Overview of analytical models of permanent magnet electrical machines for analysis and design purposes

Hugquette Tiegna, Yacine Amara *, Georges Barakat

GREAH, Université du Havre, 25 rue Philippe Lebon, B.P. 1123, 76063 Le Havre cedex, France

Received 25 October 2011; received in revised form 18 October 2012; accepted 2 December 2012
Available online 21 January 2013

Abstract

Generally, accurate modelling of electrical machines requires the use of finite-element method. However, FE analysis is too time consuming, especially at firsts design stages, from the point of view of engineers working in R&D departments in the electrical machine industry. To reduce pre-design stages duration, analytical models are often preferred. Two types of analytical models are often used: magnetic equivalent circuits (MEC) and analytical models based on the formal solution of Maxwell’s equations in constant permeability regions. However, MEC method is not as generic as the finite element method. In fact, even in the case of a given structure geometry, MEC method has to be adapted if the geometric parameters vary in a large scale. Analytical models based on the formal solution of Maxwell’s equations help overcome aforementioned problem. This paper is intended as a tutorial overview based on a review of the state of the art, describing recent developments in the field of analytical modelling of permanent magnet machines.

© 2013 IMACS. Published by Elsevier B.V. All rights reserved.

Keywords: Permanent magnet machines; Analytical modelling; Electromagnetic analysis

1. Introduction

Energy efficiency is the quickest, cheapest and cleanest way to reduce energy consumption and achieve greenhouse gas emission to meet international climate policy targets.

In modern industrialized country, a large amount of electric energy is consumed by electric motors. With the worldwide trend to energy conservation, there is a need to increase the efficiency of electric motors, small and large. To reach this goal, the improvement of materials and optimization of design strategies have been tried out.

Consequently and due to their high torque/power density and high efficiency as a result of the use of high energy-product rare-earth PM materials, permanent magnet (PM) brushless machines have become one of the most popular machine technologies in many applications, ranging from domestic appliances, passing by industrial servos, electric/hybrid electric vehicles and more-electric aircrafts, to wind power generation.

To facilitate the design and analysis of PM machines, a precise computation of the magnetic field distribution in the different machine regions is required. Generally, accurate modelling of electric machines requires the use of finite-element method. However, FE analysis is too time consuming, especially for the parametric studies at the first
stages of the design process. In fact, in these first stages, it appears that, from the point of view of engineers working in R&D departments in the electrical machine industry, the use of analytical models is often preferred because they allow the exploration of the whole search space of solutions while reducing the pre-design stages duration. Two types of analytical models are often used for a design purpose: magnetic equivalent circuits (MEC) and analytical models based on the formal solution of Maxwell’s equations in constant permeability regions. At this stage, it is important to note that, even if the classical MEC method takes into account magnetic saturation as FE analyses, it is not as generic as the finite element method. Indeed, in a design process, even for the same machine geometrical structure, the permeance network of classical MEC has to be adapted if the geometric parameters vary in a large scale. To remedy to this problem some authors have proposed mesh based MEC methods [7,53,55] to compete with the finite element method. However, in contrast to FE method, it should be noticed that till now commercial software packages based on this method still not available. Analytical models based on the formal solution of Maxwell’s equations help also to overcome aforementioned problem. Some comprehensive reviews on the models for magnetic field can be found in [36,81]. This paper is intended as a tutorial overview based on a review of the state of the art, describing recent developments in the field of analytical modelling of permanent magnet machines.

Boules [19–21] presented a method suitable for the design optimization of a class of rotary radial flux permanent magnet machines. The machine model used in this method is based on solving Maxwell’s equations in low permeability regions (air-gap and permanent magnets regions). The proposed method helps overcome aforementioned problems and it has been continually improved since. A large variety of analytical models has been developed [2,3,9–14,16,18–21,25,28,34–39,44,47–49,54,56–58,61,63,70,71,74,76–81]. Some of them concern rotary machines [2,9–11,13,14,18–21,23,25,28,34,36–39,44,47–49,54,56,70,71,76–81] (Figs. 1 and 2) and others linear machines.

