x_E \gg 1$, (20) gives $H = 1/(2t)$, in the usual way, and (21) reduces to

$$\Phi = \Phi_E e^{-\Gamma(t-t_E)},$$

which agrees with [12].

From (20),

$$a \approx \left(\frac{K}{H}\right)^{\frac{1}{2}} R^{\frac{1}{4}} \left(1 + \frac{1}{x}\right)^{\frac{1}{4}}.\tag{22}$$

Now, on integrating (3), using (21) and (22), one has

$$\begin{aligned}R^{\frac{3}{4}} &= R_{\infty}^{\frac{3}{4}} \\ &- \frac{3}{8} \left(\frac{K}{\Gamma}\right)^{\frac{1}{2}} \left(x_E + \frac{3}{4}\right)^{\frac{1}{8}} \Phi_E e^{\frac{1}{2}x_E} \int_x^{\infty} x^{\frac{3}{8}} e^{-\frac{1}{2}x} dx,\end{aligned}\tag{23}$$

for $x \gg 1$, where R_{∞} is the value of R after decay is finished. The integral in (23) is an incomplete Gamma function, and can be numerically evaluated for any value of x .

The temperature is defined from

$$R = g^* \frac{\pi^2}{30} a^4 T^4,\tag{24}$$

where g^* is the effective number of relativistic degrees of freedom.

The entropy production originates from [18]

$$dS_{\phi} = -\frac{1}{T} d\Phi.\tag{25}$$

3 Entropy Production and the Boltzmann Equation for the Number Density with Scalar Decay

The effect of entropy production on decoupling is studied here with reference to the particular problem of the electron neutrino, ν , decoupling from a thermal bath of electrons, positrons, and photons, in the presence of a decaying scalar field. It is assumed that the scalar field has a very small branching ratio into neutrinos[9]. So, the main process which changes the number density n of the ν neutrinos is

$$\nu(k) + \bar{\nu}(\bar{k}) \longrightarrow \mathcal{F}(p) + \bar{\mathcal{F}}(\bar{p}), \quad (26)$$

where $\mathcal{F}, \bar{\mathcal{F}}$ are fermions, and, k is the energy-momentum 4-vector (E_k, \vec{k}) . Apart from e^-, e^+ , the relevant fermions, at the epoch of ν decoupling, are the neutrinos of the other families.

If there is no other process like scalar decay, it is usual to assume [15] that, in the absence of Fermi degeneracy, the distribution functions may be written as

$$\begin{aligned} f_\nu(k) = f'(k) &= e^{-\alpha'(t) - \beta E_k} \\ f_e(p) &= e^{-\beta E_p} \\ f_{\bar{\nu}}(\bar{k}) = \bar{f}'(\bar{k}) &= e^{-\alpha'(t) - \beta E_{\bar{k}}} \\ f_{e^+}(\bar{p}) &= e^{-\beta E_{\bar{p}}}, \end{aligned} \quad (27)$$

where $T = \frac{1}{\beta}$ is the temperature of the thermal bath, and α' is a time-dependent "pseudo chemical potential", introduced to take into account the departure of the decoupling neutrinos from equilibrium. (In (27), it has been assumed that \mathcal{F} is the electron.)

Decoupling is, in this case, governed by the integrated Boltzmann equation

$$\dot{n} + 3Hn = - \langle \sigma |v| \rangle (n^2 - n_{EQ}^2), \quad (28)$$

where n_{EQ} is the equilibrium number density,

$$n_{EQ} = \int \frac{g d^3 k}{(2\pi)^3} e^{-\beta E_k} = T^3 / \pi^2, \quad (29)$$

$$n \approx n_{EQ} e^{-\alpha'} [15], \quad (30)$$

and

$$\langle \sigma |v| \rangle = \frac{1}{n_{EQ}^2} \int dK d\bar{K} e^{-\beta E_k} e^{-\beta E_{\bar{k}}} I, \quad (31)$$

I being the invariant integral

$$I = \int dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2. \quad (32)$$

g is the number of spin degrees of freedom, $dK = g d^3 k / [(2\pi)^3 2E_k]$ etc., and $|\mathcal{M}|^2$ the spin-averaged square of the modulus of the relevant matrix element.

