ABSTRACT
Peer-to-Peer (P2P) communications are getting increasingly popular for video streaming over the Internet. This paper presents a Nash bargaining based game theoretic framework to address issues of network efficiency, fairness, utility maximization and incentives in bandwidth allocation of P2P streaming systems. We formulate the upload bandwidth allocation problem as cooperative bargaining games that reflects the cooperative strategies of the peers. By using the method of Nash bargaining solution in the games, two objectives, efficiency and fairness, can be implemented through the Pareto optimal bandwidth allocation. Considering the selfishness of peers, we introduce pricing and budget as incentive mechanism. It encourages the peer to share the resource as much as possible to obtain the sufficient budget so as to maximize the utility in bandwidth allocation. Then, a distributed algorithm based on dual-decomposition is proposed. Finally, we verify the performance of the proposed algorithm by numerical results.

Categories and Subject Descriptors
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Peer-to-Peer networks, video streaming, bandwidth allocation, Nash bargaining solution

1. INTRODUCTION
Peer-to-Peer (P2P) video streaming is widely considered as a promising platform for delivering high quality video content at the global scale. In such network settings, peers are expected to operate without control and coordination by a central agent. They self-organize through local interaction with surrounding neighbors and get into virtual proximity of each other through overlay structures, over which they exchange video content. A common problem in a P2P system is that a peer wishes to achieve upload bandwidth from its neighbors as much as possible, while avoid sharing the resource with other peers. This egotistic behavior, also known as free-riding problem, leads to the degradation of the system performance, with respect to the overall network utility and the peer fairness. For a healthy P2P system, the peer requires to provide a fair upload bandwidth allocation among its neighbors and to be encouraged to contribute its resource to the system.

In recent years, game theory has been widely utilized for solving resource allocation problems. In particular, many studies in the literature have focused on modeling bandwidth allocation problem for P2P streaming by noncooperative game theory. For instance, Guo et al. in [4] proposed an upload bandwidth auction algorithm, in which the peer uses its own resource as payment for the required upload bandwidth. Zou et al. in [12] developed a content-aware bandwidth auction model for scalable streaming in P2P networks, which formulates multi-overlay multi-layer bandwidth request and allocation problems as auction games. However, the solutions with noncooperative game theory ignore the issue of peer fairness.

In this paper, we shift our focus to Nash bargaining solution (NBS). As a branch of cooperative game theory, NBS has been extensively employed in wireless networks to achieve an effective tradeoff between the overall network efficiency and the user fairness. We refer our readers to [1],[9],[2] and the references herein for some existing works in wireless networks. However, Nash bargaining solutions for wireless networks cannot be directly applied to P2P networks, as each peer plays as server and client at the same time, and faces the dilemma of devoting the resource to its own benefit by requesting contents from neighbors, versus acting altruistically by providing contents to others. Therefore, some incentive mechanisms should be considered to encourage peers to share information with others. To the best of our knowledge, this paper is the first work that employs NBS with incentives in designing bandwidth allocation strategies for P2P streaming.

Monetary payment, one main class of incentive mechanisms, has been studied in the literature for mitigating the free-riding problem in P2P networks. With monetary payment incentive, as can be seen in [10],[11],[7], the peers are...
charged for their download and compensated for their upload. The payment can be made by either virtual currency or real currency. In this study, we introduce a budget-based incentive scheme to encourage cooperation, in which the peer increases its budget by sharing the contents and uses the available budget as the payment for its download service.

In this paper, we propose a Nash bargaining based game theoretic framework to address issues of network efficiency, fairness, utility maximization and incentives in bandwidth allocation of P2P streaming systems. Our main contributions are as follows: First, the upload bandwidth allocation problem is formulated as cooperative bargaining games that reflects the cooperative strategies of the peers. Using the method of the NBS in the games, two objectives, efficiency and fairness, can be implemented through the Pareto optimal bandwidth allocation. Next, considering the selfishness of peers, we introduce pricing and budget as incentive mechanism. It encourages the peer to share the resource as much as possible to obtain the sufficient budget so as to maximize the utility in bandwidth allocation. Finally, a distributed algorithm based on dual-decomposition is proposed for implementing the optimal and fair bandwidth allocation in a decentralized manner.

