Airborne Radar for Measuring Snow

Thickness over Sea Ice

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ABSTRACT

Sea ice has proven to be a crucial element in the climate and overall heat budget of the globe. The behavior of sea ice is dependent on the presence of snow cover due to its impact on the thermal insulation and albedo of the system. An airborne FMCW radar was created to measure the depth of the snow over sea ice. It was designed with a 6 GHz bandwidth for a small range resolution and high PRF for coherent sampling at high velocities. During field testing in Alaska and Greenland, the system was shown to have problems related to the linearity of the chirp. After lengthening the sweep time and reducing the PRF, data were successfully collected in coordination with satellite passes and ground samples. Two concepts were explored for reducing the beamwidths of the antennas to minimize the received clutter. Simulations were performed to investigate the issues in the PLL responsible for the nonlinearities seen, and recommendations were made for improvement.
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Chapter 1

INTRODUCTION

1.1 Importance of Sea-Ice

The total area of sea ice that melts and reforms with the seasons each year is roughly equivalent to the size of the United States [14]. This large amount of ice has an impact on the global climate in a variety of ways [15]. First, the formation of sea ice increases the salt flux into the ocean, which increases the thermohaline circulation of the waters. Second, the presence of sea ice provides thermal insulation between the cold arctic air and the warm ocean waters, reducing their interaction. Because of these issues, obtaining a comprehensive knowledge of the sea ice is a key step to understanding the heat budget in the Polar Regions [16].

The presence of the ice can also have a direct effect on life in these areas. The sea ice provides a habitat for polar bears, seals and penguins whose success can be related to its extent. Furthermore, the amount of photosynthetic light available at the ocean floor is limited by the presence of sea ice, affecting life within the waters.

1.2 Effects of Snow Cover

The existence of snow on the surface of the sea ice can provide additional effects on the climate system [16]. Snow possesses a thermal conductivity that is around an order of magnitude less than that of sea ice. As a result of this, even shallow layers of snow can drastically reduce the heat transfer at the surface. Snow
also has a very large albedo, meaning that it will reflect nearly all of the energy incident upon it from the sun. It is feared that a reduction in the snow-covered sea ice formed each year would result in greater energy absorption for the planet. This additional heat could lead to a further reduction in sea ice, creating a dangerous feedback response. Finally, a large amount of snow cover can depress the surface of the ice below sea level, allowing flooding and changing the behavior of the system.

1.3 History

In May of 2002 NASA launched a satellite named AQUA for the purpose of collecting data about the Earth’s water cycle [7]. AQUA includes six instruments, including the Advanced Microwave Scanning Radiometer for EOS (AMSR-E). The system collects brightness temperatures at six unique frequencies with both vertical and horizontal polarizations. An algorithm has been developed to calculate snow depths from these passive microwave data [17]. The use of this information can provide a much better understanding of the polar climates.

In order to verify to capabilities of this algorithm while quantifying the errors involved, large-scale data are required to coordinate with the large footprint of the radiometer. Existing snow depth data have been recorded using meter sticks on the ground and using estimates taken from ships [15]. These data are not sufficient because the small sample sizes do not account for the large variances seen in the snow depths over short ranges. An airborne radar should be able to provide a much larger volume of data to be used in analysis of the AMSR-E snow depth algorithm.
Previously, a ground based system was created at KU to test the concept of an ultra-wideband FMCW radar for collecting snow depth information [15]. The radar featured a bandwidth of 2-8 GHz, as in the design presented here, but with a much slower sweep time of 10 ms. During an experiment in Antarctica in September of 2003 the system was shown to accurately measure snow depths from 5 to 85 cm.

1.4 Objectives

To make the transition from a ground based system to an airborne one, several changes must be considered. First, the sweep time must be shortened to allow for coherency within the pulses while traveling at a much higher velocity. This involves a redesign of the chirp generating section of the radar. Next, power considerations must be made to assure that the radar will possess the loop sensitivity necessary to overcome the additional range added by the use of an aircraft. Finally, the antennas employed need to be reconsidered because the additional range will create a very large footprint if a wide beamwidth is used. This could potentially introduce an unacceptable amount of clutter.