Considering the process (26), and, assuming CP-invariance and the absence of Fermi degeneracy, the Boltzmann equation for the neutrino number density n may be written[15, 18]

$$\frac{1}{a^3} \frac{\partial}{\partial t} (a^3 n) = - \sum_{\mathcal{F}} \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}_{\mathcal{F}}|^2 (f' \bar{f}' - f \bar{f}). \quad (33)$$

f' corresponds to the ν neutrino and f to the fermion \mathcal{F} .

Then,

$$\begin{aligned} \frac{1}{a^3} \frac{\partial}{\partial t} (a^3 n) &= - \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 (f' \bar{f}' - f \bar{f}) \\ &- \sum_i \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}_i|^2 (f' \bar{f}' - f_i \bar{f}_i), \end{aligned} \quad (34)$$

where, on the RHS, the first term refers to the process

$$\nu + \bar{\nu} \longrightarrow e^- + e^+, \quad (35)$$

and the second to the processes

$$\nu + \bar{\nu} \longrightarrow \nu_i + \bar{\nu}_i, \quad i = \mu, \tau, \quad (36)$$

f_i being the ν_i distribution function.

The injection of entropy will cause changes in the distribution functions. One way to tackle the situation is to use the Boltzmann equation for the ν distribution function, instead of the integrated Boltzmann equation for the number density, include $\nu + e^- (e^+) \rightarrow \nu + e^- (e^+)$ processes, and proceed numerically [9]. However, the integrated Boltzmann equation for the number density will be used here to study decoupling at late times, when the entropy generation rate is small, in the following way.

It has been assumed[9], in accordance with current models, that the ϕ does not decay into neutrinos. But, ref.[9] finds that, for reheat temperatures $< 7MeV$, the neutrino distribution function is materially affected by the decay. How can entropy injected into the e, e^+, γ sector affect neutrino distribution function and, hence, neutrino density? The transfer of entropy must occur via process (35), and this must lead to extra terms on the RHS of (28) if n is to be affected. From (33), this can only occur if f , the electron distribution function, changes due to entropy injection.

The electromagnetic interactions, faster than the expansion, will tend quickly to thermalise the e, e^+, γ (i.e. establish kinetic and chemical equilibrium in this sector). But recent work[19, 20] has shown that this thermalisation of ϕ decay products is not nearly instantaneous. A massive ϕ , as it goes on decaying, continually injects highly energetic light particles, with energies much above the thermal mean. There is no possibility of inverse decay and so the term from scalar decay in the Boltzmann equation for, say, the electron distribution function opposes the equilibrating effect of the electromagnetic interaction. As long as the decay does not become negligible, complete thermalisation of the distribution

functions of the decay products is not assured. How quickly complete thermalisation will occur, i.e. the epoch of thermalisation, depends, not only on the relative rapidity of the electromagnetic interactions and the expansion, but also on the parameters of ϕ . In fact, the requirement of thermalisation before a specific epoch (e.g. BBN) has been used to derive bounds on these parameters[19, 20, 11].

The thermalisation of rapidly interacting, light decay products, of a heavy boson was considered in ref.[15]. There, the progress, in time, of the light particle distribution function g was directly considered. A solution $g = e^{-\alpha-\beta E}$ of the Boltzmann equation was mooted, where α was a time-dependent "pseudo chemical potential", although, the actual calculation was done with a δ function distribution, because the problem was simplified by assuming that all the heavy bosons decayed at one instant. Following this lead, it will be supposed, in this paper, that the injection of entropy into the bath, due to scalar decay, can be taken care of by introducing a small time-dependent "pseudo-chemical potential" $\alpha(t)$ into the electron and positron distribution functions, in addition to the potential $\alpha'(t)$ already introduced in the $\nu, \bar{\nu}$ distribution functions.

Of course, the electromagnetic interactions, being so fast, will cause α to be very small.

