

# Novel Scaling Relation of the Energy Spacing Distribution in Quantum–Hall Systems

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## Abstract

The shape analysis of the energy spacing distribution  $P(s)$  obtained from numerical simulation of two dimensional disordered electron systems subject to strong magnetic fields is performed. In the present work we reanalyze the data obtained in a previous publication. Special moments of the  $P(s)$  function related to Rényi–entropy differences show a *novel* scale invariant relation that is attributed to the presence of one–parameter scaling. This relation seems to show both deviations and universality depending on which Landau–band is considered and whether the disorder is correlated or uncorrelated. Furthermore, our analysis shows the existence of an *huge, however, irrelevant length scale* in the case of the second lowest Landau–band and no disorder correlations that completely disappears with the introduction of disorder correlations on the range of one magnetic length.

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The study of the statistical properties of spectra of disordered mesoscopic systems attracted much attention during the past decade. One of the main issues of this research became the study of the disorder induced localization–delocalization transition in  $d \geq 2$ . In the metallic regime while the correlation length is much larger than the system size the states are expected to be delocalized producing spectral fluctuations that are well described by the random matrix theory (RMT). [1] The most prominent feature of this theory is the presence of level repulsion depending on the global symmetry of the system (orthogonal, unitary or symplectic). On the other hand in the insulating regime the states are exponentially localized therefore level clustering occurs. In the critical regime, however, the states are multifractal objects still showing similarities to RMT behavior although new statistics is expected reflecting the heavy spatial fluctuations of the eigenstates. The main purpose of the present work is to see how spectral statistics change when approaching the transition in quantum–Hall systems.

One of the tools to analyze the spectral statistics is the study of the nearest neighbor energy spacing distribution function  $P(s)$ . This function tends to  $P_P(s) = \exp(-s)$  in the insulating regime corresponding to Poisson statistics and to  $P_W(s) = (32/\pi^2)s^2 \exp(-4s^2/\pi)$  corresponding to Wigner statistics that is a good description of the RMT behavior in the metallic regime. In the critical regime the form of  $P(s)$  is yet unknown.

The system we study is a simplified version of the model of electrons moving in two dimensions subject to a perpendicular, strong magnetic field. We refer to all details to Ref. [2]. Here we only mention that  $N \times N$  random Landau–matrices were diagonalized and the unfolded spectra  $\{x_i\}$  were calculated for many samples in order to ensure good statistics. The size of the matrix  $N$  corresponds to the system size  $L = \sqrt{2\pi N}\ell$ , where  $\ell = \sqrt{\hbar/eB}$  is the magnetic length. Both the lowest and the second lowest Landau–band are considered. In either case the effect of Gaussian disorder correlations are studied with a correlation length  $\zeta = 0$ , or  $\ell$ .

Based on the general understanding of the integer Quantum–Hall Effect (QHE) [3] the Landau–bands contain localized states except in the critical region in the band–center that

reduces to an extended level at  $E_c$  in the thermodynamic limit. In the vicinity of  $E_c$  one-parameter scaling is expected

$$Z(E, L) = f(L/\xi), \quad \xi \sim |E - E_c|^{-\nu} \quad (1)$$

where  $Z$  is quantity describing the shape of  $P(s)$  functions and  $\xi$  is the localization length,  $\nu$  is its exponent. Due to the relation between  $N$  and  $L$  in the representation of the unfolded levels the scaling relation reads as

$$Z(x, N) = \tilde{f}(x/N^\delta), \quad \nu = \frac{1}{2(1-\delta)} \quad (2)$$

Such quantities that obey the scaling law (2) have been successfully applied in the case of the three dimensional Anderson model [4]. They are differences of generalized Rényi-entropies

$$-\ln(q) = -\ln\left(\frac{\mu_1^2}{\mu_2}\right) \quad \text{and} \quad S_{str} = \frac{\mu_S}{\mu_1} + \ln\left(\frac{\mu_2}{\mu_1}\right) \quad (3)$$

