Side-payment profitability depending on sensitivity of linear-demand to price of access and content

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***Abstract*—This note is concerned with the issue of side-payments between content providers (CPs) and Internet service providers (ISPs) in a potentially not-neutral Internet. Previously, we found that side-payments were not profitable under communal demand response for different types of providers. Here, we consider different (though coupled) linear demand response to usage based prices applied by each provider type, and find conditions on demand sensitivities for overall profitability. The model is extended to account for competition among similar providers of the same type using a simple model of user loyalty to their existing provider.**

I. Introduction

Network neutrality continues to debated as its core economic issues as described in, *e.g.*, [7] have not been resolved. The debate concerns all participants in the enormous and growing Internet economy: Internet service (access) providers (ISPs), content providers (CPs, including providers of computing services), end-user consumers, and government regulators.

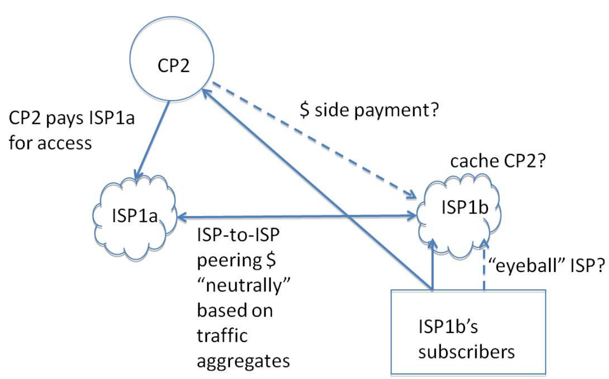

Fig. 1. Side-payments issue

In Figure 1, the subscribers of ISP1b directly pay ISP1b and content provider CP2. CP2's Internet service provider, ISP1a, is different from ISP1b. The financial arrangements at the peering points where the two ISPs exchange traffic are typically "neutral" in the sense that they are based on traffic aggregates and not on the (application) type of traffic[1] [5], [1] or its origin. Another question of neutrality is whether CP2 also pays ISP1b for access to its subscribers. In the absence of such a "side payment", ISP1b might be disincentivized to locally cache "remote" content providers like CP2, potentially gaining revenue at the ISP-to-ISP peering points at the expense of additional downloading delays experienced by ISP1b's subscribers.

In the following, we will not consider alliances of CPs and ISPs wherein revenues may need to be dispersed by use of Shapley values [9], [10] (an "eyeball" ISP may also earn revenue as a content provider). Also, we do not consider the role of advertising revenue [11] in Figure 1. Indeed, instead of directly subscribed revenues, CPs may earn advertising revenue, *e.g.*, [1] (obviously, the subscribers ultimately pay by buying the advertised merchandise or services). Note that ISPs have pointed out that end-users do not necessarily "request" of the CPs the advertising which the CPs insert into content, and have argued for sharing of advertising revenue either through side-payments from CPs or for the right to insert their own ads into downloaded content (broadcast television service may be analogous to the latter scenario). Additionally, we do not extend the model of Figure 1 to study content exchanged peer-to-peer, often in violation of copyright. The enormous traffic volume associated with p2p activity was one of the original motivators of the neutrality debate [3], and issues of piracy continue to be developed, *e.g.*, [13]. Finally, we do not consider multiple types of content as in, *e.g.*, [5].

In this paper, we consider the simpler model of side-payments depicted in Figure 1 for one type of content on an end-user platform ([8], [11] developed games involving end-users and content providers on an ISP platform). We review the case of a communal demand response for providers of both types in Section II. The case of different, though coupled, demand responses is given in Section III for one provider of each type. In Section IV, we discuss the case of multiple identical ISPs and multiple identical CPs, and give some numerical results.

[1]Note that, differentiated service (diffserv) among application types is "neutral" if requested by end-users, whereas application diffserv implemented unilaterally by an ISP is not application neutral (whether implemented at ISP peering points or at end-user access points, and even if for altruistic purposes, *e.g.*, to give more bandwidth for putative real-time applications).