1.5 Organization

Chapter 2 will describe some important concepts including the behavior of an FMCW radar and the use of the radar equation to predict the performance of the system. Chapter 3 describes the reconstruction and use of a new airborne system, and discusses its shortcomings. Chapter 4 describes two ideas for improving the antenna
system used with the radar. Chapter 5 explores the issues discovered with the Phase-locked loop used in the chirp. Finally, Chapter 6 provides a summary of the work along with some suggestions for improvement in future iterations.
Chapter 2

BACKGROUND

2.1 FMCW Radar

A Frequency-Modulated Continuous-Wave (FMCW) radar is distinguishable by two main characteristics. First, rather than utilizing pulses of a single-frequency tone, a FMCW radar modulates the signal during the duration of the pulse to achieve a desired bandwidth. Second, rather than transmitting short pulses with the system always transmitting and receiving separately, a FMCW radar transmits long pulses so that it is transmitting and receiving simultaneously. The behavior of FMCW radar is described using the following figure.

![Diagram of FMCW radar behavior](image)

During each pulse of duration $T$, the frequency of the transmitted signal is linearly increased from the starting frequency by a bandwidth, $B$. While propagating and reflecting off the target, the signal will incur a delay of $\tau$ before returning to the
receiver. This delay will result in a difference in frequency between the transmitted and received signals for the duration of the chirp. This is known as the beat frequency, \( f_b \), and it is the purpose of the receiver to capture this signal. The chirping will repeat with a period of \( T_M \), the modulation period, with the time in between chirps used to reset the system.

The process of uncovering the beat frequency in the receiver can be described in the time domain beginning with the transmitted chirp in terms of voltage, with \( f_0 \) as the starting frequency and \( \theta_0 \) as the initial phase of the chirp.

\[
V_{TX}(t) = A \cos \left( 2\pi f_0 t + \frac{\pi B}{T} t^2 + \theta_0 \right)
\]  

(2.1)

Upon returning to the radar, this signal will not only be delayed by a time \( \tau \), but will also be modified by the frequency dependent reflection coefficient of the target, \( \Gamma(f(t)) \) [4].

\[
V_{RX}(t) = A |\Gamma(f(t))| \cos \left( 2\pi f_0 (t - \tau) + \frac{\pi B}{T} (t - \tau)^2 + \theta_0 + \angle \Gamma(f(t)) \right)
\]

(2.2)

The two signals are multiplied using a mixer in the receiver, producing both a sum and difference term. The summation term will then be filtered out.

\[
A^2 |\Gamma(f(t))| \cos \left( 2\pi \frac{B \tau}{T} + 2\pi f_0 \tau - \frac{\pi B \tau^2}{T} - \angle \Gamma(f(t)) \right)
\]

(2.3)

Now this equation can be rewritten in terms of the desired, ideal beat frequency signal.

\[
A_b \cos (2\pi f_b t + \phi_b)
\]

(2.4)
This leads to the following solution for the beat frequency that will be recovered by the receiver.

\[ f_b = \frac{B \tau}{T} \]  \hspace{1cm} (2.5)

Since the bandwidth and sweep time are known, the delay can be extracted from the beat frequency.

The objective of this system is to gather two returns, one from the air-snow interface and one from the snow-ice interface, with each producing its own unique beat frequency. The difference between the two represents the additional time delay associated with the transmitted signal traveling through the snow and reflecting off of the ice back toward the radar. Thus, the snow depth is derived as:

\[ D = \frac{1}{2} \Delta \tau \frac{c}{\sqrt{\varepsilon_r}} \]  \hspace{1cm} (2.6)

with \( \Delta \tau \) as the time delay between the two returns, \( c \) as the speed of light, and \( \varepsilon_r \) as the permittivity of the snow. This allows, with an accurate estimation of the snow’s permittivity, for the snow depth to be recovered from the beat frequencies of the two returns.

Additionally, it is important to understand how shallow the snow can be before the system is no longer able to resolve both interfaces. This is known as the range resolution and is inversely related to the bandwidth of the chirp.
2.2 Radar Equation

The radar equation is used to find the expected power level of a desired return. Before examining the expected returns, the physical model is described.

The model above assumes a large distance of air, followed by the depth of snow, followed by a half-space of sea ice. For the reflection off of an idealized specular interface, using the air/snow interface as an example, the radar equation will be:

\[ P_{r1} = P_t G_t \frac{1}{4\pi (2R)^2} |r_1|^2 G_s \lambda^2 \frac{1}{4\pi} = \frac{P_t G_t^2 \lambda^2}{(4\pi)^2 (2R)^2} |r_1|^2 \]  

(2.8)

The received power, \( P_{r1} \), is found by starting with the transmitted power, \( P_t \). Next, the gain of the transmit antenna, \( G_t \), is included, along with the spherical spreading loss associated with traveling twice the range to the target, \( R \). While
traveling this distance the signal will experience a loss from the reflection at the interface. This is accounted for using the reflection coefficient $\Gamma_1$. Finally, the effective aperture area of the receive antenna is included in terms of the receive antenna gain $G_r$. The equation simplifies by assuming that identical antennas are used for transmitting and receiving with a gain $G$.