where  $\mu_k = \int s^k P(s) ds$  and  $\mu_S = -\int s \ln s P(s) ds$ . Obviously for the  $P(s)$  by definition  $\mu_1 = 1$ . These quantities have been originally used in the context of wave function shape analysis (cf. [5]) but they are appropriate to study shapes of distribution functions in general, e.g. the  $P(s)$  [6]. Our parameters may be appropriately rescaled to

$$\tilde{Q} = \frac{-\ln(q) + \ln(q_W)}{-\ln(q_P) + \ln(q_W)} \quad \text{and} \quad \tilde{S} = \frac{S_{str} - S_W}{S_P - S_W} \quad (4)$$

where  $P$  and  $W$  refer to the corresponding limiting cases the Poisson- and the Wigner-statistics. In this way we obtain  $0 \leq \tilde{Q}, \tilde{S} \leq 1$ , where the value 0 corresponds to the Poisson-limit and 1 to the Wigner-limit. In Fig. (1) we present the results already in terms of the rescaled variable  $x/N^\delta$ . For the lowest Landau-band ( $n = 0$ ) the critical exponent  $\nu = 2.4$  is consistent with the literature, on the other hand for  $n = 1$  it is 7.1. The latter value is obviously wrong as it has been calculated assuming a simple one-parameter scaling. In the figure our parameters show almost full metallic behavior ( $\tilde{S} \approx \tilde{Q} \approx 1$ ) for  $n = 1$  and  $\zeta = 0$ . This effect should be attributed to the existence of a huge, albeit irrelevant

length scale. [7] It is also clear that the introduction of disorder correlations makes this pathological behavior disappear. The critical indices obtained from  $\delta$  in Fig. (1) are in the range of 1.7–2.4 and show similar trends.

Figure (2) shows the data on a  $\tilde{S}$  vs.  $\tilde{Q}$  plot. As a comparison we also plot the evolution of our parameters for the interpolating  $P_\alpha(s)$

$$P(s) = c(1 + \alpha) s^{2\alpha} \exp(-c s^{1+\alpha}), \quad (5)$$

where  $c$  is obtained from  $\int sP(s)ds = 1$ , as  $\alpha$  runs from 0 (Poisson) to 1 (Wigner) limits.

It is remarkable that the data for  $n = 0, 1$  and  $\zeta = 0$  fall on more or less the same curve, and so it does for  $\zeta = \ell$ , however, these two cases ( $\zeta = 0, 1$ ) seem to deviate at least in the band-tails. On the other hand as for the band-center, the data close to  $\tilde{Q} \rightarrow 1$  and  $\tilde{S} \rightarrow 1$  fall on the curve corresponding to Eq. (5). It seems that further analytical as well as numerical work is required in order to clarify this kind of correlation.

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## FIGURES

FIG. 1. The rescaled quantities  $\tilde{S}$  (solid symbols) and  $\tilde{Q}$  (open symbols) as a function of the rescaled variable  $x/N^\delta$  for different Landau-bands ( $n = 0, 1$ ) and disorder correlations  $\zeta = 0, \ell$ . The data represent different system sizes:  $N = 200$  (squares), 400 (triangles), 600 (diamonds), and 800 (circles). (a) is for  $n = 0$  and  $\zeta = 0$ , (b) is for  $n = 0$  and  $\zeta = \ell$ , (c) is for  $n = 1$  and  $\zeta = 0$ , (d) is for  $n = 1$  and  $\zeta = \ell$

FIG. 2. Correlations between  $\tilde{S}$  and  $\tilde{Q}$ . The symbols represent different system sizes as in Fig. 1. Open (solid) symbol stands for the lowest (second) Landau-band. (a) no disorder correlations, (b) with  $\zeta = \ell$  disorder correlations. The continuous curve represents the relation for the  $P(s)$  in Eq. (5).

Figure 1. Varga et al.