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II. Communal demand response of an end-user platform

Suppose are two providers, one content (CP indexed 2) and the other access (ISP indexed 1). Also, the *communal* consumer demand [6] is

$$D \quad = \quad D^{\max} - d_1 p_1 - d_2 p_2,$$

where d_k is demand sensitivity to price set by provider k, and $D^{\max} > 0$ is the demand at zero usage based price[2]. The revenue of the ISP is

$$U_1 \quad = \quad D(p_1 + p_s), \tag{1}$$

where $p_s \geq 0$ is the side payment from content to access provider. Similarly, the revenue of the CP is

$$U_2 \quad = \quad D(p_2 - p_s). \tag{2}$$

Consider a game played by the CP and ISP adjusting their prices, respectively p_2 and p_1, to maximize their respective revenues, with all other parameters fixed.

The question whether the ISP will profit from side payments is not trivial because the side payment will naturally cause the CP to raise its prices thus lowering overall demand. The proof of following result is from [2], [5] and is included here in order to subsequently relate it to a principal result of this paper. We assume that the ISP will decide that it profits from side payments only if this profit is increasing at $p_s = 0$.

Theorem 1. *At an interior Nash equilibrium requiring*

$$-\frac{D^{\max}}{2d_2 + d_1}, \; < \; p_s \; < \; \frac{D^{\max}}{2d_1 + d_2}, \tag{3}$$

the ISP will decide side payments are profitable if and only if $d_2 > d_1$.

Proof: The conditions for Nash equilibrium are:

$$\begin{aligned} 0 &= \partial U_1/\partial p_1 = (D^{\max} - d_1 p_s) - 2d_1 p_1 - d_2 p_2, \\ 0 &= \partial U_2/\partial p_2 = (D^{\max} + d_2 p_s) - d_1 p_1 - 2d_2 p_2. \end{aligned}$$

So, the Nash equilibrium plays are

$$\begin{aligned} p_1^* &= \frac{1}{3d_1}(D^{\max} - (2d_1 + d_2)p_s) \;\text{ and} \\ p_2^* &= \frac{1}{3d_2}(D^{\max} + (2d_2 + d_1)p_s). \end{aligned}$$

Based on the previous two displays, we can see the requirement of (3) for an interior Nash equilibrium (*i.e.*, $p_1^*, p_2^* > 0$ and $D^* > 0$).

The ISP's revenue at this Nash equilibrium is

$$U_1^* = \frac{1}{9d_1}(D^{\max} - (d_2 - d_1)p_s)^2.$$

So, U_1^* is increasing at $p_s = 0$ (and for all $p_s \geq 0$) if and only if $d_2 > d_1$. □

Note that if $d_2 < d_1$, U_1^* is increasing in p_s if and only if $p_s \geq d_1 - d_2$.

[2]Note that ISPs are continuing to depart from pure flat-rate pricing (based on access bandwidth) for unlimited monthly volume, *e.g.*, [12], [4].

III. Different demand response depending on provider type

In [5], we considered the model of the previous section for the case of multiple providers of each type. Several other previous works have used a communal demand function for both content and access bandwidth even though these are two different commodities with different dimensions. This said, demand for content and access bandwidth may be coupled in that pricing of one will affect the other.

So, consider two different, coupled demand responses for providers of type $k \in \{1, 2\}$:

$$\begin{aligned} D_1 &= D_1^{\max} - d_{11} p_1 - d_{12} p_2, \\ D_2 &= D_2^{\max} - d_{21} p_1 - d_{22} p_2. \end{aligned}$$

A. Side-payments as a function of consumed content

Suppose the side-payments based on content are factored as follows:

$$U_1 = D_1 p_1 + D_2 p_s, \qquad U_2 = D_2(p_2 - p_s). \tag{4}$$

In this setting, we wish to find conditions on the of demand sensitivities d_{ij} and usage-based side-payment price p_s such that side payments are of value to the ISP.

Theorem 2. *The introduction of content-based side-payments (4) will profit the ISP if and only if*

$$\begin{aligned} &[4\, d_{11} {d_{22}}^2 d_{12} + 4\, d_{11} {d_{22}}^2 d_{21} + d_{12} {d_{21}}^2 d_{22}] D_1^{\max} \\ &+ [-2\, d_{11} d_{22} d_{12}^2 + 2\, d_{11} d_{22} d_{21} d_{12} - {d_{12}}^2 {d_{21}}^2 - 8\, {d_{11}}^2 {d_{22}}^2] D_2^{\max} \\ &< \; 0. \end{aligned}$$

