Abstract—In this work, we consider the device-to-device (D2D) communications underlaying cellular networks to support local communication needs. In particular, we focus our attention on the design of an optimal resource allocation and mode selection algorithm for both cellular and D2D users. In the design, communication mode selection for D2D users is also taken into account so that a D2D source-destination pair has an option to either directly communicate or indirectly communicate through the base station (BS). On the other hand, it is necessary and important to provide a certain level of Quality of Service (QoS) to users. The users can be further differentiated by assigning different weighting factors and QoS requirements. To this end, we formulate a problem of maximizing the weighed sum rate of all the users constrained by their power and QoS requirements. The tool that we use to handle this problem is the primal-dual technique which transforms the original problem into the equivalent problem showing lower computational complexity. The simulation results verify that our proposed scheme outperforms the previous heuristic scheme by jointly optimizing the power and resource allocation along with the communication mode selection for the D2D communication pairs.

I. INTRODUCTION

By 2020, it is estimated that there will be approximately seven trillion wireless devices serving billions of people [1][2][3]. Such an explosive increase in the number of devices is mainly attributed to the advent of new device types such as wearable and machine type communication (MTC) devices. Given the limited availability of bandwidth and marginal potential improvement on the spectral efficiency, capacity provisioning for this enormous number of devices through the conventional cellular communication connecting all of them to the base stations (BSs) would not be conceivable. By noting that the communication needs for these new devices are mostly with another device or devices within close proximity, the notion of direct device-to-device (D2D) communication has drawn wide attention recently.

There have been active research and development to enable the D2D communication within the existing radio access technology. The D2D functionality was introduced in the Long Term Evolution (LTE) system as part of 3rd Generation Partnership Project (3GPP) Release 12 [4]. On the other hand, Wi-Fi Direct was developed by the Wi-Fi Alliance to support the D2D communication [5]. Bluetooth was inherently designed for the short-range D2D communication. Unlike the Bluetooth or the Wireless Local Area Network (WLAN) based D2D technology, LTE D2D operates in licensed spectrum where the licensee has exclusive right to use the spectrum and, thereby, allowing more reliable communications under controlled interference environment. On the other hand, geographical separation of the D2D links allows to spatially reuse the resources, which can significantly boost the system performance. In the conventional cellular communication, however, all the traffic needs to be routed through the BS, which is a single point of transmission and reception requiring orthogonality in the resource usage.

When compared to the conventional cellular communications, the gains of D2D communication can be listed as follows [6]. The first one is proximity gain, where a source-destination pair in a short distance can enjoy high data rate and low latency at low power consumption. The second one is hop gain, where a source-destination pair within a communication distance of each other can directly communicate rather than communicating through the BS, i.e., one hop vs. two hops. The third one is the spatial resource reuse again as explained before. The last one is pairing gain, where a source-destination pair can choose to directly communicate or communicate through the BS depending on their radio propagation environment.

To put our contributions in perspective, let us discuss some background on the resource allocation for D2D communication. In [7], the authors considered a sum rate maximization problem given the constraints on the individual link quality for a simple setup comprised of a single cellular link and a single D2D link. In [8], the results in [7] was extended to multiple cellular link and multiple D2D links via a maximum weighted bipartite matching formulation. However, still at most one D2D link is allowed to share the cellular resource. More aggressive resource sharing between D2D links and cellular links were considered in [9] and [10]. In particular, a two-stage resource allocation framework was proposed in [11] using the column generation method. It needs to be mentioned, however, that these approaches are suboptimal. More importantly, we emphasize that all the above-mentioned work on D2D resource allocation neglected fairness concern amongst users. From network operator’s perspective, prioritizing the subscribers and providing differentiated QoS to users are essential elements of policy and charging control.
In this work, we consider D2D communications underlaying cellular networks using orthogonal frequency division multiple access (OFDMA) technology. To be more specific, we aim at optimizing the overall performance of both cellular and D2D users while enforcing a certain level of fairness between the individual users. The differentiation amongst users are done via assigning different weighting factors and QoS requirements. The primal-dual approach is chosen to tackle the formulated problem of minimal network power consumption while guaranteeing the QoS requirements. The system-level performance evaluation confirms the superiority of our proposed scheme to previous heuristic approach.

