Design and Analysis of Fuzzy Morphological Algorithms for Image Processing

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Abstract—A general paradigm for lifting binary morphological algorithms to fuzzy algorithms is employed to construct fuzzy versions of classical binary morphological operations. The lifting procedure is based upon an epistemological interpretation of both image and filter fuzzification. Algorithms are designed via the paradigm for various fuzzifications and their performances are analyzed to provide insight into the kind of liftings that produce suitable results. Algorithms are discussed for three image processing tasks: shape detection, edge detection, and clutter removal. Detailed analyses are given for the effect of noise and its mitigation owing to fuzzy approaches. It is demonstrated how the fuzzy hit-or-miss transform can be used in conjunction with a decision procedure to achieve word recognition.

Index Terms—Binary morphological algorithms, fuzzy algorithms.

I. INTRODUCTION

A mathematical morphology is grounded on the subset relation: erosion, the fundamental operation of mathematical morphology, is defined as the point set for which a translated structuring element is a subset of the input image. The fuzzy morphology introduced in [3] relaxes strict subsethood, thereby making erosion less susceptible to perturbations within the observed image. A more general approach to fuzzification of mathematical morphology depends on a formal fuzzification of the subset relation itself. This having been successfully accomplished in [4] via an axiomatization of subset fuzzification in conjunction with a proposed form for a subsethood indicator function, a more general theory of fuzzy morphology has been characterized in [5]. The general theory encompasses an infinite class of fuzzy morphologies characterized by the axioms of subset fuzzification. The goal of axiomatic characterization is twofold: first, to rigorously define the extent of fuzzy morphology (under the axioms) and second, to provide a framework for algorithm development and analysis. As in any mathematical system, the axioms determine the meaning of the system and define what can be accomplished within the system. Operations are defined within the context of the axioms and their properties and meanings are derived therefrom. Should one wish to change the meaning of fuzzification or limit further the choices for subset fuzzification, one can select a different set of axioms; however, given the axioms of [5] (which we believe convey desirable properties of subset fuzzification relative to morphological analysis), the algorithm framework is determined.

Since fuzzy mathematical morphology provides a fuzzified binary mathematical morphology, an inherent paradigm exists for taking classical binary algorithms of (binary) mathematical morphology and lifting them to fuzzy morphological algorithms. Lifting must be accomplished in such a way that the binary character of an algorithm is maintained, except insofar as it now operates in a fuzzy environment and with an eye on subsequent high-level decision procedures. Starting from the processing—decision relation—the present paper describes a general paradigm in which classical binary morphological algorithms can be lifted to fuzzy morphological algorithms. It is reasonable to expect that algorithm lifting is not unique. Not only are there an infinity of fuzzy morphologies in which lifting can take place, but there are many ways in which binary operations such as union, intersection, and complement can be fuzzified [6]. If ψ denotes a binary algorithm and \( \{\phi_1, \phi_2, \ldots, \phi_n\} \) is a collection of fuzzy algorithms lifted from ψ (the precise notion of lifting to be presented shortly), then the choice of which \( \phi_i \) to select is application dependent, in the sense that one desires a lifted algorithm that performs well relative to the task at hand. Ultimately, one would like to develop a theory of optimality relative to lifted algorithms analogous to statistical optimality criteria. Here we take a first step in this direction by analyzing liftings of three important binary morphological algorithms: hit-or-miss shape detection, edge detection via the morphological gradient, and opening-based clutter removal. Algorithm analysis includes possible fuzzification choices, analytic characterization of algorithm performance, and the effects of noise. The three algorithms have been extensively studied in the crisp binary setting and their sensitivities to noise and image variability have been investigated (hit-or-miss shape recognition [7]–[9], morphological gradient in edge detection [10], [11], clutter removal [12]–[15]). Fuzzifications of the algorithms have been applied previously and certain rules of thumb developed [16]; our purpose here, however, is to structurally analyze the algorithms within the proposed central design paradigm and to provide a prototype for how a fuzzy morphological algorithm can be used in conjunction with a decision procedure to solve a pattern recognition problem (fuzzy morphological character recognition).
Other fuzzy set theory-based frameworks have been proposed for mathematical morphology [17]–[20]. This is not surprising, given the lattice-theoretic structure of both fuzzy set theory and mathematical morphology [21]–[23]. The proposed frameworks preserved the nuances of fuzzy set theory but failed to preserve essential characteristics of mathematical morphology (see [5]). The axiomatic approach taken in [5] introduces intrinsic fuzziness into the system. Intrinsic fuzziness can also be introduced analytically [24], [25].

The fuzzy morphology of [4] and [5] has been motivated by the desire to “soften” the effect of fitting structuring elements. With regard to the hit-or-miss transform (and erosion), rank-order filters can be employed: $E$ is declared to fit inside $A$ if some number of points in $E$ are elements of $A$ [26], [27]. The most extensively developed rank-order approach concerns soft morphological filters [28]. While each of these approaches softens the fitting requirement, each does so following binarization; that is, softening follows the loss of image content due to binarization. The fuzzy approach is different: it operates on gray-scale realizations of image processes and by doing so postpones binary decisions to a later point. There is an inherent assumption that salient image content is binary and this inherent binary content is somehow corrupted (or simply hidden) in the gray-scale realization. One might argue that this limits the model. No doubt it does, but no less than an approach that binarizes at the outset, for what else could justify initial binarization?

One might prefer to automatically design optimal morphological algorithms [29], [30], however, optimization is difficult and finding an optimal filter depends on finding appropriate image and degradation models, statistical analysis of random-process realizations, massive computation, and/or mathematical tractability of optimization criteria. Although some fuzzy morphologies fit into the framework of gray-scale computational morphology [31], [32], modeling, computational, and intractability problems associated with application of the optimization theory to fuzzy morphology have yet to be overcome. Moreover, if they are overcome, they will most likely be overcome for particular models and not in any grand implementation theory. Indeed, a salient reason for postulating a developmental paradigm and analyzing key fuzzy morphological algorithms is to try to arrive at satisfactory algorithms without statistical design. In a related development, it is interesting to note that fuzzy adaptive resonance theory has been applied to automatically generate fuzzy morphological structuring elements for hit-or-miss detection [33].

