Automated Linear Function Submission-based Double Auction for Emergent Real-Time Pricing in a Regional Smart Grid

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Abstract—This paper presents a clarification of the relationship between a real-time pricing (RTP) algorithm, which we derived on the basis of a dual decomposition framework, and a linear function submission-based double auction (LFS-DA) algorithm for regional smart grids. The double auction technique involves the use of linear demand and supply functions by multi-agents when they submit bids and asks to an auctioneer in a regional electricity market. In this paper, we introduce a concise LFS-DA method, which is able to achieve an exact balance between electricity demand and supply for each time slot throughout learning phase. We proved that the price profile dynamics of the LFS-DA are equal to those achieved by RTP controlled by the algorithm derived from the dual decomposition framework after deriving the RTP algorithm. This means that the price controller emerging as a result of the use of the double auction technique can maximize the social welfare of the regional smart grid. A simulation experiment was used to examine the validity of the proposed mechanism.

Index Terms—Decentralized optimization, double auction, microgrid, real-time pricing, and smart grid

I. INTRODUCTION

DOUBLE auction is a technique that has been used for designing multi-agent software systems to manage regional smart grids \cite{1}–\cite{5}. Double auction involves the submission of bids by potential buyers and the simultaneous submission of asks by potential sellers to an electricity market. If an adequate price is determined, the auctioneer clears the market. The determined price allows the participants to buy or sell electricity on the basis of their submitted bids or asks.

One example of a double auction-based multi-agent software system for automated electricity transactions that has been implemented successfully, is the PowerMatcher \cite{1}, \cite{2}, \cite{6}. Several field tests proved the ability of the PowerMatcher to allocate energy resources efficiently in a decentralized manner. Multi-agent software systems offer advantages over humans when it comes to the implementation of double auction in markets. Agent software can submit continuous demand and supply functions directly to an auctioneer in a market. Several multi-agent systems for realizing smart grids employ such a function submission-based double auction method (FS-DA) \cite{1}, \cite{2}, \cite{5}–\cite{7}.

On the other hand, a number of studies have been devoted to the use of RTP in smart grids \cite{4}, \cite{8}–\cite{16}. These studies have mainly used game theory and/or control theory to develop RTP algorithms and to prove their optimality and/or robustness. In particular, the dual decomposition framework provides a sophisticated explanation and theoretical foundation for RTP algorithms \cite{17}.

The use of double auction leads to the automatic generation of a time-varying price profile, which means the technique has an intrinsic real-time pricing (RTP) mechanism. However, the optimality and the performance of LFS-DA have not been proved from a control theoretic viewpoint. Especially, the relationship between the LFS-DA and the dual decomposition-based RTP is not clear.

In this paper, we describe a linear function submission-based double auction (LFS-DA) method for a day-ahead power market, in which an adequate price profile emerges to control the demand and supply, and show the LFS-DA essentially equal to the dual decomposition-based RTP from the viewpoint of the dynamics of price profile. Especially, our main contributions are as follows.

- We propose a concise LFS-DA mechanism for a regional smart grid consisting of electricity prosumers. The mechanism is able to achieve an exact balance between electricity demand and supply at each moment during iteration.
- We prove that the optimal price profile obtained by the LFS-DA method becomes the same as that obtained by the RTP derived from the dual decomposition framework.
- We show that the LFS-DA practically outperforms the RTP through a simulation experiment.

The remainder of this paper is organized as follows. Section \textsection{II} provides the background to our research. Section \textsection{III} describes the problem definitions and basic assumptions of the target electricity network used in our research, which is a regional smart grid consisting of many prosumers. Section \textsection{IV} describes an RTP algorithm derived from the dual decomposition framework. Section \textsection{V} describes our proposed LFS-DA method and provides proof that the LFS-DA is intrinsically equivalent to dual decomposition-based RTP. Section \textsection{VI} contains details of a simulation experiment and its results. Finally, Section \textsection{VII} concludes this paper.

II. BACKGROUND

A. Penetration of distributed energy resources and smart grid

Distributed energy resources (DER), e.g., distributed energy generators and storage facilities, have gradually been introduced into households, plants, and offices. Such distributed
generation (DG) is mostly performed by using renewable energy resources, including solar, wind, and hydroelectric power. Conventionally, large power generation companies have generated electricity centrally with thermal electric power stations and nuclear power plants, which they then transmit to consumers on a unilateral basis. The situation differs from country to country. Specifically, in Japan, each power supplier has traditionally been a regional monopolizer, obliged by law to completely fulfill consumers’ electricity demands.

However, the increasing penetration of DERs and the emergence of smart grid technologies are changing the requirements of power networks. The wide-scale introduction of DERs may cause a reverse power flow to the conventional power grid, which would be rendered increasingly unstable and confused. Therefore, it is important to develop a regional electricity network capable of mitigating the effects of a reverse power flow. In this context, several power network concepts, including a smart grid [18], micro grid [19], and a virtual power plant (VPP) [20], have been studied with the aim of the smooth introduction of DERs to the grid.