The rest of the paper is organized as follows. In Section II, we describe the system model and formulate our problem. In Section III, we propose our jointly optimal subchannel and power allocation algorithm along with the communication mode selection for D2D pairs. In Section IV, we describe the simulation setup and discuss the results. Finally, we draw some conclusions in Section V.

II. System Model and Problem Formulation

A. System Model

This paper studies an optimal power control and resource allocation problem in a cellular network with underlaying D2D communications. More particularly, the study focuses on a downlink cellular network so the D2D communication reuses cellular downlink resources. There are two types of user equipments (UE) defined in the system, namely cellular UE (CUE) and D2D UE (DUE). CUE is defined as a UE that communicates with another UE located in a different cell while DUE is defined as a UE that communicates with another UE located in the same cell. The two communicating UEs in the same cell together are called a DUE in order to facilitate the mathematical expressions and analysis in following. The system in total has \( K \) resources blocks (RBs), indexed by the set \( \mathcal{K} = \{1, \ldots, K\} \). RB is the scheduling and channel feedback granularity in this study. There are in total \( L \) UEs, indexed by the set \( \mathcal{L} = \{1, \ldots, M, M+1, \ldots M+N\} \). Among \( L \) UEs, there are \( M \) CUEs, indexed by \( \mathcal{M} = \{1, \ldots, M\} \), and \( N \) DUEs, indexed by \( \mathcal{N} = \{1, \ldots, N\} \). \( \mathcal{L} = \mathcal{M} \cup \mathcal{N} \).

Furthermore, the system supports three communication modes, namely CUE cellular mode or C-C mode, D2D cellular mode or D-C mode, and D2D mode or D-D mode. In the D-C mode, the communication consists of two hops, first uplink from one D2D UE to BS and then downlink from BS to another D2D UE. In the D-D mode, communication between two D2D UEs (together as one DUE) in a close proximity can be directly carried out between each other in one hop. Consequently a DUE needs to do mode selection before the real communication starts and mode selection is an essential part of the problem formulation in this paper. We further use \( \tau \) to numerically represent different communication mode, i.e., \( \tau = 0 \) represents the C-C mode, \( \tau = 1 \) represents the D-C mode, and \( \tau = 2 \) represents the D-D mode. Illustrations for three communication modes are shown in Fig.1.

Extensive studies have shown that inter-cell interference can be effectively mitigated with inter-cell interference control mechanisms such as power control or radio resource management. Joint resource and power allocation can be performed to achieve the most efficient spectral usage and power saving in a cellular network by fully exploiting the knowledge channel state information (CSI). In a network with underlay DUEs, radio resources can be shared among CUEs and DUEs in a non-orthogonal mode or DUE and CUE are allocated resources in an orthogonal mode. In this paper, our study assumes an orthogonal resource sharing mode. The channel model between any two communication parties. i.e. between BS and UE or between two UEs) consists of path loss, shadowing and fast fading. The channel gain for CUE \( i \) on \( k \)th RB can be expressed as

\[
\frac{g_{i,k}}{\tau = 0} = g_0\mu_{i,B}^k \phi_{i,B}^k d_{i,B}^{-\alpha},
\]

where \( g_0 \) is a system dependent constant, \( \mu_{i,B}^k \) is the fast fading channel gain with an exponential distribution, \( \phi_{i,B}^k \) is the slow fading gain with a log-normal distribution, \( \alpha \) is the path loss exponent, \( d_{i,B} \) is the distance between CUE \( i \) and the BS. Similarly, the channel gain for DUE \( j \) can be expressed as \( g_{j,k}^\tau, \tau = 0 \) or 1.