The remainder of this paper is organized as follows: The system model and the concept of the NBS is presented in Section II. In section III, we formulate the peer selection and incentive mechanism, and then discuss the bandwidth optimization allocation algorithm based on NBS. Section IV presents the simulation results. Finally, we conclude the paper in section V.

2. SYSTEM MODEL

2.1 Modeling of A P2P System

Assume that there are \( N \) peers (nodes), represented by \( V_1, V_2, \ldots, V_N \), in the system. The topology can be modeled as a directed graph \( G = (V, E) \), where \( V \) is the set of peers and \( E \) is the set of links between peers. Let \( C_i \) represent the available upload capacity of node \( V_i \). Each peer acts as server and client simultaneously. When a peer works as a server, it serves other peers independently, and makes upload bandwidth allocation decision independently. When a peer plays as a client, it chooses the server and the amount of bandwidth it required from the server. Fig. 1 shows a network model of a small P2P system, where peer \( V_3 \) gets bandwidth from peer \( V_1 \), meanwhile, it allocates bandwidth to peer \( V_4 \) and peer \( V_5 \).

2.2 Nash Bargaining Solution

Nash bargaining solution is one method of axioms bargainings. It allows us to find an unique and fair bandwidth allocation among many peers. Under the context of bandwidth allocation in a network system, the game can be described as follows:

Assume that \( M \) users are competing for the bandwidth of the servers. Let \( x_{ij} \) refer to the amount of the bandwidth that server \( V_i \) allocates to user \( V_j \). Each user \( V_j \) has a utility, denoted as \( U_{ij}(x_{ij}) \), which is non-empty for the allocated bandwidth \( x_{ij} \), and \( J \) is the set of users. Each user has a minimum desired utility \( U^{min}_{ij} \), called the disagreement point, which is the minimum utility that the user can obtain without cooperation. We have \( U^{min}_{ij} = U_{ij}(x^{min}_{ij}) \), where \( x^{min}_{ij} \) is referred to as the minimum rate requirement.

Let \( X_i = \{x_{1i}, \ldots, x_{ji}, \ldots, x_{Mi}\}, j \in M \), be the vector of the available bandwidth allocation space of server \( i \) for \( M \) users, and \( U_i = \{U_{i1}(x_{1i}), \ldots, U_{ij}(x_{ji}), \ldots, U_{iM}(x_{Mi})\}, j \in M \), be the achieved utility set for \( M \) users. Note that \( U_i \) is a feasible subset that is convex, nonempty and closed. Correspondingly, the disagreement point set is defined as:

\[
U^0_i = \{U_{i1}(x^{min}_{1i}), \ldots, U_{ij}(x^{min}_{ji}), \ldots, U_{iM}(x^{min}_{Mi})\}
\]

Definition 1. A mapping \( S(U_i, U^0_i) \) is called a Nash bargaining solution if it satisfies these axioms: individual rationality, independence of irrelevant alternatives, independence of linear transformations, symmetry and Pareto optimality [9].

Suppose \( U_i \) is a convex and compact set and \( J \) is not empty, we can derive the following theorem:

Theorem 1. There exists a bargaining solution. The vector \( X \) of the solution set solves the following optimal problem \( P1 \):

\[
\max \prod_{j \in J} [U_{ij}(x_{ij}) - U_{ij}(x^{min}_{ij})], x_{ij} \in X_i
\]

In the Nash bargaining game, two or more players enter the game with an initial utility as well as an utility function. They cooperate in the game to achieve a win-win solution, in which the social utility gains (represented by the Nash product in (1)) are maximized. Suppose \( U_i \) is a convex and compact subset and \( J \) is not empty, we can derive the following theorem.