Fig. 1. Rotary radial flux machines. (a) Classical surface mounted PM machine, (b) hybrid excitation machine [10], (c) PM flux switching machine [34] and (d) rotating radial flux magnetic gear [48].
Most of developed models do not take into account slotting in an explicit manner [36,81]. Ittstein [38] and Ackermann and Sottek [2] presented an exact analytical description of magnetic field in idealized surface mounted permanent magnet structure which does take into account stator slotting. In [2], rotary radial flux machines and flat linear machines are considered but not tubular linear machines. The developed models, in [38,2], only concern field distribution due to permanent magnet source; they have been used to calculate cogging torque. Recently, several publications have proposed analytical models for different radial flux rotating structures (Fig. 1). Extensive research works is being done on the development of more accurate models for more realistic structures geometries [34,37,39].

The same trends can also be observed for rotating axial flux machines [9,28,76] and linear machines [2,3,16,35,57,58,61,63,74]. Azzouzi et al. [9] presented an exact analytical description of magnetic field in idealized rotary surface mounted permanent magnet axial flux machines which takes into account stator slotting in an explicit manner. Wang et al. [61] presented a general framework for the analysis and design of slotless tubular linear permanent magnet machines. Recently, Amara et al. [3] presented an analytical model, which takes into account stator slotting, of tubular linear machines under open circuit and on load conditions.

Fig. 2. Rotary axial flux machine.

Fig. 3. Linear machines. (a) Planar linear PM machine and (b) tubular linear PM machine.
In this paper, an example of a classical surface mounted PM machine model considering slotting effect, in an explicit manner, is first presented. The assumptions on which the model is based are presented. Then, numerical problems related to the use of this kind of models are discussed. Methods to overcome some limitations inherent to simplifying assumptions are also discussed. Finally, methods for the calculation of global quantities necessary for the design optimization are presented.

2. Analytical modelling

The basic steps to obtain an analytical model based on the formal solution of Maxwell’s equations are described in this section. Then, numerical problems related to the use of this kind of models are discussed.

2.1. Model construction

Fig. 4 shows different regions (slots (I), air-gap (II), permanent magnets (III), region under magnets (IV)) where the exact analytical solution is established for a classical surface mounted PM machine. Region IV is only considered in case of a non-magnetic permanent magnets supporting armature. Permeability of all ferromagnetic parts is assumed to be infinite. The permeability of permanent magnets is assumed to be equal to that of air.

The governing field equations, in terms of the Coulomb gauge, $\nabla \times A = 0$, are:

\[
\begin{align*}
\nabla^2 A &= -\mu_0 J, & \text{in region I} \\
\nabla^2 A &= 0, & \text{in regions II and IV} \\
\nabla^2 A &= -\mu_0 \nabla \times M, & \text{in region III}
\end{align*}
\]

(1)

$A$ only has $A_z$ component which is independent of $z$ (infinitely long machine in axial direction). $J$ is the armature current density and $M$ is the magnetization.

Combining Eq. (1) with boundary conditions, and using separation of variables method, help establish a set of linear equations ($N_H \times N_H$) (where $N_H$ is the number of considered harmonics), where coefficients of magnetic vector potential solution in region III are the unknown. Solving these linear equations and using interface conditions give coefficients of magnetic vector potential in other regions. Obtained linear equations can be solved using Gaussian elimination method.

Fig. 5 shows comparisons of flux density components space distribution in air-gap region (region II) obtained by FE and analytical methods. Table 1 gives some characteristics of machine to which both methods (analytical technique and finite element) have been applied. Finite element calculations are done considering that stator and rotor cores are
linear with a very high relative permeability ($\mu_r = 1e5$). More precisely, finite element computations are done under same assumptions on which the analytical model is based.

As can be seen, the predictions from analytical model are in good agreement with finite element calculations. The slotting effect on magnetic field components space distribution is clearly visible in Fig. 5. The accuracy is reduced as the number of considered harmonic is lowered. Fig. 6 shows relative error $\delta$ estimated using:

$$\delta = \frac{|x_{AMrms} - x_{FErms}|}{x_{FErms}}$$

Fig. 6. Relative error $\delta$ for both flux density components. (a) Flux density radial component and (b) flux density circumferential component.
for both flux density components. $x_{\text{AMrms}}$ and $x_{\text{FErms}}$ represent the root mean square values of flux density components obtained respectively from analytical and finite element methods. It can be seen that, even by choosing a relatively low number of harmonics (25 harmonics), at least for the studied example, the relative error for both components is smaller than 10%. The accuracy and computational time depend on the highest spatial harmonic orders considered in the different machine’s regions. The higher the harmonic orders are, the higher the accuracy is, but the longer time the computation takes [69].