In effect, then, the scheme in (27) will be changed, in the absence of Fermi degeneracy, to

$$\begin{aligned} f_\nu(k) = f'(k) &= e^{-\alpha'-\beta E_k} \\ f_e(p) = f(p) &= e^{-\alpha-\beta E_p} \\ \bar{f}' &= e^{-\alpha'-\beta E_{\bar{k}}} \\ \bar{f} &= e^{-\alpha-\beta E_{\bar{p}}}. \end{aligned} \tag{37}$$

The entropy density s_j , contributed by a particle j with a distribution function $h_j(q)$, may be defined, in the absence of degeneracy, as

$$s_j = - \int g_j \frac{d^3q}{(2\pi)^3} (h_j \ln h_j - h_j).$$

Following [15], the covariant divergence of the entropy density current for a component j , in a process like that described by (35), can be shown to be

$$\frac{1}{a^3} \frac{\partial}{\partial t} (s_j a^3) \approx - \int g_j \frac{d^3q}{(2\pi)^3} \frac{C_j}{E_j} \ln h_j,$$

where C_j is the relevant collision integral as defined ^{b,3} in [18]. For example, for the electron,

$$\begin{aligned} \frac{1}{a^3} \frac{\partial}{\partial t} (s_e a^3) &= - \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\ &\times \ln f(f' \bar{f}' - f \bar{f}). \end{aligned}$$

³This differs from the definition in [15] by a factor of E_j .

It can immediately be seen that the covariant divergence of the total entropy density current for the process (35) is, assuming energy conservation,

$$\begin{aligned}
\frac{1}{a^3} \frac{\partial}{\partial t} (s_{tot} a^3) &= \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\
&\quad \times 2(\alpha' - \alpha)(e^{-2\alpha} - e^{-2\alpha'}) e^{-\beta(E_k + E_{\bar{k}})} \\
&\approx \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\
&\quad \times 2\alpha'(1 - e^{-2\alpha'}) e^{-\beta(E_k + E_{\bar{k}})} \\
&\quad + \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\
&\quad \times (-2\alpha)(1 - e^{-2\alpha} + 2\alpha') e^{-\beta(E_k + E_{\bar{k}})}. \tag{38}
\end{aligned}$$

Here, α , being the effect of entropy injection, is assumed to be small at late times. Squares and higher powers of α have been neglected.

The first term on the RHS of (38) is the covariant divergence of the entropy density current for $\alpha = 0$, and the second term is proportional to α . So, it can be concluded that the second term measures, approximately, to first order in α , the contribution of ϕ -decay to the covariant divergence of the entropy density current, corresponding to process (35), viz.

$$\begin{aligned}
\epsilon \frac{1}{a^3} \frac{\partial S_\phi}{\partial t} &= -2\alpha(1 - e^{-2\alpha'} + 2\alpha') \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\
&\quad \times e^{-\beta(E_k + E_{\bar{k}})}, \tag{39}
\end{aligned}$$

where S_ϕ is defined in (25) and ϵ is the fraction of the entropy current from ϕ -decay which contributes to process(35).

Again, in the presence of ϕ -decay,

$$\begin{aligned}
\frac{1}{a^3} \frac{\partial}{\partial t} (a^3 n) &= - \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 \\
&\quad \times (e^{-2\alpha'} - e^{-2\alpha}) e^{-\beta(E_k + E_{\bar{k}})} \\
&\quad - \sum_i \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}_i|^2 (f' \bar{f}' - f_i \bar{f}_i) \tag{40}
\end{aligned}$$

$$\begin{aligned}
&= -(n^2 - n_{EQ}^2) < \sigma |v| > \\
&\quad - 2\alpha \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}|^2 e^{-\beta(E_k + E_{\bar{k}})} \\
&\quad - \sum_i \int dK d\bar{K} dP d\bar{P} (2\pi)^4 \delta^4(p + \bar{p} - k - \bar{k}) |\mathcal{M}_i|^2 (f' \bar{f}' - f_i \bar{f}_i) \tag{41}
\end{aligned}$$

to first order in α , using (30), (31), (37) and (34). $< \sigma |v| >$ refers to (35).

Now, the finer point that, in this epoch, the electron neutrino has both charged and neutral current interactions, while the other neutrinos have only neutral current interactions

(muons and taons having already decoupled), will be neglected, and it will be assumed that the charged and neutral current interactions fall out of equilibrium together. Also, neutrinos of all types will be assumed to be massless. Then, apart from their contributions to ρ_R as separate species, the neutrinos of the different types may be treated as identical, and it may be supposed that they decouple together. In this approximation, the difference between f' and f_i may be neglected, and the terms within the summation sign dropped, in comparison to the other terms on the RHS of (41).