In the development of a regional electricity network on the basis of DERs, demand side management (DSM) becomes an important design element. In contrast with conventional power generation, which is based on the use of fossil fuels, the amount of electricity produced by DG using photovoltaic cells and wind power generators is difficult to control, because it fully depends on the weather conditions. However, in an electricity network, balancing the demand and supply at each moment is crucially important. Therefore, achieving a balance between the demand and supply requires the power demand, rather than the power supply, to be controlled. A comprehensive survey of research relating to DSM was conducted by Alizadeh et al. [8]. Moreover, many field tests demonstrating the effectiveness of DSM have already been performed (e.g., [6], [15], [21]).

B. Demand side management

The realization of efficient, robust, and applicable DSM has led to the study of market-based methods [8], which have used price-based incentives, e.g., time-of-use (TOU) pricing, real-time pricing (RTP), and peak-time rebates, to control consumers’ electricity demand. The approaches used in game and control theory have provided the theoretical foundations for DSM methodologies.

A comprehensive survey of the application of game theory to smart grid systems was provided by Saad et al. [4]. Vytelingum et al. analyzed the Nash equilibrium for an electricity grid to which distributed micro-storage devices are connected. Their multi-agent simulation showed that this had the effect of flattening the electricity price profile [22]. Mohsenian-Rad et al. proposed a distributed algorithm for autonomous DSM [16]. They provide analytical proof to show that their proposed algorithm guarantees a significant reduction in the total energy cost of the system.

Control theory forms the theoretical foundation of RTP on the basis of a dual decomposition framework [9]–[14]. A comprehensive tutorial of dual decomposition was compiled by Palomar et al. [17]. In a dual decomposition-based approach, the problem of maximizing the social welfare of consumers and/or a power producer under several constraints is defined as the primal problem. This problem is transformed into a dual problem by applying a Lagrange relaxation and decomposition methods which have historically been used in optimization problems in electric power systems [23]. The dual problem obtained in this way can be divided into many sub-problems and a master problem. Each sub-problem corresponds to a selfish profit maximization problem of a consumer or producer, whereas the master problem corresponds to a problem involving a search for an optimal price profile. In a dual decomposition approach, the introduced Lagrange multipliers are interpreted as “prices”. In this way, the dual decomposition method provides a mathematical foundation based on RTP and derives each pricing algorithm of DSM depending on each condition of the target electric power network. On the basis of the dual decomposition framework and related concepts, various RTP methods have been proposed [9]–[14], [24]–[26].

C. Multi-agent system and double auction

However, most of the previous studies relating to RTP control theory assume that power producers and consumers exist separately in an electric power network as shown in Figure 1. In contrast, in our work, we focus on an electricity network consisting of electricity prosumers as shown in Figure 2. The term prosumer refers to a person or entity who is both a producer and a consumer at the same time [27]. If a consumer installs a PV system at his/her house, he/she not only becomes a consumer, but also a power producer. The electric power network considered in this work is therefore one in which prosumers participate. The structure of an energy network in which regional prosumers participate would differ completely
from that of a conventional unilateral power grid. Such a network has been studied in the context of multi-agent system-based microgrid.

In recent times, multi-agent systems using the double auction method for managing electric power networks have been gaining attention \cite{19, 28}. Kok et al. developed the PowerMatcher Smart Grid Technology \cite{6}, a multi-agent-based distributed software system for the market integration of small and medium sized DER units. In a PowerMatcher cluster, many agents interact with each other and the equilibrium price for electricity is determined automatically by using the double auction approach. The effectiveness of dynamic pricing using the PowerMatcher has been tested in simulations and field experiments \cite{1, 2, 6}. Vytelingum et al. proposed a market-based mechanism on the basis of Continuous Double Auction (CDA), and the use of an efficient trading agent strategy. They showed that the mechanism and the agents were able to cope with unforeseen demand or increased supply capacity in real time \cite{3}. Taniguchi et al. proposed an inter-intelligent renewable energy network (i-Rene), which is automatically managed by using double auction \cite{5}. Related papers reported the capability of their methods to achieve a reduction in peak consumption and/or the efficient utilization of energy \cite{3, 5, 6}. However, they were unable to prove that the double auction-based approaches maximize the social welfare of the network from an analytical point of view. This contrasts with the previously mentioned dual decomposition-based methods, all of which have an analytical foundation that guarantees the increase of the social welfare.

