Competition among Experts via Face-to-Face and Online Channels

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ABSTRACT

Online experts increasingly compete against traditional, face-to-face experts by offering many consulting and customer services. In turn, some traditional high-quality experts consider offering online services to defend their markets. In this research, we investigate how a traditional high-quality expert should modify its business model to respond to the new competition arising due to the Internet. We find that the Internet impacts the high-quality traditional expert more adversely than the low-quality traditional expert due to losses in the second opinion market. If the online transaction costs are low, the online expert with an intermediate quality charges a lower price and obtains a higher profit compared to the traditional high-quality expert.

Keywords

INTRODUCTION

Thirty-seven percent of the adult Internet users are willing to pay to communicate online with their physicians, corresponding to a $4 billion market in 2002 (Harris Interactive Report, 2002; Parker-Pope, 2003; Parker-Pope, 2003; Schatz, 2003). In addition to medical consultations, a growing number of consumers seek legal and tax related second opinions before committing to a course of action. While the convenience of online consultations is a benefit for consumers, the absence of physical inspection and face-to-face dialogue may diminish experts’ ability to solve consumers’ problems. Still, several experts offer online access, differentiate themselves based on the provided opinion type (first or second), and charge as high as $2,800 depending on the service (Parker-Pope 2003).

Individual level decisions can be quite involved as consumers try to reduce their risks. Also, in addition to diagnostic uncertainties, there can be significant variations in the way experts handle a problem, which further creates risks for the consumers, increases the value of expertise, and may justify seeking additional opinions before committing to a course of action. In this research we focus on the optimal business models for the traditional and online experts in a dual-channel environment. Specifically, we ask the following research questions: How should a traditional high-quality expert respond to the new competition arising due to the Internet? How should an online expert compete against traditional face-to-face experts? Under which conditions should a high-quality traditional expert consider offering online consultations, and how would this affect his face-to-face profits?

Research on private information goods that improve consumer decision-making is limited. Previous information systems research examines public information goods, such as software and music, which require a significant sunk cost to produce but only a negligible cost to reproduce (for example, see Chen and Png, 2003; Bakos and Brynjolfsson, 1999; Dewan, Freimer and Seidmann, 2000). Such goods appeal to many consumers once they are ready for sale. In contrast, a private-value information good is produced to improve a specific consumer’s decision making. Bouckaert and Degryse (2000) and Emons (2000) study the pricing and quality competition in private information markets, focusing on the face-to-face channel only. Sarvary (2002) studies the pricing of private information goods and shows that high-quality information sellers may specialize in second opinions. In this research we extend the private information goods literature to a multi-channel setting. In order to focus on the channel management problem, however, we abstract away from certain information problems that beset the relationship between an expert and his customers, such as the expert’s incentive to provide the right service or the right amount of service quality (Darby and Karni, 1973; Pitchik and Schotter, 1987; Wolinsky, 1993; Taylor, 1995; Emons,
There is no asymmetric information involved in the model. We do not underestimate the importance of the missing information problems. However, these problems have already been discussed in the literature and their manner in which they work is relatively well understood.

Our main contribution to the literature is the analysis of optimal business models of experts in a dual-channel environment. We show that the emerging competition from the Internet impacts the high-quality traditional expert more adversely than the low-quality traditional expert due to losses in the second opinion market. As a result of the online expert’s entry, the traditional high-quality expert focuses more on the customers who visited other experts before. If the online transaction costs are low, the online expert with an intermediate quality charges a lower price and obtains a higher profit compared to the traditional high-quality expert.

**MODEL FORMULATION**

The setting involves a linear spatial market of unit length. Consumers are distributed uniformly along this market and have a problem that needs to be fixed. This problem can be personal, or it could be about a product that needs repair. Consumers obtain the value \( r \) for getting the problem fixed through a consultation process. There are three experts with varying abilities to fix the problem. The marginal cost of production of all experts is normalized to zero. The low-quality expert is located at the left extreme of the interval \((x = 0)\), sells his service at price \( p_l \), and fixes the problem successfully with probability 0.5. The high-quality expert is located at the other extreme of the interval \((x = 1)\), charges \( p_h \), and always successfully solves the problem. Consumers incur transportation costs at a linear rate \( t \) per unit distance when visiting the low- or high-quality expert. These costs can include the opportunity cost of time, the real cost of travel, and the implicit cost of inconvenience. Transportation cost serves to differentiate experts’ services with respect to ease of shopping.