II. EPISTEMOLOGY FOR FUZZY MORPHOLOGICAL ALGORITHMS

Binary morphological algorithms often operate on binary images that have been derived from gray-scale realizations by means of some binarization procedure such as thresholding. From a modeling perspective, there is an ideal binary image and captured gray-scale images represent obscured or corrupted realizations of the ideal image. Such a modeling perspective can be applied, even when not dealing with a binary image corrupted with noise. For instance, we might postulate the existence of an ideal binary edge image and view an observed gray-scale image inducing the edge image as an obscuring realization.

From a crisp-set binary perspective, pixel $v \in \overline{A}$ if and only if $\mu_{\overline{A}}(v) = 1$, where $\mu_{\overline{A}}$ is the characteristic function of set $\overline{A}$. Epistemologically, we view this to mean that $\mu_{\overline{A}}(v) = 1$ if we are certain $v \in \overline{A}$ and $\mu_{\overline{A}}(v) = 0$ if we are certain that $v \notin \overline{A}$. The valuation space of a fuzzy set’s characteristic function is the interval $[0, 1]$. If we assume there exists some randomization process $\rho$ affecting the ideal binary image $\overline{A}$ and we observe a gray-scale image $f_{\overline{A}} \in \rho(\overline{A})$, then a fuzzy set results from some operation $\zeta$ designed to provide degree of certainty. Specifically, $\overline{A} = \zeta(f_{\overline{A}})$ is a fuzzy set for which $\mu_{\zeta(f_{\overline{A}})}(v)$ indicates the degree of certainty that $v \in \overline{A}$. One choice for $\zeta$ is a simple linear range normalization. If the foreground is white and the background black, then a natural interpretation is that the image capture $\rho$ degrades the crisp foreground–background separation and we are more certain that bright pixels lie in the foreground than do dark pixels. If the capture process $\rho$ produces an illumination gradient, then an appropriate choice for $\zeta$ might be an adaptive threshold followed by linear range normalization. Choice of $\zeta$ depends on knowledge of $\rho$.

To formalize these notions, consider a collection $B$ of (idealized) binary images, a class $R$ of gray-scale realizations, and a randomization operator $\rho$. Even if $\rho$ can be modeled, in general, it is multivalued, so that $\rho(\overline{A}) \subseteq R$. Algebraically, given $\rho : B \to 2^R$, we would like to find a binarization $\beta : R \to B$ so that for any $\overline{A} \in B$ and $A' \in \rho(\overline{A})$, $\beta(A') = \overline{A}$. For $\beta$ to be well defined, we must assume that two distinct images in $B$ cannot possess the same realization. Even if there exists such a $\beta$, finding it is a prohibitive task.

Suppose one wishes to perform a binary morphological operation $\overline{\psi}$ on the ideal images and the output of this operation is to be subjected to a decision process $\overline{A}$. In Fig. 1, $D$ denotes the set of all decisions. In practice, one tries to find an optimal $\beta$ based on the decision process, that is, one wishes to find a $\beta$ which maximizes the probability of obtaining $\Delta[\overline{\psi}(\overline{A})] = \Delta[\overline{\psi}(\beta(\rho(\overline{A})))]$. The more ill-defined $\rho$, the less will be the probability of the correct decision. Furthermore, if the function $\beta$ is not extremely simple, then the requirements imposed upon $\beta$ by the optimization may be very constraining.

![Fig. 1. Paradigm for fuzzy morphological algorithms.](image-url)
We consider a paradigmatic example concerning clutter removal by opening to illustrate the problem with the binarization-first approach. Subsequently, we will show how the proposed fuzzy set theory-based framework can mitigate the problem. For simplicity, we work in the context of one-dimensional (1-D) signals. The ideal gray-scale shapes are essentially binary, their only gray-scale characteristic being their single gray value. Phenomenologically, they can be considered binary. The ideal gray-scale shape can be easily thresholded to yield a perfect \( \{0, 1\} \)-binarization. The problem, however, is that signal capture is less than perfect in most situations and demands imposed upon algorithms by the “dark copy” and “light copy” realizations are quite different and often conflicting.

Consider the situation depicted in Fig. 2 where there are two bright 1-D objects, one signal, and one clutter. To facilitate transparent algebra, the gray-scale objects have been modeled as trapezoids with the angle between the sides and the vertical being \( \theta \) and the height of both signal and clutter over their regions of highest intensity being one. The smaller \( \theta \), the sharper the edge. The clutter and signal lengths at their highest intensities are \( L \) and \( L + 2a \), respectively. If the image is thresholded at \( t < 1 \), then the lengths of the thresholded clutter and signal are

\[
C(t, \theta) = L + 2(1 - t) \tan \theta \\
S(t, \theta) = L + 2a + 2(1 - t) \tan \theta
\]

respectively. For the binarized clutter to be eliminated and the binarized signal not be eliminated by opening, the length \( \ell \) of the structuring element must be between \( C(t, \theta) \) and \( S(t, \theta) \). Thus, we must have

\[
0 < \frac{\ell - L}{2(1 - t)} - \tan \theta < \frac{a}{1 - t}.
\]

The problem is that \( \theta \) is random and, unless \( \theta \) is sufficiently constrained, there will be no \( \ell \) satisfying the inequality for all \( \theta \). For instance, suppose we let \( \ell = L + a \), which is midway between the high-intensity lengths. Then solution of the preceding inequality shows that the correct decision is made when \( \theta < \arctan(a/(2(1 - t))) \). Thus, the probability of a correct decision (keeping the signal while eliminating the clutter) is

\[
P(\text{Correct Decision}) = P\left( \theta < \arctan\left(\frac{a}{2(1 - t)}\right) \right).
\]

For \( \theta \) too large, the clutter will pass. A similar situation results if \( \ell = L + 2a \). Should \( \ell = L + 3a \), then \( \theta \) must satisfy

\[
0 < \tan \theta - \frac{a}{2(1 - t)} < \frac{a}{1 - t}.
\]

For \( \theta \) too small, neither signal nor clutter is passed; for \( \theta \) too large, both are passed. In either case, \( \theta \) outside the required range yields an incorrect “binary” decision. From the foregoing inequalities, it can be easily seen that given the distribution of \( \theta \), an optimal choice of \( \ell \) can be obtained; nevertheless, there will still be a positive probability for obtaining an incorrect decision. If it happens that \( a \) is also random, the analysis remains valid, except that the probability statements must be reformulated in terms of the joint distribution of \( a \) and \( \theta \); for intuition sake, we have chosen to remain in a univariate setting.