Most of the multi-agent-based approaches employ double auction in the regional electric power network to determine the price and the amount of electricity transacted. In double auction, bids (i.e., buy orders) and asks (i.e., sell orders) have to be matched to enable a certain amount of electricity to be transacted and the price to be determined. However, various agents submit bids and asks at various prices for various units of electricity. Therefore, the market matches demand and supply by using iterative communications. To reduce the communication overhead, Kok et al. simplified the communication by using only demand functions and prices \cite{6}. Agents in the network submit individual supply and/or demand functions to the market and if all of the agents submit continuous functions, the market is able to calculate adequate pricing and suitable transactions by searching for the point at which the aggregate demand and supply functions intersect. In this paper, we refer to the double auction method in which each agent submits demand and supply functions to the market as the function submission-based double auction (FS-DA) method. In the Agent-based Modeling of Electricity Systems (AMES) test bed model developed by Li et al. and in the i-Rene developed by Taniguchi et al., the submitted functions are also restricted to linear functions \cite{5, 7}. We call such FS-DA method using only linear functions the linear function submission-based double auction (LFS-DA) method.

In contrast with the traditional top-down approach, in which price profiles are determined by a public utility or an RTP generator (see Figure \ref{fig:grid}), our double auction approach determines price profiles dynamically through interaction between the market and if all of the agents submit continuous functions, the market is able to calculate adequate pricing and suitable transactions by searching for the point at which the aggregate demand and supply functions intersect. In this paper, we introduce a concise LFS-DA method for a network of prosumers, show that this method is essentially equal to the dual decomposition-based RTP method from the viewpoint of price profile dynamics and show the optimality.

\section{III. Problem Definition}

\subsection{A. Basic assumptions}

In the work presented in this paper, we define a target smart grid, to which regional prosumers are connected, as follows (a schematic diagram is shown in Figure \ref{fig:grid}). In our regional electricity network, we do not distinguish between electricity suppliers and consumers; rather, we assume that all people on the grid are capable of producing and consuming electricity as prosumers. Thus, it is assumed that each house is equipped with a generator powered by renewable energy, e.g., photovoltaic cells or wind power generators, and a storage device, e.g., a battery. Furthermore, each house is considered to have a smart meter running agent software, which automatically trades electricity on behalf of the people living in the house. In addition, the agent can also manage its battery, transmit electricity to other houses, and communicate with other information systems. The agent attempts to optimize its trading rule so as to maximize the house owner’s welfare. When the amount of power from its generator, its trading transactions, and its battery cannot satisfy the particular consumer’s demand, the agent buys electricity from an outside grid. Agents are able to buy electricity from the outside grid at the fixed price at any time, and can also sell surplus electricity to the outside grid at the fixed price, which is set to be low to reduce reverse power flow. We assume the outside grid is a conventional grid. Hereafter, we refer to a household, including the prosumers living in the house and the software agent, simply as an agent. Suppose there are \( N \) agents in the i-Rene and a set of these agents is represented by \( \mathcal{N} := \{1, 2, \ldots, N\} \). An agent can consume, generate, charge, discharge, buy, and sell electricity through its smart meter during every one of the \( T \) time slots. In addition, the number of time slots for transactions is shared by all of the agents in the grid. The set of time slots is defined as \( \mathcal{T} := \{1, 2, \ldots, T\} \). The \( i \)-th agent can determine the amount it consumes \( b_{i}^t \).
generates \( l_i^- \), charges \( b_i^+ \), discharges \( b_i^- \), sells to \( m_i^+ \), and buys from the regional market \( m_i^- \), and sells to \( g_i^+ \), and buys from the outside grid \( g_i^- \) during each time slot \( t \). The variables of the \( i \)-th agent \((i \in \mathcal{N})\) at time \( t \in \mathcal{T} \) are defined as shown in Table I. The relationship among the variables is also schematically shown in Figure 3. Each variable has its individual lower limit and/or upper limit owing to the limited capability of each apparatus or system. The superscripts \(+\) and \(-\) represent the direction from the viewpoint of a smart meter, which is at the center of electric energy flows in an agent’s house. Specifically, \(+\) and \(-\) represent outflow and inflow, respectively. We assume that the energy flow through the smart meter adheres to the law of the conservation of energy for each time slot \( t \) as follows.

\[
l_i^- - l_i^+ + b_i^+ - b_i^- + m_i^+ - m_i^- + g_i^+ - g_i^- = 0 \quad (1)
\]

The amount of electricity demanded has to be balanced against the supply by the regional electricity network for each time slot \( t \). We assume

\[
\sum_{i \in \mathcal{N}} (\gamma m_i^+ - m_i^-) = 0 \quad (2)
\]

where \( \gamma \in [0, 1] \) represents the electricity transmission efficiency. If \( \gamma = 1 \), there is no electricity energy loss during transmission. In addition, the storage efficiency \( \eta_i \in [0, 1] \) must be taken into consideration. If \( \eta = 1 \), the charged electricity can be fully discharged without any loss. The storage profile \( s_i^t \) represents the state of charge (SOC) of the \( i \)-th agent’s storage device at time \( t \) and is expressed by the following equation.

\[
s_i^t := s_i^{t-1} + \eta_i b_i^+ - b_i^- = s_i^{\text{init}} + \sum_{t \in \mathcal{T}} (\eta_i b_i^+ - b_i^-). \quad (3)
\]

where \( s_i^{\text{init}} \) is the initial SOC of the \( i \)-th agent’s battery.

