Decentralized control of a mobile sensor network for deployment in corridor coverage

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Abstract—This paper addresses the problems of barrier coverage and sweep coverage in a corridor environment by a network of self-deployed mobile autonomous sensors. Using the ideas of nearest neighbor rules, we propose a decentralized motion coordination algorithm for the sensors to solve the coverage problems. Numerical simulations illustrate the effectiveness of the proposed algorithm. The results in this paper demonstrate that such simple motion coordination rules can play a significant role in addressing the issue of coverage in a mobile sensor network.

Index Terms—Mobile sensor networks; sensor deployment; barrier coverage; sweep coverage

I. INTRODUCTION

Due to a rapid development of communication and microelectronics technologies, it is natural to consider deploying wireless sensor networks for performing tasks in geographically-vast and dangerous working areas, such as ballistic missile tracking, hazardous environment monitoring, or border surveillance. To meet coverage requirements and also reduce the cost of operating a sensor network, mobile or movement-assisted sensors are commonly deployed [19], [20]. In fact, one of the important issues in a wireless sensor network is sensing coverage and it is usually treated as a measure of quality of service in a sensor network.

By using a network of sensing nodes, three types of coverage were defined by Gage [5], namely: 1.) blanket coverage-for achieving a static arrangement of nodes that maximizes the detection rate of targets appearing in the sensing field [4], [7], [19], [20]; 2.) barrier coverage-for achieving a static arrangement of nodes that minimizes the probability of undetected intrusion through the barrier [10], [11], [17]; and 3.) sweep coverage-for moving a number of nodes across a sensing field, such that it addresses a specified balance between maximizing the detection rate of events and minimizing the number of missed detections per unit area [3].

In this paper, we focus on the problems of barrier coverage and sweep coverage in a corridor environment. As indicated in [5], these two coverages have enormous potential applications in, but not limited to, military; for example, barrier coverage can be applied to mine deployment and sentry duty; whereas sweep coverage can be used in multi-agent minesweeping [1], [6], reconnaissance, maintenance inspection, ship hull cleaning. Our barrier coverage problem is to deploy a group of mobile autonomous sensors forming a barrier that detects intruders from entering a protected region in a corridor. On the other hand, the sweep coverage problem is to deploy the sensors so that every point in a specific region in a corridor is detected by the mobile sensors as they move along the region.

We consider the mobile sensors that are self-deployed as the optimal placement of sensors may not be achieved at the initial deployment in the barrier coverage and also the sensors are required to maintain a certain formation in sweep coverage for maximum detection. In addition, each mobile sensor could have severe detection constraints and can sense only a small portion of its environment. Hence, we are particularly interested in self-deployed mobile sensors that work in a distributed unsupervised mode for our coverage problems.

The control of such self-deployed mobile sensors falls within the domain of decentralized control, but the unique aspect of it is that mobile sensors or agents in a group are dynamically decoupled, the motion of one agent does not directly affect any of the other agents. The study of decentralized control laws for groups of autonomous agents has emerged as a challenging research area recently [2], [8], [9], [13]. In this control framework, the motion of each self-deployed mobile sensor is coordinated using information about coordinates or velocities of several other sensors that are closest neighbors of the sensor at a given time.

To develop these local motion coordination rules, researchers were inspired from animal aggregations, such as schools of fish, flocks of birds or swarms of bees, that are believed to use simple, local motion coordination rules at the individual level, but result in complex intelligent behaviors at the group level. To explain these behaviors, Vicsek et al. [18] proposed a simple discrete-time model of a system of several autonomous agents and each agent’s motion is updated using a local rule based on its own state and the state of its “neighbors”. This simple but interesting model was then analyzed by a number of researchers, e.g., [9], [15], [21].

