A moving zone of quiet for narrowband noise in a one-dimensional duct using virtual sensing

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(Dated: December 5, 2006)

A frequent problem in active noise control is that the zone of quiet created at the error sensor tends to be very small. This means that the error sensor generally needs to be located close to an observer’s ear, which might not always be a convenient or feasible solution. Virtual sensing is a method that can move the zone of quiet away from the error sensor to a desired location that is spatially fixed. This method has been investigated previously, and has shown potential to improve the performance of an active noise control system. However, it is very likely that the desired location of the zone of quiet is not spatially fixed. An active noise control system incorporating a virtual sensing method thus has to be able to create a moving zone of quiet that tracks the observer’s ears. This paper presents a method for creating a moving zone of quiet based on the LMS virtual microphone technique. To illustrate the proposed method, it is implemented in an acoustic duct and narrowband control results are presented. These results show that a moving zone of quiet was effectively created inside the duct for narrowband noise.

I. INTRODUCTION

The aim of a local active noise control system is to create a zone of quiet at a specific location, for instance at the passenger’s ear inside a vehicle cabin. Generally, the greatest noise reduction is achieved at the error sensor location, and the created zone of quiet tends to be very small. Elliott et al.1 have shown both analytically and experimentally that in a diffuse sound field, the zone of quiet in which the noise is reduced by 10 dB or more typically has the shape of a sphere with a diameter of one-tenth of an acoustic wavelength. This means that the error sensor usually has to be placed close to an observer’s ear, which might not always be a convenient or feasible solution. Virtual sensing2–5 is a method that has been developed in order to overcome these problems that are often encountered in local active noise control systems. This method requires a non-intrusive sensor which is placed remotely from the desired location of maximum noise attenuation. The non-intrusive sensor is used to provide an estimate of the pressure at the desired location, which is spatially fixed. The estimated pressure can then be minimized by a local active noise control system such that the zone of quiet is moved away from the physical location of the transducers to the spatially fixed desired location of maximum attenuation, such as a person’s ear.

The concept of virtual sensing has been shown to improve the performance of a local active noise control system2–20. However, it is very likely that the desired location of maximum attenuation is not spatially fixed. For instance, a passenger inside a vehicle cabin will move their head, thereby changing the desired location of the zone of quiet. This means that an active noise control system incorporating a virtual sensing method has to be able to create a moving zone of quiet that tracks the passenger’s head. Although active noise control at a spatially fixed virtual location has been investigated previously by a number of authors, the concept of creating a zone of quiet at a moving virtual location based on virtual sensing has not been investigated to the authors’ knowledge.

Elliott and David2 were the first to suggest a virtual sensing method called the virtual microphone arrangement. An important assumption made in this method is that the primary pressures at the physical and virtual microphones are equal. Furthermore, a preliminary system identification step is required in which models of the transfer paths between the control source and the physical and virtual microphones are estimated. The assumption of equal primary pressures and knowledge of these transfer paths allow the pressure at the virtual microphone to be estimated. The virtual microphone arrangement has been thoroughly investigated by a number of authors3,10.

A virtual sensing method called the remote microphone technique was suggested by both Popovich11, and Roure and Albarrazin14. The remote microphone technique requires the estimation of two transfer paths and one filter, which are usually estimated in a preliminary system identification step. The two transfer paths are the secondary transfer paths also needed in the virtual microphone arrangement2. However, an additional filter needs to be estimated in the remote microphone technique, that computes an estimate of the primary pressure at the virtual microphone from the primary pressure at the physical microphone. The virtual microphone arrangement assumes
this filter to be unity, and is therefore a simplified version of the remote microphone technique.

Cazzolato\textsuperscript{5} suggested a virtual sensing method based on forward difference prediction techniques. In this approach, the pressure at the virtual microphone is estimated by summing the weighted pressures from a number of microphones in an array. The weights for each of the elements in the microphone array are determined using forward difference prediction techniques. Kestell\textsuperscript{12} and Munn\textsuperscript{13} investigated the forward difference prediction approach for simple sound fields, namely an acoustic duct and a free field. Experiments showed that a linear prediction method, which uses a two-microphone array, proved to be better in practice than a quadratic prediction method, which uses a three-microphone array\textsuperscript{14-16}. The reason for this was the high sensitivity of the quadratic prediction method to short wavelength extraneous noise. In an effort to overcome this problem, higher-order virtual microphone arrays, which were thought to be able to spatially filter out the extraneous noise, were investigated\textsuperscript{13}. A higher number of microphones than the order of the prediction algorithm is used in this method, which results in an over-constrained problem that is solved by a least squares approximation. Unfortunately, the accuracy of these higher-order prediction algorithms was shown to be very much affected by the phase and sensitivity mismatches and relative position errors between the microphones in the array\textsuperscript{17}. These mismatches and position errors are generally unavoidable, especially if the number of microphones used is large.

Instead of using forward difference prediction techniques to determine the weights for each of the elements in the microphone array, Cazzolato\textsuperscript{18} suggested that the optimal values for the weights could be determined using the LMS algorithm\textsuperscript{21}. A similar method was also suggested by Gawron and Schaaf\textsuperscript{19}, who applied it to local active noise control inside a car cabin. The adaptive LMS virtual microphone technique involves placing a microphone at the virtual location, after which the microphone weights are adapted by the LMS algorithm so as to optimally predict the sound pressure at this location. After the weights have converged, the microphone is removed from the virtual location and the weights are fixed to their optimal value. The adaptive LMS virtual microphone technique was theoretically investigated for an acoustic duct\textsuperscript{13}. The results indicated that the adaptive LMS algorithm was able to completely compensate for relative position errors and sensitivity mismatches between the microphone elements, and partly compensate for phase mismatches between the microphone elements. The adaptive LMS virtual microphone technique proved to outperform the forward difference prediction technique in real-time control experiments conducted in an acoustic duct\textsuperscript{13,20}.

The virtual sensing methods discussed so far were all developed with the aim to create a zone of quiet at a spatially fixed virtual microphone. The primary aim of this paper is to present a method for creating a zone of quiet at a moving virtual microphone inside an acoustic duct based on the adaptive LMS virtual microphone technique\textsuperscript{18}. In previous work\textsuperscript{13,18}, this technique has only been applied to the case of using one spatially fixed virtual microphone, and it is therefore initially extended here to the case of using multiple spatially fixed virtual microphones. The analysis is based on a state-space model of the active noise control system under consideration, because this allows an easy extension to the case of a moving virtual microphone instead of a spatially fixed virtual microphone. The developed moving virtual microphone method is then combined with the filtered-x LMS algorithm\textsuperscript{22}. To illustrate the proposed method, it is implemented in a one-dimensional acoustic duct with the aim to create a moving zone of quiet for narrowband noise. Experimental results are presented and compared to experimental results for narrowband active noise control at a spatially fixed virtual microphone, and a spatially fixed physical microphone.