Two elements are often present in developed analytical models based on the formal solution of Maxwell’s equations: (1) authors always make the assumption that magnetic materials (iron and permanent magnets) have a constant finite (iron and permanent magnets) or infinite (iron) relative permeability and; (2) most of authors use separation of variables method along with the use of Fourier series for solving Maxwell’s equations. However, some authors [28] have used the free-space Green’s function method.

It should be emphasized that the presented analytical modelling approach helps, in a first step, to explore rapidly the search space of potentially optimal prototypes. Obviously, the chosen potentially optimal prototypes issued from this first step have to be refined using a finite element based optimization procedure, which acts near the global optimum and then save a large amount of time. The confidence to have on the first design stage, if based on analytical modelling approach, should be put in perspective in case the magnetic saturation plays an important role in the designed actuator operation. In case design constraints imply a highly saturated actuator operation, finite element method should be preferred, even at early design stages. Methods to include magnetic saturation in this kind of models are discussed in the third section of the paper.

### 2.2. Numerical problems

Numerical problems discussed in this section only concern analytical models which use Fourier series. Numerical techniques, as finite element (FE) method, will have a limited accuracy related to the density of the mesh. The analytical models based on the use of Fourier series exhibit a similar problem in the frequency domain [36]. Both methods (Numerical and analytical (or semi-analytical) methods) results in a set of linear Eq. (3) which should be solved to obtain magnetic field distribution in the different machine regions.

$$[A] \cdot [X] = [B]$$  \hspace{1cm} (3)

[A] ($N_H \times N_H$) is the topological matrix where the elements are depending on the geometrical shape of the limits between the different regions of the machine; [B] ($N_H \times 1$) is the source vector, elements of which are related to geometry distribution and physical properties of magnetic field sources (magnetic remanence and current density distributions) and [X] ($N_H \times 1$) is the unknowns vector whose are the series coefficients of the vector potential in a chosen machine region.

The accuracy of magnetic field solution depends on two parameters which can become contradictory: the number of considered harmonics and the conditioning of the obtained linear equations system. Increasing the number of considered harmonics ($N_H$) increases in a certain extent the accuracy of magnetic field solution as long as the linear equations system still well conditioned. Beyond a certain number of harmonics the system becomes ill conditioned and the results inaccurate [36]. A discussion on the numerical limitations of such analytical models is presented in [36].

Effect of numerical problems can be reduced by two means: (1) using scaling techniques [36] and/or, (2) using approximation techniques [2]. A discussion on different approximation techniques is presented in [2].

In [36], Gysen et al. proposed scaling techniques to improve the conditioning of the equations system to be solved. While for analytical models defined in Cartesian and polar coordinates it is easy to apply scaling techniques, authors mentioned that scaling techniques cannot be applied for Bessel functions, making problems in the cylindrical coordinate system difficult, if not impossible, to scale. In [2], Ackermann et al. used approximation techniques to ease the solving of the equations system. Authors used separation of variables method along with the use of Fourier series for solving Maxwell’s equations. They then developed an approximate closed form solution of magnetic field Fourier series coefficients. The aim of their approach was to speed-up the computation time, but it helps, in the same time, to overcome some numerical problems because it avoid the need of inverting the matrix [A]. The approach consisted in neglecting the off-diagonal terms in the matrix [A], which helps the authors to derive a closed form solution of the Fourier series coefficients of the magnetic field analytical solution. Authors obtained good agreement between
results taken from measurements, finite element computations, the exact and the approximate analytical models, for the studied machine.

3. Limits and applications of analytical modelling

In many practical designs, some physical phenomenon, as magnetic saturation, neglected as a consequence of adopted assumptions cannot be ignored. This section discusses methods to overcome some limitations inherent to simplifying assumptions. Applications of analytical models for analysis and design optimization of electric machines are also discussed.

3.1. Limits and capabilities of analytical modelling

In this part some limitations inherent to physical phenomenon and machines geometric structures are discussed. Methods used to take into account magnetic saturation are first presented. Then, limitations due to the geometric structure of electric machines are considered. Machines geometric structures thematic covers aspects related to the consideration of slotting or not, end effects and 3D effects, and finally some faults modelling, i.e., rotor eccentricity.