By exploiting such a powerful coordination rule, the objective of this paper is to develop a set of decentralized control laws for a group of self-deployed mobile sensors to perform barrier coverage and sweep coverage tasks in a corridor environment. The paper [17] studied the barrier coverage with mobile sensors in a rectangular environment using a virtual force model approach. Only simulation results were presented in [17] and the theoretical validation of its algorithm was absent. However, in this paper, we propose decentralized control laws for motion coordination of mobile
sensors for both barrier coverage and sweep coverage in a corridor environment and, most importantly, we provide a detailed theoretical development of these control laws.

The rest of the paper is organized as follows. In Section II, we formulate the problem of decentralized control of mobile sensors for barrier coverage and sweep coverage. Section III addresses a barrier coverage problem in an 1-dimensional space and the results will be useful for the development of the main results. Next, we study a 2-dimensional barrier coverage problem in Section IV. Then, the results of barrier coverage will be extended to sweep coverage and presented in Section V. Finally, Section VI presents some simulations results to demonstrate the effectiveness of the proposed algorithm. Due to the page constraint, we leave the proofs of the results in the full version of this paper.

II. PROBLEM FORMULATION

Our sensor deployment problem is to develop a distributed motion coordinated algorithm for coverage in a region with autonomous mobile sensors. We consider a 2-dimensional region \( C \subset \mathbb{R}^2 \) between two parallel lines \( W_1 \) and \( W_2 \), referred to as a corridor. Let \( t = [l_1, l_2]^T \) be a given vector and let \( d_1 \) and \( d_2 \) be given scalars associated with the lines \( W_1 \) and \( W_2 \), respectively. A corridor \( C \) is defined as the intersection of the two regions defined by the lines \( W_1 \) and \( W_2 \), namely, if \( d_1 > d_2 \), \( C := \{(x, y) \in \mathbb{R}^2 : l^T x \leq d_1 \} \cap \{(x, y) \in \mathbb{R}^2 : l^T y \geq d_2 \} \). For a given corridor \( C \), let \( B \subset C \) be a line segment connecting \( W_1 \) and \( W_2 \), and \( \theta_0 \) be the angle of the corridor \( C \) with respect to the x-axis.

A. Barrier Coverage

The first self-deployment objective is to have a network of autonomous mobile sensors to cover a line \( B \) across a given corridor \( C \). This kind of coverage forms a sensor barrier that can be used to detect penetrating objects (intruders) in a corridor \( C \). At the time of initial deployment, the mobile sensors scatter around \( C \) and they may not detect intruders moving along \( C \), see Fig 1(a). To meet the self-deployment objective, the mobile sensors must move autonomously to cover the line segment \( B \) so that the sensor network guarantees that any intruder will be detected when it crosses the line segment \( B \), see Fig 1(b). According to the terminology introduced by Gage [5], the first deployment problem studied in this paper is a barrier coverage problem.

In this paper, we consider a mobile sensor network consisting of \( n \) autonomous sensors labeled 1 through \( n \). Let \( v_i(t) \) and \( \theta_i(t) \) be the linear velocity and heading of the sensor \( i \), respectively. The kinematic equations of the sensors are given by:

\[
\begin{align*}
\dot{x}_i(t) &= v_i(t) \cos(\theta_i(t)), \\
\dot{y}_i(t) &= v_i(t) \sin(\theta_i(t))
\end{align*}
\]

for \( i = 1, 2, \ldots, n \), where \((x_i(\cdot), y_i(\cdot)) \in \mathbb{R}^2\) be the Cartesian coordinates of the sensor \( i \) and \( \theta_i(\cdot) \in \mathbb{R} \) be its heading with respect to the x-axis in the counter-clockwise direction. The velocity \( v_i \) and the heading \( \theta_i \) satisfy \( |v_i(t)| \leq v_{\max} \) and \( \theta_i(t) \in [0, \pi) \) for \( i = 1, 2, \ldots, n \) and all \( t \geq 0 \).