In a practical application, an important issue that would need to be addressed is how to determine the desired location of maximum attenuation, i.e. the moving virtual location. This could for instance be done using a 3D head tracking system based on camera vision, or a head tracking system based on ultrasonic position sensing such as the Logitech\textsuperscript{®} head tracker. However, the issue of determining the desired location of the zone of quiet is beyond the scope of this paper, and the focus here is on the development of a method for creating a zone of quiet at a moving virtual location inside an acoustic duct, while assuming that an exact determination of this location is available.

In Section II, the acoustic duct arrangement used to illustrate the proposed method is introduced. In Section III, the LMS virtual microphone technique is extended to include multiple spatially fixed virtual microphones, given a state-space model of the considered active noise control system. In Section IV, the theory discussed in Section III is extended to develop a moving virtual sensing algorithm that computes an estimate of the error signal at a moving virtual microphone. In Section V, it is explained how the moving virtual sensing algorithm is combined with the filtered-x LMS algorithm to create a zone of quiet at a moving virtual microphone. In Section VI, the proposed algorithm is implemented in real-time on an acoustic duct arrangement, and narrowband control results are presented.

II. PROBLEM DEFINITION

A. Acoustic duct arrangement

Fig. 1 shows a schematic diagram of the one-dimensional acoustic duct arrangement considered here. The rigid-walled acoustic duct has a length $L$, a primary source located at $x_p$, and a control source located at $x_c$. Moving zone of quiet inside duct 2
The primary source is excited by the disturbance source signal $x(n)$, and the control source by the control signal $u(n)$. Both signals are first passed through a DA-converter and a reconstruction filter before being sent to their respective sources. It is assumed here that the disturbance source signal $x(n)$ is a stationary process, and that the primary sound field created inside the acoustic duct is therefore stationary as well.

The aim of the active noise control system is to minimize the virtual error signal $e_v(n)$ at a moving virtual microphone located inside the acoustic duct, indicated by the black dot in Fig. 1. This moving virtual microphone tracks the desired location of the zone of quiet defined by the moving virtual location $x_v(n) = x_M + v(n)$, with $v(n)$ the *moving virtual distance* shown in Fig. 1. It will be assumed that the moving virtual microphone stays within a confined region inside the acoustic duct called the *target zone*, which has been indicated in Fig. 1 by the grey box. The virtual error signal at the moving virtual microphone is given by

$$e_v(n) = d_v(n) + y_v(n),$$

where $d_v(n)$ is the virtual primary disturbance that needs to be attenuated, and $y_v(n)$ is the virtual secondary disturbance created by the control source, such that

$$y_v(n) = G_{vu}(n)u(n),$$

with $G_{vu}(n)$ the *linear time-varying* virtual secondary transfer path. This transfer path is time-varying due to the movement of the virtual microphone.

The virtual primary disturbance $d_v(n)$ is a *non-stationary* process due to the movement of the virtual microphone, even though it is assumed that the primary sound field inside the acoustic duct is stationary. When the primary disturbance is non-stationary, the common approach in active noise control is to use an adaptive control algorithm. Adaptive control algorithms are able to track the changes in the statistical properties of the primary disturbance and adjust the controller accordingly. Here, the filtered-x LMS algorithm is used to adapt an FIR control filter that generates the control signal $u(n)$ in Fig. 1. A feedforward control approach is adopted as it is assumed that a feedforward reference signal $x(n)$ is available for control purposes. As illustrated in Fig. 1, this signal is equal to the disturbance source signal that excites the primary source.

### B. Implementing the filtered-x LMS algorithm

If the virtual error signal $e_v(n)$ was directly measured by a physical microphone, and the virtual location was spatially fixed, the filtered-x LMS algorithm could be implemented as

$$w(n+1) = w(n) - \mu r_v(n)e_v(n),$$

where $\mu$ is the convergence coefficient, $w(n) \in \mathbb{R}^I$ is a vector containing the filter coefficients

$$w(n) = [w_0(n) \; w_1(n) \; \cdots \; w_{I-1}(n)]^T,$$

and $e_v(n)$ is the virtual error signal directly measured by the spatially fixed physical microphone. The vector $r_v(n) \in \mathbb{R}^I$ in Eq. (3) is given by

$$r_v(n) = [r_v(n) \; r_v(n-1) \; \cdots \; r_v(n-I+1)]^T,$$

where $r_v(n) = G_{vu}x(n)$ is the virtual filtered-reference signal generated by filtering the reference signal $x(n)$ with the *linear time-invariant* virtual secondary transfer path $G_{vu}$. However, the virtual error signal $e_v(n)$ is not directly measured in the active noise control system considered here, and the virtual microphone is not spatially fixed but moving, such that the virtual secondary transfer path is linear time-varying, as indicated in Eq. (2). To implement the filtered-x LMS algorithm, these two issues thus need to be addressed.

### 1. Moving virtual sensing algorithm

As indicated by Eq. (3), the filtered-x LMS algorithm needs the feedback information contained in the virtual error signal $e_v(n)$ to update the control filter coefficients $w(n)$. This signal is not directly measured by a physical microphone in the problem considered here. A *moving virtual sensing* method thus needs to be developed that is able to compute an estimate $\hat{e}_v(n)$ of the virtual error signal at the moving virtual location $x_v(n)$. As illustrated in Fig. 1, the moving virtual sensing algorithm uses information about the sound field provided by a physical microphone array, and knowledge of the moving virtual location $x_v(n)$ to compute an estimate $\hat{e}_v(n)$ of the virtual error signal. The physical microphone array, indicated by the grey dots in Fig. 1, measures the physical

![FIG. 1: Schematic diagram of a the one-dimensional acoustic duct arrangement under consideration.](image-url)
error signals
\[ e_p(n) = [e_1(n) \ e_2(n) \ \cdots \ e_{M_p}(n)]^T, \]  
(6)
at the spatially fixed physical locations
\[ x = [x_1 \ x_2 \ \cdots \ x_M]^T. \]  
(7)
The physical error signals \( e_p(n) \) are first passed through an anti-aliasing filter and an AD-converter before being used by the moving virtual sensing algorithm to compute an estimate \( \hat{e}_v(n) \) of the virtual error signal. As illustrated in Fig. 1, instead of the unavailable virtual error signal \( e_v(n) \), the filtered-x LMS algorithm uses this estimate to update the control filter coefficients.