We propose a paradigm to address the problems inherent in the binarization-first approach. One works directly with realizations in \( \mathbb{R} \) and the aim is to mirror the intent of the process that would have been performed on ideal binary images. We first normalize the gray-value range to \([0, 1]\) to obtain the set of images \( F \). Suppose the ideal binary image is \( \overline{A} \) and its normalized realization (in \( F \)) is \( A \). We treat \( A \) as a fuzzy (binary) image: the membership value \( \mu_A(v) \) reflects the certainty with which \( v \in \overline{A} \). The intent is to define a fuzzy morphological operation \( \psi : F \to F \) and a decision procedure \( \Delta : F \to D \) such that \( \Delta[\psi(A)] = \overline{\Delta[\overline{A}]} \).

\( \psi \) must preserve the nuances of fuzzy set theory (in so far as modeling of spatial uncertainty is concerned) and of the binary morphological operation \( \overline{\psi} \). The paradigm of Fig. 1
provides considerable latitude in defining \( \psi \) and is in the spirit of other artificial intelligence methodologies. For example, in granulometric analysis, one wishes to find the area of the opened image (that is, \( \psi \equiv \text{opening} \) and \( \Delta \equiv \text{area} \)). It is possible, though not necessarily desirable, to define \( \Delta \) and \( \psi \) such that \( \psi(A) \) and \( \Delta \psi(A) \) are different images but the area of \( \psi(A) \) is \( \Delta \psi(A) \).

The process of defining \( \psi \) based on knowledge of \( \psi \) is called lifting. Some liftings are more natural than others and some are more appropriate than others. The appropriateness of a lifting is hard to generalize since it is application specific. Assume algorithm \( \psi \) is composed of operations in the binary Minkowski algebra: union, intersection, complement, spatial translation, reflection, erosion, and dilation. As shown in [5], each operation of the binary Minkowski algebra lifts directly to a fuzzy operation so that \( \psi \) at once induces a fuzzy operator that can be employed for \( \psi \). This is not to say that there cannot be other candidates for \( \psi \), but certainly direct lifting of the operations is a natural choice; we term the resulting \( \psi \) the natural lifting of \( \psi \). Every binary morphological algorithm possesses a natural lifting; indeed, under suitable subset fuzzifications, the Minkowski algebra itself lifts [5]. For any lifting, the meaning of \( \psi \) as an operator on ideal objects is lifted to \( \psi \) as an operator on fuzzy realizations of the ideal image. This is a key point for requiring an axiomatic formulation of subset fuzzification: as a marker, binary erosion takes its meaning from fitting (subsetting) and naturally lifting it to multiscaled realizations of binary images requires only that the meaning of fitting (subsetting) be preserved insofar that it is relevant to the Minkowski algebra.

### III. FUZZY MORPHOLOGY

For completeness, we summarize the basic definitions and results from [5] required for algorithm development. The general analytic expressions for erosion, dilation, and opening are somewhat complicated. We refer to [5] for fuller explanations and the relevant theory.

We let \( \mathcal{U} \) denote the image tessellation, which, in all our examples, is either the set of integers \( \mathbb{Z} \) or the Cartesian grid \( \mathbb{Z} \times \mathbb{Z} \). Images are treated as fuzzy subsets of \( \mathcal{U} \); that is, images are functions from \( \mathcal{U} \) into \([0, 1]\). We define three operations on fuzzy sets: reflection, (spatial) translation, and range translation. Reflection of fuzzy or crisp set \( X \) (denoted as \( \neg X \)) is defined pointwise as \( \mu_{\neg X}(v) = \mu_X(-v) \). Translation of a set \( X \) by vector \( \tau \) (denoted \( T(X; \tau) \)) is defined pointwise as \( \mu_{T(X; \tau)}(v) = \mu_X(v - \tau) \). The range translation operator \( \triangleright \) \( [0, 1]^U \times [-1, 1] \to [0, 1]^U \) is defined pointwise as \( \mu_X \triangleright \alpha(v) = \min\{1, \max(0, \alpha + \mu_X(v))\} \). If \( \alpha < 0 \), then the operation \( X \triangleright \alpha \) lowers down the graph of the membership function of \( X \) by the amount \( |\alpha| \); the graph is lifted up by the amount \( \alpha \) when \( \alpha > 0 \).

For motivation, we begin with an intuitive explanation of the original fuzzy erosion of [3]. Relative to the general theory (about to be outlined) in terms of the \( \lambda \) function, the original theory of [3] involves \( \lambda(x) = 1 - x \). If \( A \) and \( B \) are fuzzy sets, then the fuzzy erosion for \( \lambda(x) = 1 - x \) of \( A \) by the structuring element \( B \) is defined at pixel \( v \) via the characteristic function by \( \mu_{\varepsilon(A, B)}(v) = 1 \) if \( \mu_T(B; v) \leq \mu_A \) and \( \mu_{\varepsilon(A, B)}(v) = 1 - \alpha \) otherwise, where \( \alpha \) is the minimum amount \( \mu_T(B; v) \) needs to be decreased (via \( \triangleright \alpha \)) to fit beneath \( \mu_A \). Should \( A \) and \( B \) be crisp sets, then \( \mu_T(B; v) \leq \mu_A \) if and only if \( T(B; v) \) is a crisp subset of \( A \) so that \( \mu_{\varepsilon(A, B)}(v) = 1 \) and when the inequality is not satisfied, \( \alpha = 1 \) and \( \mu_{\varepsilon(A, B)}(v) = 0 \). Hence, for \( A \) and \( B \) crisp, the fuzzy definition reduces to classical binary erosion. Such a straightforward conception is not possible in the general axiomatic theory, nor for all \( \lambda \) functions and the resulting indicators; nevertheless, the manner in which the original fuzzy erosion softens fitting is reflected in the general axiomatic theory. Erosion measures the degree to which the translated structuring element fits within the image.