B. Problem Formulation

In this paper we study the mode selection, resource allocation and power control altogether in one problem to realize the maximum system spectral efficiency. As primary users, CUEs are guaranteed with a minimum QoS. Let \( p_{i,k} \) and \( r_{i,k} \) denote the transmission power and rate of user \( i \) on RB \( k \), respectively. We have the following expressions for the achievable rate at each RB for different UEs supported in different communication modes. In our study, the size of one RB is normalized into 1, which makes the rate of each RB the...
same as spectral efficiency. Rate of UE $l$ in the C-C mode:
\[ r_{l,k}^{\tau=0} = \log_2(1 + \frac{p_{l,k}^{\tau=0} g_{l,k}^{\tau=0}}{\sigma^2}), \quad \forall l \in \mathcal{M}. \] (2)

Rate of UE $l$ in the D-C:
\[ r_{l,k}^{\tau=1} = \frac{1}{2} \log_2(1 + \frac{p_{l,k}^{\tau=1} g_{l,k}^{\tau=1}}{\sigma^2}), \quad \forall l \in \mathcal{N}. \] (3)

Rate of UE $l$ operating in the D-D mode:
\[ r_{l,k}^{\tau=2} = \log_2(1 + \frac{p_{l,k}^{\tau=2} g_{l,k}^{\tau=2}}{\sigma^2}), \quad \forall l \in \mathcal{N}. \] (4)

Obviously the combination of $\tau$ and UE $l$ is only valid for certain values. For example, $\tau = 1$ and $l \in \mathcal{M}$ or $\tau = 0$ and $l \in \mathcal{N}$ cannot be valid combinations. These constraints will be considered in the problem formulation. For the D-D communication mode, we assume its uplink channel and downlink channel are symmetric. It takes 2 RBs of the serving cell, one for uplink and one for downlink, to deliver one-round communication. Thus in one RB the supported rate is cut half, as shown in Equation (3). As a comparison, communication modes 0 and 2 take only one RB in the serving cell to deliver one-round communication. $\sigma^2$ is the additive white Gaussian noise on each RB, $\Gamma$ defines the SNR gap between the ideal Shannon channel rate and a more practical rate based on a selected modulation and coding scheme. For example, if the M-ary quadrature amplitude modulation (QAM) applied in the system, $\Gamma = \frac{Q^{-1}(\frac{6 \bar{B} R}{\bar{B}^2 R})}{3}$, where $Q^{-1}(x)$ is the inverse Q-function. The overall system optimization problem ($\mathbb{P}_1$) can be formulated as follows.

\[ \mathbb{P}_1 : \max_{p_{l,k}} \left\{ \sum_{l \in \mathcal{L}} \omega_l \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} r_{l,k}^{\tau} x_{l,k}^{\tau} \right\}, \] (5)

subject to
\[ \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} r_{l,k}^{\tau} x_{l,k}^{\tau} \geq R_c, \quad \forall l \in \mathcal{M}, \] (6)
\[ \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} r_{l,k}^{\tau} x_{l,k}^{\tau} \geq R_d, \quad \forall l \in \mathcal{N}, \] (7)
\[ \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} p_{l,k}^{\tau} x_{l,k}^{\tau} \leq P_D, \quad \forall l \in \mathcal{N}, \] (8)
\[ \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} p_{l,k}^{\tau} x_{l,k}^{\tau} \leq P_B, \] (9)
\[ \sum_{l \in \mathcal{L}} \sum_{\tau \in \{0, 1, 2\}} x_{l,k}^{\tau} \leq 1, \quad \forall k \in \mathcal{K}, \] (10)
\[ x_{l,k}^{\tau} \in \{0, 1 \} \quad \forall l \in \mathcal{L}, \tau \in \{0, 1, 2\}. \] (11)

The above problem aims to maximize the sum of a weighted system spectral efficiency by assigning a set of weighting factors $\omega_l, \{l \in \mathcal{L}\}$, to each UE. $\omega_m > \omega_n$ gives CUEs a higher weight thus a higher priority service than DUEs. Constraints (6) and (7) enforce the minimum QoS requirements for CUEs and DUEs, respectively. The total DUE power consumption, including modes 1 and 2, is constrained to a maximum amount $P_{m_D}$ in (8). In constraint (9), the total base station power consumption, including transmission power to CUEs in mode 0 and transmission power to DUEs in mode 1, is limited to $P_B$. $P_{m_D}$ and $P_B$ are defined as overall energy consumption caps. It should be noted that we assume a symmetric channels for uplink and downlink in mode 2 and thus the same rate is transmitted on the uplink and downlink channels. Constraint (10) denotes an exclusive RB assignment rule, i.e., a single RB can be assigned to only one UE, either CUE or DUE. Constraint (11) ensures the right combination of mode $\tau$ and UE $l$.