2. Generating the virtual filtered-reference signal

The second issue that needs to be addressed to implement the filtered-x LMS algorithm in Eq. (3) is that the virtual microphone is moving. For a spatially fixed virtual microphone, the virtual secondary transfer path \( G_{vu} \) is linear time-invariant, and the virtual filtered-reference signal can be generated as usual, such that \( r_v(n) = G_{vu}x(n) \). However, for a moving virtual microphone, the virtual secondary transfer path is linear time-varying, and the virtual filtered-reference signals are then generated as
\[ r_v(n) = G_{vu}(n)x(n). \]  
(8)
To generate the virtual filtered-reference signal, a model of the time-varying virtual secondary transfer path \( G_{vu}(n) \) is thus needed in a practical implementation for every sample \( n \). This is generally not possible in a practical situation, and a method for generating estimates of the virtual filtered-reference signals is therefore presented in Section V.

C. State-space model of system

In this section, a state-space model\(^{21}\) of the acoustic duct arrangement in Fig. 1 is introduced. This state-space system models the transfer paths between the input signals into the primary and control sources, and the output signals at the physical microphones located at \( x_\) the moving virtual microphone located at \( x_v(n) \), and a number of spatially fixed virtual microphones located within the target zone at
\[ x_v = [x_{v1} \ x_{v2} \ \cdots \ x_{vM_v}]^T. \]  
(9)
The state-space system that models these transfer paths is given by
\[
\begin{align*}
z(n + 1) &= A z(n) + B_u u(n) + B_x x(n) \\
e_p(n) &= C_p z(n) + v_p(n) \\
\hat{e}_v(n) &= C_v z(n) + v_v(n) \\
e_v(n) &= C_v(n) z(n) + v_v(n),
\end{align*}
\]  
(10)
with \( z(n) \in \mathbb{R}^N \) the state of the plant, \( u(n) \) the control signal, \( x(n) \) the disturbance source signal, \( e_p(n) \in \mathbb{R}^{M_p} \) the physical error signals at the spatially fixed physical microphones, \( \hat{e}_v(n) \in \mathbb{R}^{M_v} \) the virtual error signals at the virtual microphones spatially fixed at \( x_v \) within the target zone, with
\[ e_v(n) = [\hat{e}_v1(n) \ \hat{e}_v2(n) \ \cdots \ \hat{e}_vM_v(n)]^T, \]  
(11)
and \( e_v(n) \) the virtual error signal at the moving virtual location \( x_v(n) \). The signals \( v_p(n) \in \mathbb{R}^{M_p} \), \( v_v(n) \in \mathbb{R}^{M_v} \), and \( v_v(n) \) in Eq. (10) are the measurement noise signals on the physical microphones, spatially fixed virtual microphones, and moving virtual microphone, respectively. The state-space matrices in Eq. (10) are real-valued and of appropriate dimensions.

One may wonder why measurement noise on the virtual microphones has been included in Eq. (10). The reason is that physical microphones are usually located at the virtual locations in a preliminary identification stage of the plant, and the measurement noise on these microphones thus has to be included in the analysis.

III. SPATIALLY FIXED VIRTUAL MICROPHONE

The LMS virtual microphone technique\(^{18}\) can be used to compute an estimate of the virtual error signals \( \hat{e}_v(n) \) at the spatially fixed virtual locations \( x_v \), in Eq. (9), which are located throughout the target zone shown in Fig. 1. However, this algorithm\(^{18}\) has only been derived for the case of one spatially fixed virtual microphone. In this section, this algorithm is therefore extended to include multiple spatially fixed virtual microphones.

A. LMS virtual microphone technique

A block diagram of the LMS virtual microphone technique using multiple spatially fixed virtual microphones is shown in Fig. 2.

An estimate \( \hat{e}_v(n) \) of the virtual error signals is calculated by summing the weighted physical error signals \( e_p(n) \), which can be expressed as
\[ \hat{e}_v(n) = H^T e_p(n), \]  
(12)
where the matrix \( \mathbf{H} \in \mathbb{R}^{M_p \times M_v} \) contains the physical microphone weights given by

\[
\mathbf{H} = [ \mathbf{h}_1 \, \mathbf{h}_2 \, \cdots \, \mathbf{h}_{M_v} ]^T,
\]

with \( \mathbf{h}_i \in \mathbb{R}^{M_p} \) the weights used to compute the estimate \( \hat{\epsilon}_v(n) = \mathbf{h}_i^T \mathbf{e}_p(n) \). The physical error signals \( \mathbf{e}_p(n) \in \mathbb{R}^{M_p} \) in Eq. (12) are defined as

\[
\mathbf{e}_p(n) = \mathbf{d}_p(n) + \mathbf{y}_p(n),
\]

with \( \mathbf{d}_p(n) \) the primary physical disturbances, and \( \mathbf{y}_p(n) \) the physical secondary disturbances. The aim of the filter problem illustrated in Fig. 2 is to find a set of real weights \( \mathbf{H} \) that minimize the virtual output error \( \hat{\epsilon}_v(n) \) given by

\[
\hat{\epsilon}_v(n) = \mathbf{e}_v(n) - \hat{\mathbf{e}}_v(n).
\]

These weights will be called the optimal physical microphone weights, and are derived in the following.

### B. Modified LMS virtual microphone technique

Using Eq. (14), the estimated virtual error signal in Eq. (12) can also be written as

\[
\hat{\mathbf{e}}_v(n) = \mathbf{H}^T (\mathbf{d}_p(n) + \mathbf{y}_p(n)) = \hat{\mathbf{d}}_v(n) + \hat{\mathbf{y}}_v(n),
\]

where \( \hat{\mathbf{d}}_v(n) \) and \( \hat{\mathbf{y}}_v(n) \) are the estimated virtual primary and secondary disturbances, respectively. Eq. (16) illustrates that the microphone weights \( \mathbf{H} \) are applied to both the primary and secondary disturbances at the physical microphones. The underlying assumption made by previous researchers\(^{13} \) is therefore that the optimal microphone weights for the estimation of both \( \mathbf{d}_v(n) \) and \( \mathbf{y}_v(n) \) are equal, which might not always be true. For example, for active noise control in the near-field of the secondary source, the spatial characteristics of the primary and secondary fields can be very different\(^2 \). This difference is accounted for in the virtual microphone arrangement\(^2 \), where it is assumed that the primary field changes relatively little between the physical and virtual microphones, such that the primary disturbances at the two microphones are assumed equal. The secondary disturbances at the physical and virtual microphones are, however, not assumed equal due to the near-field properties of the secondary source. For the LMS virtual microphone technique, the described situation will result in different optimal microphone weights for the primary and secondary sound fields. In conclusion, it is thus important to find optimal microphone weights for the estimation of both the primary and secondary disturbances at the virtual microphone. If these weights are equal, the LMS virtual microphone technique can be implemented as illustrated in Fig. 2. If these weights are not equal, the virtual sensing algorithm needs to separate the physical error signals into their primary and secondary components. The optimal weights for each component can then be applied to obtain optimal estimates of \( \hat{\mathbf{d}}_v(n) \) and \( \hat{\mathbf{y}}_v(n) \), which can then be superimposed to get an optimal estimate \( \hat{\epsilon}_v(n) \) of the virtual error signal. The separation of the physical error signals \( \mathbf{e}_p(n) \) into their primary and secondary components can be achieved as illustrated in Fig. 3.