Agents can buy electricity from the outside grid at \( p_i^G \) per unit, and sell electricity to the outside grid at \( p_i^G \) per unit. The situation for which reverse power flow is completely prohibited is modeled by setting \( p_i^G = 0 \). Resale behavior is suppressed by assuming the following constraint.

\[
0 \leq p_i^G \leq p_i^G. \quad (4)
\]

### B. Social welfare

We define the cost for generating electric energy \( c_i^+ \) for the \( i \)-th agent by \( C_i^+ : \mathbb{R} \rightarrow \mathbb{R} \), where \( C_i^+ \) is a convex function of class \( C^2 \) and define the utility for consuming electric energy \( l_i^- \) for the \( i \)-th agent by \( D_i^r : \mathbb{R} \rightarrow \mathbb{R} \), where \( D_i^r \) is a concave function of class \( C^2 \). The welfare \( W_i : \mathbb{R}^{\mathcal{N} \times T} \rightarrow \mathbb{R} \) of the \( i \)-th agent is defined as follows.

\[
W_i(x_i, p) := \sum_{i \in \mathcal{N}} W_i(x_i, p), \quad (5)
\]

\[
W_i(x_i, p) := \phi_i(x_i) - p_i \gamma m_i^+ + p_i m_i^-,
\]

\[
\phi_i(x_i) := D_i^r(l_i^-) - C_i^+(l_i^+) + p_i (c_i^+ + g_i^+) + p_i (C_i^+ + g_i^-) - (7)
\]

\[
x_i := (x_i^t)_{t \in \mathcal{T}}, \quad \phi_i(x_i) := (\phi_i(x_i^t))_{t \in \mathcal{T}},
\]

\[
x_i := (l_i^+, l_i^-, b_i^+, b_i^-, m_i^+, m_i^-, g_i^+, g_i^-), \quad (9)
\]

where \( p = (p_i)_{i \in \mathcal{N}} := (p_1, ..., p_t) \) is a price profile in the regional electricity market at time \( t \in \mathcal{T} \), and \( \phi_i(x_i) \) is a utility function for the \( i \)-th agent at time \( t \) including payments to an outside grid. Next, we define the social welfare of the network by \( W(x, p) \).

\[
W(x, p) := \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} (D_i^r(l_i^-) - C_i^+(l_i^+) + p_i (c_i^+ + g_i^+) + p_i (C_i^+ + g_i^-) - (p_i \gamma m_i^+ - p_i m_i^-) \quad (10)
\]

\[
= \sum_{i \in \mathcal{N}} \left( \sum_{t \in \mathcal{T}} (\phi_i(x_i) + p_i \sum_{t \in \mathcal{T}} (\gamma m_i^+ - m_i^-)) \right) \quad (11)
\]

where \( x = (x_i)_{i \in \mathcal{N}} := (x_1, ..., x_N) \). This shows that the social welfare inside of the regional smart grid does not depend on the price profile of the internal market. Therefore, maximizing \( W(x, p) \) is the same as maximizing \( \phi(x) := \sum_{i \in \mathcal{N}} \phi_i(x_i) \) where \( \phi(x) \) is a sum of individual utility functions.

In this paper, we define the following problem as a primal problem. The primal problem has the purpose of maximizing the social welfare of the regional network.

**Problem 1 (Primal problem).**

Maximize

\[
\sum_{i \in \mathcal{N}} \phi_i(x_i), \quad (12)
\]

Subject to \( x_i \in \mathcal{X}_i \quad \forall i \in \mathcal{N}, \)

\[
\sum_{i \in \mathcal{N}} f_i^t(x_i) = 0 \quad \forall t \in \mathcal{T}. \quad (13)
\]