For a given period \( T > 0 \), the sensors gather information about their surrounding neighbors and environment at a discrete time \( t = kT \), \( k = 0, 1, 2, \ldots \). The discrete-time dynamics of the sensor \( i \) are then described by the following discrete-time dynamic model: for all \( i = 1, 2, \ldots, n \),

\[
\begin{align*}
x_i((k+1)T) &= x_i(kT) + \int_{kT}^{(k+1)T} v_i(t) \cos(\theta_i(t)) \, dt \\
y_i((k+1)T) &= y_i(kT) + \int_{kT}^{(k+1)T} v_i(t) \sin(\theta_i(t)) \, dt
\end{align*}
\]

where the heading \( \theta_i(\cdot) \) and the linear velocity \( v_i(\cdot) \) are the control inputs. Each sensor can detect other sensors in a range \( r > 0 \) at time \( kT \), \( k = 0, 1, 2, \ldots \), thus the sensor \( i \) has the ability to detect its neighbors in a disk of radius \( r \) defined by \( D_{i, r}(kT) := \{(x, y) \in \mathbb{R}^2 : (x-x_i(kT))^2 + (y-y_i(kT))^2 \leq r^2 \} \). Let \( N_i(kT) \) be the set of all sensors \( j \neq i \) at time \( t = kT \) belong to the disk \( D_{i, r}(kT) \) and \( |N_i(kT)| \) be the number of elements in \( N_i(kT) \). We describe the sensor \( i \) has \( |N_i(kT)| \) number of neighbors at time \( kT \).

As for the boundary detection, each sensor can detect the boundary of the corridor \( C (\partial C) \) in the range of \( R \).

Assumption 2.1: The sensing range \( r \) and the boundary sensing range \( R \) satisfy

\[
r < v_{\max}T/2, \quad \text{and} \quad R > r\sqrt{2}.
\]

Remark 2.1: For given \( v_{\max} \) and \( r \), the condition \( r < v_{\max}T/2 \) in (3) can be satisfied by choosing an appropriate sampling period \( T \).

Assumption 2.2: The initial headings of the sensors \( \theta_i(0) \) and the angle of the corridor \( \theta_0 \) satisfy \( \cos(\theta_i(0) - \theta_j(0)) > 0 \) for \( i, j = 0, 1, 2, \ldots, n \), where \( \theta_0(0) \equiv \theta_0 \).

Assumption 2.3: For a given width \( d > 0 \) of a corridor, the number of sensors \( n \) and the radius \( r \) of the sensing region satisfy \( (n+1)r > d / \min_{i=1,2,\ldots,n} \{\cos(\theta_i - \theta_j(0))\} \).

The relationships between neighbors can vary over time, and we will describe them by using the notion of graph. Let \( \mathcal{P} \) be the collection of simple undirected graphs defined on \( n \) vertices. For any time \( kT \geq 0 \), the relationship between neighbors are described by a simple undirected graph \( G(kT) \in \mathcal{P} \) with vertex set \( \{1, 2, \ldots, n\} \) where \( i \).
corresponds to the sensor $i$. The vertices $i$ and $j$ of the graph, where $i \neq j$, are connected by an edge if and only if the sensors $i$ and $j$ are neighbors at time $kT$.

**Assumption 2.4:** There exists an infinite sequence of contiguous, nonempty, bounded, time-intervals $[k_j, k_{j+1})$, $j = 0, 1, 2, \ldots$, starting at $k_0 = 0$, such that across each $[k_j, k_{j+1})$, the union of the collection $\{G(kT) \in \mathcal{P} : kT \in [k_j, k_{j+1})\}$ is a connected graph.

To be cost effective, each mobile sensor may have a very limited communication capability, the control laws for the mobile sensors should be distributed or decentralized in the sense that the movement of each sensor only relies on the, e.g., location and heading, information of its neighbors and itself. We assume that the information that is available to sensor $i$ is \{$(x_i(kT), y_i(kT), \theta_i(kT))$ of the sensor $j \in N_i(kT) \cup \{i\}$, $k = 0, 1, 2, \ldots$. In practice, the coordinates of neighboring sensor nodes can be estimated using robust state estimation methods; see, e.g., [12], [16].