III. AN OPTIMAL POWER ALLOCATION AND MODE SELECTION ALGORITHM

A multi-user resource allocation problem is normally formulated as a mixed-integer nonlinear programming (MINLP) problem, which is NP-complete [12]. It implies that there is no known polynomial-time algorithm to find the optimal solution, which renders it very difficult and complex to solve. Consequently a heuristic method or sub-optimal solution is normally pursued. ($\mathbb{P}_1$) is a typical MINLP, for which we propose a dual optimization framework in this paper to solve it.

A. Dual Optimization Framework

We first get rid of all binary variables $x_{l,k}^{\tau}$ in the original problem of ($\mathbb{P}_1$). The converted problem will be incorporated into the lagrangian dual function to form the new dual problem. The Lagrangian function is defined over a domain $\mathcal{D}$ as:

\[ L(\lambda, \{p_{l,k}^{\tau}\}, \{r_{l,k}^{\tau}\}) = \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} \omega_l r_{l,k}^{\tau} + \sum_{l \in \mathcal{L}} \lambda_{A,l} \left( \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} r_{l,k}^{\tau} - R_{l,(d,c)} \right) \]
\[ + \sum_{l \in \mathcal{N}} \lambda_{B,l} \left( P_D - \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0, 1, 2\}} p_{l,k}^{\tau} \right) \]
\[ + \lambda_B \left( P_B - \sum_{l \in \mathcal{M}} \sum_{k \in \mathcal{K}} \sum_{\tau = 0} \right), \] (12)

where domain $\mathcal{D}$ is defined as the set of all non-negative $p_{l,k}^{\tau}$’s $\forall l \in \mathcal{L}, k \in \mathcal{K}$ and $\tau \in \{0, 1, 2\}$ such that for each $k$, only one $p_{l,k}^{\tau}$ is positive $\forall l \in \mathcal{L}$ and $\mathcal{N}$ $\tau \in \{0, 1, 2\}$, from constraint (10). Then, the Lagrangian dual function is formed as:

\[ g(\lambda) = \max_{\{p_{l,k}^{\tau}\}, \{r_{l,k}^{\tau}\} \in \mathcal{D}} L(\lambda, \{p_{l,k}^{\tau}\}, \{r_{l,k}^{\tau}\}). \] (13)

It is observed that the dual function $g(\lambda)$ is a pointwise maximum of a family of affine functions of $\lambda = \left[ \lambda_A \right] \left[ \lambda_{B1} \right] \left[ \lambda_{B2} \right]$. Thus $g(\lambda)$ is a convex function of $\lambda$. Hence the original problem can be solved from the dual problem ($\mathbb{P}_2$).

\[ \mathbb{P}_2 : \min_{\lambda} g(\lambda). \] (14)
Actually, let \( g'(\tilde{\lambda}) \) defined in (15) be the function of \( \tilde{\lambda} \), we have the following expression:

\[
g'(\tilde{\lambda}) = \max_{(p_{l,k}^1, r_{l,k}^2) \in D} \left\{ \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} \left( \omega_l r_{l,k}^\tau + \lambda_{A,l} r_{l,k}^\tau - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau = 0,0} \sum_{k \in \mathcal{K}} \lambda_{B,l} \cdot p_{l,k}^\tau - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau = 0,1} \sum_{k \in \mathcal{K}} \lambda_{B,l} \cdot p_{l,k}^\tau \right) \right\},
\]

(15)

\[
= \max_{k \in \mathcal{K}} \max_{(p_{l,k}^1, r_{l,k}^2) \in D} \left\{ \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau = 0,1} \left( \omega_l r_{l,k}^\tau + \lambda_{A,l} r_{l,k}^\tau \right) \right\} - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau = 0,1} \sum_{k \in \mathcal{K}} \lambda_{B,l} \cdot p_{l,k}^\tau.
\]

(16)

\[
= \max_{k \in \mathcal{K}} \max_{l \in \mathcal{L}} \left\{ \max_{p_{l,k}^1, r_{l,k}^2 \in D} \left\{ f_1(p_{l,k}^{\tau=0}), f_2(p_{l,k}^{\tau=1}), f_3(p_{l,k}^{\tau=2}) \right\} \right\}.
\]

(17)

Functions \( f_1, f_2, f_3 \) are respectively defined as:

\[
f_1(p_{l,k}^{\tau=0}) = (\omega_l + \lambda_{A,l}) r_{l,k}^{\tau=0} - \lambda_{B,l} p_{l,k}^{\tau=0},
\]

\( \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \).