Where

\[
\mathcal{X}_i := \{ x_i \in \mathbb{R}^{\mathcal{T}} \mid \}
\]

\[
h_i^j(x_i) \leq 0 \quad \forall t \in \mathcal{T}, j \in \{1, ..., 16\}, \quad (15)
\]

\[
h_i^{17}(x_i) = 0 \quad \forall t \in \mathcal{T}, \}
\]

\[
h_i^{18}(x_i) := l_i^+ - l_i^- \quad (16)
\]

\[
h_i^{19}(x_i) := b_i^+ \quad (17)
\]

\[
h_i^{20}(x_i) := -b_i^- \quad (18)
\]

\[
h_i^{21}(x_i) := -m_i^+ \quad (21)
\]

\[
h_i^{22}(x_i) := -m_i^- \quad (22)
\]

\[
h_i^{23}(x_i) := -g_i^+ \quad (23)
\]

\[
h_i^{24}(x_i) := -g_i^- \quad (24)
\]

\[
h_i^{25}(x_i) := l_i^- - l_i^{\text{max}} \quad (25)
\]

\[
h_i^{26}(x_i) := b_i^+ - b_i^{\text{max}} \quad (26)
\]

\[
h_i^{27}(x_i) := b_i^+ - b_i^{\text{max}} \quad (27)
\]

\[
h_i^{28}(x_i) := b_i^- - b_i^{\text{max}} \quad (28)
\]
In the above equations, \( h^1, \ldots, h^{14} \) are constraints for the domains of the variables shown in Table I and \( x_{i}^{\max} \) is the maximum capacity of the \( i \)-th agent’s storage device.

The primal problem is solved by the solver, which has to control each agent’s variable \( x_i \). However, no electric power companies, utilities, or governments can directly control the behavior of any of the agents. Therefore, a decentralized optimization method using price information would be of interest in the context of smart grid management.

IV. DUAL DECOMPOSITION AND REAL-TIME PRICING

Problems such as the primal problem are solved by using the dual decomposition technique [17]. This technique decomposes a primal problem into independent sub-problems representing agents’ selfish optimization behaviors, and a master problem that optimizes prices to balance the demand and supply. Each of the decomposed sub-problems can be solved independently. The solution of the primal problem and that of the dual problem are known to be the same if Slater’s theorem is satisfied [29]. In addition to that, the Lagrange multipliers, which appear through dual decomposition, can be interpreted as a “price” in our economic system. Within the context of the studies of smart grids and power systems, the dual decomposition technique provides the mathematical basis of RTP. The dual problem corresponding to the primal problem becomes

**Problem 2** (Dual problem).

\[
\begin{align*}
\min_{\lambda \in \mathbb{R}^n} & \quad g(\lambda) \\
= & \sum_{i \in N} \sup_{x_i \in \mathcal{X}_i} L_{\lambda i}(x_i), \\
L_{\lambda i}(x_i) & = \phi_i(x_i) + \sum_{j \in \mathcal{G}} \lambda_j f_j^i(x_i).
\end{align*}
\]

Generally, it is known that the Lagrange multiplier \( \lambda := (\lambda_i)_{i \in \mathcal{G}} \) represents the “price” of goods, which is constrained by an equation relaxed by \( \lambda \). A comparison between (38) and (6) provides us with a clear understanding of this relationship. In this problem, \( \lambda_i \) represents the price of electricity traded in the local electricity market at time \( t \). The simultaneous solution of the sub-problems and the master problem results in the solution of the dual problem and this is achieved in practice by adopting an iterative optimization technique.

**Algorithm 1** Iterative solution of dual problem

\( k \leftarrow 0 \)

**repeat**

Each agent solves its sub-problem (39), and obtains its solution \( x_i^{(k+1)} \) for \( \lambda^{(k)} \).

A central utility solves the master problem (41) under the condition that \( x_i = x_i^{(k+1)} \).

Update \( \lambda^{(k+1)} \) as \( \lambda^{(k)} - \theta_k \xi(\lambda^{(k)}) \),

\( k \leftarrow k + 1 \)

**until** a predefined stopping criterion is satisfied.

**return** Transact \( \{x_i^{(k)}\}_{i \in N} \) with \( \lambda^{(k)} \) as price profile

A. Sub-problems

If the Lagrange multipliers \( \lambda \) are given, the objective function (37) of the dual problem can be divided into \( N \) objective functions. Each objective function for each agent can be solved independently.

\[
\max_{x_i \in \mathbb{R}^T} L_{\lambda i}(x_i), \quad \text{subject to } x_i \in \mathcal{X}_i.
\]  

This problem is referred to as a sub-problem of the dual problem (39). From the viewpoint of game theory or micro-economics, if each agent can be regarded as rational, i.e., selfish, each sub-problem is expected to be solved autonomously by each agent. If the price of electricity in the market is given as \( \lambda \), one of the optimal solutions \( x_i^{*}(\lambda) \) to the problem (39) can be obtained as follows.

\[
x_i^{*}(\lambda) = \arg \max_{x_i \in \mathcal{X}_i} L_{\lambda i}(x_i)
\]

where \( x_i^{*} = (t_i^{+*}, t_i^{-*}, b_i^{*}, \ldots, m_i^{*}, s_i^{*}) \) and \( x_i^{*} = (x_i^{(k)})_{k \in \mathcal{G}} \).