The only factor in Equation (2.8) not under control of the radar operator is the reflection coefficient at the air/snow interface [4].

$$\Gamma_1 = \frac{\sqrt{\varepsilon_{r_2}} - \sqrt{\varepsilon_{r_1}}}{\sqrt{\varepsilon_{r_2}} + \sqrt{\varepsilon_{r_1}}} \times 2 \sin \left( \frac{2\pi d}{\lambda_m} \right) \quad (2.9)$$

Here, $\varepsilon_{r_1}$ and $\varepsilon_{r_2}$ are the dielectric constants of the snow and ice layers respectively, $d$ is the thickness of the snow layer, and $\lambda_m$ is the wavelength within the snow. The first part of this equation accounts for the dielectric contrast between the air and the snow. The second part of the equation takes into account the destructive interference occurring with reflections from the snow/ice interface below. This function is periodic in frequency for a given snow thickness, oscillating between zero and one and resulting in a 6 dB power loss for the return.

Calculating the signal reflected from the snow/ice interface involves first finding the signal transmitted through the air/snow interface. Next the reflection coefficient at the snow/ice interface, which looks like Equation (2.9) without the second interference term, is used. Finally the transmission back through the snow/air interface must be included. The resulting power return is shown below.
\[ P_{r2} = \frac{P_i G^2 \lambda^2}{(4\pi)^2 (2R)^2} |\Gamma_1|^2 |\Gamma_2|^2 |\Gamma_i|^2 \] (2.10)

This is a highly simplified interpretation of the actual environment, as both the snow and ice will show various degrees of layering. These equations can be used, however, to provide an estimate of the expected return power.

2.3 Dielectric Constants

In order to achieve a strong reflected signal, it is important to assure that there is a large dielectric contrast between the layers. The three materials of interest here are air, snow and ice. The dielectric constant of air will be close to one under all conditions and need not be further considered. For the case of snow and sea ice, however, things are more complex.

Dry snow behaves as a mixture of air and ice, resulting in a permittivity that can vary from 1 to 3.15. The permittivity is virtually independent of temperature, but is dependent on the density, as shown.
This behavior of the real part of the dielectric constant of dry snow can be approximated as shown [6].

\[ \varepsilon_{ds} = (1 + 0.51\rho_s)^3 \]  \hspace{1cm} (2.11)

Although the imaginary part of the dielectric constant has not been accounted for, it is always much smaller than the real part and will not have a large effect on the reflection coefficient. This experiment involves measuring the current year’s fresh snowfall, so a moderate snow density is expected, with a dielectric constant of around 1.5 for dry snow.
As illustrated in the following figures, both the real and imaginary part of the dielectric constant of snow increase with the snow wetness. This means it could become difficult to distinguish between wet snow and ice in warmer areas.

Figure 2.4. Affect of wetness, \( m_w \), on the permittivity of snow [6].

Figure 2.5. Affect of wetness, \( m_w \), on the loss factor of snow [6].
The dielectric constant of sea ice exhibits a very complex behavior. This is due to the multitude of factors at play including the salinity and volume fraction of brine, the shape and orientation of brine inclusions, temperature and frequency. The figures below illustrate some of this behavior. However, since it is known that the real part of the dielectric constant is bound within the range of 2.5-8 below 40 GHz [6], there should be no problem detecting the interface between dry snow and the sea ice.

![Figure 2.6](image)

Figure 2.6. Illustration of the large variations in the dielectric constant of sea ice [6].

2.4 Surface and Volume Scattering

There are some inaccuracies associated with the model assumed earlier. First, the surfaces of the snow and sea ice will not be perfectly smooth, resulting in surface
scattering, and second, the ice is not homogeneous, resulting in volume scattering.

The effect of surface scattering is felt due to the finite beamwidth of the antennas used. When the transmitted signal hits the air/snow interface off nadir, a portion of that signal will be reflected back toward the system if the surface is rough. This clutter can appear at the same range as the nadir return from the snow/ice interface, potentially masking it.