Theorem 2. Under the definition of Theorem 1, problem \( P1 \) is equivalent to \( P2 \):

\[
\max \sum_{j \in J} [U_{ij}(x_{ij}) - U_{ij}(x^{min}_{ij})], x_{ij} \in X_i
\]

Problem \( P2 \) is a convex optimization problem and has an unique solution. The solution of \( P2 \) is the NBS.

3. OPTIMAL BANDWIDTH ALLOCATION

In this section, we address the optimal and fair bandwidth allocation problem in P2P video streaming systems on the basis of NBS.

3.1 Peer Selection and Incentive Mechanism

A peer can request resource from many neighbors. However, which neighbor is the best? The peer has to take a choice.

Assume each peer \( V_j \) has a highest acceptable price \( b_j \). When peer \( V_j \) requests upload bandwidth from the servers,
it chooses a set $E_i$ of the peers in which each peer $V_i$ asks for a unit price $p_i$, with $p_i$ no larger than $b_i$. That is

$$E_{ij} = \begin{cases} 1 & p_i \leq b_j \\ 0 & p_i > b_j \end{cases}$$

$E_{ij} = 1$ represents that peer $V_j$ wants to request bandwidth from peer $V_i$; $E_{ij} = 0$ denotes that $V_j$ does not choose $V_i$ as its server.

In order to encourage the peers to share their resources with others, we assume that each peer has a certain initial amount of the budget $B_i$, $i \in N$. Each time peer $V_i$ receives the bandwidth $x_{ij}$ from peer $V_j$, it pays budget $p_i x_{ij}$ to $V_i$. In this way, the peers are required to act as the servers of others for increasing their own budgets.

### 3.2 Definition of Utility Function

The rate-distortion model used in this paper is given by [8]

$$D = \frac{\theta}{R - R_0} + D_0, \quad R \geq R_0, \quad D_0 \geq 0, \quad \theta > 0 \tag{3}$$

where $D$ is the video sequence distortion, measured as the mean square error (MSE), and $R$ is the rate of video sequence. $\theta$, $R_0$ and $D_0$ are the parameters, which are dependent on the video sequence characteristics and delay constraints. With the received bandwidth $x_{ij}$ from peer $V_j$, we can define the utility function of peer $V_j$ as:

$$U_{ij}(x_{ij}) = \frac{c}{D_{ij}} = \frac{c(x_{ij} - R_0)}{D_{ij}(x_{ij} - R_0)} + p_i x_{ij}$$

where $c$ is a nonnegative constant.

### 3.3 Bandwidth Allocation with NBS

When a peer is a server, it charges to the peers who get resources from it. When peer $V_i$ allocates bandwidth to its downstream nodes at a unit price $p_i$, the utility it achieves is:

$$\sum_{j \in E(i,j)} p_i x_{ij} \tag{4}$$

s.t. $\sum_{j \in E(i,j)} x_{ij} \leq C_i$.

Meanwhile, peer $V_i$ updates its budget by:

$$B_i = B_i + \sum_{j \in E(i,j)} p_i x_{ij}$$

Given the servers’ prices, peer $V_i$ decides the peers from which they request resource, and determines its minimum bandwidth $x_{ij}^{\min}$ and maximum bandwidth $x_{ij}^{\max}$ request from peer $V_i$, $i \in E(i,j)$.

1) The minimum rate requirement: The minimum rate requirement $x_{ij}^{\min}$ represents that the allocated rate below the minimum requirement rate is deemed unacceptable by the peer. The minimum rate requirement makes sure that the server must allocate enough bandwidth to the peer to satisfy its minimum requirement, or else the peer would not join the cooperation. Therefore, the bandwidth $x_{ij}$ peer $V_i$ assigns to peer $V_j$ should satisfy $x_{ij} \geq x_{ij}^{\min}$. Also, the aggregate minimum rate requirements of all the users can’t exceed the upload capacity of the server, i.e., we have $\sum_{j \in E(i,j)} x_{ij}^{\min} \leq C_i$.

