An Improved Model to Obtain Some Statistical Properties of Surface Slopes via Remote Sensing Using Variable Reflection Angle

Josué Álvarez-Borrego and Beatriz Martín-Atienza

Abstract—Relationships to obtain information concerning some statistical properties of sea-surface slope from statistical properties of the sea-surface intensities in remotely sensed images, considering an improved model with variable detector line-of-sight angle, are shown. A new expression between the variance of the intensity in the image and the variance of the surface slopes is derived. The association between correlation functions of the intensities in the image and surface slopes is numerically analyzed. A 1-D case is considered, and the results are presented graphically.

Index Terms—Glitter pattern, random processes, sunglint.

I. INTRODUCTION

TO MEASURE the sea-wave motion, the use of radar images and optical processing of aerial photographs have been used. The significance of wave data is manifold; one element is the inherent interest in the directional spectra of waves and how they influence the marine environment and the coastline. These wave data can be readily and accurately collected by aerial photographs of the wave sunglint patterns, which show reflections of the sun and skylight from the water and thus, offer high-contrast wave images.

Several authors [1], [2] have studied this problem considering a sea surface illuminated by a continuous skylight with no azimuthal variations in sky radiance. Different models of skylight have been used, emphasizing the existence of a nonlinear relationship between the slope spectrum and the corresponding wave image spectrum [3], [4]. Simulated sea surfaces have been analyzed by optical systems to understand the optical technique in order to obtain better qualitative information of the spectrum [5], [6].

Cox and Munk [7], [8] studied the distribution of intensity or glitter pattern in aerial photographs of the sea. One of their conclusions was that, for constant and moderate wind speed, the probability density function of the slopes is approximately Gaussian. This could be taken as an indication that in certain circumstances, the ocean surface could be modeled as a Gaussian random process.

Fuks and Charnotskii [9] derived the joint probability density function of surface height and partial second derivatives for an ensemble of specular points at a randomly rough Gaussian isotropic surface at normal incidence.

Fuks et al. [10] derived the joint probability density function of travel-time fluctuations and intensities of the first two arrivals of a wave backscattered from a 3-D statistically rough Gaussian surface. They use a source that emits a short pulse with spherical wavefront.

Álvarez-Borrego [11]–[13] derived the equation which describes the glitter pattern in one and two dimensions. The relationship between surface slopes and the intensities of the image can be represented by this function. With the glitter function, this author found the relationship between the correlation function of the intensities in the image and the correlation function of the surface heights. As a by-product, it is possible to obtain the variance of the surface heights from the variance of the intensities in the image.

Álvarez-Borrego [14] considered the problem of retrieving spatial information of the statistical properties of random rough surfaces from images via remote sensing. He obtained expressions between variance of the intensity in the image and surface heights. He considered a physical situation where the detector D (Fig. 1, [14]) is fixed in the zenith, i.e., the detector line of sight angle $\theta_d$ is constant for each point on the surface. Cureton et al. [15] considered the same physical situation to obtain similar expressions for the variances. Álvarez-Borrego [14] and Cureton et al. [15] used the particular case $\theta_d = 0$ as an illustrative example.

In this paper, we propose an improved model considering that $\theta_d$ is variable for each point in the surface (Fig. 1). Thus, this...
model does not impose a restriction on the sensor field of view. A new expression between variance of the intensity in the image and surface slopes is derived, and a new relationship between the correlation function of the intensities in the image and the correlation function of the surface slopes is obtained.

II. GEOMETRY OF THE MODEL

The improved model, considering \( \theta_d \) as variable, is shown in Fig. 1. The surface \( \zeta(x) \) is illuminated by a uniform incoherent source \( S \) of limited angular extent, with wavelength \( \lambda \). Its image is formed in \( D \) by an aberration-free optical system. The incidence angle \( \theta_i \) is defined as the angle between the incidence direction and the normal to the mean surface and represents the mean angle subtended by the source \( S \). \( (\theta_d)_i \) corresponds to the angle subtended by the optical system of the detector with the normal to point \( i \) of the surface, i.e.,

\[
(\theta_d)_i = \tan^{-1} \left( \frac{i \Delta x}{H} \right)
\]

where \( H \) is the height of the detector and \( \Delta x \) is the interval between surface points. We can see that in this more realistic physical situation, angle \( \theta_d \) is changing with respect to each point in the surface. It is worth noticing that a variable \( \theta_d \) does not restrict the sensor field of view.

\( \alpha_i \) is the angle subtended between the normal to the mean surface and the normal to the slope for each \( i \) point in the surface

\[
\alpha_i = \frac{\theta_s + (\theta_d)_i}{2}
\]

\[
\alpha_i = \frac{\theta_s}{2} + \frac{1}{2} \tan^{-1} \left( \frac{i \Delta x}{H} \right).
\]

The apparent diameter of the source is \( \beta \). Light from the source is reflected on the surface for just one time, and, depending on the slope, the light reflected will or will not be part of the image. Thus, the image consists of bright and dark regions that we call a glitter pattern.