3.1.1. Magnetic saturation

At a first glance, magnetic saturation is always neglected in developed analytical models, in machine’s regions where the analytical solution is sought, as a consequence of adopted assumptions (linear magnetic materials or with infinite relative permeability). Nevertheless, main magnetic saturation can be accounted, in an iterative way, by adapting the geometrical length of the air-gap [19,37,58].

Boules, in [19], applied this method to surface mounted radial flux machines using an analytical model which does not take into account stator slotting, in an explicit manner, by coupling it with a very simple reduced order lumped parameter magnetic circuit model. Wang et al. [58] applied the same approach to tubular linear machines. Ilhan et al. [37] used a similar approach for the modelling of flux switching PM machines.

However, it should be noticed that only main magnetic saturation is taken into account in these papers [19,37,58]. No examples of analytical models based on the formal solution of Maxwell’s equations which take into account local magnetic saturation are available to date.

In order to illustrate the idea behind this first approach of considering magnetic saturation; a simple comparison study aiming to highlight the effect of magnetic saturation has been carried out. In this study, open circuit air-gap magnetic flux density components distributions obtained from calculations without and with considering magnetic saturation are compared. The goal here is not to thoroughly explore the effect of magnetic saturation, which depends on several parameters: operating conditions (open circuit or on-load), rotor/stator relative position, iron core material, etc., but to describe through this comparison study how main magnetic saturation can be considered in a first approximation.

Fig. 7 compares flux density amplitude distribution in two machines. Both machines share dimensions given in Table 1. For the machine shown in Fig. 7(a) (Machine 1), the stator back iron thickness is set equal to 1.5 × tooth width (equalization of magnetic flux cross sections), which means that statoric teeth and back iron regions should roughly have same magnetic flux densities levels. For the machine shown in Fig. 7(b) (Machine 2), the stator back iron thickness is set equal to 0.25 × tooth width, which implies a highly saturated stator back iron (Fig. 7(b)). Finite element analyses
Fig. 8. Comparison of air-gap flux density components for both machines. Radial component (Machine 1), (b) circumferential component (Machine 1), (c) radial component (Machine 2) and (d) circumferential component (Machine 2).

are done under open circuit conditions for both machines. Fig. 8 compares air-gap flux density components distributions for both machines. As can be seen, while magnetic saturation has a negligible effect for the first machine (Fig. 8(a) and (b)), the effect of magnetic saturation is clearly visible on flux density components distribution for second machine (Fig. 8(c) and (d)). Fig. 9 compares harmonic content for both magnetic flux density components when magnetic saturation is considered (non-linear case, $B–H$ curve of laminated material M270-35A) and not considered (linear case, $\mu_r = 1e5$), for the second machine.

It can be noted that magnetic saturation will induce a reduction of the amplitude of the first harmonic of the flux density radial (normal) component (Fig. 9(a)). The normal component of air-gap flux density is often the one of interest when it comes to the estimation of some global quantities (EMF and mean value of load force or torque).

Increasing air-gap thickness will have same effect on the amplitude of the fundamental harmonic of the flux density radial (normal) component. It is because of this similarity, that some authors proposed to take into account main magnetic saturation by adapting the geometrical length of the air-gap [19,37,58].

Examples where analytical models, based on the formal solution of Maxwell’s equations, are involved and where the local magnetic saturation is taken into account are situations where the analytical models are either combined with finite element method [1,31,46], or coupled with magnetic equivalent circuit (MEC) method [29,30,51].
The analytical solution of magnetic field is established in very low permeability regions (air-gap) and numerical solution (FE), or semi-analytical solution (MEC), is sought in regions where magnetic saturation cannot be neglected (stator and rotor iron cores). More recently, Zhang et al. [75] used the same approach for modelling two rotating permanent magnet radial flux machines (one with inner rotor and the other with an external rotor).