To better understand this problem, a composite scattering coefficient can be calculated, taking into account the snow surface scattering, \( \sigma_{ss}^0 \), the snow volume scattering, \( \sigma_{sv}^0 \), and the ice surface scattering, \( \sigma_{is}^0 \) [5].

\[
\sigma^0(\theta) = \sigma_{ss}^0(\theta) + T_s(\theta) \left[ \sigma_{sv}^0(\theta') + \frac{1}{L^2(\theta')} \sigma_{is}^0(\theta') \right]
\]

Here \( \sigma^0 \) is the composite scattering coefficient, \( T_s \) is the surface temperature of the ice, \( L \) is the one way loss, \( \theta \) is the incident angle, and \( \theta' \) is the angle of refraction in the snow. This composite scattering coefficient can be used in Equation (2.8) in place of \( |\Gamma|^2 \) to find the backscattered power at each angle. This is useful in determining whether or not ice return masking will be an issue.

Another issue associated with volume clutter is the additional loss it can create as the signal propagates through the snow. Typically this loss is minimal and not of serious consequence. With wet snow, however, the loss can be substantial. This should not be a major issue as the system is not designed to be used in wet snow conditions.
Chapter 3

SNOW RADAR SYSTEM

3.1 Key Specifications

The radar was initially designed with the following specifications.

Table 3.1. Initial system specifications.

<table>
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<tr>
<th>Specification</th>
<th>Value</th>
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<tbody>
<tr>
<td>Operating Frequency</td>
<td>2-8 GHz</td>
</tr>
<tr>
<td>Radar Height</td>
<td>450 m</td>
</tr>
<tr>
<td>Radar Velocity</td>
<td>100 m/s</td>
</tr>
<tr>
<td>PRF</td>
<td>2 kHZ</td>
</tr>
<tr>
<td>Chirp Length</td>
<td>250 µs</td>
</tr>
<tr>
<td>Transmit Power</td>
<td>20 dBm</td>
</tr>
<tr>
<td>Antennas</td>
<td>2 Collocated Horns</td>
</tr>
<tr>
<td>IF Sampling Frequency</td>
<td>160 MHz</td>
</tr>
<tr>
<td>Receiver Noise Figure</td>
<td>10.7 dB</td>
</tr>
</tbody>
</table>

The 6 GHz bandwidth translates to a maximum potential range resolution of about 2 cm, using Equation (2.7) and assuming the permittivity of snow to be 1.5. This allows the radar to distinguish between the two required returns when they are only a few centimeters apart. Thus, the system should be able to provide data from areas with very thin snow cover.

A high PRF is desired to assure that each pulse remains coherent. This limitation is established by the following equation, where R is the height of the radar, and λ is the minimum wavelength of operation [2].

26
\[ L_{\text{max}} = \frac{\sqrt{R\lambda}}{2} \]  

This provides a maximum integration length of about 2 m, meaning that any pulses occurring as the radar travels this distance can be coherently integrated. The minimum PRF necessary to maintain coherency during each modulation period, \( T_M \), is found by incorporating the velocity of the radar, \( v \).

\[ PRF > \frac{v}{L_{\text{max}}} \]  

Thus, the minimum PRF necessary to avoid a loss of SNR due to incoherency within individual pulses is 50 Hz. The target PRF of 2 kHz is 40 times higher than this, allowing the maintenance of coherency even if the height or velocity of the radar is changed. By producing multiple pulses within the integration length, the power of each individual pulse becomes less than if a single long pulse were used. This power, however, is regained by coherently integrating the data during processing.

The chirp length of 250 µs, along with the PRF of 2 kHz, provides for 250 µs of recovery time between sweeps, and a 50 % duty cycle. The lack of a 100 % duty cycle will slightly degrade the SNR of the received data, so minimizing the recovery time and maximizing the duty cycle is beneficial to the system.

The 20 dBm transmit power is chosen because it is the largest value that is easily achievable without a drastic increase in price, size and power consumption of the transmit amplifier. The broadband horn antennas used are not very ideal because of their large beamwidth, allowing the reception of additional clutter. Two alternative antenna systems were explored, and are discussed later. Separate transmit and receive
antennas are used to increase the isolation between the transmitter and receiver. This allows the use of a relatively large transmit power without saturating the receiver. The sampling frequency imposes limits on the range and sweep time, and dictates the placement of the IF low-pass filter. The noise figure represents the degradation in signal-to-noise ratio experienced by the received signal while passing through the receiver. An extremely low noise figure is not crucial in this case because a high loop sensitivity is not required for the near-surface returns desired.