2) The maximum rate requirement: Considering the limited budget of peer $V_j$, it can only ask for limited bandwidth from its servers. Specifically, a peer can find its maximum rate requirement $x_{ij}^{\max}$ by using water-filling method. For peer $V_j$, it requests the desired bandwidth to maximize the utility. The aggregate maximum rate requirements of peer $V_j$ must satisfy $\sum_{j \in E(i,j)} p_i x_{ij}^{\max} \leq B_j$.

Under the definition of $x_{ij}^{\min}$ and $x_{ij}^{\max}$, the utility maximization problem for peer $V_j$ is:

$$U_{ij}(x_{ij}) - p_i x_{ij} \tag{5}$$

s.t. $x_{ij}^{\min} \leq x_{ij} \leq x_{ij}^{\max}$

When server $V_i$ gets the minimum and maximum rate requirements of its users, it solves the bandwidth allocation problem by NBS.

Proposition 1. When $\sum_{j \in E(i,j)} x_{ij}^{\max} \leq C_i$, we have $x_{ij} = x_{ij}^{\max}$, When $\sum_{j \in E(i,j)} x_{ij}^{\max} > C_i$, peer $V_i$ will adopt the NBS to allocate the bandwidth.

When server $V_i$ uses NBS to allocate the bandwidth, its bandwidth allocation problem can be formulated as:

$$\max \prod_{j \in E(i,j)} [U_{ij}(x_{ij}) - U_{ij}^{\min}] \tag{6}$$

s.t. $x_{ij}^{\min} \leq x_{ij} \leq x_{ij}^{\max}$

where $U_{ij}(x_{ij})$ is the utility peer $V_j$ obtained from peer $V_i$, and $U_{ij}^{\min} = U_{ij}(x_{ij}^{\min})$ is $V_j$’s minimum utility requirement.

The equivalent optimization problem is:

$$\max \sum_{j \in E(i,j)} \ln[U_{ij}(x_{ij}) - U_{ij}^{\min}] \tag{7}$$

s.t. $x_{ij}^{\min} \leq x_{ij} \leq x_{ij}^{\max}$

We apply dual decomposition to problem (7), and get the following Lagrangian:

$$L(x_i, \lambda, \alpha, \beta) = \sum_{j \in E(i,j)} \ln[U_{ij}(x_{ij}) - U_{ij}^{\min}] - \lambda(\sum_{j \in E(i,j)} x_{ij} - C_i) - \sum_{j \in E(i,j)} \alpha_j (x_{ij} - x_{ij}^{\min}) - \sum_{j \in E(i,j)} \beta_j (x_{ij} - x_{ij}^{\max}) \tag{8}$$

where $\lambda$, $\alpha$, $\beta \geq 0$ are Lagrange multipliers associated with the linear bandwidth constraint.

From the concavity of the utility function, we know the Lagrangian is a concave function. The maximum value of the Lagrangian for the given $\lambda$, $\alpha$, $\beta$ is:

$$b^*(\lambda, \alpha, \beta) = \arg \max_{\lambda, \alpha, \beta \geq 0}[L(x_i, \lambda, \alpha, \beta)] \tag{9}$$

The solution of (9) is unique due to the concavity of the Lagrangian. The dual problem of (8) is defined as follows:

$$\min g(\lambda, \alpha, \beta) = L(x_i, \lambda, \alpha, \beta) \tag{10}$$

s.t. $\lambda, \alpha, \beta \geq 0$
The dual problem (10) can be solved by subgradient method with the following updates:

\[
\lambda_i(t + 1) = [\lambda_i(t) - a(\sum_{j \in E(i,j)} x_{ij} - C_i)]^+
\]

\[
\alpha_{ij}(t + 1) = [\alpha_{ij}(t) - b(x_{ij} - x_{ij}^{\min})]^+
\]

\[
\beta_{ij}(t + 1) = [\beta_{ij}(t) - c(x_{ij}^{\max} - x_{ij})]^+
\]

where \(a, b, c\) are small positive step-sizes. Nash bargaining solution can solve the problem efficiently.