The estimated virtual error signal in this figure is given by

\[
\hat{\mathbf{e}}_v(n) = \mathbf{H}^T \mathbf{e}_p(n) + (\mathbf{H}_u - \mathbf{H}_x)^T \mathbf{G}_{pu} \mathbf{u}(n),
\]

where \( \mathbf{H}_x \in \mathbb{R}^{M_v \times M_v} \) are the weights for the primary field, \( \mathbf{H}_u \in \mathbb{R}^{M_v \times M_v} \) the weights for the secondary field, and \( \mathbf{G}_{pu} \) the physical secondary transfer path matrix between the control source and the \( M_p \) physical microphones. It can be seen that the block diagram in Fig. 3 reduces to the block diagram in Fig. 2 if the weights for the primary and secondary field are equal, such that \( \mathbf{H}_x = \mathbf{H}_u \).

### C. Optimal microphone weights

In this section, an optimal solution for the physical microphone weights is derived given the state-space model in Eq. (10). By setting \( x(n) = 0 \) in this equation, optimal weights for the secondary field can be derived. Similarly, optimal weights for the primary field can be derived by setting \( u(n) = 0 \). First, consider the case where the disturbance source signal \( x(n) \) is set to zero. An estimate of the virtual secondary disturbances \( \hat{\mathbf{y}}_v(n) \) is now computed as

\[
\hat{\mathbf{y}}_v(n) = \mathbf{H}_u^T \mathbf{y}_p(n).
\]

From Eqs (10) and (18), a state-space system that models the virtual output error \( \hat{\epsilon}_v(n) = \hat{\mathbf{y}}_v(n) - \hat{\mathbf{y}}_v(n) \) is then given by

\[
\mathbf{z}(n+1) = \mathbf{A} \mathbf{z}(n) + \mathbf{B}_u \mathbf{u}(n)
\]

\[
\hat{\epsilon}_v(n) = \mathbf{C}_\varepsilon \mathbf{z}(n) + \hat{\mathbf{v}}_v(n) - \mathbf{H}_u^T \mathbf{y}_p(n),
\]

where the matrix \( \mathbf{C}_\varepsilon \in \mathbb{R}^{M_v \times N} \) is defined as

\[
\mathbf{C}_\varepsilon = \mathbf{C}_v - \mathbf{H}_u^T \mathbf{C}_p.
\]
The optimal weights $\mathbf{H}_{u_0} \in \mathbb{R}^{M_x \times M_p}$ are now defined as the weights that minimize the cost function $J_\varepsilon$ defined as

$$J_\varepsilon = \text{tr} \mathbb{E} \left[ \varepsilon_v(n) \varepsilon_v(n)^T \right].$$  \quad (21)

It is assumed that the signal $u(n)$ in Eq. (19) is a zero-mean white and stationary random process during identification of the weights. Furthermore, the measurement noise signals $\mathbf{v}_p(n)$ and $\mathbf{v}_v(n)$ are assumed to be zero-mean white and stationary random processes that are uncorrelated to $u(n)$, such that the following covariance matrices can be defined

$$E \left( \begin{bmatrix} u(n) \\ \mathbf{v}_p(n) \\ \mathbf{v}_v(n) \\ \mathbf{z}(0) \end{bmatrix} \right) \left( \begin{bmatrix} u(k) \\ \mathbf{v}_p(k)^T \\ \mathbf{v}_v(k)^T \\ \mathbf{z}(0)^T \end{bmatrix} \right) = \begin{bmatrix} Q_u & 0 & 0 & 0 \\ 0 & R_{pp} & 0 & 0 \\ 0 & 0 & R_{vv} & 0 \\ 0 & 0 & 0 & \Pi_0 \end{bmatrix} \delta_{nk},$$  \quad (22)

with $\delta_{nk}$ the Kronecker delta function, such that $\delta_{nk} = 1$ if $n = k$, $\mathbf{z}(0) \in \mathbb{R}^N$ the initial state, $\mathbf{R}_p$ the covariance matrix of the measurement noise on the physical microphones, $\mathbf{R}_v$ the covariance matrix of the measurement noise on the spatially fixed virtual microphones, and $\mathbf{R}_{pv}$ the cross-covariance matrix between the measurement noise on the physical and spatially fixed virtual microphones. In Appendix A, it is shown that the cost function in Eq. (21) can then be written as

$$J_\varepsilon = \text{tr} \left( \mathbf{H}_{u_0}^T \mathbf{R}_u \mathbf{H}_u - 2 \mathbf{H}_{u_0}^T \bar{\mathbf{p}}_u + \bar{\mathbf{c}}_u \right),$$  \quad (23)

with

$$\mathbf{R}_u = E[\mathbf{y}_p(n)\mathbf{y}_p(n)^T] = \mathbf{C}_p \bar{\mathbf{\Pi}}_u \mathbf{C}_p^T + \mathbf{R}_p,$$  \quad (24)

$$\bar{\mathbf{p}}_u = E[\mathbf{y}_p(n)\hat{\mathbf{y}}_v(n)^T] = \mathbf{C}_p \bar{\mathbf{\Pi}}_u \hat{\mathbf{C}}_v^T + \mathbf{R}_{pv},$$  \quad (25)

$$\bar{\mathbf{c}}_u = E[\mathbf{y}_v(n)\hat{\mathbf{y}}_v(n)^T] = \mathbf{C}_v \bar{\mathbf{\Pi}}_u \hat{\mathbf{C}}_v^T + \mathbf{R}_v,$$  \quad (26)

where $\bar{\mathbf{\Pi}}_u > 0$ is the solution to the discrete-time Lyapunov equation

$$\bar{\mathbf{\Pi}}_u = \mathbf{A} \bar{\mathbf{\Pi}}_u \mathbf{A}^T + \mathbf{B}_u \mathbf{Q}_u \mathbf{B}_u^T.$$  \quad (27)

The optimal weights can now be found by differentiating the cost function $J_\varepsilon$ in Eq. (23) with respect to the weights $\mathbf{H}_u$, and setting all of the resulting derivatives to zero, which results in

$$\mathbf{H}_{u_0} = \mathbf{R}_u^{-1} \bar{\mathbf{p}}_u.$$  \quad (28)

Similarly, optimal weights $\mathbf{H}_{x_0} = \mathbf{R}_x^{-1} \bar{\mathbf{p}}_x$ for the primary field can be computed, with

$$\mathbf{R}_x = E[\mathbf{d}_p(n)\mathbf{d}_p(n)^T] = \mathbf{C}_p \bar{\mathbf{\Pi}}_x \mathbf{C}_p^T + \mathbf{R}_p,$$  \quad (29)

$$\bar{\mathbf{p}}_x = E[\mathbf{d}_p(n)\hat{\mathbf{d}}_v(n)^T] = \mathbf{C}_p \bar{\mathbf{\Pi}}_x \hat{\mathbf{C}}_v^T + \mathbf{R}_{pv},$$  \quad (30)

![FIG. 4: Block diagram for deriving the optimal time-varying weights for the secondary field.](image)

where $\bar{\mathbf{\Pi}}_x > 0$ is the solution to the discrete-time Lyapunov equation

$$\bar{\mathbf{\Pi}}_x = \mathbf{A} \bar{\mathbf{\Pi}}_x \mathbf{A}^T + \mathbf{B}_x \mathbf{Q}_x \mathbf{B}_x^T.$$  \quad (31)

The above derivations show that the weights for the primary and secondary field are equal if $\bar{\mathbf{\Pi}}_u = \bar{\mathbf{\Pi}}_x$.