B. Master problem

The dual problem is solved by optimizing the price profile \( \lambda \in \mathbb{R}^n \). For RTP formulation purposes, we assume there is a central utility determining the price profile in the regional electricity market. On the basis of each agent’s optimal strategy \( \{x_i^{*}(\lambda)\}_{i \in \mathcal{N}} \), the central utility has to solve the following problem.

\[
\min_{\lambda \in \mathbb{R}^n} g(\lambda) = \sum_{i \in \mathcal{N}} L_{\lambda i}(x_i^{*}(\lambda)),
\]

This problem is referred to as a master problem and its solution corresponds to searching for an adequate price that would balance demand and supply.

C. Real-time pricing algorithm

One of the most common procedures for solving the dual problem is as shown in Algorithm 1. The price \( \lambda^{(k)} \) is updated by using a known sub-gradient method [9, 17]. If we adopt
this sub-gradient method, \( \lambda^{(k)} \) is updated as follows in the dual problem:

\[
\lambda^{(k+1)} = \lambda^{(k)} - \theta_k \xi(\lambda^{(k)}) \\
\xi(\lambda^{(k)}) := \left( \sum_{i \in \mathcal{I}} f_i'(x_i(\lambda^{(k)})) \right)_{i \in \mathcal{I}} \\
= \left( \sum_{i \in \mathcal{I}} \left( \gamma m_i^{+}(\lambda^{(k)}) - m_i^{-}(\lambda^{(k)}) \right) \right)_{i \in \mathcal{I}},
\]

where \( \theta_k > 0 \) is the learning rate of the sub-gradient method. By updating \( x_i \) and \( \lambda \) iteratively, numerical solution of the dual problem can be obtained. The solution of the dual problem is also a solution of the primal problem. Therefore, the social welfare of the network is expected to be maximized. From the viewpoint of the pricing mechanism, solving the dual problem equates to the determination of a price profile by a utility, so as to maximize the social welfare. Therefore, we refer to this algorithm as a real-time pricing (RTP) algorithm.

The RTP algorithm still has a problem although it presents a feasible solution for the decentralized energy dispatch problem. The RTP algorithm cannot achieve an exact balance between demand and supply before the algorithm converges to an optimal solution although the balance is essentially important in an electricity network. In each step, \( \delta m^t := \left( \sum_{i \in \mathcal{I}} \left( \gamma m_i^{+}(\lambda^{(k)}) - m_i^{-}(\lambda^{(k)}) \right) \right)_{i \in \mathcal{I}} \) is not exactly a zero vector in most cases. Therefore, a heuristic process should be taken when aiming to compensate \( \delta m^t \). One of the simplest methods is that the utility compensates the difference \( \delta m^t \). If \( \delta m^t \) is not zero, the utility should sell or buy \( \delta m^t \) to or from the outside grid at the price of \( p_i^G_t \) or \( p_i^F_t \), respectively. In this work, we assume that the RTP algorithm employs this heuristic procedure.

In practical situations, the iteration cannot be performed many times, as only a small number of iterations are permitted. In such cases, the effect of \( \delta m^t \) cannot be ignored. In this context, a mechanism which would be able to achieve an exact balance between the demand and supply is desirable. The LFS-DA is such a mechanism as it has the ability to balance demand and supply exactly for each time slot and every iteration step, and is guaranteed to solve the problem in the same way as the dual decomposition-based RTP algorithm.

V. THE LINEAR FUNCTION SUBMISSION-BASED DOUBLE AUCTION METHOD

The LFS-DA method functions as follows. A buyer submits an individual demand function and a seller submits an individual supply function to the market, both at the same time. Both of them must be linear functions in LFS-DA. Then, the market-clearing price is determined exactly by calculating the point at which the aggregate demand and supply functions intersect. In this section, we formulate the LFS-DA algorithm and disclose its theoretical properties.

A. Transaction with LFS-DA

In LFS-DA, each agent has a linear demand function \( m_i^{t-} = \mu_i^{t-}(p_t) \) and a linear supply function \( m_i^{t+} = \mu_i^{t+}(p_t) \) for each time slot \( t \). These two functions are determined by using two auxiliary variables \( \alpha^t_i \) and \( \beta^t_i \).

\[
\mu_i^{t+}(p_t) := \lceil \beta^t_i p_t - \alpha^t_i \rceil, \quad (45) \\
\mu_i^{t-}(p_t) := \lceil -\beta^t_i p_t + \alpha^t_i \rceil, \quad (46)
\]

where \( |x| := \max(x, 0) \). In this work, we assume \( \beta^t_i \) is a fixed positive constant, and \( \alpha^t_i \) is a flexible variable that is optimized by each agent.