Consumers can also obtain consultations from an online expert who has no “location” in the conventional sense. That is, consumers do not incur any transportation cost to obtain the online expert’s service. On the other hand, they incur the online transaction cost \( m \) for having to gather and send the personal information or to ship their good to the expert, where \( m \) is less than the highest transportation cost that can be incurred \((m < t)\). The online expert charges \( p_o \) and fixes the problem successfully with probability \( g_o \). The high-quality expert can also offer an online service with probability \( g_h \), which may be lower than what he can do face-to-face. In the case of a health problem, for example, the expert cannot physically inspect the consumer’s problem via the Internet. Personal information needs to be sent electronically, and this may adversely affect the expert’s success rate depending on the type of the health problem. \( g_h \) would be low when a physical exam is necessary to understand the problem; \( g_h \) would be high if the personal information can be stored and sent electronically, in the form of an X-ray film for instance. If the problem is due to a product that needs repair, mailing it to the expert allows physically inspection, in which case \( g_h \) equals 1. Consumers incur \( m \) also when they obtain an online consultation from the high-quality expert.

Consumers have complete information about the available products and prices. Each consumer has to choose one of the three experts to consult first. If the consumer located at \( x \) visits the low-quality expert first and the problem is solved, she obtains a surplus of \( r - p_l - tx \). If not, she then consults to the online expert and, if this expert solves the problem, the consumer obtains a surplus of \( r - p_l - p_o - tx - m \). If the problem is still not solved, the consumer visits the high-quality expert, pays the fee \( p_h \), incurs the additional transportation cost \( t(1 - x) \), and ends up with a surplus of \( r - p_l - p_h - t - m \). Alternatively, the consumer can visit the online expert first and obtain a surplus of \( r - p_l - m \) if the problem is solved. If not, she visits the high-quality expert and obtains a surplus of \( r - p_l - p_h - m - t(1 - x) \). The consumer’s third option is to visit the high-quality expert first, a strategy that results in a surplus of \( r - p_h - (1 - x)t \). See the appendix for a list of notation.

**ANALYSIS**

**Only the Face-to-Face Channel**

In the absence of the online channel, consumers can obtain a face-to-face consultation either from the low- or high-quality expert. Let \( y_0 \) denote the location of the indifferent consumer between the two options. Then,

\[
r = p_l - ty_0 - \frac{1}{2}(p_h + t(1 - y_0)) = r - p_h - t(1 - y_0).
\]
The low-quality expert provides first opinions to $y_0$ consumers, half of whom seek a second opinion from the high-quality expert. The high-quality expert provides first opinions to $1 - y_0$ consumers and second opinions to $y_0/2$ consumers as only half of the low-quality expert’s customers can’t get their problems solved. The profit of the low-quality expert is $P_l = y_0 p_1$ while that of the high-quality expert is $P_h = (1 - y_0/2)p_h$. Solving simultaneously the best response functions of the two experts yields 

$$p_1^* = \frac{7}{6}t, \quad p_h^* = \frac{11}{3}t,$$ 

$$P_l^* = \frac{49}{54}t, \quad P_h^* = \frac{121}{54}t.$$ 

Prices and profits increase with location-based differentiation, similar to the results in the literature on traditional goods. Notice also that the profit of the high-quality expert is higher than that of the low-quality expert.

**The Entry of the Online Expert**

After the online expert enters the market, consumers located in the middle of the linear city may prefer to get their first opinion from this expert to avoid the transportation cost. Let $y_1$ denote the location of the consumer who is indifferent between obtaining the first opinion from the low-quality and online expert; let $y_2$ denote the location of the consumer who is indifferent between obtaining the first opinion from the online and high-quality expert. Then,

$$r - p_t - ty_1 - \frac{1}{2}\{p_o + m + (1 - g_o)[p_h + t(1 - y_1)]\} = r - p_o - m - (1 - g_o)[p_h + t(1 - y_1)]$$