### IV. MOVING VIRTUAL MICROPHONE

In this section, the LMS virtual microphone technique introduced in Section III is extended to a moving virtual sensing algorithm. This algorithm can be used in the acoustic duct arrangement shown in Fig. 1 to estimate the virtual error signal at the moving virtual location $x_v(n)$. The LMS moving virtual microphone technique is derived in Section IVA assuming that the matrix $\mathbf{C}_v(n)$ in Eq. (10) is known at every sample $n$. This will not be the case in a practical situation, and a more practical implementation of the developed moving virtual sensing algorithm is therefore presented in Section IVB. This implementation is based on linear spatial interpolation between the estimated virtual error signals $\hat{\mathbf{e}}_v(n)$ at the spatially fixed virtual microphones located within the target zone at $x_v$.

#### A. LMS moving virtual microphone technique

In this section, the analysis of the LMS virtual microphone technique presented in Section III is extended to develop the LMS moving virtual microphone technique. Because the virtual microphone is now moving, a *time-varying* optimal solution for the physical microphone weights is derived given the state-space model in Eq. (10). Here, time-varying optimal weights for the secondary field are derived by setting $x(n) = 0$ in Eq. (10). A block diagram of the optimal filter problem for this case is shown in Fig. 4.

An estimate of the virtual secondary disturbance $\hat{y}_v(n)$ at the moving virtual location $x_v(n)$ is now computed as

$$\hat{y}_v(n) = \mathbf{h}_u(n)^T \mathbf{y}_p(n),$$  \quad (32)

with $\mathbf{h}_u(n) \in \mathbb{R}^{M_p}$ the time-varying weights for the secondary field. Using Eqs (10) and (32), a state-space system that models the virtual output error $\varepsilon_v(n) = \ldots$
The time-varying optimal weights \( h_{uo}(n) \) are defined as the weights that minimize the cost function \( J_e(n) \) at every sample \( n \), with

\[
J_e(n) = \mathbb{E} \left[ \varepsilon_v(n)^2 \right].
\]  

Following the derivations presented in Section IIIC, it can be shown that this cost function can be written, similar to Eq. (23), as

\[
J_e(n) = h_u(n)^T R_u h_u(n) - 2 h_u(n)^T p_u(n) + c_u(n),
\]  

with \( R_u \) as defined in Eq. (24), and

\[
p_u(n) = \mathbb{E}[y_p(n) y_v(n)^T] = C_p \Pi_u C_v(n)^T + R_{pv},
\]

\[
c_u(n) = \mathbb{E}[y_v(n) y_v(n)^T] = C_v \Pi_u C_v(n)^T + R_v,
\]

where \( \Pi_u > 0 \) is the solution to the discrete-time Lyapunov equation in Eq. (27), \( R_v \) is the covariance of the measurement noise on the moving virtual microphone, and \( R_{pv} \) the cross-covariance matrix between the measurement noise on the physical microphones and the moving virtual microphone, such that

\[
R_{pv} = \mathbb{E}[v_p(n) v_v(n)] , \quad R_v = \mathbb{E}[v_v(n)^2].
\]

The time-varying optimal weights can now be found by differentiating the cost function \( J_e(n) \) in Eq. (36) with respect to the weights \( h_u(n) \), and setting all of the resulting derivatives to zero at every sample \( n \), which results in

\[
h_{uo}(n) = R_u^{-1} p_u(n).
\]

Similarly, time-varying optimal weights for the primary field can be computed as \( h_{xo}(n) = R_x^{-1} p_x(n) \), with \( R_x \) as defined in Eq. (29), and

\[
p_x(n) = \mathbb{E}[d_p(n) d_v(n)] = C_p \Pi_x C_v(n)^T + R_{pv},
\]

where \( \Pi_x > 0 \) is the solution to the discrete-time Lyapunov equation in Eq. (31).

To implement the derived moving virtual sensing algorithm, the cross-covariance vectors \( p_u(n) \) and \( p_x(n) \) in Eqs (37) and (41) need to be known at every sample \( n \). This is generally not possible in practice, and a more practical implementation of the developed moving virtual sensing algorithm is therefore presented in the next section.

\[\text{FIG. 5: Block diagram of the practical implementation of the LMS moving virtual microphone technique based on spatial interpolation, using } M_p \text{ physical microphones, } M_v \text{ virtual microphones spatially fixed at } x_{uo}, \text{ and one moving virtual microphone located at } x_v(n).\]

B. Practical implementation using spatial interpolation

The assumption that the moving virtual location \( x_v(n) \) stays within the target zone shown in Fig. 1 is now used to arrive at a more practical implementation of the moving virtual sensing algorithm derived in the previous section. A block diagram of the proposed implementation based on spatial interpolation is shown in Fig. 5.

The approach taken here is to first compute an estimate \( \hat{e}_v(n) \) of the virtual error signals at each of the \( M_v \) spatially fixed virtual microphones located at \( x_v \), defined in Eq. (9). These spatially fixed virtual microphones are positioned throughout the target zone in Fig. 1, and the estimate of the virtual error signals at each of these locations can be computed using the LMS virtual microphone technique described in Section III. As shown in Fig. 5, an estimate \( \hat{e}_v(n) \) of the virtual error signal at the moving virtual location \( x_v(n) \) is then computed using linear spatial interpolation between the error signals \( \hat{e}_v(n) \). Fig. 5 shows that the information needed to compute the linear spatial interpolation are the estimated virtual error signals \( \hat{e}_v(n) \), the spatially fixed virtual locations \( x_v \), at which these virtual error signals are estimated, and the moving virtual location \( x_v(n) \), which is assumed known here.