3.2 Operational Description

A block diagram of the system is shown below. An overview of the system will be given first, before describing each section individually.

![Block Diagram](image)

Figure 3.1. Simple system block diagram.

A 12-18 GHz chirp is first produced using a phase-locked loop to upconvert a digital 10-15 MHz reference chirp. Next, this signal is mixed down to the desired
band of 2-8 GHz using a mixer with a 10 GHz phase-locked oscillator at the LO port. A 2-8 GHz band-pass filter attenuates the high frequency terms leaking through the mixer, before a limiting amplifier provides gain while smoothing the chirp’s power level across the band. A portion of the chirp is passed down to the receiver by a directional coupler, and another amplifier raises the power of the signal to be transmitted. Another band-pass filter assures that no unaccounted for harmonics remain before the chirp is finally transmitted by a horn antenna.

The return signal enters the radar through the other identical horn and a final band-pass filter eliminates any out of band signals that may have been received. A low-noise amplifier adds gain to the return, limiting the noise figure of the receiver. Next, a mixer is used to multiply the coupled down portion of the outgoing chirp with the return. This creates the beat frequency signal that is needed. A high-pass filter attenuates the strong antenna feed-through signal, which will appear near DC. Next, the IF amplifier provides the gain necessary for the beat frequency to be properly captured by the analog-to-digital converter. Finally, a low-pass filter eliminates any undesired signals that would be undersampled and aliased onto the expected returns.

### 3.3 Chirp Generator

The PLL circuit used to produce the 6 GHz chirp was created previously [8] with the only modification being a change in the loop filter design to allow for faster sweeping. The design of this circuit will be further explored in Chapter 5. The coupler seen in Figure 3.1 is a part of the PLL design, providing the necessary feedback. The
phase-locked oscillator used is the THOR-10000-02 from EM Research. It provides a 12 dBm signal at 10 GHz, which is locked to a 10 MHz reference. This signal acts as the local oscillator for the M/A-COM M88C mixer, which is designed for a 13 dBm input power. The mixer produces an 8 dB conversion loss before passing the chirp into the transmitter.

### 3.4 Transmitter

Following the mixing process, there will still be residual signal power at 12-18 GHz that leaks through the mixer. To prevent this from propagating through the system, a KW Microwave BPF-L-15355 band-pass filter is used. The filters provide 72 dBc and 36 dBc attenuation at 1.2 GHz and 8.8 GHz respectively, with 1.5 dB of insertion loss. Because of the large, multi-octave bandwidth, the signal created in the chirp generator can be unacceptably nonlinear in frequency. The Ciao Wireless CLA28-4001 limiting amplifier is used to compensate for this. The amplifier accepts an input power in the range of -21 to 10 dBm and outputs a range of 14 to 18 dBm, with a flatness of +/- 1.5 dB. The output of this amplifier will always have a known power level and flatness regardless of changes in the performance of the Chirp Generator, simplifying the remaining design work.

At this point the signal is split using a Narda 4244-6 coupler. The coupler directs one signal, 6 dB down from the input, toward the transmit amplifier, while the other signal, 2 dB down from the input, is sent to the mixer in the receiver. The smaller signal is kept in the transmitter because there is more than enough gain in the
chain to reach the maximum output power achievable. This output power is reached using the Minicircuits ZVE-8G amplifier. It provides 30 dB of gain with a compression point of 30 dBm. After adding attenuators and the long cable required to reach the transmit antenna, as well as leaving additional room to assure the avoidance of saturation, the actual power provided to the antenna is closer to 20 dBm.

3.5 Receiver

The output of the receive antenna is first fed through another identical band-pass filter. This filter is required for eliminating unwanted signals generated by other instruments operating onboard the aircraft. Next, a low-noise amplifier, the Hittite HMC462LP5, provides 14 dB of gain, with a noise figure of 3 dB, limiting the noise figure of the receiver. It also has a reverse isolation of 37 dB to assure that any unwanted signal power leaking out of the receive antenna is well below the power level of the chirp leaving the transmitter.

As the received signal reaches the MITEQ DB0218LA1 mixer, it is mixed with the portion of the transmit signal mentioned previously. The mixer operates with a 7-13 dBm LO drive power, which is easily achieved by the signal passed down from the transmitter. It has a 7 dB conversion loss and a 5 dBm compression point. The isolation between the transmitter and receiver, measured through the antennas and including both of the long cables used to reach them, is measured to be at least 27 dB once installed. Using this value, the gain and loss values of the other components
in the chain, and the mixers compression point, it is found that the transmit amplifier must be limited to 28 dBm to avoid saturating the mixer.