More generally, fuzzy erosion is defined via its membership function by \( \mu_{\varepsilon(A, B)}(v) = \tau \) if and only if \( T(B; v) \) is a subset of \( A \) to the “degree” \( \tau \). Degree of subsethood is defined via a \( \lambda \) function plays a key role: choosing \( \lambda \) of the “degree” based on knowledge of \( \lambda \). We define three families of functions give a variety of functions to choose from:

\[
\begin{align*}
\lambda(x) &= 1 - x^n, \quad n \geq 1 \\
\lambda(x) &= \frac{1 - x}{1 + nx}, \quad 0 \geq n > -1 \\
\lambda(x) &= \frac{1}{1 + n^x} - \frac{x}{2}, \quad n \geq \frac{\ln(3)}{\ln(2)} = 1.5849 \ldots.
\end{align*}
\]

We have the following fitting characterization of fuzzy erosion:

\[
\forall v \in \mathcal{U}, \quad \mu_{\varepsilon(A, B)}(v) = \sup\{1 - \alpha \in [0, 1]: T(B; v) \trianglerIGHT \alpha \subseteq \gamma(A')\}
\]

(6)

where the unary operation \( \gamma : [0, 1]^U \to [0, 1]^U \) is range induced by \( \lambda \)

\[
\forall v \in \mathcal{U}, \quad \mu_{\gamma(X)}(v) = \lambda(\mu_X(v)).
\]

Suppose \( \alpha \) is the smallest number in \([0, 1]\) such that if we lower-down the set \( T(B; v) \) by amount \( \alpha \), then its membership function will fit beneath the membership function of \( \gamma(A') \). The certainty of pixel \( v \) belonging to the eroded image is \( 1 - \alpha \). The \( \lambda \) function plays a key role: choosing different \( \lambda \) functions can alter numerical certainties in the eroded image. For \( \lambda(x) = 1 - x \), (6) reduces to

\[
\forall v \in \mathcal{U}, \quad \mu_{\varepsilon(A, B)}(v) = \sup\{1 - \alpha \in [0, 1]: T(B; v) \trianglerIGHT \alpha \subseteq A\}
\]

(7)

the fitting characterization described earlier.
Fig. 3. Illustration of fuzzy hit-or-miss transform. (a)–(c) The aim is to detect the letter “L” in the image shown—the hit-and-miss structuring elements. (d) The output of hit-or-miss transform.

**Dilation** is defined in terms of erosion via duality \( D(A, B) = \mathcal{E}(A^c, -B)^c \). The fitting characterization can be stated as
\[
\forall v \in \mathcal{U}, \quad \mu_{D(A, B)}(v) = \inf \{ \alpha \in [0, 1] : -T[\gamma(B^c); v] \triangleright (-\alpha) \subseteq \gamma(A) \}. \tag{8}
\]

Suppose \( \alpha \in [0, 1] \) is the smallest number such that when we lower-down the reflection of \( T[\gamma(B^c); v] \) by amount \( \alpha \), its membership function fits beneath the membership function of \( \gamma(A) \). Then the certainty of pixel \( v \) belonging to the dilated image is \( \alpha \). For \( \lambda(x) = 1 - x \), (8) reduces to
\[
\forall v \in \mathcal{U}, \quad \mu_{D(A, B)}(v) = \inf \{ \alpha \in [0, 1] : -T(B; v) \triangleright (-\alpha) \subseteq A^c \}. \tag{9}
\]

**Opening** is defined iteratively by \( \mathcal{O}(A, B) = D[\mathcal{E}(A, B), B] \). Opening is translation invariant and increasing. If \( \lambda(x) = 1 - x \), then opening is also anti-extensive and idempotent [5], making it an opening in the sense of [21]. In the present paper, we only consider the case \( \lambda(x) = 1 - x \) in which case the fitting characterization of opening is
\[
\mathcal{O}(A, B) = \bigcup_{T(B; z) \triangleright (-\alpha) \subseteq A, \forall z \in \mathcal{U}, \forall \alpha \in [0, 1]} T(B; z) \triangleright (-\alpha) \big]. \tag{10}
\]

so that \( \mathcal{O}(A, B) \) is a fuzzy union of all lowered down translates of \( B \) that fit inside \( A \). There is a strong similarity to the binary fitting condition for opening; however, here the maximum value of \( \mu_{\mathcal{O}(A, B)} \) can be no greater than the maximum value of \( \mu_B \).

Based on experimentation, certain heuristics have been developed regarding fuzzy morphological algorithms. When using (3) and (5) to define \( \lambda \), we must use very small values of \( \eta \) (usually, less than two) if high noise sensitivity is required and slightly larger values of \( \eta \) (usually, more than five) if very low noise sensitivity is required. When \( \eta \) is large, the indicator \( I \) cannot distinguish between “signal” and “noise.” The converse holds for \( \lambda \) defined via (4): the closer \( \eta \) is to \( 1 \), the more noise sensitive \( I \) will be and vice versa.

There are two obvious liftings for an algorithm involving set subtraction:
\[
\mu_{X \setminus Y}(v) = \max[0, \mu_X(v) - \mu_Y(v)] \tag{11}
\]
\[
\mu_{X \setminus -Y}(v) = \min[\mu_X(v), 1 - \mu_Y(v)] \tag{12}
\]

In general, algorithms employing “\( \setminus \)” are more robust with respect to perturbations in the values of \( \eta \) in (3) through (5). Thus, we do not consider (12).

**IV. SHAPE DETECTION BY FUZZY HIT-OR-MISS TRANSFORM**

The (binary) hit-or-miss transform detects shapes via fitting inside and outside with a “hit” structuring element \( H \) and a “miss” structuring element \( M \), respectively
\[
A \odot \langle H, M \rangle = (A \odot H) \cap (A^c \odot M), \tag{13}
\]

A point \( z \) is in the hit-or-miss output if and only if \( H \) translated to \( z \) fits inside \( A \) and \( M \) translated to \( z \) fits outside \( A \). \( H \) and \( M \) must be disjoint. The naturally lifted algorithm is
\[
S \odot \langle H, M \rangle = \mathcal{E}(S, H) \cap \mathcal{E}(S^c, M), \tag{14}
\]

where \( H \) and \( M \) are fuzzy sets. With the fuzzy hit-or-miss transform shapes can be detected (with some degree of certainty) in light images because \( H \) can still fit in \( S \) to some degree and in dark images because \( M \) can still fit within \( S^c \) to some degree.
To analyze detection via the fuzzy hit-or-miss transform, consider detection of the following 1-D discrete pulse \( A \) of length \( 2\ell + 1 \)

\[
\mu_A(k) = \begin{cases} 
\frac{b+p}{b}, & \text{if } 0 \leq |k| \leq \ell \\
0, & \text{otherwise}
\end{cases}
\]

with \( b + p \leq 1 \), \( p > 0 \), and \( \infty > \ell \geq 0 \). Let \( H \) and \( M \) be defined by

\[
\mu_H(k) = \begin{cases} 
\frac{h}{h}, & \text{if } 1 \leq |k| \leq \ell \\
0, & \text{otherwise}
\end{cases},
\]

\[
\mu_M(k) = \begin{cases} 
m, & \text{if } \ell + 1 \leq |k| \leq \ell + w \\
0, & \text{otherwise}
\end{cases}
\]

where the windowing coefficient \( w \geq 1 \) and it is permissible to let \( w = \infty \). The values of \( h \), \( m \), and \( w \) are to be determined based on application-specific details. Then (see equation at the bottom of this page).