It can be shown that an equivalent estimate of the virtual error signal can be computed as

\[
\hat{e}_v(n) = \hat{h}_{xo}(n)^T e_p(n) + (\hat{h}_{uo}(n) - \hat{h}_{xo}(n))^T G_{pa} u(n),
\]

where the weights \( \hat{h}_{uo}(n) \in \mathbb{R}^{M_p} \) and \( \hat{h}_{xo}(n) \in \mathbb{R}^{M_v} \) are estimates of the optimal time-varying weights defined in the previous section. These estimates are obtained using linear spatial interpolation between the weights \( \bar{H}_{uo} \) and \( \bar{H}_{xo} \) derived in Section IIIC for the spatially fixed virtual locations \( x_v \). As an example, the \( m \)th component \( \hat{h}_{uo}^{m}(n) \) of \( h_{uo}(n) \) is computed as

\[
\hat{h}_{uo}^{m}(n) = \frac{x_v(n) - x_v(n+1)}{x_v(n) - x_v(n+1)} \bar{H}_{uo}^{m,i} + \frac{x_v(n) - x_v(n)}{x_v(n) - x_v(n+1)} \bar{H}_{uo}^{m,i+1},
\]

with the moving virtual location \( x_v(n) \) such that \( x_v(n) \leq x_v(n) \leq x_v(n+1) \), and with \( x_v \) the spatially fixed virtual...
locations defined in Eq. (9). If the weights for the primary and secondary field are equal, Eq. (44) reduces to

$$\hat{e}_v(n) = \hat{h}_{uo}(n)^T e_p(n), \quad (44)$$

with $\hat{h}_v(n) = \hat{h}_{uo}(n) = \hat{h}_{xv}(n)$. This is the implementation used in the experiments presented in Section VI.

V. FILTERED-X LMS ALGORITHM

A. Implementation

As discussed previously, to implement the filtered-x LMS algorithm in Eq. (3), the virtual filtered-reference signals $r_v(n) = G_{vu}(n)x(n)$ need to be generated, with $G_{vu}(n)$ the time-varying virtual secondary transfer path. In a practical situation, this transfer path will not be available for every sample $n$, and a method for generating an estimate $\hat{r}_v(n)$ of the virtual filter-reference signal needs to be developed. Here, an estimate of the virtual filtered-reference signal is generated as

$$\hat{r}_v(n) = \hat{h}_{uo}(n)^T r_p(n), \quad (45)$$

with $\hat{h}_{uo}(n)$ the time-varying optimal weights defined in the previous section, and $r_p(n) \in \mathbb{R}^{M_p}$ the vector of filtered-reference signals for the physical microphones, which is generated as

$$r_p(n) = G_{pu}x(n), \quad (46)$$

with $G_{pu}$ the physical secondary transfer path between the control source and the physical microphones. Substituting Eq. (46) into Eq. (45) gives

$$\hat{r}_v(n) = \hat{G}_{vu}(n)x(n), \quad (47)$$

with $\hat{G}_{vu}(n) = \hat{h}_{uo}(n)^T G_{pu}$. Effectively, this method thus provides an estimate of the time-varying virtual secondary transfer path $G_{vu}(n)$. The filtered-x LMS algorithm can now be implemented as

$$w(n + 1) = w(n) - \mu \hat{r}_v(n)\hat{e}_v(n), \quad (48)$$

with $\hat{e}_v(n)$ the estimated virtual error signal at the moving virtual location $x_v(n)$, and $\hat{r}_v(n) \in \mathbb{R}^{I}$ given by

$$\hat{r}_v(n) = [\hat{r}_v(n) \hat{r}_v(n-1) \ldots \hat{r}_v(n-I+1)]^T. \quad (49)$$

This is the implementation used in the experiments presented in Section VI.

B. Tracking

The aim of the filtered-x LMS algorithm in Eq. (48) is to track the changes in the statistical properties of the estimated virtual primary disturbance at the moving virtual location $x_v(n)$, and adjust the control filter coefficients $w(n)$ accordingly. The amount and speed of tracking needed is dependent on both the temporal rate of change of the moving virtual location, and the spatial rate of change of the relative magnitude and phase between the primary and secondary sound field over the target zone. This spatial rate of change determines how much the filter coefficients need to be adjusted over the spatial region through which the virtual microphone is moving. This information, together with the temporal rate of change of the moving virtual location, determines the amount and speed of the tracking that is needed to successfully create a moving zone of quiet. However, as stated by Haykin [24], the tracking details of a time-varying system are very problem specific. As a result, the tracking behaviour of the suggested algorithm will be problem specific as well, and general statements on how fast the virtual microphone can be moved cannot be made.

As discussed by Haykin [24], the convergence rate and the tracking capability of an adaptive algorithm are generally two different properties. Whereas convergence is a transient phenomenon, tracking is a steady-state phenomenon. For an adaptive algorithm to exercise its tracking capability, it must therefore first pass from the transient mode to the steady-state mode [24]. In the experiments presented in the next section, the performance obtained with the proposed method was measured after the filtered-x LMS algorithm in Eq. (48) had passed from the transient mode to the steady-state mode. Thus, in the experiments presented in the next section, it is the tracking capability of the filtered-x LMS algorithm that was investigated.

VI. ACOUSTIC DUCT EXPERIMENTS

The algorithms introduced in the previous sections were implemented on the acoustic duct arrangement shown in Fig. 1. The experimental arrangement is now described in more detail, after which experimental results are presented and discussed.

A. Experimental arrangement

The rectangular acoustic duct in Fig. 1 that was used in the real-time experiments was of length $L = 4.830$ m, width 0.205 m, and height 0.205 m. A loudspeaker located at $x_p = 4.730$ m was used as a primary source, and another loudspeaker located at $x_s = 0.500$ m as a control source. The primary loudspeaker was excited by a tonal disturbance source signal $x(n)$. As shown in Fig. 1, this signal was also used as a feedforward reference signal in the filtered-x LMS algorithm. A physical microphone array consisting of two electret microphones was located at $x = [1.425 \quad 1.475]$ m. A traversing microphone mounted on a cable wrapped around pulleys at each end of the duct was located inside the duct. This cable was wound onto a 0.150 m diameter spool, which was mounted on the
shaft of a DC servo-motor encoder unit. This encoder provided dual track TTL output signals of 500 pulses per revolution, which enabled accurate position control of the traversing microphone. In a preliminary identification procedure, the traversing microphone was placed at a number of spatially fixed virtual locations \( x_v \) in order to determine optimal microphone weights for these locations. Furthermore, the traversing microphone was position controlled to measure the primary and controlled sound pressure at the moving virtual location \( x_v(n) \). To implement the developed algorithms in real-time, a host-target software program called xPC TARGET® was used. A sampling frequency of \( f_s = 4 \text{ kHz} \) was employed in the real-time experiments.