Our first aim is to design a family of coordinated control laws that steer the sensors so that they will eventually distribute along a line $B$ in the corridor $C$. The vector $l$ describing the corridor $C$ is defined by $l := [\sin(\theta_0) - \cos(\theta_0)]^T$. Using this, we define a line $L_0$ by

$$L_0 := \{(x, y) \in \mathbb{R}^2 : x\cos(\theta_0) + y\sin(\theta_0) = F_0\}$$

where $F_0$ is some scalar. The line $L_0$ is perpendicular to a corridor $C$ with an angle $\theta_0$.

Next, we define $n$ points $h_i \in \mathbb{R}^2$ on $L_0$ by (see Figure 2(a))

$$h_i = h_0 + i\left(\frac{d_1 - d_2}{n + 1}\right)l, \quad i = 1, 2, \ldots, n,$$  

(5)

where

$$h_0 := L_0 \cap \{(x, y) \in \mathbb{R}^2 : \sin(\theta_0)x - \cos(\theta_0)y = d_2\}.$$  

Define $p_i(kT) = (x_i(kT), y_i(kT))$ be the position of the sensor $i$ at time $kT$, for $i = 1, 2, \ldots, n$ and $k = 0, 1, 2, \ldots$.

**Assumption 2.5:** Assume that the initial positions of $n$ mobile sensors $p_i(0) = (x_i(0), y_i(0))$, $i = 1, 2, \ldots, n$, are uniformly distributed in a bounded set with Lebesgue measure (see e.g., [14]).

**Definition 2.1 (Barrier Coverage):** Given a corridor $C$ with angle $\theta_0$, a decentralized control law is said to be an optimal corridor-barrier-coverage coordinated control for the mobile sensors if for almost all initial sensor positions, there exists a permutation of the set $\{1, 2, \ldots, n\}$ denoted by $\{z_1, z_2, \ldots, z_n\}$ such that the following condition holds:

$$\lim_{k \to \infty} \|p_{z_i}(kT) - h_i\|_2 = 0, \quad i = 1, 2, \ldots, n.$$  

(6)

**Remark 2.2:** Using the definition introduced in [11], a given corridor will be $K$-barrier covered by applying the optimal corridor-barrier-coverage coordinated control to the mobile sensors where $K := \lfloor (n + 1)r/d \rfloor$ and $\lfloor x \rfloor$ is the largest integer that is less than or equal to $x$.

**Remark 2.3:** A condition holds for almost all initial conditions means that it holds for all possible initial conditions except for a set of initial conditions that has measure zero.

### B. Sweep Coverage

The second self-deployment problem we address in this paper is to deploy a network of mobile sensors such that they move in a formation and detect every point in a specified region. Again, at the initial deployment, the sensors are randomly scattered in a corridor $C$. One way to achieve the deployment objective is that the sensors autonomously move in a line formation and scan the specified region as they go along. This second coverage problem belongs to the sweep coverage as defined by Gage [5].

The sweep coverage problem to be defined here is an extension to the corridor barrier coverage we mentioned in the previous subsection, except that the sensors are required to move along a corridor at a desired speed. We let $v_0$ be the desired speed that the sensors sweep along the corridor. Assume that this desired speed is known to all the sensors.

To accommodate the sweeping speed, we impose the following assumption for sweep coverage.

**Assumption 2.6:** The sweeping speed $v_0$ satisfies

$$|v_0| \leq \min\left\{\frac{v_{\text{max}} T}{2}, (\sqrt{R^2 - r^2} - r)\right\}/T.$$  

(7)

In sweep coverage, the line that the sensors converge to is not stationary; it is in contrast to the case of barrier coverage (c.f. (4)). Therefore, for a given $v_0$, we define a moving line $L_0(kT)$ as follows:

$$L_0(kT) := \{(x, y) \in \mathbb{R}^2 : \cos(\theta_0) + y\sin(\theta_0) = F_0 + kTv_0\}, \quad k = 0, 1, 2, \ldots,$$  