So, to first order in α , the integrated Boltzmann equation can be written, in the presence of ϕ -decay, as

$$\begin{aligned} \frac{1}{a^3} \frac{\partial}{\partial t}(a^3 n) &= \dot{n} + 3Hn \\ &= -(n^2 - n_{EQ}^2) \langle \sigma |v| \rangle + \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{1}{a^3} \frac{\partial S_\phi}{\partial t}, \end{aligned} \quad (42)$$

from (39) and (41). (42) will replace (28) in the presence of scalar decay.

4 Effect of Scalar Decay on Neutrino Decoupling

Decoupling will be supposed to set in when the RHS of (42) becomes less than $3Hn$. To analyse the two terms on the RHS, radiation domination is first assumed. A change of variables is introduced:

$$x = \frac{\Gamma}{H}, Y = nx^{\frac{3}{2}}, Y_{EQ} = n_{EQ} x^{\frac{3}{2}}. \quad (43)$$

Also, (20) gives, in the era of neutrino decoupling,

$$H = \frac{4.461 \times 10^{-19}}{GeV} T^2 \left(1 + \frac{1}{x}\right)^{\frac{1}{2}}, \quad (44)$$

taking $g^* = 10.75$. The LHS of (42), then, becomes $(2\Gamma/x^{\frac{3}{2}})dY/dx$. Near the start of the decoupling regime, one can put

$$Y = Y_{EQ} + \Delta,$$

where Δ is very small, and it is possible [18, 21, 22] to set

$$\begin{aligned} \frac{d\Delta}{dx} &= 0, \text{ and,} \\ \frac{dY}{dx} &= \frac{dY_{EQ}}{dx}. \end{aligned} \quad (45)$$

(43), (44) and (29) are used to find that $dY_{EQ}/dx > 0$, i.e. the LHS of (42) is positive. Use of (30) shows that each term on the RHS of (42) is of the same sign. So, for the equation to hold,

$$\alpha' > 0, \quad (46)$$

and each of the terms on the RHS of (42) is positive. This means, that when decoupling sets in, each term must be separately less than $3Hn$.

$T_D(\text{Mev})$	α'
0.75	4.33275
1	3.4705
1.25	2.8038
1.5	2.264
1.75	1.8174
2	1.4470
2.25	1.1438
2.5	0.901
2.75	0.7109
3	0.5644

Table I: Neutrino α' values at decoupling for different decoupling temperatures without scalar decay

The value of α' chosen will determine the temperature of decoupling. This choice is fixed as follows. In the absence of ϕ -decay, there is only the first term on the RHS of (42). By setting this term equal to $3Hn$, a set of values for α' is obtained for neutrino decoupling temperatures T_D between 0.75 and 3 Mev, a range which more or less safeguards BBN. It may be mentioned that, at present, observed ${}^4\text{He}$ abundance values, Y_p , fall between 0.234 [23] and 0.244 [24]. A rough estimate, as indicated in [18], gives corresponding decoupling temperatures ~ 1 Mev.

Next, for these values of α' , the full RHS of (42) is put equal to $3Hn$, and the decoupling temperatures, corresponding to different values of the scalar decay constant Γ , are worked out.

The first step uses

$$-\frac{(n^2 - n_{EQ}^2) \langle \sigma|v| \rangle}{3Hn} \leq 1. \quad (47)$$

As already discussed, in the evaluation of $\langle \sigma|v| \rangle$, it is enough to consider the process (35), in the s-channel with Z exchange and the t-channel with W exchange. Evaluating (32), and, then, (31), using (29),

$$\begin{aligned} \langle \sigma|v| \rangle &= \frac{8}{\pi} G_F^2 [(C_{Ve} + 1)^2 + (C_{Ae} + 1)^2] T^2 \\ &= \frac{4.112 \times 10^{-10}}{\text{Gev}^4} T^2. \end{aligned} \quad (48)$$

The mass of the electron has been neglected. G_F is the Fermi constant.

Taking $H = \frac{4.461 \times 10^{-19}}{\text{Gev}} T^2$, in the absence of ϕ -decay, and using (29), (30), and (48), one obtains the values of α' shown in Table I.