Our analysis involves three random processes: the surface profile \( \zeta(x) \), its surface slopes \( M(x) \), and the image \( I(x) \). The glitter function can be expressed as

\[
B(M_i) = \text{rect} \left[ \frac{M_i - M_{oi}}{(1 + M_{oi}^2)^{\beta/2}} \right]
\]

where

\[
M_{oi} - \left( 1 + M_{oi}^2 \right)^{\beta/4} \leq M_i \leq M_{oi} + \left( 1 + M_{oi}^2 \right)^{\beta/4}
\]

\[
M_i = \tan(\alpha_i)
\]

\[
M_{oi} = \tan \left( \frac{\theta_s + (\theta_d)_i}{2} \right).
\]

The interval characterized by (5) defines a specular band where certain slopes generate bright spots in the image. This band has now a nonlinear slope due to the variation of \( (\theta_d)_i \) with respect to each \( i \) point of the surface (see Fig. 2). Combining (5)–(7), the slope interval, where a bright spot is received by the detector, is

\[
\frac{\theta_s}{2} + \frac{1}{2} \tan^{-1} \left( \frac{i \Delta x}{H} \right) - \frac{\beta}{4} \leq \alpha_i \leq \frac{\theta_s}{2} + \frac{1}{2} \tan^{-1} \left( \frac{i \Delta x}{H} \right) + \frac{\beta}{4}.
\]

III. RELATIONSHIPS AMONG THE VARIANCES OF THE INTENSITIES IN THE IMAGE AND SURFACE SLOPES

The mean of the image \( \mu_I \) may be written as [16]

\[
\mu_I = \langle I(x) \rangle = \int_{-\infty}^{+\infty} B(M_i) p(M_i) dM_i
\]

where \( B(M_i) \) is the glitter function defined by (4), \( p(M_i) \) is the probability density function, where a Gaussian function is considered in one dimension. Substituting in (9) the expressions for \( B(M_i) \) and \( p(M_i) \), we have

\[
\mu_I = \langle I(x) \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\sigma_M(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \text{rect} \left[ \frac{M_i - M_{oi}}{(1 + M_{oi}^2)^{\beta/2}} \right] \times \exp \left( -\frac{M_i^2}{2\sigma_M^2} \right) dM_i.
\]
The variance of the intensities in the image \( \sigma_I^2 \) is defined by \([14]\)

\[
\sigma_I^2 = \langle I^2(x) \rangle - \langle I(x) \rangle^2 = \int_{-\infty}^{+\infty} [B(M_i) - \mu_I]^2 p(M_i) \, dM_i. \tag{12}
\]

However, \( B(M_i) = B^2(M_i) \), then \( \langle I(x) \rangle^2 = \langle I(x) \rangle \); therefore

\[
\sigma_I^2 = \langle I^2(x) \rangle - \langle I(x) \rangle^2 = \mu_I (1 - \mu_I). \tag{13}
\]

Substituting the expression of \( I^2(x) \), \( (11) \), in \( (13) \), we have

\[
\sigma_I^2 = \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{2} \left[ \text{erf} \left( \frac{b_i}{\sqrt{2} \sigma_M} \right) - \text{erf} \left( \frac{a_i}{\sqrt{2} \sigma_M} \right) \right] \right. \\
- \frac{1}{4N} \left[ \text{erf} \left( \frac{b_i}{\sqrt{2} \sigma_M} \right) - \text{erf} \left( \frac{a_i}{\sqrt{2} \sigma_M} \right) \right]^2 \right\}, \tag{14}
\]

which is the required relationship between the variance of the intensities in the image \( \sigma_I^2 \) and the variance of the surface slopes \( \sigma_M^2 \).

Equation \( (14) \) is shown in Fig. 3 as a smooth line, for an angle \( \theta_s = 30^\circ \), using the geometry described earlier, when \( H = 100 \, \text{m} \) and the \( x \)-axis length is 327.68 m. The detector is located as shown in Fig. 1, and the subtended angle by the source is \( \beta = 0.68^\circ \). The noisy line is the numerical calculation from the data when we have one realization only.

The dashed line in Fig. 4 shows the same case, but we have used 500 realizations to perform the numerical calculation. We can see that the noise is reduced significantly, and we observe a very good match between the theoretical and the numerical calculation.

The relationship between the variance of the surface slopes and the variance of the intensities of the image for different incidence angles \((10^\circ - 50^\circ)\) is shown in Fig. 5.

We can see that the relationship between \( \sigma_I^2 \) and \( \sigma_M^2 \) in this paper, compared with the relationship obtained in Álvarez-Borrego \([14]\) and Cureton \textit{et al.} \([15]\), is totally different. Our results show that \( \sigma_I^2 \) increases for higher \( \theta_s \) values. This new result can be explained by Fig. 6.