3.6 IF Section

The first component of the IF section is a Gaussian high-pass filter. The purpose of this filter is to attenuate strong low-frequency signals created by the mixer, the most serious of these being the antenna feed-through signal. A Gaussian filter design is chosen because it minimizes any ringing of the filter [8], which would have undesirable effects on the data. There is a large range of acceptable cutoff frequencies, because of the large difference in range between any internally created signals and the first return from the ground.

The IF amplifier was created using inverting amplifiers made with Texas Instruments OPA847 op-amps. This allowed for adjustment of the gain by changing the value of the feedback resistance on each amplifier. The gain achievable by one section is limited to 34 dB by the op-amps 3.9 GHz gain-bandwidth product along with the maximum IF frequency of 80 MHz set by the sampling frequency. A cascade of three inverting amplifiers is used to assure that enough gain can be achieved. The IF amplifier is designed for a theoretical gain of 84 dB. A peak gain of 74 dB is achieved by the entire IF board, including filters and attenuators.

The Minicircuits SCLF-65 is chosen as the low-pass filter for the IF section. It has a cutoff frequency of 71 MHz, allowing the capture of signals nearing the 80
MHz limit, while achieving 20 dB of attenuation at 86 MHz to prevent aliased signals from masking the return.

Due to its lower frequency of operation, the IF section was assembled out of surface mount components on a printed circuit board. The schematic and layout are included in the appendix.

3.7 Timing

All of the signals in the system are referenced off of a single 10 MHz iridium source to keep the frequency terms coherent. A second PCB was designed to input the 10 MHz reference and output the two other required frequencies. These include a 50 MHz signal, which is required by the digital arbitrary waveform generator that creates the low-frequency reference chirp, and a 160 MHz signal, which sets the sampling frequency of the analog-to-digital converter. The programming required for one of the two PLL chips is accomplished using a CPLD. The schematic and layout for this board are included in the appendix.

3.8 Antennas

The two antennas used are the Model 3115 Double Ridged Guide Horn from ETS-Lindgren. The antenna provides around 10 dB of gain from 2-8 GHz with a VSWR not exceeding 2.4. The half-power beamwidth across the relevant bandwidth varies from about 40 to 60 degrees.
3.9 System Analysis

In order to understand the performance of the radar, it is useful to first find the thermal noise floor at the input of the receiver:

\[ N_{in} = kTB \quad (3.3) \]

with \( k \) as Boltzmann’s constant (1.38E-23 J/K), \( T \) as room temperature (290 K), and \( B \) as the bandwidth of the radar stated previously. The signal power present at the input of the receiver can be defined as \( S_{in} \), leading to the receiver’s input signal-to-noise ratio.

\[ SNR_{in} = \frac{S_{in}}{N_{in}} \quad (3.4) \]

As the signal and noise propagate through the receiver, they will both be affected by additional terms. First, there is the gain of the receiver, \( G_{RX} \), which accounts for the gains and losses of the varying components in the receiver chain. This will affect the power levels of both the signal and the noise. Next, there is a compression gain, \( G_{comp} \), provided during the mixing process. This gain represents the utilization of the 6 GHz bandwidth in compressing the pulse length, and applies only to the signal power [9].

\[ G_{comp} = B\tau \quad (3.5) \]

An integration gain could also be included here to account for coherent integrations made digitally, but since the number of coherent integrations achievable is expected to be minimal, that factor is not included. Finally, the noise floor must also be multiplied by the noise figure, \( F \), accounting for the degradation in signal-to-noise ratio.
ratio that occurs in the receiver chain. Taking all of this into account, the signal-to-
noise ratio at the output of the receiver is calculated.

\[
SNR_{\text{out}} = \frac{S_{\text{in}} G_{\text{Rx}} G_{\text{comp}}}{N_{\text{in}} G_{\text{Rx}} F} = \frac{S_{\text{in}} G_{\text{comp}}}{N_{\text{in}} F} = \frac{S_{\text{in}} G_{\text{comp}}}{N_{\text{out}}} \tag{3.6}
\]

Note that \(N_{\text{in}} F\) has been redefined as the output noise, \(N_{\text{out}}\).