$$r - p_o - m - (1 - g_o)[p_h + t(1 - y_2)] = r - p_h - t(1 - y_2).$$

The low-quality expert provides first opinions to $y_1$ consumers, half of whom seek a second opinion from the online expert. The online expert provides first opinions to $y_2$ consumers and second opinions to $y_1/2$ consumers. The high-quality expert provides first opinions to $1 - y_2$ consumers, second opinions to $(1 - g_o)(y_2 - y_1)$ consumers, and third opinions to $(1 - g_o)y_1/2$ consumers. Consequently, the profits of the experts are $P_l = y_1 p_t$, $P_o = (y_2 - y_1/2)p_o$, and $P_h = [1 - g_o y_2 - (1 - g_o)y_1/2]p_h$. Solving simultaneously the best response functions of the three experts yields the following optimal prices:

$$p_t^* = \frac{(g_t^2 t - 9g_o t - mg_o + 6m + 12t)(7 + 2g_o - g_o^2)}{8(37g_o - 18g_o^2 + 2g_o^3 + 9)}$$

$$p_o^* = \frac{-18m + 91g_o t + 2g_o^2 t - 2mg_o^3 + 19mg_o^2 - 17g_o^3 t - 39mg_o + 12g_o^2 t}{4g_o t (37g_o - 18g_o^2 + 2g_o^3 + 9)}$$

$$p_h^* = \frac{(g_o^2 t - 9g_o t - mg_o + 6m + 12t)(11 - 3g_o)}{4(37g_o - 18g_o^2 + 2g_o^3 + 9)}.$$

The optimal profits are

$$P_l^* = \frac{(g_t^2 t - 9g_o t - mg_o + 6m + 12t)(7 + 2g_o - g_o^2)^2}{32r(37g_o - 18g_o^2 + 2g_o^3 + 9)(3 - g_o)}$$

$$P_o^* = \frac{(6 - g_o)(-18m + 91g_o t + 2g_o^2 t - 2mg_o^3 + 19mg_o^2 - 17g_o^3 t - 39mg_o + 12g_o^2 t)^2}{32g_o t (37g_o - 18g_o^2 + 2g_o^3 + 9)^2 (3 - g_o)}$$

$$P_h^* = \frac{(g_o^2 t - 9g_o t - mg_o + 6m + 12t)(11 - 3g_o)^2(1 + 4g_o - g_o^2)}{32r(37g_o - 18g_o^2 + 2g_o^3 + 9)(3 - g_o)}.$$
The online channel cost $m$ restrains the online expert’s entry. The higher $m$ is, the less attractive is the online expert’s offering. This restraint as well as low $g_o$ values benefit the traditional high- and low-quality experts. On the other hand, transportation cost plays a different role. Although a high $t$ gives the online expert an edge over the traditional experts, high- and low-quality experts also benefit from it due to the location-based differentiation. Since the online expert does not change the market dynamics much for low $g_o$ values, an increase in $t$ has a more positive effect on the profits of the traditional experts. For high $g_o$ values, however, the online expert benefits more from a high transportation cost. Figure 1 illustrates these points. Notice that the online expert can make a higher profit than the high-quality expert while charging a lower price (see Figure 1e).

![Figure 1. Profits and prices in Case 2](image)

**Proposition 1.** Prices and profits of the high- and low-quality experts increase (decrease) whereas the profit of the online expert decrease (increase) with $m$ ($g_o$). Prices and profits of all experts increase with $t$.

Figure 2 illustrates the effect of ($g_o$) on the market shares of the high-quality and online experts. Interestingly, both the number and proportion of the online expert’s second opinion customers increase as his success rate deteriorates. This is because the success rate sets the nature of competition. For low success rates the online expert does not create much competition at all, and since consumers can buy consultations multiple times, the online expert’s market share increases with $g_o$. But a high $g_o$ hurts the high-quality expert and forces him to decrease price and compete for a larger share of the first opinion market. Hence, the profit of the high-quality expert plummets with $g_o$ while that of the online expert only slightly increases with it (see Figure 1a). Taking the ability of the online expert as given, this observation suggests that the difficulties related with fixing consumers’ problems from a distance can force experts to focus more on second opinion services.