(18)

\[
f_2(p_{l,k}^{\tau=1}) = (\omega_l + \lambda_{A,l}) r_{l,k}^{\tau=1} - (\lambda_{B,l} + \lambda_{B,l}) p_{l,k}^{\tau=1},
\]

\( \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \).

(19)

\[
f_3(p_{l,k}^{\tau=2}) = (\omega_l + \lambda_{A,l}) r_{l,k}^{\tau=2} - \lambda_{B,l} p_{l,k}^{\tau=2},
\]

\( \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \).

(20)

Equation (16) is derived since power and rate variables can be separable across different RBs and Equation (17) is satisfied because an RB is exclusively assigned to a UE with the selected mode. Thus the original problem can be decomposed into the \( K \) independent optimization problems, each of which is a per-RB optimization problem and computation complexity is prominently decreased.

Also we have

\[
g(\lambda) = g'(\tilde{\lambda}) - \sum_{l \in \mathcal{L}} \lambda_{A,l} R_l + \sum_{n \in \mathcal{N}} \lambda_{B,n} P_n^0 + \lambda_{B,2} P_B.
\]

(21)

It is not difficult to verify that \( f_1(p_{l,k}^{\tau=0}), f_2(p_{l,k}^{\tau=1}), f_3(p_{l,k}^{\tau=2}) \) are all concave functions of \( p_{l,k}^{\tau=0}, p_{l,k}^{\tau=1}, p_{l,k}^{\tau=2} \), respectively. By evaluating the first derivative of each of them, we can attain the optimal power allocation for each user with the best mode selection.

\[
p_{l,k}^{\tau=0*} = \left[ \frac{1}{\ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] = \left[ \frac{1}{\ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] + \frac{\Gamma \sigma^2}{g_{l,k}}, \quad \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.
\]

(22)

\[
p_{l,k}^{\tau=1*} = \left[ \frac{1}{2 \ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] = \left[ \frac{1}{2 \ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] + \frac{\Gamma \sigma^2}{g_{l,k}}, \quad \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.
\]

(23)

\[\tau=2^*\]

\[
p_{l,k}^{\tau=2*} = \left[ \frac{1}{\ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] = \left[ \frac{1}{\ln 2} \frac{\omega_l + \lambda_{A,l}}{\lambda_{B,l}} \right] + \frac{\Gamma \sigma^2}{g_{l,k}}, \quad \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.
\]

(24)

\([\cdot]^+\) denotes the nonnegative value function. The optimal select mode for UE \( l \) on RB \( f \) is:

\[
[l, \tau]_l^* = \arg \max_{m \in \mathcal{M}, n \in \mathcal{N}} \left\{ f_1(p_{m,k}^{\tau=0}), f_2(p_{n,k}^{\tau=1}), f_3(p_{n,k}^{\tau=2}) \right\},
\]

(25)

The optimal values for binary variables can be determined in the following.

\[
(x_{l,k}^*) = \begin{cases} 1 & \{l, \tau\} = \{l, \tau\}_l^*, \quad \forall k \in \mathcal{K}; \\
0 & \text{otherwise.}
\end{cases}
\]

(26)

Corresponding optimal rate allocations \( r_{l,k}^{\tau=0*}, r_{l,k}^{\tau=1*}, r_{l,k}^{\tau=2*} \) can be evaluated by inserting above optimal power expressions and best mode selection to Equations (2)-(4).