The loop sensitivity, \(S_{\text{loop}}\), can be defined as the maximum attenuation
experienced between the transmitter and receiver that still allows detection of the
signal. It is found by first setting a minimum output signal-to-noise ratio required for
detection, \(SNR_{\text{out.min}}\). The chirp will be transmitted at a given power level, \(P_t\), then
will experience a loss represented by the loop sensitivity, \(S_{\text{loop}}\), and must return to the
receiver with a minimum power level, \(S_{\text{in.min}}\), that will satisfy the required output
signal-to-noise ratio. This process is expressed in the following equation.

\[
P_t \times \frac{1}{S_{\text{loop}}} = S_{\text{in.min}} = \frac{SNR_{\text{out.min}} N_{\text{out}}}{G_{\text{comp}}} \tag{3.7}
\]

In the above, \(S_{\text{in.min}}\) has been substituted for using Equation (3.6) with \(SNR_{\text{out.min}}\) set.

\(S_{\text{loop}}\) is then solved for in dB.

\[
S_{\text{loop}} (dB) = P_t (dBm) - N_{\text{out}} (dBm) + G_{\text{comp}} (dB) - SNR_{\text{out.min}} (dB) \tag{3.8}
\]

Next, the radar’s actual values are included to complete the calculation, assuming a
minimum signal-to-noise ratio of 10 dB.

\[
P_t = 20dBm
\]

\[
N_{\text{out}} = kTBF = -96dBm
\]
\[ G_{\text{comp}} = B\tau = 62dB \]

\[ \text{SNR}_{\text{out}_\text{min}} = 10dB \]

\[ S_{\text{loop}} = P_T - N_{\text{out}} + G_{\text{comp}} - \text{SNR}_{\text{out}_\text{min}} = 168dB \]

Now that the loop sensitivity is known, the maximum signal loss expected must be calculated so that a comparison can be made. Using Equation (2.10) at 8 GHz with a 450 m range, with the dielectric constants of air, snow and ice as 1, 1.5 and 3.2 respectively, and with an additional 10 dB of worst-case attenuation through the snow produces the following.

\[
\frac{P_r^2}{P_t} = \frac{10^2 \cdot 0.0375^2}{(4\pi)^2 (2 \times 450)^2} \left| 0.9949 \right|^2 \left| 0.1872 \right|^2 \left| 0.9949 \right|^2 \times 0.1 = -114dB
\]  

This shows that the maximum expected loss is well below the loop sensitivity, meaning that the radar should perform well in the expected conditions.

The minimum expected signal power at the output of the receiver can be found with the addition of the receiver gain, which is 65 dB.

\[ S_{\text{out}_\text{min}} = P_t - 114dB + G_{\text{Rx}} = -29dBm \]  

The input power range for the 12 bit ADC used is -70 dBm to 4 dBm [4]. Since our expected return is within these limits, the ADC should be able to capture it. Any changes in the expected return power can be easily countered by adjusting the gain of the IF amplifier to assure that the desired signals are captured by the ADC.
3.10 Experiment

The system was integrated into a single aluminum box. The two PCBs were each mounted into their own enclosures that were then fastened onto the bottom surface of the box. The connectorized components were mounted into the box as well, and everything was connected using Minibend coaxial cables. Power supplies were also included to run the active components. Connectors were included on the front panel to allow timing signals to be exchanged with the digital system, as well as to provide the transmitter and receiver interfaces with the antennas, and a final IF output for the ADC.

Figure 3.2. Completed system – front view.
Figure 3.3. Completed system – top view.

The radar was connected to the digital system and run in the lab. A beat frequency was created using a short, coaxial transmission line, and was viewed on a spectrum analyzer. Because the transmission line was not long enough to place the beat frequency above the high-pass cutoff frequency, the signal had to be viewed before the IF section at the output of the mixer. The shape of the beat frequency was verified to be as expected. Additionally, the receiver was run with no input and
captured by the ADC. This allowed verification that the ADC was working properly, and showed that the noise characteristics were as expected.

The radar was installed onboard a NASA P3 aircraft in March of 2006. The antennas were installed in the bomb bay of the aircraft and connected to the radar through two low-loss SMA cables. Flight lines were made over the outskirts of Alaska and Greenland to coincide with satellite passes made over the same areas. Field teams were also in place to take ground data for later comparison.

When initially run, the radar was unable to clearly resolve the surface of the ice, as seen below.

![Figure 3.4. Poor initial radar results from Alaska.](image-url)
This problem was eventually countered by increasing the length of the chirp to 2500 us. This involved an adjustment in the loop filter of the PLL to tighten up the bandwidth. Additionally, with the beat frequencies now coming in much lower than originally expected, the IF high-pass filter had to be replaced with a single capacitor. The resulting data looked much better as evidenced by the following figures.