(8)

where $F_0$ is some scalar. Again, using $L_0(kT)$ (8), we define $n$ points $h_i(kT)$ on the line $L_0(kT)$ (see Figure 2(b)) by

$$h_i(kT) = h_0(kT) + i\left(\frac{d_1 - d_2}{n + 1}\right)l, \quad i = 1, 2, \ldots, n,$$  

(9)

for $k = 0, 1, 2, \ldots$, where

$$h_0(kT) := L_0(kT) \cap \{(x, y) \in \mathbb{R}^2 : \sin(\theta_0)x - \cos(\theta_0)y = d_2\}.$$  

**Definition 2.2 (Sweep Coverage):** Given a corridor $C$ with angle $\theta_0$ and a desired sweeping speed $v_0$, a decentralized
control law is said to be an optimal corridor-sweep-coverage coordinated control with sweeping speed \(v_0\) for the mobile sensors if for almost all initial sensor positions, there exists a permutation of the set \(\{1, 2, \ldots, n\}\) denoted by \(\{z_1, z_2, \ldots, z_n\}\) such that the following condition holds:

\[
\lim_{k \to \infty} \|p_{z_i}(kT) - h_i(kT)\|_2 = 0, \quad i = 1, 2, \ldots, n. \tag{10}
\]

III. BARRIER COVERAGE IN 1-DIMENSIONAL SPACE

In this section, we propose a decentralized control law for the mobile sensors for them to deploy in a 1-dimensional space such that there will be an equal distance between neighboring sensors. Here, without any loss of generality, we assume that the number of mobile sensors have already aligned on the \(x\)-axis between two fixed points \(x_0\) and \(x_{n+1}\), and their initial positions are assumed to be

\[
x_0 < x_1(0) < x_2(0) < \ldots < x_n(0) < x_{n+1}. \tag{11}
\]

For sensor \(i, i = 1, 2, \ldots, n\), we introduce the following update rule:

\[
\text{if } x_i(kT) - x_{i-1}(kT) \leq r \text{ and } x_{i+1}(kT) - x_i(kT) \leq r,
\]

\[
\text{then } x_i((k+1)T) = x_i(kT) + \frac{r}{2} + \frac{x_{i+1}(kT) - x_{i-1}(kT)}{2}.
\]

\[
\text{if } x_{i+1}(kT) - x_i(kT) \leq r \text{ and } x_i(kT) - x_{i-1}(kT) > r,
\]

\[
\text{then } x_i((k+1)T) = x_i(kT) + \frac{x_{i+1}(kT) - x_{i-1}(kT)}{2} + \frac{r}{2}.
\]

\[
\text{if } x_i(kT) - x_{i-1}(kT) > r \text{ and } x_{i+1}(kT) - x_i(kT) > r,
\]

\[
\text{then } x_i((k+1)T) = x_i(kT).
\]

for \(k = 0, 1, 2, \ldots, n\), \(x_0(0) = x_0\) and \(x_{n+1}(kT) = x_{n+1}\).

Lemma 3.1: Consider \(n\) mobile sensors with sensing range \(r > 0\). Suppose that their initial locations satisfy the condition (11) and also that \(n\) and \(r\) satisfy \((n+1)r > L := x_{n+1} - x_0\). Then the update rule (12) gives

\[
\lim_{k \to \infty} x_i(kT) = x_0 + \frac{iL}{n+1} \tag{13}
\]

for \(i = 1, 2, \ldots, n\).

Proof: The proof of Lemma 3.1 will be given in the full version of the paper. ■

IV. CORRIDOR BARRIER COVERAGE

We define the average heading of sensor \(i\) and its neighbors at time \(kT\), \(k = 0, 1, 2, \ldots\), as

\[
\mathcal{H}_i(kT) := \frac{1}{1 + |\mathcal{N}_i(kT)|} \left( \sum_{j \in \mathcal{N}_i(kT)} \theta_j(kT) \right).
\tag{14}
\]