\textbf{Proposition III.1.} For the dual optimization problem \( (\mathbb{P}_2) \) with a Lagrangian dual function \( g(\lambda) \) defined in (13), the following vector \( S \) is a subgradient for \( g(\lambda) \):

\[
S = \begin{bmatrix} \mathcal{S}_{1,l} & \mathcal{S}_{2,l} & P_B - \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} p_{l,k}^\tau \end{bmatrix},
\]

(27)

where

\[
\mathcal{S}_{1,l} = \sum_{k \in \mathcal{K}} r_{l,k}^{\tau=0} - R_c, \quad \text{if} \quad l \in \mathcal{M},
\]

(28)

\[
\mathcal{S}_{1,l} = \sum_{k \in \mathcal{K}} r_{l,k}^{\tau=0} - R_d, \quad \text{if} \quad l \in \mathcal{N},
\]

(29)

\[
\mathcal{S}_{2,l} = P_B - \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} p_{l,k}^\tau, \quad l \in \mathcal{N}.
\]

(30)

\( p_{l,k}^\tau \) and \( r_{l,k}^\tau \) optimize the maximization.

\textbf{Proof.} For any \( \bar{\nu} = [\bar{\nu}_A, \bar{\nu}_B, \nu_B] \geq 0 \), we have the following.

\[
g(\bar{\nu}) = \max_{(p_{l,k}^1, r_{l,k}^2) \in D} L(\bar{\nu}, \{p_{l,k}^0, \{r_{l,k}^1\}, \{r_{l,k}^2\}\}) \geq L(\bar{\nu}, \{p_{l,k}^0, \{r_{l,k}^1\}, \{r_{l,k}^2\}\})
\]

\[
= g(\lambda) + \sum_{l \in \mathcal{L}} (\nu_{A,l} - \lambda_{A,l}) \cdot \left( \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} r_{l,k}^\tau - R_l \right) + \sum_{l \in \mathcal{L}} (\nu_{B,l} - \lambda_{B,l}) \cdot \left( \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} p_{l,k}^\tau \right) + (\nu_{B,2} - \lambda_{B,2}) \cdot \left( P_B - \sum_{k \in \mathcal{K}} \sum_{\tau \in \{0,1\}} p_{l,k}^\tau \right)
\]

\[
= g(\lambda) + S \cdot (\bar{\nu} - \lambda)^T.
\]

(31)

\( \square \)

\textbf{B. Ellipsoid Method Based Optimal Search}

Theoretically, the solution for the dual problem \( \mathbb{P}_2 \) just provides an upper bound for the original primary problem if the primary problem is not a convex problem, which is the case for the primary problem \( \mathbb{P}_1 \). However, it has been proved in [13] that when the total number of users is becoming larger enough (> 8) as in our case, the duality gap is approaching
zero. Thus we can solve the dual problem and find the optimal dual variables $\lambda^*$. Then by substituting them back into the primary problem, the optimal values for the primary variables $p_{l,k}^*$ and $r_{l,k}^*$ can be found. Through the above analysis, a joint optimal resource allocation and mode selection based on a dual framework is developed and the detailed algorithm flow is illustrated in Table I.

### Table I: Optimal Resource Allocation and Mode Selection Algorithm

| Algorithm 1 Optimal Resource Allocation and Mode Selection Algorithm |
|------------------------|------------------|
| **Input:** $R_m$, $R_n$, $P_D^m$, $P_B$ and $\omega_l$, $\forall m \in M$, $\forall n \in N$ |
| **Output:** $p_{l,k}^*$, $r_{l,k}^*$, $x_{l,k}^*$, $\forall l \in L$, $\forall k \in K$, $\forall r \in \{0, 1, 2\}$ |
| 1: Initialize $\lambda_{A,l}, \lambda_{B1,n}$ and $\lambda_{B2}, \forall l \in L$, $\forall n \in N$ |
| 2: Initialize parameter for Ellipsoid search |
| 3: while $g(\lambda)$ has not converge do |
| 4: for all $l \in L$, $k \in K$, $\tau \in \{0, 1, 2\}$ do |
| 5: calculate optimal power $p_{m,k}^*$, $p_{n,k}$, $\tau = 0$ from (22)–(24), and $r_{m,k}^*$, $r_{n,k}$, $\tau = 2$ correspondingly. |
| 6: end for |
| 7: for all $k \in K$ do |
| 8: Calculate function $f_1$, $f_2$, $f_3$ in (18) – (20), respectively |
| 9: Update sub-channel allocation $x_{l,k}^*$ |
| 10: end for |
| 11: Evaluate subgradient $S$ in (27) |
| 12: Update $\lambda_{A,l}, \lambda_{B1,n}$ and $\lambda_{B2}$ using ellipsoid method |
| 13: Evaluate Lagrangian function $g(\lambda)$ in (21) |
| 14: end while |