When each agent submits the linear function to the market, an adequate price \( p_t \) is determined that would suffice to clear the market by searching for the point at which the aggregate demand and supply functions intersect. The constraint for balancing demand and supply \( (32) \) becomes

\[
\sum_{i \in \mathcal{I}} f_i'(x_i, p_t) = \gamma \sum_{i \in \mathcal{I}} \lceil \beta^t_i p_t - \alpha^t_i \rceil - \sum_{i \in \mathcal{I}} \lfloor -\beta^t_i p_t + \alpha^t_i \rfloor = 0. \quad (47)
\]

The price \( p_t \) divides the agents into two groups. If for the \( i \)-th agent, \( \alpha^t_i \leq p_t \), the agent becomes a seller, i.e., \( m_i^{t+} > 0 \) and \( m_i^{t-} = 0 \). In contrast, if for the \( i \)-th agent \( \frac{\alpha^t_i}{p_t} > p_t \), the agent becomes a buyer, i.e., \( m_i^{t+} = 0 \) and \( m_i^{t-} > 0 \). We define the set of suppliers by \( I^+_t(p_t) := \{ i : \alpha^t_i \leq p_t \} \), and the set of consumers by \( I^-_t(p_t) := \{ i : \frac{\alpha^t_i}{p_t} > p_t \} \).

In this case, the constraint \( (47) \) becomes

\[
0 = \gamma \sum_{i \in I^+_t(p_t)} (\beta^t_i p_t - \alpha^t_i) - \sum_{i \in I^-_t(p_t)} (-\beta^t_i p_t + \alpha^t_i) \quad (48)
\]

\[
\Leftrightarrow p_t = \frac{\gamma \sum_{i \in I^+_t(p_t)} \alpha^t_i + \sum_{i \in I^-_t(p_t)} \beta^t_i}{\gamma \sum_{i \in I^+_t(p_t)} \alpha^t_i + \sum_{i \in I^-_t(p_t)} \beta^t_i}, \quad (49)
\]

where \( \alpha^t_i := \sum_{i \in I^+_t(p_t)} \alpha^t_i, \quad \alpha^t_i := \sum_{i \in I^-_t(p_t)} \alpha^t_i, \quad \beta^t_i := \sum_{i \in I^+_t(p_t)} \beta^t_i, \quad \text{and} \quad \beta^t_i := \sum_{i \in I^-_t(p_t)} \beta^t_i \). The price \( p_t \), as obtained by \( (49) \), exactly balances demand and supply, and fulfills the constraint in \( (47) \).

In the LFS-DA, when the market receives \( (\alpha^t_i, \beta^t_i)_{i \in \mathcal{I}, t \in \mathcal{I}} \) from all of the agents, the market determines the price \( p_t \) by using \( (49) \). Based on the price \( p_t \), each agent either transacts \( m_i^{t+} = \mu_i^{t+}(p_t) \) or receives \( m_i^{t-} = \mu_i^{t-}(p_t) \). This is the concise form of the LFS-DA mechanism. After determining the amount of \( (m_i^{t+}, m_i^{t-}) \), each agent has to reconfigure its \( x_i^t \) for the given \( (m_i^{t+}, m_i^{t-}) \) so as to satisfy the constraint \( x_i \in \mathcal{X}_i \).

B. LFS-DA as an emergent RTP method

If a price profile \( \{ p_t \}_{t \in \mathcal{I}} \) is observed, each rational agent tries to maximize its welfare.

\[
\begin{align*}
\text{maximize} & \quad \phi(x_i) + \sum_{t \in \mathcal{I}} p_t (\gamma m_i^{t+} - m_i^{t-}) \\
\text{subject to} & \quad x_i \in \mathcal{X}_i,
\end{align*}
\]

(50)

This corresponds to a sub-problem of the dual problem introduced in the previous section. One of the optimal solutions \( x_i^t(p_t) \) to the problem \( (50) \) can be determined as

\[
x_i^t(p_t) = \arg\max_{x_i \in \mathcal{X}_i} \left( \phi(x_i) + \sum_{t \in \mathcal{I}} p_t (\gamma m_i^{t+} - m_i^{t-}) \right). \quad (51)
\]
Algorithm 2 Iterative dynamics of LFS-DA

\[ k \leftarrow 0 \]

\[ \text{Initialize the price profile } p^{(k)} \]

\[ \text{repeat} \]

\[ \text{Each agent solves its sub-problem } \text{(50)}, \text{and obtains its solution } x_i^* = x_i^*(p^{(k)}). \]

\[ \alpha_i^t = \beta_i^t p_i^t + (m_i^t - m_i^t) \]

\[ \text{Each agent submits } (\alpha_i^t, \beta_i^t) \text{ to the market.} \]

\[ \text{Update } p^{(k+1)} \leftarrow \text{market clearing } ((\alpha_i^t, \beta_i^t) \forall i, k \in \mathcal{T}), \]

\[ \text{Update } x_i^{(k+1)} \leftarrow \text{proj}^m(x_i^t | p^{(k+1)}), \text{ for each } i \]

\[ k \leftarrow k + 1 \]

\[ \text{until a predefined stopping criterion is satisfied.} \]

\[ \text{return Transact } (x_i^{(k)}) \forall i \text{ with } p^{(k)} \text{ as price profile} \]