In Fig. 6, we calculate the glitter pattern, considering the special case of a horizontal mirror surface, for different incidence angles \((10^\circ - 50^\circ)\). The geometry is the same as the one considered in Fig. 1. We see that the glitter pattern is narrower for lower values of incidence angle because the detector angle \( (\theta_d) \) varies with the distance \( x \). Thus, the variance increases with the incidence angle.

In Fig. 5, we also can observe that, for big incidence angles \((40^\circ - 50^\circ)\) and small values of variance of the surface slopes,
it is possible to obtain bigger values in the variance of the intensities in the image. From (14), we can see that this relation depends on the probability density function of the surface slopes and the geometry of the experiment only.

In certain cases, Fig. 5, if we have data corresponding to only one \( \theta_s \) value, it is not possible to obtain the variance of the surface slopes \( \sigma_M^2 \). To solve this problem, it is necessary to analyze images which correspond to two or more incidence angles and to select a slope variance value which is consistent with all these data [17].

IV. RELATIONSHIP BETWEEN THE CORRELATION FUNCTIONS OF THE INTENSITIES IN THE IMAGE AND OF THE SURFACE SLOPE

As mentioned before, our analysis involves three random processes: the surface profile \( \zeta(x) \), its surface slopes \( M(x) \), and the image \( I(x) \). Each process has a correlation function, and it was shown in [11] that these three functions are related.

The relationship between the correlation function of the surface slopes \( C_M(\tau) \) and the correlation functions of the intensities in the image \( C_I(\tau) \) is given by

\[
C_I(\tau) = \frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{+\infty} \frac{B(M_{1i})B(M_{2i})}{2\pi \sigma_M^2 [1 - C_M^2(\tau)]^{1/2}} \times \exp \left[ -\frac{M_{1i}^2 + M_{2i}^2 - 2C_M(\tau)M_{1i}M_{2i}}{2\pi \sigma_M^2 [1 - C_M^2(\tau)]} \right] dM_{1i}dM_{2i}.
\]

Although it is possible to obtain an analytical relationship for the first integral, for the second integral, the process must be numeric.

In order to avoid computer memory problems, the 16,384-data-point profile was divided into 17 consecutive intervals. The value of \( \theta_d \) varies point to point in the profile. For each interval and for each \( \theta_s \) value, the relationship between the correlation functions \( C_I(\tau) \) and \( C_M(\tau) \) was calculated. Then, the 17 computed relationships for each \( \theta_s \) value were averaged.

Then, to compare our results with those of Álvarez-Borrego [14] and Cureton et al. [15], we used a value of \( \sigma_M = 0.2121 \). It is necessary to normalize the correlation functions, by using the corresponding variances. A theoretical variance \( \sigma_i(\tau) \) can be calculated from (14). It is also possible to estimate the variance by averaging the theoretical variances calculated for each of the 17 intervals. We compare both variances in Table I.

We can see that the variance values are similar up to three values to the right of the decimal point. The difference in the next values is possibly due to numerical error.

Fig. 7 shows graphically the relationship between the normalized correlation function of the surface slopes \( [C_M(\tau)]_n \) and the normalized correlation function of the intensities of the image \( [C_I(\tau)]_n \). The relationship between \( [C_I(\tau)]_n \) and \( [C_M(\tau)]_n \) in this paper is totally different from the relationship proposed by Álvarez-Borrego [14] and Cureton et al. [15].

**TABLE I**

<table>
<thead>
<tr>
<th>( \theta_s )</th>
<th>Theoretical variance of the image</th>
<th>Mean theoretical Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0033431</td>
<td>0.0032175</td>
</tr>
<tr>
<td>20</td>
<td>0.0046773</td>
<td>0.0045003</td>
</tr>
<tr>
<td>30</td>
<td>0.0061677</td>
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</tr>
<tr>
<td>40</td>
<td>0.0075672</td>
<td>0.0074548</td>
</tr>
<tr>
<td>50</td>
<td>0.0085148</td>
<td>0.0084664</td>
</tr>
</tbody>
</table>
our results, this relationship increases for higher values of $\theta_s$. This new result can be explained by Fig. 6.

V. CONCLUSION

We have derived the variance of the surface slopes from the variance of the intensities of remotely sensed images, considering a glitter function given by (4) in the unidimensional case, considering a geometrically improved model with variable detector line of sight angle, as shown in Fig. 1. In addition, we discussed the determination of the correlation function of the surface slopes from the correlation function of the image intensities.

Our new model is more widely applicable than the previous approach by Álvarez-Borrego [14] and Cureton et al. [15]. However, we obtained an opposite relationship between the normalized correlation function of the surface slopes $[C_M(\tau)]_n$ and the normalized correlation function of the intensities of the image $[C_I(\tau)]_n$. In our results, this relationship increases for higher values of $\theta_s$ because, for higher incidence angles, the glitter pattern is wider.

REFERENCES


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