The coupling of analytical solution with MEC method [29,30,51], seems to give the best accuracy to computational time ratio as compared to the other coupling method (coupling analytical solution with FE) [1,31,46] and to FE method. The coupling method presented in [29,30,51], consists of solving Maxwell’s equations analytically in regions where magnetic saturation can be neglected (mainly the air-gap) and of constructing a permeance network in the ferromagnetic parts of the machine. The coupling of MEC method (magnetic scalar potential) and analytical solution (magnetic vector potential) is obtained by equalizing magnetic filed components at the air-gap/ferromagnetic parts boundaries. The series coefficient of magnetic vector potential and the magnetic fluxes of the permeance network constitute the unknowns vector of the nonlinear system obtained by equalizing magnetic filed components at the air-gap/ferromagnetic parts boundaries.

Furthermore, coupling of air-gap flux density analytical solution and ferromagnetic parts permeance network could be an interesting improvement for permeance network simulation with moving parts.

3.1.2. Geometric structures, end effects and 3D effects

In addition to the two recurrent assumptions, described in Section 2.1, all analytical modelling techniques consider idealized geometries to approach the actual machines geometric structures. For radial field rotating machines full or partial cylindrical surfaces are used [2]. For linear (flat and tubular) and axial flux machines rectangular surfaces are considered [2].

While, in the past, most of analytical models considered slotting effects via a permeance function [78,81], analytical techniques which explicitly consider slotting have been extensively studied these last years [9,13,25,49,81]. However, it should be noticed that analytical models with due account of slotting have been introduced in early nineteen nineties [2,38]. More recently, many authors have considered more realistic slots geometries by considering the tooth tips [12,14,50,56,70,71]. Explicit consideration of more realistic slots geometries helps to obtain more accurate estimations of air-gap magnetic field solution as compared to models which do consider slotting indirectly. Analytical models, which explicitly consider slotting effects, allows, among other thinks, accurate estimation of open circuit and armature reaction flux linkages, cogging forces and torques, forces and torques ripples, and unbalanced magnetic forces. In [72,73], Wu et al. compared several analytical models which can be classified in two groups: analytical models which indirectly consider stator slotting, and these which explicitly consider the stator slotting. Authors arrived to the conclusion that
analytical models, which explicitly consider the stator slotting, are most accurate for predicting the electromagnetic performance although much time consuming for computation.

Before discussing the methods to take into account end effects and 3D effects, it should be noticed that for some electrical machines topologies, end effects and 3D effects can be clearly distinguished while for other machines topologies the distinction is less obvious. 3D effects can be defined as a topology variation in the direction perpendicular to the modelling plane for a given relative position of moving armature as regards to fixed armature; the skewing of permanent magnets is an example of 3D effects, axial flux machines have an inherent 3D structure, longitudinal end effects in classical rotating radial flux machines can be considered as 3D effects, transverse end effects in plane linear machine (Fig. 10) can also be considered as 3D effects. Most of the time end effects cannot be distinguished from 3D effects. However, longitudinal end effects in linear machines (plane and tubular linear machines) can clearly be distinguished from the 3D effects (Fig. 10).

3D effects (permanent magnets or stator teeth skewing and variation of machine topology in radial direction of axial flux machines) can be taken into account using classical multi-slice technique [52]. Fig. 11 illustrates the principle of the quasi-3D model (multi-slice model) for axial flux machines. The axial flux machine is divided into a certain number of annular slices in the radial direction. The analytical model based on the solution of Maxwell equations is established at the average radius of each slice (Fig. 11(b)). $R_i$ and $R_e$ in Fig. 11, are respectively the inner and outer radii of the axial flux machine.

In [9,76] authors proposed a quasi-3D analytical model for axial flux machines. In these papers authors used a modulation function in order to consider variation of magnetic field in the radial direction. In [76], Zhilichev used
Hankel transformation in order to calculate the modulation function; while in [9], authors derived the modulation function from finite element analyses. However, some authors [28] have used the free-space Green’s function method and the method of images to derive directly a semi-analytical 3D solution for axial flux machines. In [28], Furlani combined results from a previous study [27], with the method of images, to derive a semi-analytical solution of only axial field component. In [76], it is also the axial component of magnetic field which has been derived by Zhilichev. In [9], Azzouzi et al. estimated both axial and circumferential components of magnetic field. In all these references [9,28,76], magnetic field analytical solution is estimated under open circuit condition. Axial component of magnetic field can be used to estimate armature flux linkage and by the way electromotive-force. It can also been used to estimate normal (axial) attraction force between rotor and stator armatures. When both axial and circumferential components of magnetic field are estimated, as in [9], it is possible to compute the cogging torque. While stator slotting is explicitly considered in [9], models presented in [28,76] concerned slotless structures. For axial field machines, the quasi-3D analytical models give more accurate results as compared to mean radius 2D models.