One might wonder why the high-quality expert’s first opinion market share increases with the online expert’s success rate ($g_o$). The reason is the change in the high-quality expert’s focus. In the absence of the online expert, the high-quality expert’s optimal strategy is to serve a small number of first timers given the stream of consumers coming from the low-quality expert. Since a high-quality online expert disrupts that stream, the focus of the high-quality expert shifts to capturing a larger...
segment of the remaining first opinion market. A high $m$ adversely affects the online expert’s competitiveness, increases the high-quality expert’s first opinion market share and reduces his second opinion market share. Consequently, $m$ has a negative effect on the proportion of second opinions served by the high-quality expert. Similarly, an increase in $t$ (relative to $m$) adversely affects the high-quality expert’s competitiveness, increases the online expert’s market share, and therefore increases the proportion of second opinions provided by the high-quality expert. Proposition 2 summarizes these findings.

Proposition 2. The overall market share of the online expert increases with $g_o$ when $g_o$ is low, decreases with $g_o$ when $g_o$ is high.

We next compare the size of the first and second opinion markets for the high-quality expert in the benchmark case (Case 1) and after the entry of the online expert (Case 2). The parameter values in the upper shaded region result in corner solutions in which the online expert either does not offer any first opinions or he is inactive. No solution in pure strategies exists in the lower shaded region. The high-quality expert provides more second/third opinions in Case 2 than in Case 1 for the parameter values in Region I: he provides fewer first opinions in Region I and II. In Region III, he provides more first opinions but fewer second/third opinions. The success rate of the online expert has a significant impact on the profit of the high-quality expert. As the success rate improves, the number of second/third timers that visit the high-quality expert drop dramatically. The loss of these customers forces the high-quality expert to enter into a price competition with the online expert for the first opinion market.

Proposition 3. As a result of the online expert’s entry (Case 2 versus Case 1), the high-quality expert provides fewer second opinions when $g_o$ is high. He provides more first opinions if $m$ is also high, fewer otherwise. When $g_o$ is low, the high-quality expert provides fewer (more) first opinions and more (fewer) second opinions for low (high) values of $m$. 
The Channel Strategy of the High-Quality Expert

In this section we study the high-quality expert’s incentive to acquire the online expert and offer his own online service. In this scenario, consumers who can’t get their problem fixed with the low-quality expert obtain an online consultation from the high-quality expert. However, some consumers located away from both experts may seek their first consultation from the high-quality expert via the Internet. In case the high-quality expert cannot fix a problem online, he invites the consumer to his office without further charge. Let $y_3$ denote the location of the consumer who is indifferent between obtaining the first consultation from the low-quality expert and the high-quality expert (via the Internet), and let $y_4$ denote the location of the consumer who is indifferent between obtaining the first consultation via the Internet or face-to-face from the high-quality expert. Then,

$$r - p_l - ty_3 - \frac{1}{2} \{ p_h + m + (1 - g_h)(1 - y_3) \} = r - p_h - m - (1 - g_h)(1 - y_3),$$

and

$$r - p_h - m - (1 - g_h)(1 - y_4) = r - p_h - f(1 - y_4).$$

The low-quality expert provides first opinions to $y_3$ consumers, half of whom seek an online second opinion from the high-quality expert. The high-quality expert provides first opinions to $1 - y_3$ consumers and second opinions to $y_3 / 2$. The allocation of these consultations between the two channels depends on the online success rate of the expert. The high-quality expert successfully fixes the problem of $g_h y_3 / 2$ second opinion customers and $g_h (y_4 - y_3)$ first opinion customers online; the rest are served face-to-face. The profits of the experts are $P_l = y_3 p_l$ and $P_h = [1 - y_3 / 2] p_h$. Solving simultaneously the best response functions of the experts yields the following optimal outcomes:

$$y_3 = 1 - \frac{2r - m}{3t (3 - g_h)}, y_4 = 1 - \frac{m}{tg_h},$$

$$p_l = \frac{7t}{6} - \frac{g_h t}{2} + \frac{m}{6}, p_h = \frac{11t}{3} - \frac{g_h t}{3} - \frac{m}{3},$$

and

$$P_l = \frac{(7t + m - 3g_h t)^2}{18t (3 - g_h)}, P_h = \frac{(11t - m - 3g_h t)^2}{18t (3 - g_h)}.$$