For each neighbor of sensor \(i\), we define the following scalar at time \(kT\), \(k = 0, 1, 2, \ldots\):

\[
c_{i,j}(kT) = [\cos(\mathcal{H}_i(kT)) \sin(\mathcal{H}_i(kT))] \left[ \begin{array}{c} x_j(kT) \\ y_j(kT) \end{array} \right], \tag{15}
\]

for \(j \in \mathcal{N}_i(kT)\). Similar to the average heading (14), we define the average for the scalar \(c_{i,j}(\cdot), j \in \mathcal{N}_i(kT) \cup \{i\}\) of sensor \(i\) as follows:

\[
\mathcal{M}_i(kT) := \frac{1}{1 + |\mathcal{N}_i(kT)|} \left( c_{i,i}(kT) + \sum_{j \in \mathcal{N}_i(kT)} c_{i,j}(kT) \right), \tag{16}
\]

for \(k = 0, 1, 2, \ldots\). Also, for each sensor \(i\) at time \(kT\), \(k = 0, 1, 2, \ldots\), we define a scalar \(\mathcal{F}_i(kT)\) by

\[
\mathcal{F}_i(kT) = \left[ \cos(\mathcal{H}_i(kT)) \sin(\mathcal{H}_i(kT)) \right] \left[ x_i(kT) \ y_i(kT) \right]. \tag{17}
\]

Let \(\partial \mathcal{C}\) be the boundary of the corridor \(\mathcal{C}\). At time \(kT\), \(k = 0, 1, 2, \ldots\), when sensor \(i\) detects the boundary of the corridor \(\partial \mathcal{C}\), i.e., \(\partial \mathcal{C} \cap D_{i,k}(kT) \neq \emptyset\), the heading \(\theta_i(t)\) of sensor \(i\) will be fixed to \(\theta_0\) for \(j = k+1, k+2, \ldots\).

Next, we define a line \(\mathcal{L}_i(kT)\) for sensor \(i\) as follows:

\[
\mathcal{L}_i(kT) = \{(x, y) \in \mathbb{R}^2 : x \cos(\mathcal{H}_i(kT)) + y \sin(\mathcal{H}_i(kT)) = \mathcal{M}_i(kT)\} \tag{18}
\]

for \(k = 0, 1, 2, \ldots\). We also let \(q_i^j(kT)\) be the projection of the position of the sensor \(j \in \mathcal{N}_i(kT) \cup \{i\}\) on the line \(\mathcal{L}_i(kT)\) at time \(kT\), and it is given by

\[
q_i^j(kT) = \sin(\mathcal{H}_i(kT)) x_j(kT) - \cos(\mathcal{H}_i(kT)) y_j(kT).
\]

Define \(\alpha, \beta \in \mathcal{N}_i(kT)\) such that \(q_{\alpha}^i(kT)\) and \(q_{\beta}^i(kT)\) are immediately next to \(q_i^j(kT)\) and \(q_{\alpha}^i(kT) < q_{\beta}^i(kT)\). It is obvious that such \(\alpha\) or \(\beta\), or both, may not exist since at instance \(kT\), the set \(\mathcal{N}_i(kT)\) for sensor \(i\) could be \(N_i(kT) = 1\) or even \(N_i(kT) = 0\).

Using the coordinates \(q_i^j(kT), j \in \mathcal{N}_i(kT) \cup \{i\}\) at time \(t = kT\), we introduce a function \(Q_i(kT)\) for sensor \(i\) as follows: for \(k = 0, 1, 2, \ldots, i = 1, 2, \ldots, n\),

\[
Q_i(kT) = \begin{cases} q_{\alpha}^i(kT) + q_{\beta}^i(kT) \, \frac{2}{Q_i(kT) - r + q_{\beta}^i(kT)} \, \frac{2}{Q_i(kT) + q_{\alpha}^i(kT) + r}, & \text{if } \alpha \text{ and } \beta \text{ exist}; \\
q_{\alpha}^i(kT) \, \frac{2}{Q_i(kT) - r + q_{\beta}^i(kT)} \, \frac{2}{Q_i(kT) + q_{\alpha}^i(kT) + r}, & \text{if only } \beta \text{ exists}; \\
q_{\alpha}^i(kT), & \text{if only } \alpha \text{ exists}; \\
q_{\alpha}^i(kT), & \text{if both } \alpha \text{ and } \beta \text{ do not exist}. \end{cases} \tag{19}
\]

Next, we will introduce a set of decentralized control laws that solve our proposed problem by using \(\mathcal{H}_i(kT), \mathcal{M}_i(kT)\) and \(Q_i(kT)\).