When peer \(V_j\) gets the bandwidth from its servers, it pays to them, and updates its budget by:

\[
B_j = B_j - \sum_{i \in E(i,j)} p_i x_{ij}
\]

4. SIMULATION RESULTS

In this section, we conduct simulations to evaluate the performance of our proposed algorithm. We adopt peer bandwidth distribution as in Table 1. The default number of peers in the system is 200. We choose Coastguard CIF video sequence as the experiment video sequence. The frame rate of the video sequence is chosen at 30 frames per second.

We consider two different optimization goals, the MaxSum [5] and the NBS. In MaxSum, the system maximizes the system performance without consideration of the fairness. While in NBS, the peer fairness is taken into account. We define the fairness index of the system as [6]:

\[
\beta = \frac{\left(\sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}\right)^2}{K \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}}
\]

(12)

where \(K\) is the number of links.

The index \(\beta\) ranges from 0 to 1. \(\beta\) equals to 1 when all the peers obtain the same amount of the bandwidth and tends to 1\(/K\) when the bandwidths received by different peers are quite different.

Fig. 2 shows the fairness index for different scales of networks. We can observe that the NBS outperforms the MaxSum with respect to the fairness performance of the system.

Fig. 3 shows the utilization of upload capacity. The peers with a low capacity have a high average utilization, and the utilization of some may reach 1. If a peer with a low capacity has an appropriate price, the users requirements can be larger than its capacity. In this case, it can make full use of its bandwidth. For the peers with a high capacity, their average utilization is about 0.33, since these peers are more inclined to assign the bandwidth directly. From Fig. 3, we observe some peers’ utilizations are 0. This is because that their prices are too high to be accepted by others. As for the peers with a moderate capacity, their bandwidth utilizations are more likely to locate between 0.5 and 1.

Fig. 4 shows the average PSNR for different types of peers.

To evaluate the performance of our proposed algorithm, we observe the average PSNR for different types of peers. In Fig 4 the average PSNR for different types is various. But we can figure out that when we don’t consider the impact of fairness, peers can get the better performance. From Fig. 2 and Fig. 4, we can conclude that through the sacrifice of a small number of system performance, the algorithm of our propose gets a better performance in terms of fairness. This is consistent with our idea. And in Fig 4, different types of peers have different PSNR, the PSNR of peers is not associated with the types. From Fig 4, we can know that

<table>
<thead>
<tr>
<th>Peer Types</th>
<th>Upload Capacity(Kbps)</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-bw peers</td>
<td>1000</td>
<td>15%</td>
</tr>
<tr>
<td>Middle-bw peers</td>
<td>584</td>
<td>39%</td>
</tr>
<tr>
<td>Low-bw peers</td>
<td>256</td>
<td>46%</td>
</tr>
</tbody>
</table>

Table 1: Peers Distribution
even if a peer has high upload bandwidth, it may not get a good performance. This is because that when a peer allocates its bandwidth to its users, the server doesn’t consider the capacity of its users, it only considers the fairness and efficiency.

5. CONCLUSIONS

In this paper, we formulated the optimization problem of bandwidth allocation in P2P video streaming system. In the model, the peer selection and incentive mechanism are considered. With peer selection, peers can request more resources. The optimization problem is based on Nash bargaining solution. By analysis, we can conclude there is a tradeoff between fairness and performance. In our current analysis, we only considered single-layer video and we didn’t consider the impact of price. How to define optimum prices is our future work. And we plan to investigate the optimization problem in layered video streaming systems.

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7. REFERENCES