The search for dual variables $\lambda^*$ is done by using the ellipsoid method, which has a better performance compared with the subgradient method [14]. Ellipsoid method converges in $O(n^2)$ steps where $n$ is the number of variables. In our problem, overall optimization needs $O((L + N + 1)^2)$ runs of an optimization problem with a complexity of $O((L + N)K)$. Hence, $O(K(L + N)^2)$ executions are required to find the optimal solutions by using the proposed algorithm. As an example of one constraint parameter setting, convergence of some of the Lagrangian multipliers and the dual objective function is shown in Fig. 2. Here the convergence of $\lambda_{B2}$ and $\lambda_{A,m}$ in the figure implies the rate constraint (6) and power constraint (9) are all satisfied in our algorithm.

### IV. Numerical Analysis

In this section, performance of the proposed algorithm is evaluated in simulations by considering a single cell network with a radius $R = K m$, where regular CUEs are uniformly distributed in the cell. The two UEs of a DUE are randomly located in a circle with a radius of $R_d = 100 m$ and DUES are uniformly distributed in the cell. In total 64 RBs are considered in the system. The wireless channels are modeled by using path loss, shadow fading and Rayleigh fading. A distance-based path loss of each wireless link is modeled as $PL = 128 + 40 \log_{10}(d)$, where $d$ is the distance of the link in kilometer. The variance for log-normal shadow fading is $8 dB$. The noise power spectral density is set to be $-174 dBm/Hz$. Transmission power limit for DUEs is $P_B^m = 24 dBm$ and the peak transmission power for a BS is set as $P_B = 46 dBm$.

In Fig. 2, we first show the convergence of the proposed optimization algorithm. 5 CUEs and 5 DUEs are uniformly distributed around the cell. QoS requirements for CUEs and DUEs are set to be $R_m = 10$ and $R_n = 5$, respectively. The convergence of the Lagrangian dual function is shown in Fig. 2. To further verify the performance of the algorithm, we compare our dual optimization approach with a heuristic scheme based on the greedy water-filling approach proposed in [15] in a similar system environment. we set $\omega = [1 1 \cdots 1]$, so that weighting factors for all CUEs and DUEs are the same. The system objective is to maximize the total system spectral efficiency. In the approach defined in [15], exclusive RB allo-
and DUEs are set to be assigned to DUEs and less to CUEs. It should be noted that the weighting factor for DUEs increases, more RBs would be total weighted system throughput increases. Similarly, when will be assigned to CUEs and less RBs to DUEs, so that the value of the weighting factor for CUEs increases, more RBs between D2D UEs is set to DUEs under different weighting factors. The average distance between D2D users in a pair increases.

In Fig. 4, we investigate the average rates for CUEs and DUEs under different weighting factors. The ratio between a CUE weighting factor and a DUE weighting factor, \( r = \frac{\omega_c}{\omega_d} \), changes from 1:8 to 8:1. When the weighted factor become favor to DUE or CUE, their rate increment or decrement is diminishing. This is because both cellular user and D2D users are constrained by BS or their own transmission power and QoS requirement.

V. CONCLUSION

In this paper, a joint optimal resource/power allocation and mode selection scheme is proposed in a cellular with underlay D2D commutation. The problem is a NP-complete mixed integer nonlinear programming problem. We develop a dual optimization framework to solve the problem with a reasonable computational complexity of \( O(K(L + N)^3) \). Simulation results show that the D2D users can intelligently the select transmission mode, either through D2D direct transmission or through two-top transmission via base station depending on the channel condition and resource constraint. The comparison with other schemes show that the proposed scheme can achieve a much higher total system throughput.

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