For \(t \in (kT, (k+1/2)T]\), \(k = 0, 1, 2, \ldots\), the control inputs \(\theta_i(\cdot)\) and \(v_i(\cdot)\) are chosen as:

\[
\theta_i(t) = \begin{cases} \mathcal{H}_i(kT) + \pi/2, & \text{for } \mathcal{H}_i(kT) < \pi/2 \\
\mathcal{H}_i(kT) - \pi/2, & \text{for } \mathcal{H}_i(kT) \geq \pi/2 \end{cases} \tag{20}
\]

\[
v_i(t) = \begin{cases} 2(Q_i(kT) - q_{\alpha}^i(kT)) \, \frac{T \gamma(\mathcal{H}_i(kT))}{T \gamma(\mathcal{H}_i(kT))} \, q_{\beta}^i(kT), & \text{for } \mathcal{H}_i(kT) < \pi/2 \\
1, & \text{for } \mathcal{H}_i(kT) \geq \pi/2 \end{cases} \tag{21}
\]
On the other hand, for \( t \in ((k + 1/2)T, (k + 1)T], k = 0, 1, 2, \ldots \), we choose the heading and velocity control inputs \( \theta_i(t) \) and \( v_i(t) \) as
\[
\begin{align*}
\theta_i(t) &= \begin{cases} 
H_i(kT), & \text{for } \partial C \cap D_{i,R}(kT) = \emptyset \\
\theta_0, & \text{for } \partial C \cap D_{i,R}(kT) \neq \emptyset 
\end{cases} \\
v_i(t) &= 2 \left( \mathcal{M}_i(kT) - \mathcal{F}_i(kT) \right) / T,
\end{align*}
\] (22)
for \( i = 1, 2, \ldots, n \), with \( H_i(-T) := \theta_i(0) \).

Intuitively, the control law (20) drives the sensor \( i \) parallel to the line (18) during the time interval \((kT, (k + 1/2)T] \) for equalizing their projected distances between adjacent sensors on (18); whereas the control law (22) steers the sensor \( i \) to the line (18) during the time interval \(( (k + 1/2)T, (k + 1)T] \). Since over the time interval \((kT, (k + 1/2)T], (k = 0, 1, 2, \ldots) \) the control law (20) only drives the sensor \( i \) parallel to the line (18), the control laws (22) and (20) gives the future heading at time \((k + 1)T \) and the scalar \( F_i \) at time \((k + 1)T \) as follows: for \( i = 1, 2, \ldots, n \),
\[
\begin{align*}
\theta_i((k + 1)T) &= \begin{cases} 
H_i(kT), & \text{for } \partial C \cap D_{i,R}(kT) = \emptyset \\
\theta_0, & \text{for } \partial C \cap D_{i,R}(kT) \neq \emptyset 
\end{cases} \\
F_i((k + 1)T) &= \mathcal{M}_i(kT).
\end{align*}
\] (23)

Now we are in position to present the main result of this section.

**Theorem 4.1:** Consider \( n \) mobile sensors described by the equations (2). Suppose that Assumptions 2.1–2.6 are satisfied. Then the decentralized control law (20) and (22) is an optimal corridor-barrier-coverage coordinated control for the mobile sensors.