In [32], authors used an analytical approach based on separation of variables method with Fourier series solution to study the fringing effect in permanent magnet machines. To do so, authors consider an infinitely long machine comprising an infinite number of finite length static armatures. Provided that the spacing between adjacent armatures is much greater than the armature dimensions, the influence of neighbouring armatures on the fringing field at the ends of each armature will be negligible [60]. It is the same approach which has been used in [60] to study the longitudinal end effect in tubular linear machines. In [24], authors used same approach along with the conformal transformation to derive an analytical solution of magnetic field in a permanent magnet linear actuator with cylindrical magnets. Their study is divided in two parts. In the first part they used the same technique as in [32,60] to derive the magnetic field solution without taking into account the ends effects. They then used conformal transformation to derive a modulation function to take into account the transverse end effect, in the second part.

Unbalanced magnetic forces (UMF) in rotating electric machines generate vibrations and noises and cause the wear of bearings. Unbalanced magnetic forces should be taken into account at the design stage. One of the major sources of UMF in rotating electric machines is the rotor eccentricity (Fig. 12).

Two main approaches are often used for the calculation of UMF due to rotor eccentricity: one common way is to derive the air-gap permeance using indirect approaches and useful simplifications; the second method consists of directly derive the magnetic field in the faulty machine. A comprehensive review on the calculation of UMF methods can be found in [17]. Due to its relative simplicity, the first approach has attracted more attention in scientific literature [17,33,45,62]. The second approach has been first reported in scientific literature by Kim et al. in [41,42]. Authors used perturbation method for predicting the instantaneous magnetic field distribution in the air-gap region of permanent magnet motors with rotor eccentricity. This method needs long mathematical developments and is
rather complex. This second approach is more related to the subject discussed in this paper. Kim et al. used this method in [43], for studying magnetic force imbalance and cogging torque in permanent-magnet motors under both static and dynamic rotor eccentricity. While Kim et al., in [42], considered slotting effect via a relative permeance function, more recently; Fu and Zhu [26] proposed an analytical model, of PM machines with rotor eccentricity, where slotting effect is explicitly taken into account. In both studies perturbation theory is used to consider rotor eccentricity.

Borisavljević et al. [17] highlighted the fact that the effectiveness of the different approaches has rarely been explored in literature, with an exception of [68]. It is then difficult to compare objectively the different modelling methods for calculation of UMF. In [68], although authors used a simple approach (first method), they obtained results with a sufficient engineering precision as compared with experimental measurements. However, it can be expected that methods which derive directly the air-gap magnetic field solution [26,41–43] should be more accurate than approaches which use indirect approaches and useful simplifications, as long as same assumptions on physical phenomenon, e.g., magnetic saturation, are adopted in compared models.

3.2. Analytical models applications

The aim of discussed analytical modelling is to facilitate and speed-up the optimal design process of electrical machines. The analytical field solution allows the analytical prediction of machines performances, trough calculation of global quantities, which, in turn, facilitates the characterization of the machines, and provides a basis for design optimization and system dynamic modelling [64]. Indeed, calculation of global quantities (EMF, self and mutual inductances, cogging and on load torques or forces, electromagnetic losses) allows evaluating machine performances. Furthermore, global quantities can be used for the coupling of developed models with electric circuit equations [11,23]. Doing so, these models can then be used to study the behaviour of PM machines when connected to power converters and so for sizing and optimization purposes.

Several authors reported design optimization studies based on the use of the analytical field solution [23,58,64–67]. In [58,64], Wang et al. presented analysis and design optimization studies of tubular permanent-magnet machines. In [64], authors used the analytical approach to maximize the force capability of a slotless tubular linear PM structure. While the drive system was not considered in [64], authors, in [58], used the analytical field solution to optimize both the machine and its driving system (power converter). Authors concluded that the power factor has a significant influence on the system efficiency and cost. In [23], Chebak et al. presented an optimal design study of a high-speed slotless permanent magnet synchronous generator with a rectifier load. In their conclusion, authors highlighted the reduced convergence time of proposed design approach. Albeit there is a commonly shared consensus that finite element method is satisfactory to validate analytical modelling approach, Wang et al., in [65], presented a design optimization study of a linear drive, based on a tubular permanent magnet generator, which has also been experimentally verified. In [66], Wang et al. presented a design optimization study of a tubular linear machine equipped with quasi-Halbach magnetized magnets. An experimental verification was also conducted. In [67], Wang et al. presented a design optimization study, aiming to reduce the cogging force associated with the finite length of the ferromagnetic armature core of tubular linear machines, where an experimental validation was again achieved.