B. Experimental results

The filtered-x LMS algorithm discussed in Section V was implemented in the acoustic duct arrangement to create a moving zone of quiet inside the acoustic duct. Only \( I = 2 \) filter coefficients were used in the experiments presented here, since the duct was excited by a tonal excitation signal\(^{22} \). For broadband noise, more filter coefficients need to be used. The aim of the experiments was to estimate and minimize the virtual error signal \( e_v(n) \) at a moving virtual location \( x_v(n) = x_2 + v(n) \) that changed sinusoidally with time, with \( v(n) \) the moving virtual distance. The expression governing the desired position of the virtual microphone is given by

\[
v(n) = 0.070 + 0.050 \sin \left( \frac{2\pi n}{T_v f_s} \right), \quad (50)
\]

where \( T_v \) is the period of the sinusoidally time-varying moving virtual distance \( v(n) \) in Fig.1. The virtual microphone is thus moving sinusoidally between a virtual distance bounded by 0.020 m and 0.120 m. The performance at the moving virtual distance was measured for two excitation frequencies \( f \) of 213 Hz and 249 Hz. These frequencies correspond to the sixth and seventh resonance frequencies of the acoustic duct. For these excitation frequencies, the performance at the moving virtual distance \( v(n) \) was measured for three different values of \( T_v \) in Eq. (50) given by 10 s, 5 s, and 2.5 s. The maximum amplitude of the sinusoidally time-varying velocity of the moving virtual microphone is thus given by \( 5 \text{cm} \times 2\pi/2.5 \text{s} \approx 12.6 \text{cm/s} \). This was considered to be representative of the likely motion of a head in the intended applications. In total, six experiments were thus conducted in order to measure the performance of the implemented algorithms for various speeds of the moving virtual microphone, and various spatial characteristics of the sound field through which the virtual microphone is moving.

1. Preliminary identification procedure

In a preliminary identification procedure, the optimal microphone weights \( \mathbf{H}_{m} \) for the secondary field given in Eq. (28) were determined for \( M_v = 16 \) spatially fixed virtual locations \( x_v = x_2 + v \), with \( v \) given by

\[
v = \left[ 0.000 \quad 0.010 \quad 0.020 \ldots \quad 0.150 \right] \text{m}. \quad (51)
\]

These spatially fixed virtual distances \( v \) were evenly positioned throughout the target zone located within a virtual distance range of 0.000–0.150 m. The acoustic duct was excited with band-pass filtered white noise in the frequency range of 50–500 Hz while determining the optimal microphone weights. For each of the spatially fixed virtual distances in Eq. (51), the optimal weights were calculated using Eq. (28) based on 30 s of data obtained from the two physical microphones and the traversing microphone positioned at the spatially fixed virtual distance of interest. The results of this procedure are shown in Fig. 6, where the optimal weights for the secondary sound field have been plotted against \( v \).

As discussed in Section III, it is necessary to verify if the identified optimal weights for the primary and secondary sound fields are equal. Optimal microphone weights \( \mathbf{H}_{zo} \) for the primary sound field were determined in a similar way, and it was observed that the resulting weights for both cases were almost identical. The LMS virtual microphone technique as shown in Fig. 2 was therefore implemented in the acoustic duct.

Using the optimal microphone weights shown in Fig. 6, the LMS virtual microphone technique was implemented in the acoustic duct for a number of spatially fixed virtual distances. The tonal attenuations achieved at these virtual distances were measured for excitation frequencies of 213 Hz and 249 Hz. The results are shown in Fig. 7, which confirms previous experimental results presented by other researchers\(^{13,20} \), and indicates that good performance can be achieved at these fixed virtual distances.

Moving zone of quiet inside duct
2. Relative spatial change of primary and secondary sound fields over target zone

In Section V, it was discussed that the amount and speed of the tracking needed from the filtered-x LMS algorithm to successfully create a moving zone of quiet is dependent on the spatial rate of change of the relative magnitude and phase between the primary and secondary sound field over the target zone, and the temporal rate of change of the moving virtual location $x_v(n)$. The measured spatial rate of change of the relative magnitude and phase between the primary and secondary sound field inside the acoustic duct over the target zone has been plotted in Fig. 8, for the excitation frequencies of 213 Hz and 249 Hz. In this figure, the relative magnitude and phase between the primary and secondary sound fields have been normalized against the relative magnitude and phase at a virtual distance of $v = 0.000$ m.

For an excitation frequency of 213 Hz, the relative magnitude and phase change by about 0.58 dB and $4.2^\circ$ over the target zone, respectively. For an excitation frequency of 249 Hz, the relative magnitude and phase change by about 0.77 dB and $13.0^\circ$ over the target zone, respectively. Thus, when the virtual location is moving through the target zone located between a virtual distance $v$ bounded by 0.000 m and 0.150 m, the filtered-x LMS algorithm has to account for these changes in the relative magnitude and phase between the primary and secondary sound field by adjusting the filter coefficients, such that tracking of the non-stationarities in the estimated virtual primary disturbance is achieved. In the experimental results presented next, the performance at the moving virtual location was measured after convergence of the filtered-x LMS algorithm, such that the tracking capability of the adaptive algorithm was investigated.

3. Performance at moving virtual location

To illustrate the increase in local control performance that can be obtained at a moving virtual location when using the suggested method, the performance at the moving virtual location $x_v(n) = x_2 + v(n)$ was also measured for the case of active noise control at a spatially fixed physical microphone located at $v = 0.000$ m, and active noise control at a spatially fixed virtual microphone located at $v = 0.020$ m. For the spatially fixed physical microphone, the physical error signal was directly measured by the physical microphone located at $x_2 = 1.475$ m, and was minimized using the standard formulation of the filtered-x LMS algorithm.$^{22}$ For the spatially fixed virtual microphone, the virtual error signal at $v = 0.020$ m was estimated using the LMS virtual microphone technique described in Section III. The estimate was then minimized using the filtered-x LMS algorithm. Unlike the currently proposed method, both of these active noise control systems cannot account for the fact that the desired location of the zone of quiet is not spatially fixed.

The performance of the three active noise control systems that each employ a different sensing method was measured at the moving virtual location $x_v(n) = x_2 + v(n)$, where $v(n)$ is defined by Eq. (50) with $T_v$ given by 10 s, 5 s, and 2.5 s, respectively. The results of these measurements are illustrated in Fig. 9, where the average tonal attenuation measured at the moving virtual location has been plotted against time for excitation frequencies of 213 Hz and 249 Hz. In Fig. 9, the dashed line is the average tonal attenuation measured using active
noise control at the spatially fixed physical microphone located at $v = 0.000 \text{ m}$, the dash-dotted line the average tonal attenuation measured using active noise control at the spatially fixed virtual microphone located at $v = 0.020 \text{ m}$, and the solid grey line the average tonal attenuation measured using active noise control at the moving virtual microphone that tracks the moving virtual distance $v(n)$. Each of these lines was generated by averaging the results of 30 data-sets of 10 s measured with the traversing microphone, which was position controlled to track the moving virtual distance $v(n)$. The resulting position of the traversing microphone as measured by the encoder is plotted against time in the bottom parts of the subfigures in Fig. 9. Furthermore, the average tonal attenuations in decibels were low-pass filtered in order to prevent noisy plots in Fig. 9.