Consider the choice of \( w \). Suppose signal \( B \) is such that \( B \cap [-\ell - w, \ell + w] = A \); that is, when restricted to the interval \([-\ell - w, \ell + w]\), the signals \( A \) and \( B \) are identical. We find that \( \mu_{A \otimes H, M}(k) = \mu_{B \otimes H, M}(k) \) for all \(|k| \leq 2\ell + w\). Thus, a large \( w \) is beneficial for avoiding confusion. In the similar vein if it is known that no proper subset of signal \( A \) can coexist with \( A \) then a “sparse” hit structuring element can be employed

\[
\mu_H(k) = \begin{cases} 
h, & \text{if } k = \pm \ell \\
0, & \text{otherwise}
\end{cases}
\]

There is no change in the outcome of the hit-or-miss transform for the signal \( A \).

Regarding \( h \) and \( m \), suppose we wish to detect all realizations of the pulse \( A \) and that these realizations have different values for \( p \) and \( b \) only. Let us also suppose a priori that \( b \in [\alpha^-, \alpha^+] \) and \( p + b \in [\beta^-, \beta^+] \). The conditions \( \lambda(h) + \lambda(1 - b) < 1 \) and \( \lambda(m) + \lambda(b + p) < 1 \) must now be guaranteed for all possible values of \( b \) and \( p \). Since \( \lambda \) is nonincreasing, we let \( \lambda(h) + \lambda(1 - \alpha^+) < 1 \) and \( \lambda(m) + \lambda(\beta^-) < 1 \). Such a choice is always possible as we can let \( h = m = 1 \). When \( \lambda(x) = 1 - x \), the two conditions are \( h > \alpha^+ \) and \( m > 1 - \beta^- \).

The value of the transform depends linearly on the values of \( \lambda(h) \) and \( \lambda(m) \). If we desire a “clean” output, we must choose \( h = m = 1 \) so that for \( b = 0 \) we have

\[
\mu_{A \otimes H, M}(k) = \begin{cases} 
\lambda(1 - p), & \text{if } k = 0 \\
0, & \text{otherwise}
\end{cases}
\]

As \( h, m \to 1 \), the certainty at the peak in the hit-or-miss transform gets sharper vis-a-vis the certainties at the pixels in the neighborhood around the peak. For \( \lambda(x) = 1 - x \)

\[
\frac{\mu_{A \otimes H, M}(0)}{\mu_{A \otimes H, M}(1)} \approx 1 + \frac{2p}{3 - h - m - p - |1 + h - m - p - 2|}.
\]

Thus, the ratio can be increased by increasing \( h + m \) and/or \(|h - m|\).

On the down side, choosing \( h \approx 1 \) and \( m \approx 1 \) may result in vastly different certainties for different realizations, especially when \( \alpha^+ = \alpha^- \) and \( \beta^+ = \beta^- \) are large. For example, let \( \alpha^- = 0.2, \alpha^+ = 0.6, \beta^- = 0.6, \) and \( \beta^+ = 1 \). If \( \lambda(x) = 1 - x \), then \( h > 0.6 \) and \( m > 0.4 \). For \( h = m = 1 \) we find that the certainty at the peak is in the range \([0.4, 0.5] \) for \( h = 0.65 \) and \( m = 0.45 \) the corresponding range is \([0.95, 1] \). The complexity of detecting the peak at the origin, however, is the same in both cases.

Experiments with noisy images have shown that a slightly fuzzy subset is preferable: choose \( H \) and \( M \) with constant membership values of 0.8 or more. If \( a \) priori data is available on the background and foreground, then a more precise choice can be made along the previous lines. The value of \( \eta \) should be very small: \( n \leq 1.75 \) in (3), \( n \geq -0.4 \) in (4), and \( n \leq 4 \) in (5). The performance of the \( \lambda \) function given by (5) is the best (especially if the optimal value of \( \eta \) is unknown).

We now present an example to illustrate the manner in which the fuzzy hit-or-miss transform leads to fuzzy sets whose membership values are indicative of object location. The intent is to focus on key design parameters, examine the effects of different \( \lambda \) functions, and consider effects of image degradation. In all examples, \( \zeta \) is linear range normalization. Fig. 3(a) shows an out-of-focus camera image of the text “GENERAL.” Suppose we wish to identify the letter “L.” The structuring elements \( H \) and \( M \) should facilitate differentiation of “L” from the letters “A,” “E,” “G,” “N,” and “R.” Were the image truly binary and perfectly formed, \( H \) and \( M \) could be the perfect “L” and outer boundary of the perfect “L,” respectively. A more computationally efficient approach would be to use \( H \) and \( M \) in Fig. 3(b), where their common origin is the marked pixel. Used alone as an erosion marker, \( H \) finds “L” and “E”; the pair \( (H, M) \) locates “L” but not “E” [Fig. 3(b)]. Taking a fuzzy approach for the test image of Fig. 3(a), we employ the sparse structuring element \( H \) of Fig. 3(c). The gray values of \( H \) taper off to accommodate the fuzziness of “L”; otherwise, the gray values of both fuzzy structuring elements are 0.9. The fuzzy hit-or-miss transform with \( \lambda(x) = 1 - x \) is shown in Fig. 3(d). The detector is slightly confused (certainty 0.3) at the letter “G” and about the exact position of the letter “L.” A 3 × 3 neighborhood about the actual position has the following certainties:

\[
\begin{pmatrix}
0.38 & 0.80 & 0.54 \\
0.40 & 0.88 & 0.54 \\
0.34 & 0.48 & 0.49
\end{pmatrix}.
\]
These confusions arise from the extreme sparseness of the chosen miss structuring element and also because of the presence of fuzz around the letter “L.”