In the absence of consumer differentiation with respect to the services of the two experts, Bertrand-type price competition would ensue if both the high quality and the online expert offered online consultations simultaneously. The strategy of buying out the online expert can also be interpreted as the high-quality expert preempting in offering the online channel to discourage the entry of the online expert.
The profit of the high-quality expert in Case 3 is greater than that in Case 2 when
\[ \frac{11r - m - 3g_h t}{(g_o^2 t - 9g_o t - mg_o + 6m + 12r)(11 - 3g_o)} > \frac{9(1 + 4g_o - g_o^2)(3 - g_h)}{16(37g_o - 18g_o^2 + 2g_o^3 + 9)(3 - g_o)}. \]

Based on \( g_o \) and \( g_h \) values, Figure 4 illustrates the high-quality expert’s optimal strategy about when to acquire the online expert. The high-quality expert prefers offering online consultations for all middle and high \( g_o \) values. But for very low \( g_o \) values he is better off with the online expert’s entry, especially when \( g_h \) is high. A high \( g_h \) makes the Internet the high-quality expert’s prime service channel where he fixes problems, and since the online channel eliminates the location-based differentiation between experts, profits of both the high- and low-quality experts suffer when the high-quality expert offers online consultations. Hence, the high-quality expert may not provide online consultations in order to maintain the location-based differentiation.

**Figure 4. The online channel strategy of the high-quality expert.**

**Proposition 4.** The high-quality expert does not acquire the online expert when \( g_o \) is very low. This strategy is further supported by high \( g_h \) values.

The total number of second opinions the high-quality expert offers, \( y_3/2 \), is decreasing in \( g_h \). Said differently, the high-quality expert sells more second opinions when his online success rate is low. While the expert serves fewer second opinions as his success rate improves, he can still serve more than what he does in Case 2, primarily because of the competition induced by the online expert in that scenario. Figure 5 compares the resulting composition of the high-quality expert’s customers in Cases 2 and 3 where \( g_h \) equals \( g_o \) for illustrative purposes. Note that consumers seek online first opinions from the high-quality expert only if \( y_4 > y_3 \), or, equivalently, \( \frac{m}{r} < \frac{2g_h}{9 - 2g_h} \). This happens for low online channel cost (\( m \)) and high online success rate (\( g \)), below the dashed line \( \frac{m}{r} = \frac{2g}{9 - 2g} \) in the figure. Compared to Case 2, the high-quality expert offers more first opinions in Region I and II, while he offers more second opinions in Region II and III. Thus, the high-quality expert can provide more second opinions by capturing the online channel, especially for relatively high online success rates. Low \( g \) values in Figure 5 indicate low values for both \( g_h \) and \( g_o \), in which case the high-quality expert offers more second opinions in Case 2. Furthermore, since the partial derivative of \( y_3/2 \) with respect to \( m \) is positive, the number of second opinions served in Case 3 increase with \( m \). That is, the number of second opinions served by the high-quality expert increases with the online transaction cost. On the other hand, \( g \) and \( m \) affect the total number of first opinions in opposite directions.
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Proposition 5a. In Case 3, the total number of first (second) opinions served by the high-quality expert increases (decreases) with $g_h$ and decreases (increases) with $m$. Compared to Case 2, however, the high-quality expert serves more first (second) opinions if online success rates are low (high) and the online channel cost is low (high). He offers online first opinions only when $\frac{m}{t} < \frac{2g_h}{9-2g_h}$.

The comparative static results presented in Proposition 5a present the overall effect of $g_h$ and $m$ on the number of first and second opinions served by the high-quality expert. While $m$ affects the number of online and face-to-face consultations similarly, the effect of $g_h$ is slightly different. The number of online first opinions increases with $g_h$ as the online success rate directly affects the value of the high-quality expert's service. However, the number of online second opinions also increases. The reason is that, although the number of consumers seen via the Internet for the second consultation decreases, the number of “successful” cases increases with $g_h$, resulting in a positive net effect.

Proposition 5b. The numbers of both online first opinions and online second opinions served by the high-quality expert increase with his online success rate. The number of online first (second) opinions decrease (increase) with the online transaction cost.