Fig. 9 indicates that using a spatially fixed virtual microphone gives better performance at the moving virtual location than using a spatially fixed physical microphone. However, the best results are obtained when using a moving virtual microphone that tracks the desired location of the zone of quiet defined by the moving virtual distance $v(n)$. When the moving virtual distance is at $v = 0.020 \text{ m}$ in Fig. 9, the system that uses a moving virtual microphone gives similar results to the system that uses a spatially fixed virtual microphone, indicating that the filtered-x LMS algorithm is able to provide sufficient tracking of the non-stationarities in the estimated virtual primary disturbance. When the moving virtual distance is not at $v = 0.020 \text{ m}$, however, Fig. 9 shows that the active noise control system that uses a moving virtual microphone provides the best performance. The results in this figure indicate that a moving zone of quiet has effectively been created at the moving virtual location in the acoustic duct, for all of the six narrowband experiments that were conducted. When using a moving virtual microphone, the average tonal attenuation at the moving virtual location does not fall below 42 dB for all values of $T_e$ and an excitation frequency of 213 Hz. For the spatially fixed physical and virtual microphones, the average tonal attenuation at this frequency reduces to 24 dB and 25 dB, respectively, at a virtual distance of $v = 0.120 \text{ m}$. For an excitation frequency of 249 Hz, the average tonal attenuation at the moving virtual location does not fall below 36 dB for all values of $T_e$. For the spatially fixed physical and virtual microphones, the average tonal attenuation at this frequency reduces to 17 dB and 18 dB, respectively, at a virtual distance of $v = 0.120 \text{ m}$. These results indicate that for the narrowband acoustic duct experiments presented here, the filtered-x LMS algorithm is able to provide sufficient tracking of the non-stationarities in the estimated virtual primary disturbance. As a result, a moving zone of quiet is effectively created inside the acoustic duct for narrowband noise.

VII. CONCLUSIONS

The LMS virtual microphone technique is a virtual sensing method that can be used to estimate the error signal at a virtual microphone that is spatially fixed. In this paper, this technique has been extended to a moving virtual sensing algorithm that is able to estimate the error signal at a moving virtual microphone which is tracking the desired location of the zone of quiet. The proposed algorithm can be combined with the filtered-x LMS algorithm to create a moving zone of quiet. The filtered-x LMS algorithm can provide the tracking needed to account for the non-stationarities in the primary disturbance, which are caused by the movement of the virtual microphone. A practical application of the suggested method is the creation of a moving zone of quiet that tracks a person’s head. Here, the developed algorithm was implemented in a one-dimensional acoustic duct, and results of narrowband control experiments were presented to validate the proposed method. These results showed that the proposed algorithm was able to create a moving zone of quiet inside an acoustic duct for narrowband noise. This resulted in an increase in local control performance compared to using a spatially fixed virtual microphone or a spatially fixed physical microphone. The experimental results indicate that the developed algorithm has the potential to improve the scope of successful local active noise control applications. The proposed algorithm can also be used for broadband active noise control, and in three-dimensional sound fields. Ongoing research is currently being conducted to investigate if the filtered-x LMS algorithm can provide the necessary tracking for broadband noise, and to analyse the performance of this suggested method in a three-dimensional sound field.

Acknowledgments

The author gratefully acknowledges the University of Adelaide for providing an ASI scholarship, and the Australian Research Council for supporting this research. Thanks also to the editor, Dr Kenneth A. Cunefare, for his constructive comments on the original manuscript.

FIG. 9: (Bottom) Moving virtual distance $v(n)$ plotted against time. (Top) Average tonal attenuation at moving virtual distance plotted against time for active noise control at: -- physical microphone spatially fixed at $v = 0.000\,\text{m}$; --- virtual microphone spatially fixed at $v = 0.020\,\text{m}$; and — moving virtual microphone at $v(n)$. 

(a) $f = 213\,\text{Hz}$ and $T_v = 10\,\text{s}$ (b) $f = 249\,\text{Hz}$ and $T_v = 10\,\text{s}$ 

(c) $f = 213\,\text{Hz}$ and $T_v = 5\,\text{s}$ (d) $f = 249\,\text{Hz}$ and $T_v = 5\,\text{s}$ 

(e) $f = 213\,\text{Hz}$ and $T_v = 2.5\,\text{s}$ (f) $f = 249\,\text{Hz}$ and $T_v = 2.5\,\text{s}$
APPENDIX A: DERIVATION OF COST FUNCTION

Eqs. (23) and (27) can be derived as follows. First, the state covariance matrix $\Pi_u(n)$ is defined as

$$\Pi_u(n) = E[z(n)z(n)^T]. \quad (A1)$$

From Eq. (10), the state covariance matrix satisfies the recursion

$$\Pi_u(n + 1) = A\Pi_u(n)A^T + AE[z(n)u(n)^T]B_u^T + B_uE[u(n)z(n)^T]A_u^T + B_uQ_uB_u^T. \quad (A2)$$

It can be shown, using Eqs. (10) and (22), that the current state $z(n)$ is uncorrelated to the current and past inputs $\{u(k), k = 1, \ldots, n\}$, such that

$$E[z(n)u(n)^T] = 0. \quad (A3)$$

This can be seen by deriving the following expression from Eq. (10)

$$z(n) = A^n z(0) + \sum_{m=1}^{n} A^{n-m} B_u u(m-1). \quad (A4)$$

The state $z(n)$ is thus a linear combination of the initial state $z(0)$ and the past inputs $\{u(k), k = 1, \ldots, n-1\}$. From Eq. (22), the input $u(n)$ is uncorrelated to all of these variables, thereby arriving at Eq. (A3). Eq. (A2) now reduces to

$$\Pi_u(n + 1) = A\Pi_u(n)A^T + B_uQ_uB_u^T. \quad (A5)$$

When the state $z(n)$ reaches its mean steady state value, the state covariance matrix $\Pi_u(n + 1) = \Pi_u(n) = \bar{\Pi}_u$ in Eq. (A5), and solving the discrete-time Lyapunov equation in Eq. (27) thus gives the steady state solution $\bar{\Pi}_u$.

The expression for the cost function $J_\varepsilon$ given in Eq. (23) is derived next. Using a similar reasoning that was used to derive Eq. (A3), it can be shown that the current state $z(n)$ is uncorrelated to the current and past measurement noise signals $\{v_p(k), v_v(k), k = 1, \ldots, n\}$, such that

$$E[z(n)v_p(n)^T] = 0, \quad E[z(n)v_v(n)^T] = 0. \quad (A6)$$

Using Eqs. (22), (A3) and (A6), the cost function in Eq. (21) can now be written as defined in Eq. (23).