To study the effect of choosing different \( \lambda \) functions, let the \( 3 \times 3 \) neighborhood about the actual position of “L” be denoted as

\[
\begin{pmatrix}
  x_{11} & x_{12} & x_{13} \\
  x_{21} & x_{22} & x_{23} \\
  x_{31} & x_{32} & x_{33}
\end{pmatrix}.
\]

For a given family of \( \lambda \) functions, we plot the following quantities as functions of \( \eta \):

1) \( p(n) = (x_{12} + x_{22})/2 \) denotes the strength of peak;
2) \( q(n) = (x_{11} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} + x_{33})/7 \) denotes the average confusion about the exact location of the peak;
3) \( r(n) \), the maximum certainty at character “G” denotes the strength of false peak;
4) \( s(n) \) is the average background.

Ideally, we would like \( p(n) \approx 1, q(n) \approx 0, r(n) \approx 0 \), and \( s(n) \approx 0 \).

Figs. 4(a)–(c) show the results for different \( \lambda \) functions belonging to the families (3), (4), and (5), respectively. In all these plots, we find that \( p(n) \rightarrow 1 \) extremely fast. The values of \( q(n), r(n), \) and \( s(n) \) also increase as we increase \( n \) [decrease \( n \) for \( \lambda \) given by (4)]. The rate of change is high for \( q(n) \) and low for \( s(n) \). In fact, the value of \( q(n) \) approaches one extremely fast for the \( \lambda \) functions given by (3) and (4). The \( \lambda \) function given by (5) behaves the best.

V. EDGE DETECTION BY FUZZY MORPHOLOGICAL GRADIENT

Binary edge detection is often performed by the morphological gradient, which, for an isotropic structuring element \( B \), usually the \( 3 \times 3 \) square is defined by

\[
\nabla(A) = (A \oplus B) \setminus (A \ominus B),
\]

The gradient locates the edge by expanding the image (via dilation), shrinking it (via erosion), and then taking the set-theoretic difference. The binary morphological gradient is lifted by means of the fuzzy algorithm

\[
\nabla(A) = D(A,B) \setminus E(A,B),
\]

This lifting is not the only one. Since \( A \ominus B \subseteq A \) when \( B \) contains the origin, one can rewrite (15) as \( \nabla(A) = (A \oplus \overline{B}) \setminus [A \cap (A \ominus \overline{B})] \). This can be lifted to

\[
\nabla(A) = D(A,B) \setminus [A \cap E(A,B)],
\]

In the fuzzy case, \( E(A,B) \subseteq A \) is true only under rather stringent conditions. Like the ordinary gradient, the fuzzy gradient of (17) gives a (fuzzy) band about the image marking its edge. Unless otherwise stated, we employ the gradient given by (17). An image example of fuzzy morphological gradient can be seen in [16].

In choosing a structuring element, \( B \) should be a fuzzy subset of an isotropic image and should not be much larger than \( 3 \times 3 \); otherwise, the bands around the edges will be thick and small edge segments will not be identified. Consider the structuring element \( S(\ell, h) \), which is a square of edge-length \( \ell \) and height \( h \). Let \( \ell \approx 3, h \) must be chosen to
Fig. 5. Noise-sensitivity analysis of fuzzy edge detection. Here, $\lambda$ function is defined by (3). The graphs obtained when $\sigma = 0.10$, $\sigma = 0.25$, and $\sigma = 0.50$, respectively.

Fig. 6. Noise-sensitivity analysis of fuzzy edge detection. Here $\lambda$ function is defined by (4). The graphs obtained when $\sigma = 0.10$, $\sigma = 0.25$, and $\sigma = 0.50$, respectively.
Fig. 7. Noise-sensitivity analysis of fuzzy edge detection. Here λ function is defined by (5). The graphs obtained when σ = 0.10, σ = 0.25, and σ = 0.50, respectively.

facilitate edge detection. Since \( \gamma(B^c) = S(\ell, 1 - \lambda(h)) \) and \( \gamma(B^c) = S(\ell, \lambda(1 - h)) \), the sets \( \gamma(B^c) \) and \( \gamma(B^c) \) increase as \( h \) increases. From (6) it follows that \( \mathcal{E}(A, B)(v) \) will decrease for all pixels \( v \) as \( h \) increases. Similarly, from (8) it follows that \( D(A, B)(v) \) increases for all pixels \( v \) as \( h \) increases. Thus, \( \nabla(A) \) increases as \( h \) increases. This implies that we should let \( h \approx 1 \). Hence, we choose the structuring element to be \( S(3,1) \).

Let us illustrate the action of the fuzzy morphological gradient on 1-D signals. Consider a “fuzzy” generalization of the unit step function

\[
\nu(k) = \begin{cases} 
    b + p, & \text{if } k \geq 0 \\
    b, & \text{otherwise}. 
\end{cases}
\]

Of course, \( b + h \leq 1 \) and \( b, h \geq 0 \). Intuitively, \( p \) determines the degree to which an edge exists at the origin. Consider the following isotropic structuring element \( B \):

\[
B(k) = \begin{cases} 
    h, & \text{if } |k| \leq 1 \\
    0, & \text{otherwise}.
\end{cases}
\]

with \( h \approx 1 \), as suggested by our earlier discussions.
The analysis for arbitrary \( \lambda \) function is very complicated. We make simplifying assumptions that render the analysis manageable. Assume that \( \lambda(x) \geq 1 - x \) for all \( x \in [0, 1] \). Further assume that the values of \( b, p, \) and \( h \) are such that \( \lambda(h) + \lambda(1 - b - p) \leq 1 \) and \( \lambda(b) \leq \lambda(1 - h) \) (conditions easily satisfied when \( h \approx 1 \)). A lengthy but straightforward argument involving the preceding three inequalities yields (18), shown at the bottom of the page. Any negative value for the gradient should be treated as zero. It follows that \( \nabla(A)(1) \geq \nabla(A)(0) \). As is expected, when \( p \to 0 \), \( \nabla(A) \to \emptyset \). Since \( \lambda \) is nonincreasing, it also follows that \( \nabla(A) \) increases as \( p \) increases.

Regarding our previous general remarks about \( h \), as \( h \) increases, \( \nabla(A) \) increases (due to the increase in the value of \( \lambda(1 - h) \)). Thus, for more effective edge detection, \( h \) should be close to one. \( \nabla(A)(1) \) and \( \nabla(A)(0) \) are maximized when \( h = 1 \) and \( p + b = 1 \). The edge intensity is then \( 1 - b \) at pixel \(-1\).