Compared to Case 1, the high-quality expert is always worse off in Case 3 because of two reasons. First, the Internet eliminates the location-based differentiation that allows the experts to extract consumer surplus. Second, consumers incur the transportation cost in addition to the online transaction cost when the online consultation is not successful, which reduces the value of the expert’s service. Consequently, after acquiring the online expert, the only rationale for the high-quality expert to offer online consultations is to deter entry. Proposition 6a outlines this result. The proof is in the appendix.

Proposition 6a. The high-quality expert makes the same profit in Case 1 and Case 3 for $g_h = 1$ and $m = 0$. For all other values he makes less profit in Case 3 than in Case 1.

The high-quality expert’s total first opinion market share in Case 3, $\frac{2t - m}{3t (3 - g_h)}$, is greater than that in Case 1, $\frac{2}{9}$, when $g_h > \frac{3m}{2t}$. The parameter values that satisfy this inequality lie in Region I in Figure 6. The high-quality expert does not provide any online first opinions in this region and in Region IIa. In Region Ib the expert serves more online second opinions in Case 3 than the number of face-to-face second opinions he serves in Case 1. In Region IIc the expert provides more online first opinions in Case 3 than the number of face-to-face first opinions he serves in Case 1. The expert’s price in Case 3 is always lower than that in Case 1.
Figure 6. Comparison of the number of first and second opinions served by the high-quality expert in Cases 3 and 1.

**Proposition 6b.** Compared to Case 1, the high-quality expert’s first (second) opinion market share in Case 3 is larger (smaller) when $g_h > \frac{3m}{2r}$. His price, on the other hand, is always lower.

**DISCUSSION AND CONCLUSIONS**

We present a model that studies the optimal business model of experts in a dual-channel market, and analyze how a high-quality expert should respond to the competition arising from the Internet and when he should consider offering his online service.

We find that the entry of the online expert affects the profit of the high-quality expert more adversely than that of the low-quality one. As the online expert’s success rate improves, less and less consumers visit the high-quality expert for a second or third consultation, significantly reducing an important revenue stream. A less profitable second/third consultation market forces the high-quality expert to focus more on first consultations and compete directly with the online expert. Interestingly, we show that the online expert may obtain a higher profit compared to the high-quality expert while charging a lower price. These results indicate that the Internet has a significant potential to transform competition in the expert markets. However, such a transformation critically depends on the effectiveness of the online consultation technologies. Further empirical research is necessary to validate these findings.

The simplifying assumptions made in the model pose some limitations. First, the model allows for a maximum of three opinions. Second, the consumers are assumed to pay the full price, and experts may be able to charge higher prices when consumers pay only a fraction (e.g., due to insurance). Third, the experts do not price discriminate based on consumers’ search history. And fourth, the success rates of experts are assumed to be independent. We believe our qualitative results would still hold without these limitations. We also believe that the current results warrant further investigation.
APPENDIX

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$r$</td>
<td>The value consumers receive for having their problem fixed</td>
</tr>
<tr>
<td>$i$</td>
<td>Expert type: $l$ = low-quality, $h$ = high-quality, $o$ = online</td>
</tr>
<tr>
<td>$p_i$</td>
<td>The price of expert $i$ (l, h, o)</td>
</tr>
<tr>
<td>$g_i$</td>
<td>The online success rate of expert $i$ (l, h, o)</td>
</tr>
<tr>
<td>$t$</td>
<td>Transportation cost parameter</td>
</tr>
<tr>
<td>$m$</td>
<td>Online transaction cost</td>
</tr>
<tr>
<td>$p_i$</td>
<td>The profit of expert $i$ (l, h, o)</td>
</tr>
</tbody>
</table>

Table 1. List of Notation

Proof of Proposition 6a. The high-quality expert’s profits in Case 1 and Case 3 are $\frac{121t}{54} + \frac{1}{18(1 - \frac{3g_h}{gh})}t$, respectively. The latter expression is maximized with respect to $m$ when $m = 0$. Taking $m = 0$, Case 3 profit is less than Case 1 profit if $121(3 - g_h)^3 \cdot 3(11 - 3g_h)^2$. Solving this inequality gives $0 \leq g_h \leq \frac{77}{27}$. Since the domain of $g_h$ is a subset of this interval, Case 3 profit is always less than Case 1 profit, except when $m = 0$ and $g_h = 1$, in which case the two profits are equal.

REFERENCES