3.2.1. Electromotive force and inductances calculation

Two different techniques are used for calculation of EMF and inductances (self and mutual); one is based on the winding function theory [40] and the other one is based on Stokes theorem and uses vector potential in slots [3,10]. The first technique is always used for analytical models which do not take into account stator slotting in an explicit manner. The second technique can be used in analytical models of slotless structures [61], and in analytical models which do take into account stator slotting in an explicit manner.

However, analytical models which do not take into account stator slotting in an explicit manner cannot be used to estimate the total self or mutual inductances since slot-leakage inductances cannot be evaluated. These models can only be used to estimate air-gap self (magnetizing) or mutual inductances. The slot leakage-inductance can only be estimated using a separate model [59].
3.2.2. Forces and torques calculation

Forces and torques calculation is often done using the Maxwell stress tensor method. This method is applied on the surface of permanent magnets at the interface between regions II and III (Fig. 4) to obtain the torque (Eq. (4)):

\[
\Gamma = \frac{L_a R_1^2}{\mu_0} \int_0^{2\pi} B_\psi^{(1)}(\varphi, R_1)B_\varphi^{(1)}(\varphi, R_1) d\varphi
\]

(4)

where, \(L_a\) is the machine active length, and \(B_\psi^{(1)}\) and \(B_\varphi^{(1)}\) are respectively the normal (radial) and circumferential components of magnetic field in region (II).

It should be noticed that virtual work method can also be used [22].

For calculation of cogging force or torque, open circuit field components are used and for on load force or torque, on load magnetic field components are used. Calculated force density distributions at the stator inner surface and the rotor outer surface can be used for predicting the acoustic noise and vibration of studied machines [80].

3.2.3. Electromagnetic losses calculation

In addition to the calculation of classical electric machines global quantities (EMF, inductances, torques and forces), many authors have used analytical models for the prediction of electromagnetic losses [4–6,8]. In [5,8], authors used analytical models for the prediction of eddy current loss in the permanent magnets. In [4], authors described a technique for calculation of eddy current loss in armature windings. In these papers authors consider that the eddy current losses are resistance limited. Thus, the eddy current density can be obtained from the following:

\[
J_e(r, \varphi, t) = -\sigma \frac{\partial A_x}{\partial t} + C(t)
\]

(5)

where, \(\sigma\) is the conductivity of the region where the eddy current density is estimated. \(C(t)\) is a function of time which is introduced to insure that total current flowing in the considered region is equal to source current. Once eddy current density is estimated, eddy current losses can easily be calculated.

In [6], authors presented an analytical technique for calculation of stator iron loss in tubular linear PM synchronous machines. The iron loss density is calculated using the experimentally verified iron loss density model defined in [15]. Also, it is important to note that coupling MEC method and Maxwell type analytical solution could help to improve the estimation of iron loss.

4. Conclusion and perspectives

This paper has review the state of the art of analytical modelling, based on the formal solution of Maxwell’s equations, of PM electrical machines. Methods to overcome some limitations inherent to simplifying assumptions were presented and discussed. Recent developments in the field of analytical modelling of permanent magnet machines have been described.

The analytical tools, based on this approach, should be useful for comparative design studies, and aid design optimization at the first stages of the design process. In this context, analytical models constitute a complementary tool to finite elements method. The presented analytical modelling helps, in a first step, to explore rapidly the search space of potentially optimal prototypes. Obviously, the chosen potentially optimal prototypes issued from this first step have to be refined using a finite element based optimization procedure which acts near the global optimum and then save a large amount of time.

In some cases where magnetic saturation can be integrated to the analytical model as presented above, Maxwell type analytical models become an appreciable alternative to the finite elements method especially for 3D problems.

References