**Proof:** The proof of Theorem 4.1 will be given in the full version of the paper.

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**V. CORRIDOR SWEEP COVERAGE**

To solve our corridor sweep coverage problem, we define the control \( v_i(\cdot), i = 1, 2, \ldots, n \), in (2) as follows:
\[
v_i(t) = \begin{cases} 
\bar{v}_i(t), & \text{for } t \in (kT, (k + 1/2)T] \\
\bar{v}_i(t) + 2v_0, & \text{for } t \in ((k + 1/2)T, (k + 1)T],
\end{cases}
\] (24)

where \( v_0 \) is the desired sweeping speed and \( \bar{v}_i \) is a control input to the following system:
\[
\begin{bmatrix} 
x_i((k + 1)T) \\
y_i((k + 1)T)
\end{bmatrix} = \begin{bmatrix}
\bar{x}_i(kT) + \int_{kT}^{(k + 1)T} \bar{v}_i(t) \cos(\theta_i(t)) dt \\
\bar{y}_i(kT) + \int_{kT}^{(k + 1)T} \bar{v}_i(t) \sin(\theta_i(t)) dt
\end{bmatrix} + w_i(kT)
\] (25)

with \( \bar{x}_i(0) = x_i(0) \) and \( \bar{y}_i(0) = y_i(0) \), and the vector \( w_i(kT) \) defined by
\[
w_i(kT) := 2v_0 T \times \int_{(k + 1/2)T}^{(k + 1)T} \begin{bmatrix}
\cos(\theta_i(t)) - \cos(\theta_0) \\
\sin(\theta_i(t)) - \sin(\theta_0)
\end{bmatrix} dt.
\]

**Theorem 5.1:** Consider \( n \) mobile sensors described by the equations (2). Suppose that Assumptions 2.1–2.6 are satisfied. Then the decentralized control law (20), (22) and (24) is an optimal corridor-sweep-coverage coordinated control with sweeping speed \( v_0 \) for the mobile sensors.

**Proof:** The proof of Theorem 5.1 will be given in the full version of the paper.

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**VI. SIMULATION RESULTS**

In this section, we present some simulation results to illustrate and validate the proposed algorithm. First, we consider six mobile sensors and employ them for the barrier coverage in a straight corridor. Figure 3(a) shows their position at the initial deployment. Initially, they are stationary and the arrows indicate their headings. Using the proposed algorithm, the locations of the sensors are shown in Figure 3(b) at time \( t = 100T \), where \( T = 1 \). It is clear that the sensors coverage to a stationary line that is orthogonal to the corridor, again the arrows indicate their headings at that time. The sensors are evenly distributed in the line segment between the walls of the corridor. Since the sensing range of the sensor is \( r = 2 \) and the sensors form a barrier that can detect any passing intruder. Next, we employ the six mobile sensors for the sweep coverage in the same corridor and their initial positions are shown in Figure 4(a). Our proposed sweep coverage algorithm steers the sensors into a line formation and they maintain this formation as they sweep along the corridor. As a result, any stationary intruders or objects that are located with \( y \geq 1 \) can be detected. Here the sensors sweep along the corridor at a speed of \( v_0 = 0.09 \).

![Fig. 3. Barrier coverage with a 90° straight-line corridor (r = 2, R = 3.5): (a) Initial deployment; (b) final deployment.](image)

![Fig. 4. Sweep coverage with a 90° straight-line corridor (r = 2, R = 3.5, v0 = 0.09): (a) Initial deployment; (b) sensor trajectories.](image)

Our proposed algorithm was developed with an assumption that the corridors are straight. However, the algorithm
can also be applied to smooth curved corridors. Using the algorithm, Figures 5–6 show the simulation results of barrier coverage in a circular corridor and an arbitrarily-curved corridor. Sweep coverage can also be achieved in these corridors as shown in Figures 7–8. Similar to the case with a straight corridor, the sensing ranges \( r = 2 \) and \( R = 3.5 \) were used. For the circular corridor, we chose \( v_0 = 0.2 \); whereas \( v_0 = 0.6 \) was chosen for the arbitrarily-curved corridor. We point out that \( v_0 \) is not the actual sweeping speed along a curved corridor. It is because the forward speed of the sensors are different as they maneuver along a curved corridor.

**References**