Now, the second term on the RHS of (42), i.e. the contribution of scalar decay, is

considered. Using (21), this term becomes

$$\begin{aligned} \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{1}{a^3} \frac{\partial S_\phi}{\partial t} &= - \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{\dot{\Phi}}{T a^3} \\ &= \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{\Gamma \Phi_E \left(\frac{4x_E+3}{4x+3}\right)^{\frac{1}{8}} e^{-\frac{1}{2}(x-x_E)}}{T a^3}. \end{aligned} \quad (49)$$

This term, for $x \gg 1$, will be dominated by the exponential. So, rather drastic assumptions will be made to estimate the pre-exponential, in the absence of phenomenological information about x_E and Φ_E . x_E refers to any epoch sufficiently early in the regime of incomplete radiation domination, and is set equal to 1, so that $\Phi_E = R_E/a_E$. This is quite an approximation, because (21) is faithful to $\Phi \sim e^{-\Gamma t}$ provided $x_E, x \gg 1$. With this approximation, T_E is, then, estimated by setting $x = 1$ in (44). Next, one puts $R_E = (\pi^2/30)g_E^* a_E^4 T_E^4$ and $g_{SE}^* a_E^3 T_E^3 = g_S^* a^3 T^3$. This last is the weakest assumption because it is only true if there is conservation of entropy, with $a \propto T^{-1}$ behaviour. But, entropy is being produced, and, in fact, in the regime of Φ domination, $T \propto a^{-\frac{3}{8}}$ [12]. In the early part of the regime of incomplete radiation domination, too, one expects a deviation from $a \propto T^{-1}$. However, the expectation is that the exponential dominates the entropy current and its effect on ν decoupling, and one can take the view that the pre-exponential is being estimated only to order of magnitude. Neglect of the deviation from $a \propto T^{-1}$ behaviour in the pre-exponential, and the other approximations, should not make too much difference to an order of magnitude calculation of lower bounds on the scalar decay constant.

With all these assumptions, the condition of neutrino decoupling in the presence of scalar decay becomes, using (44),

$$\begin{aligned} - \frac{(n^2 - n_{EQ}^2) < \sigma |v| >}{3Hn} + \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{1}{a^3} \frac{\dot{S}_\phi}{3Hn} &= \\ \frac{208.945 \sinh \alpha' \Gamma_0^{\frac{3}{2}}}{x^{\frac{1}{4}}(x+1)^{\frac{5}{4}}} + \frac{\epsilon 10.4927 x^{\frac{5}{4}}(1+x)^{\frac{1}{4}} e^{-\frac{1}{2}(x-1)}}{(x + \frac{3}{4})^{\frac{1}{8}}(1 - e^{-2\alpha'} + 2\alpha')e^{-\alpha'}} &\leq 1, \end{aligned} \quad (50)$$

where

$$\Gamma_0 = \frac{\Gamma}{10^{-22} GeV}.$$

It must be remembered that $x > 1$ in (50), corresponding to radiation domination.

4.1 Numerical Results

Three values of α' are used from Table I, namely, those corresponding to decoupling in the absence of scalar decay at $T_D = 1, 2, 3 Mev$. In each case, the RHS of (50) is put equal to 1, and the resulting equation solved for $x = x_D$, taking different values of Γ_0 . The corresponding decoupling temperatures T'_D are found from (44), using $x = \Gamma/H$ to rewrite it as

$$\frac{T}{Mev} = \frac{14.9718 \sqrt{\Gamma_0}}{(x^2 + x)^{\frac{1}{4}}}. \quad (51)$$

$\frac{T_D=1Mev}{\alpha'=3.4705}$			$\frac{T_D=2Mev}{\alpha'=1.4470}$			$\frac{T_D=3Mev}{\alpha'=0.5644}$		
Γ_0	x_D	T'_D	Γ_0	x_D	T'_D	Γ_0	x_D	T'_D
100	22414	1.00	100	5603	2.00	100	2490	3.00
10	2241	1.00	10	560	2.00	10	248	3.00
1	223	1.00	1	55.2	2.01	1	24.2	3.01
0.1	22.6	0.985	0.25	16.3	1.83	0.5	15.4	2.65
0.02	16.5	0.513	0.1	13.7	1.26	0.1	12.9	1.30
-	-	-	0.02	13.0	0.576	0.02	12.7	0.583

Table II: Decoupling Temperatures T'_D in Mev for different values of the Scalar Decay parameter $\Gamma_0 = \Gamma/(10^{-22}Gev)$

ϵ would be 1 if the entire entropy current of decay contributed to process (35). There are two points here. The decay modes of ϕ are unknown, and it is assumed that decay products, more massive than the electron, rapidly decouple, and almost all the entropy current of decay ends up first in the e, e^+, γ sector and, then, goes on to contribute to the entropy current of process (35) via a non-zero α . Also, $\alpha \neq 0$ means that there will be a small contribution of the entropy current from ϕ -decay to the e, e^+, γ processes also. But, because α is so small, ϵ has been put equal to 1.