When \( h = 1 \) and \( b = 0 \), we obtain the edge intensity as \( 1 - \lambda(p) \) at the pixels 0 and \(-1\). Thus, there is some detection no matter which particular \( \lambda \) function we employ as long as \( \lambda(x) \leq 1 - x \) for all \( x \in [0, 1] \). This statement does not hold in general; that is, it is possible to obtain \( \nabla(A) = \emptyset \) even when \( p > 0 \). To see this, let \( h = 1, b = 0, p = 0,2, \) and \( \lambda(x) = 1 - x^2 \).

To study the fuzzy morphological gradient given by (17) with respect to different noise levels and different \( \lambda \) functions, we considered a synthetic image containing random rectangles and suspect we wish to identify all edges. We first created a binary image containing ten (not necessarily disjoint) rectangles of lengths uniformly distributed in the integer range \([5, 14] \) and then added uniform random noise in the range \([-\sigma, \sigma] \) to each pixel. (\( \sigma \) will be referred to as the noise level.) Any value below zero in the noisy image was clipped at zero and the resulting image was normalized to yield pixel values in the range \([0, 1] \). Even though subtractive noise has no effect on the background, it can drastically alter the foreground surface.

Let \( S(3,1) \) be the structuring element and the \( \lambda \) function be in the family of (3). For each value of \( n \), we are interested in extracting the following four values from the fuzzy morphological gradient image:

- \( p(n) \): the smallest certainty present in the pixels that should belong to the edge;
- \( q(n) \): the average certainty present in the pixels that should belong to the edge;
- \( r(n) \): the largest certainty present in the pixels that should not belong to the edge;
- \( s(n) \): the average certainty present in the pixels that should not belong to the edge.

Intuitively, \( p(n) \) denotes the threshold value to identify edge pixels. If \( r(n) \geq p(n) \), then thresholding will yield pixels that do not belong to the edge. If the value of \( q(n) < r(n) \), then the thresholding may yield a highly unreliable result. On the other hand, if \( p(n) \gg r(n) \) then the thresholding will yield a very reliable result. The ratio \( q(n)/s(n) \) can be used to determine the faith in edge information obtained from thresholding.

Fig. 5(a) shows the values of these quantities as a function of \( n \) for the \( \lambda \) function of (3) and \( \sigma = 10\% \). In the figure, \( p(n) \) and \( q(n) \) decrease almost linearly with \( n \) with the rate of decrease being faster for \( p(n) \). \( r(n) \) remains more-or-less constant at 0.1 (which is much smaller than \( p(n) \)). \( s(n) \) quickly becomes almost zero. Fig. 5(b) and (c) shows graphs for \( \sigma = 25\% \) and \( \sigma = 50\% \). The trends are the same as before except the rates of decrease in \( p(n) \) and \( q(n) \) are much faster. For \( \sigma = 50\% \), the value of \( p(n) \) becomes less than \( r(n) \) almost immediately. For \( n \geq 5 \), we find that \( p(n) \approx s(n) \). We also find that the (more-or-less constant) value of \( r(n) \) increases as the noise level increases; however, there is no appreciable difference in \( s(n) \).

Figs. 6 and 7 show the corresponding values for the \( \lambda \) functions defined via (4) and (5). A similar but less drastic trend can be observed in these plots. In particular, we find that the gradient based on the \( \lambda \) function defined via (4) is least susceptible to both the perturbations in the value of \( n \) and the noise levels. Practically, this implies that it should be much easier to arrive at heuristics for selecting the threshold value when the gradient employs the \( \lambda \) function defined via (4).

\[

\nabla(A)(k) = \begin{cases} 
\lambda(1 - h) - \lambda(b + p) - b, & \text{If } k = -1 \\
\lambda(1 - h) - \lambda(b + p) - \min[b + p, \lambda(h) + \lambda(1 - b)], & \text{If } k = 0 \\
0, & \text{Otherwise}
\end{cases}
\]
VI. FILTERING CLUTTER BY FUZZY OPENING

Binary opening can be used to filter a binary image corrupted by union clutter noise. There is signal $S$ and clutter $N$, the noisy image is $S \cup N$, and the task is to filter $S \cup N$ to restore $S$. For opening filtering, if the structuring element does not fit within the clutter components not contiguous with $S$, then these are eliminated. If $S$ and $N$ are normalized fuzzy realizations of the ideal binary images $\overline{S}$ and $\overline{N}$, respectively, then the fuzzy image to be processed is the fuzzy union $S \cup N$. The binary union is lifted to a fuzzy union so that the fuzzy filter is defined by $\psi(S \cup N) = \mathcal{O}(S \cup N, B)$. An image example of fuzzy opening can be seen in [16].

To analyze the fuzzy approach, we return to the clutter removal problem discussed in Section III (Fig. 2) and let $\lambda(x) = 1 - x$. Choose $\ell = L + 2a$ and $h = 1$ for the structuring element. Then $\psi$ fully passes the signal. If $\tan \theta < a$, then $\psi$ completely eliminates the clutter; if $\tan \theta > a$, then $\psi$ of the clutter is a trapezoidal signal with angle $\theta$, maximum intensity $1 - a/\tan \theta$ and length of maximum intensity $L + 2a$. If $\theta$ is a random variable, then the characteristic function of the fuzzy opened clutter is a random function, its probability distribution depending upon that of $\theta$. The maximum intensity of the characteristic function is a random variable with probability distribution

$$P(\text{Maximum Intensity} < r) = P\left(\theta < \arctan\frac{a}{1 - r}\right).$$

In the fuzzy approach, the clutter is either not passed with certainty or passed with some degree of uncertainty depending on the probability distribution of $\theta$.

For a second probabilistic illustration, consider the signal and two clutter trapezoids shown in Fig. 8. The signal is affected by drop-out noise of height $\eta$, the noise being (not necessarily uniformly) distributed over the range $[0, d]$. To obtain a univariate analysis, we have assumed the signal and clutter are affected by drop outs of the same intensity; however, if desired, the following probabilistic reasoning can be extended to more complicated scenarios, for instance, when the clutter is affected by drop-out noise independent of that affecting the signal. The height at which the two clutter curves intersect is $c$, and we assume that $c > 1 - d$. The structuring element is assumed to have length slightly greater than half the base of the signal curve. This is reasonable since, in the perfect binary image represented by the gray-scale model, this length is sufficient to filter out the clutter while leaving the signal. For a binarized approach, there are three alternatives for threshold selection. The first would be to select $t > 1 - d$. This would assure that the threshold is not affected by the noise; however, it would result in a binarization in which the clutter is connected and thereby passed by the binary opening. Thus, this alternative must be rejected. A second alternative is to have $1 - d < t < c$. This choice is also unacceptable because when for the noise $\eta \leq 1 - c$, again, both signal and clutter are passed and when $1 - c < \eta \leq d$ the signal is not passed. Combining the first two alternatives, we see that we must choose $t > c$, or else a correct decision is never made.
For $t > c$, the clutter is never passed and the signal is passed if $\eta < 1 - t$. Thus

$$P(\text{Correct Decision}) = P(\eta < 1 - t).$$

If the probability mass of $\eta$ is strongly concentrated about zero, then the probability for correct decision will be large if $t$ is small. The limiting probability occurs at $t = c$ so that the probability of a correct decision is maximized by letting $t = c$; nonetheless, even then there still is some probability $P(\eta > 1 - c)$ of an erroneous binary decision.