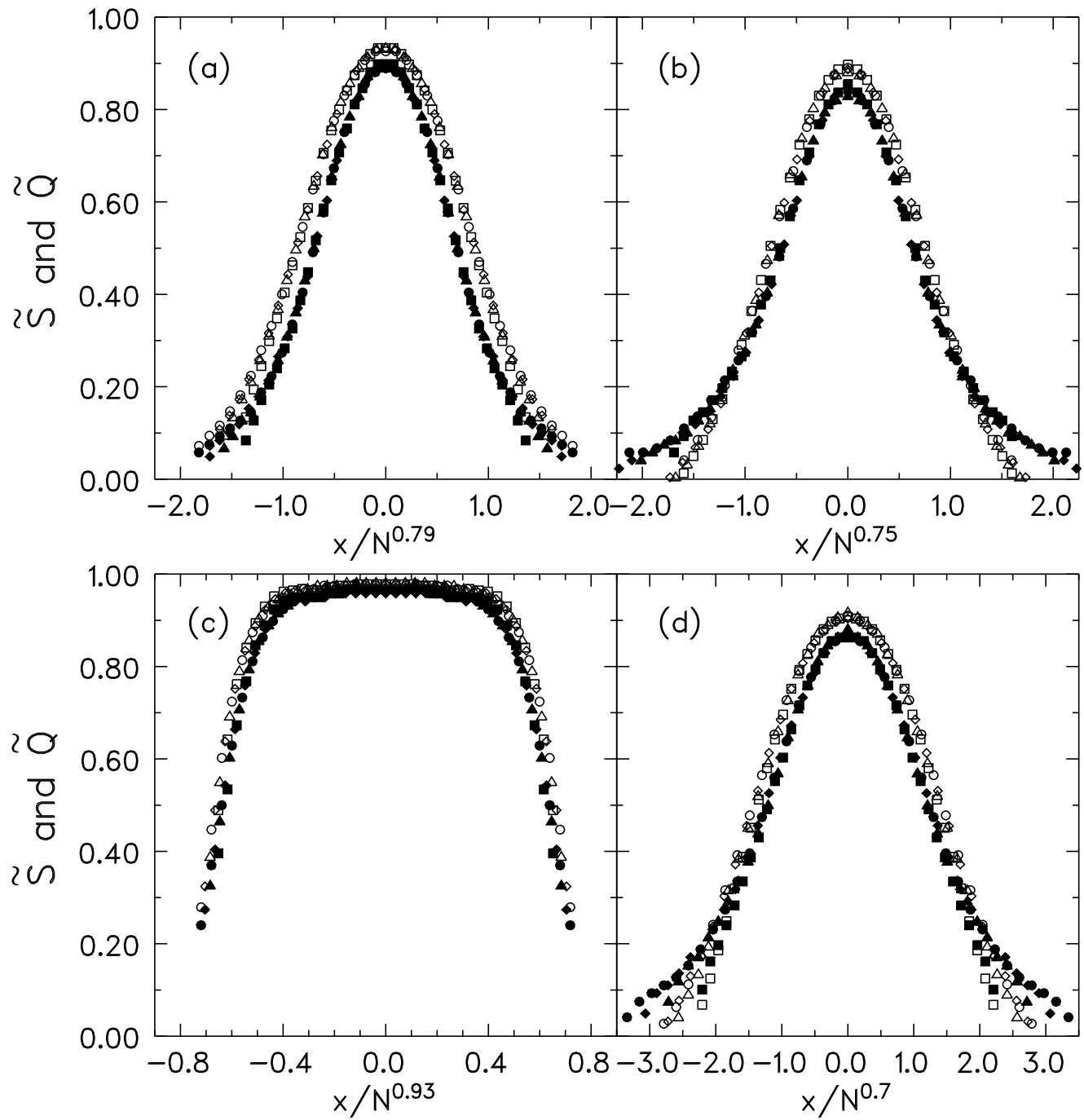


Figure 2. Varga et al.