When the LFS-DA is used, \( \alpha_i^t \) corresponds to \((m_i^t, m_i^t)\) in a bijective relation under the condition that \( \beta_i^t \) and \( p_i \) are fixed.

\[ \alpha_i^t = \beta_i^t p_i + (m_i^t - m_i^t). \]

This is satisfied on the basis of (45) and (46). Therefore, \((m_i^t, m_i^t)\) can be used on behalf of \( \alpha_i^t \) when each agent optimizes its welfare. Here, we formalize each agent’s iterative learning process as a kind of learning algorithm. In the \( k \)-th iteration, the price profile is assumed to be \( p^{(k)} := (p_i^{(k)}) \forall i \in \mathcal{T} \) by each agent. The dynamics of the price profile and agents’ behavior are determined as in Algorithm 2 where \( \text{proj}^m(\cdot | p) \) denotes a projection map from \( \mathcal{X} \) to a set of \( x \in \mathcal{X} \) satisfying \( m_i^t = \mu_i^t(p_i) \) and \( m_i^t = \mu_i^t(p_i) \) for all \( t \in \mathcal{T} \). The function \text{market clearing} represents \( 49 \). By transforming \( 49 \), we can obtain the value of \text{market clearing} as follows.

\[ p_i^{(k+1)} = \frac{\gamma \alpha_i^t}{\lambda_i^t(p_i^{(k+1)})} + \alpha_i^t \lambda_i^t(p_i^{(k+1)}) \]

\[ = \frac{\gamma \alpha_i^t}{\lambda_i^t(p_i^{(k+1)})} + \beta_i^t p_i^t \lambda_i^t(p_i^{(k+1)}) + \gamma \sum_{i \in \mathcal{K}} \alpha_i^t \lambda_i^t(p_i^{(k+1)}) \]

\[ = \frac{\gamma \sum_{i \in \mathcal{K}} \alpha_i^t \lambda_i^t(p_i^{(k+1)}) + \alpha_i^t \lambda_i^t(p_i^{(k+1)})}{\lambda_i^t(p_i^{(k+1)})} \]

\[ = \frac{\gamma \sum_{i \in \mathcal{K}} \alpha_i^t \lambda_i^t(p_i^{(k+1)}) + \alpha_i^t \lambda_i^t(p_i^{(k+1)})}{\lambda_i^t(p_i^{(k+1)})} \]

\[ = \frac{\gamma \sum_{i \in \mathcal{K}} \alpha_i^t \lambda_i^t(p_i^{(k+1)}) + \alpha_i^t \lambda_i^t(p_i^{(k+1)})}{\lambda_i^t(p_i^{(k+1)})} \]

\[ = p_i^t - (\beta_i^t)^{-1} \frac{\gamma \sum_{i \in \mathcal{K}} m_i^t - \sum_{i \in \mathcal{K}} m_i^t}{\lambda_i^t(p_i^{(k+1)})} \]

where \( \beta_i^t = \gamma \beta_i^t \lambda_i^t(p_i^{(k+1)}) \). Therefore, if we consider the determination of the price in LFS-DA as being the price update (52) in dual decomposition-based RTP, the dynamics of LFS-DA become equal to those of RTP where \( \theta_k = (\beta_i^t)^{-1} \).

VI. EXPERIMENT

This section describes our evaluation of the effectiveness of the LFS-DA method by comparing its results with those obtained for the dual decomposition-based RTP algorithm through a simulation experiment.

VII. CONCLUSION

This paper presented a clarification of the relationship between the dual decomposition-based RTP algorithm and the concise LFS-DA algorithm in which multi-agents use linear demand and supply functions when they submit bids.
or asks to an auctioneer in a regional electricity market. The LFS-DA method for managing a regional smart grid consisting of electricity prosumers is proposed. Based on the assumptions that we defined for a regional smart grid, we derived a real-time pricing (RTP) algorithm on the basis of the dual decomposition framework. We proved that the LFS-DA method is able to achieve price profile dynamics equal to those of the dual decomposition-based RTP algorithm. This means that a price controller emerges in the LFS-DA market and maximizes the social welfare of a regional electricity network.

Our proof shows the theoretical connection between research about a multi-agent based microgrid, e.g., PowerMatcher [6], and a control theoretic approach to RTP. The connection will lead the two research areas to a new collaboration and a fruitful integration. This formulation also provides us with the possibility to extend the LFS-DA method beyond its current capabilities. For example, Yo et al. introduced chance constraints to dual decomposition-based RTP [25] and a similar extension should be possible for the LFS-DA approach. In the future, we aim to develop a double auction mechanism, which could automatically control a regional smart grid even in a noisy environment.

REFERENCES