Proof: After solving the first order conditions $\partial U_k/\partial p_k = 0$, $k \in \{1, 2\}$, for the Nash equilibrium (p_1^*, p_2^*) and then substituting (p_1^*, p_2^*) into U_1, we get the maximal ISP revenue as a quadratic function of side-payment rate, $U_1^*(p_s)$. That is, the introduction of side payments ($p_s > 0$) will be profitable to the ISP if the derivative $\left.\frac{\partial U_1^*}{\partial p_s}\right|_{p_s=0} > 0$, equivalent to the (computed) condition in theorem statement. □

Generally, U_1^* is a complex nonlinear function of p_s where it is possible that $U_1^*(p_s)$ might be greater than $U_1^*(0)$ for some sufficiently large p_s notwithstanding $\frac{\partial U_1^*}{\partial p_s}(0) < 0$. We ignore this possibility in the following by assuming that side-payments will be rejected by ISPs in the event that $\frac{\partial U_1^*}{\partial p_s}(0) < 0$.

Now to simplify an interpretation of this result, let's consider the special case where $D_1^{\max} = D_2^{\max}$ and $d_{21} = d_{12}$ (despite the fact that the dimensions of these quantities may be different).

Corollary 1. *If* $D_1^{\max} = D_2^{\max}$ *and* $d_{21} = d_{12}$, *then content based side payments are profitable to the ISP if and only if*

$${d_{12}}^4 + 8\, {d_{11}}^2 {d_{22}}^2 \quad > \quad {d_{12}}^3 d_{22} + 8\, d_{11} {d_{22}}^2 d_{12};$$

a condition that obviously holds if

$$d_{11} \; > \; d_{12} \; > \; d_{22}. \tag{5}$$

Roughly, because of (1), content based side-paymemts $p_s > 0$ will, overall, give positive revenue to the ISP because user price-sensitivity to content are less than that of access bandwidth.

B. Side-payments as a function of consumed access bandwidth

Instead of (4), suppose side-payments are based on access bandwidth and factored as follows:

$$U_1 = D_1(p_1 + p_s), \qquad U_2 = D_2 p_2 - D_1 p_s.$$

Because, total revenue is $U_1 + U_2$ is a constant function of p_s, the ISP's profit is at the CP's expense, *i.e.*, $\partial U_1^* / \partial p_s > 0$ if and only if $\partial U_2^* / \partial p_s < 0$. The latter condition is similar to that of Theorem 2 with (i) the indices swapped and (ii) the resulting required condition is for the ISP to lose money.

Corollary 2. *If $D_1^{\max} = D_2^{\max}$ and $d_{21} = d_{12}$, then access bandwidth based side payments are* not *profitable to the ISP if*

$$d_{22} \;>\; d_{12} \;>\; d_{11}. \tag{6}$$

IV. Multiple identical ISPs and identical CPs

Now suppose there are $n_k \geq 1$ providers of type (ISP or CP) k, where p_{ki} is the price of the i^{th} type-k provider. All n_k type-k providers are assumed to offer a similar commodity. So, customer stickiness (inertia, loyalty) dynamics will be present. Accordingly, we assume that the fraction σ_{ki} of the end-user pool who are customers of provider ki is a decreasing function of p_{ki} such that,

$$\sum_{i=1}^{n_k} \sigma_{ki} \;=\; 1 \quad \forall k \in \{1, 2\}.$$

For the sake concreteness[3], we take the example of

$$\sigma_{ki} \;=\; \frac{1/p_{ki}}{\sum_{j=1}^{n_k} 1/p_{kj}},$$

i.e., $\sigma_{ki} \propto 1/p_{ki}$. Thus, the *average* price of providers of type k is

$$\bar{p}_k \;=\; \sum_{i=1}^{n_k} p_{ki}\sigma_{ki} \;=\; \frac{1}{\sum_{j=1}^{n_k} 1/p_{kj}},$$

i.e., the harmonic mean of the type-k provider prices.

Suppose the revenue model for content based side-payments is

$$\begin{aligned} U_{1i} &= \sigma_{1i}[D_1(p_{1i}, \bar{p}_2) - D_2(p_{1i}, \bar{p}_2)p_s] \\ &= \sigma_{1i} U_1(p_{1i}, \bar{p}_2, p_s), \\ U_{2j} &= \sigma_{2j} D_2(p_{2j}, \bar{p}_1)(p_{2j} - p_s), \\ &= \sigma_{1i} U_2(\bar{p}_1, p_{2j}, p_s), \end{aligned}$$

where $1 \leq i \leq n_1$ and $1 \leq j \leq n_2$ and the revenue functions U_k are defined above in (1) and (2). Note that by linearity of the demand functions, the total side-payment transfer is balanced for any p_s:

$$\sum_i \sigma_{1j} D_2(p_{1i}, \bar{p}_2) \;=\; \sum_j \sigma_{2j} D_2(\bar{p}_1, p_{2j}).$$

[3] The following assumption on σ leads to some degree of mathematical tractability in multiple-provider special cases with communal demand response [5].