Let us now take a fuzzy approach with $\lambda(x) = 1 - x$. The output of the signal component under the fuzzy opening is a trapezoid with the same base, but with height $1 - \eta$. Again, the fuzzy approach, when applied to the random signal, yields a random function dependent upon the distribution of $\eta$. The output of the overlapping clutter components is a trapezoid of height $c$ if $\eta \leq 1 - c$ and a trapezoid of height $1 - \eta$ if $\eta > 1 - c$. Thus, the probability of the clutter output height equaling the signal output height is $P(\eta > 1 - c)$, the same as for the decision-error probability for the optimal threshold choice in the binarized approach.

If it was possible to select the optimal threshold, there would be no advantage to the fuzzy approach; in practice, however, an optimal threshold choice is problematic because $c$ is random and is a function of the spatial distribution of the clutter, the size distribution of the clutter, and the distribution of the directional derivatives of the edge illuminations across the clutter set.

VII. APPLICATION TO WORD RECOGNITION

The present section illustrates use of the fuzzy hit-or-miss transform in conjunction with a decision procedure to classify a degraded word belonging to a word dictionary. $\lambda(x) = 1 - x$ is employed. In accordance with our model, each character represents a linearly normalized gray-scale realization of an underlying binary character. The word to be recognized is “cat” and it is in the dictionary of all three-letter English words that can be constructed from the alphabet $A = \{a, b, c, o, r, t\}$. The dictionary is the word set $D = \{cat, tab, rot, tar, rob, bat, boa, car, act, bar, cab\}$. Six realizations of “cat” are shown in Fig. 9: the original, a deformed version of the original (word 4), two warped versions of the original (words 2 and 3), and gray-scale carved and warped versions of words 2 and 3 (words 5 and 6, respectively). Sparse fuzzy hit-or-miss structuring pairs have been manually constructed for each of the characters in the alphabet from their original versions. Fig. 10 (a)–(c), and (d) shows the hit pixels and their fuzzy valuations for “t,” the miss pixels for “t,” the hit pixels for “c,” and the miss pixels for “c.” All realizations are operated upon by the six fuzzy hit-or-miss transforms. The results of the fuzzy hit-or-miss transform using the structuring pairs for “c” and “t” are graphically depicted in...
Fig. 13. Fuzzy hit-and-miss likelihoods of observed words with respect to the words in the dictionary.

Fig. 14. Rank-order fuzzy hit-or-miss outputs. (a) For “c” structuring pair. (b) For “t” structuring pair.

Fig. 15. Modes. (a) “c” rank-order hit-or-miss transform. (b) “t” rank-order hit-or-miss transform.

Fig. 16. Rank-order fuzzy hit-or-miss likelihoods of observed words with respect to the words in the dictionary.

the fuzzy morphological operator itself in the recognition algorithm; all remaining processing constitutes the artificial intelligence (AI) recognition module.

A procedure has been written to automatically extract the modes of the fuzzy membership functions. The modes corresponding to “c” and “t” hit-or-miss transforms are shown in Fig. 12(a) and (b), respectively. For each character $\alpha \in$
A and each observed character $\beta$ in Fig. 9, we define the fuzzy likelihood, $L(\alpha, \beta)$, that $\beta$ is $\alpha$, to be the mode within $\beta$ resulting from application of the hit-or-miss transform corresponding to $\alpha$. For each word $\omega = \alpha_1 \alpha_2 \alpha_3 \in D$ and each observed word $\xi = \beta_1 \beta_2 \beta_3$, we define the fuzzy likelihood that $\xi$ is $\omega$ to be the product $L(\omega, \xi) = L(\alpha_1, \beta_1) L(\alpha_2, \beta_2) L(\alpha_3, \beta_3)$. Observed word $\xi$ is classified as $\omega$ if $L(\omega, \xi)$ is maximum among $L(\omega, \xi)$ for all $\omega \in D$. Fig. 13 illustrates $L(\omega, \xi)$ for all 14 words $\omega \in D$ and for all six observed words $\xi$ in Fig. 9. $L(\text{"cat"}, \xi)$ is maximum for all observed words $\xi$.

As mentioned in the introduction, rank-order (order-statistic) binary hit-or-miss transforms have been applied to soften recognition procedures. To apply the rank-order approach in the $[\lambda(x) = 1 - x]$ fuzzy context, the definition of fuzzy erosion is changed so that when fitting a structuring element the minimum membership value in the window is ignored, thereby making the fuzzy fitting less susceptible to an extreme low membership value. More than one membership value could be ignored to further soften fuzzy erosion; however, care must be taken since the fuzzy likelihoods $L(\alpha, \beta)$ grow as more membership values are ignored. For the rank-order fuzzy hit-or-miss transform, Figs. 14–16 correspond to Figs. 11–13, respectively. Again, the word “cat” is recognized with 100% accuracy.

VIII. CONCLUSION

Fuzzy morphological algorithms lifted from binary algorithms can be useful for processing normalized gray-scale realizations of inherently binary images because they maintain the essence of the desired binary processing while at the same time being less sensitive to errors owing to early binarization. The present paper has discussed algorithmic design and has concentrated on the model-based analysis of three basic algorithms to analytically demonstrate some advantages of the fuzzy approach. From the analysis, it can be seen that fuzzy image and operator modeling is not in contradiction to binary random process modeling; rather, in the fuzzy setting, the characteristic function can be treated as a random function. Both algorithm design and analysis illustrate the manner in which fuzzy fitting (erosion) provides a softened morphology. Based upon the paradigm, full algorithm design involves a fuzzy morphological algorithm in conjunction with a decision procedure. The overall methodology has been illustrated in the context of fuzzy morphological character recognition.

REFERENCES

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