The results are shown in Table II. The results indicate that, for values of the scalar decay constant $\Gamma > 10^{-22}Gev$, the neutrino decoupling temperature is not appreciably affected by scalar decay.

It is necessary to check what happens if there is matter domination. Taking $x_I \ll 1$ in (13), extrapolating this equation to $\Phi = \Phi_E$ at $x = x_E = 1$, and evaluating Φ_E with the same set of assumptions as in the case of radiation domination, the criterion of decoupling will become, in place of (50),

$$\begin{aligned}
& - \frac{(n^2 - n_{EQ}^2) \langle \sigma|v| \rangle}{3Hn} + \frac{\epsilon}{(1 - e^{-2\alpha'} + 2\alpha')} \frac{1}{\alpha^3} \frac{\dot{S}_\phi}{3Hn} = \\
& \frac{208.945 \sinh\alpha' \Gamma_0^{\frac{3}{2}}}{x^{\frac{1}{4}}(x+1)^{\frac{5}{4}}} + \frac{17.3934x^{5/4}(1+x)^{1/4}}{(1 - e^{-2\alpha'} + 2\alpha')e^{-\alpha'}(1 + \frac{1}{3}x)^2} \leq 1. \tag{52}
\end{aligned}$$

Matter domination implies that $x < 1$ in (52). Writing the two terms on the RHS of (52) as A and B, $A + B \leq 1$. An examination of B will show that as the temporal variable x increases, B increases. So, if B predominates in an epoch, there can be no decoupling in that epoch. A little numerical work shows that for those values of Γ (greater than a critical value which depends on α') for which A predominates over B, $A+B$ does not fall below 1, for values of $x < 1$ (matter domination). So, no decoupling is possible in the epoch of matter domination.

The numerical work shows, therefore, that for neutrino decoupling to proceed without significant change, in the presence of scalar decay, there must be radiation domination, in the sense that $x = \rho_R/\rho_\phi > 1$, and the scalar decay constant Γ must be $> 10^{-22}Gev$.

This corresponds to reheating temperatures $> 8.5\text{Mev}$, taking for the reheating temperature, the definition of ref.[9] :

$$\frac{T_R}{\text{Gev}} = 0.554 \left(\frac{\Gamma}{\text{Gev}} \right)^{\frac{1}{2}} \sqrt{\frac{2.4 \times 10^{18}}{\text{Gev}}}.$$

In [9], it was found that the neutrino distribution function was distorted, and the effective number of neutrino types, N_{eff} , started to decrease below three, as the reheating temperature fell below 7 Mev, and standard BBN was endangered. So, the present results show broad agreement with those of [9] on the question of the minimum reheating temperature which safeguards standard BBN.

5 Conclusions

The effect of entropy injection due to scalar decay on neutrino decoupling has been studied here by introducing the entropy production rate term directly into the Boltzmann equation for the neutrino *number density*. The method adopted was to introduce a small "pseudo chemical potential" α into the electron distribution function, in addition to the standard introduction of such a potential in the neutrino distribution function. As the electromagnetic interactions tend to thermalise the electron distribution function, this α is bound to be very small. Its function is to transmit the entropy current, arising from scalar decay, from the e, e^+, γ sector, to the neutrino sector through processes like $e^- + e^+ \rightarrow \nu + \bar{\nu}$.

The conclusion drawn regarding the condition of validity of standard BBN from this study of neutrino decoupling, viz. reheating temperature > 8.5 Mev, agrees broadly with the conclusions drawn in the literature from calculation of the form of the neutrino distribution function and the effective number of neutrino types, by numerical integration of the Boltzmann equation for the neutrino *distribution function*[9].

It is found that, in the presence of entropy injection due to scalar decay, the universe must not be matter dominated ($\rho_\phi > \rho_R$) in the neutrino decoupling epoch, as in that case there is no decoupling; rather, the universe must have attained radiation domination in the sense that $\rho_\phi < \rho_R$. Consideration of the effect of scalar decay on decoupling temperatures in the regime of radiation domination leads to a *lower bound* of about 10^{-22} Gev on the scalar decay constant Γ .

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