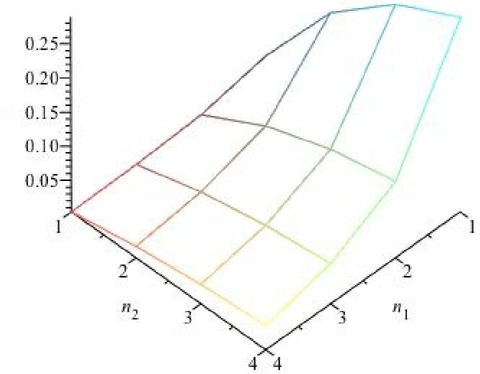

Fig. 2. Optimal per-ISP utility U_1^* at $p_s = 0$

Again, the same revenue model for providers of the same type is assumed. We are interested in the stable, "symmetric" Nash equilibria wherein all type-k providers have the same price p_k^*. As a result, the $n_1 + n_2$ first order conditions for Nash equilibria reduce to two: for $k \in \{1, 2\}$,

$$\frac{\partial}{\partial p_k}\left(\frac{p_k^{-1}}{p_k^{-1} + (n_k - 1)p_{xk}^{-1}} U_k(p_k, p_{3-k}^*, p_s) \bigg|_{p_k = p_k^* = p_{xk}} \right) = 0.$$

This further simplifies to: for $k \in \{1, 2\}$,

$$n_k p_k^* \frac{\partial U_k}{\partial p_k}(p_k^*, p_{3-k}^*, p_s) - (n_k - 1) U_k(p_k^*, p_{3-k}^*, p_s) \;=\; 0.$$

Again, this is a set of coupled quadratic equations in p_1^*, p_2^* (potentially quartic in one after eliminating the other), leading to optimal revenues

$$U_{ki}^* \;=\; \frac{1}{n_k} U_k(p_k^*, p_{3-k}^*, p_s).$$

We conducted a numerical study for this provider-symmetric case again using the assumption that $D_1^{\max} = 1 = D_2^{\max}$ and $d_{21} = d_{12}$ for simplicity. In Figure 2, Nash equilibrium per-ISP revenue U_1^* is plotted ranging over the numbers of providers (n_1, n_2) at $p_s = 0$ (no side payments) with $d_{21} = 0.35 = d_{12}$ midway between $d_{11} = 0.5$, $d_{22} = 0.2$, *i.e.*, satisfying (1). Intuitively, per-ISP revenue decreases with increasing ISP competition (n_1) and increases with increasing CP competition (n_2).

In Figure 3, the sensitivity to small side-payments at Nash equilibrium $\frac{\partial U_1^*}{\partial p_s}(0)$ is plotted ranging over the demand sensitivities $(d_{11}, d_{22}) \in [0.01, 0.61]^2$ with $d_{21} = 0.31 = d_{12}$ (again, mid range) and $n_1 = 2 = n_2$. Here, when $d_{11} = 0.01$ these sensitivities are negative (decreasing from -0.101 to

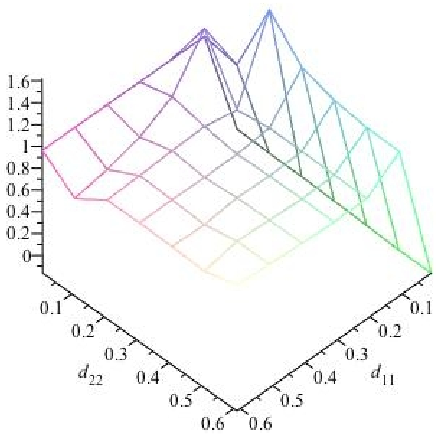

Fig. 3. Per-ISP utility sensitivity $\partial U_1^* / \partial p_s$ at $p_s = 0$

-0.164 as d_{22} increases). Also, sensitivity is unimodal in d_{11} in this range with (positive) peaks at about $d_{11} = 0.11$.

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