Figure 3.5. Improved radar results.
Chapter 4
Antenna Design

4.1 Considerations

There are a number of reasons why the horn antennas used on the snow radar experiment are not very ideal. Although the correct bandwidth was covered and the gain was adequate, the radar would benefit from beamwidth improvements in both the cross-track and the along-track directions. The requirements for each will be illustrated.

In the cross-track, the minimum detectable snow depth can be limited by the beamwidth. The process is explained using Figure 4.1.

![Figure 4.1. Illustration of masking due to a wide cross-track beamwidth [4].](image)

Figure 4.1. Illustration of masking due to a wide cross-track beamwidth [4].
Because of the finite beamwidth, a portion of the signal will be transmitted and received off-nadir. The backscattering coefficient of snow is expected to drop off while moving away from nadir [11], but it is difficult to say exactly how much. This clutter will appear at a greater range than that of the return at nadir. If the snow depth falls within this range, and the power of the clutter is greater than the power of the return from the ice, the radar will not be able to find the snow depth. The minimum snow depth that cannot be masked by clutter is calculated as:

\[ d_{\text{min}} = R(\sec(\beta_c / 2) - 1) \]

(4.1)

with \( d_{\text{min}} \) as the minimum unambiguous snow depth, \( R \) as the radar’s range to the surface and \( \beta_c \) as the cross-track beamwidth of the antenna. The snow depth in the areas of interest varies all the way down to zero, so it is a good practice to limit the cross-track beamwidth as much as possible.

When a minimization of the beamwidth is desired, it is accomplished using a large aperture. Large apertures are commonly created using an array of antennas, which will typically result in a smaller and lighter product than if a single large element were used. However, when antenna elements are spread out a great distance to create the large aperture, grating lobes become a concern. Grating lobes are new maxima, in addition to the main lobe at nadir, that appear when the element spacing exceeds one wavelength. Much like the problem created by a finite beamwidth in the cross-track, clutter returned by these grating lobes can potentially mask a return from the ice. Unlike the earlier problem, however, the closest grating lobe to nadir dictates a maximum possible snow depth that cannot be masked by clutter.
\[ \begin{align*}
  d_{\text{max}} &= R(\sec(\theta_g) - 1) \\
  \text{(4.2)}
\end{align*} \]

Here, \( d_{\text{max}} \) is the maximum unambiguous snow depth, and \( \theta_g \) is the distance of the closest grating lobe to nadir. It is clear that it is desirable to minimize the beamwidth and maximize the distance of the grating lobes. Unfortunately, improving one will diminish the other, when maintaining a constant number of elements.

In the along-track the limitations must be thought of in a different way. Regardless of the actual antenna beamwidth, the use of coherent averaging in the processing will allow the synthesis of a very small beam, negating any off-nadir clutter. However, this only works to a certain degree. As the returns move off-nadir in the along-track direction, they will begin to experience a Doppler shift in frequency before returning to the radar. This shift must be effectively sampled by the PRF of the radar in order to avoid the reception of any clutter. A given PRF will set a limitation on how far off-nadir returns can exist and still be sampled properly. The along-track beamwidth should be kept within this range. If the beamwidth is greater than this, than clutter returns will be aliased back within the properly sampled range during processing. This again leaves a potential for overlapping ranges and a masking of the return from the ice. In order to avoid aliasing, the limitation imposed on the PRF is found to be:

\[ \begin{align*}
  \text{PRF}_{\text{min}} &= 4fv\sin(\beta_a / 2) / c \\
  \text{(4.3)}
\end{align*} \]

with \( f \) as the frequency of the radar signal, \( v \) as the velocity of the radar, and \( \beta_a \) as the along-track beamwidth. It is difficult to achieve a high PRF for such a wideband
FMCW radar, so a small beamwidth can be necessary in high velocity airborne applications.

For this specific application, the antenna requirements can be calculated as follows. Assuming a range of 450 m and a desire to record all depths past 0.5 m, Equation (4.1) provides that a beamwidth of around 5° is required. Although this may be impossible to achieve across the entire bandwidth, it would be the goal. Next, assuming a maximum snow depth of 5 m and the same range of 450 m, the grating lobes can fall within 9° of the main lobe. This result should allow a lot of flexibility in the spacing of the array, as this is much closer than grating lobes would traditionally be allowed. Finally, using Equation (4.3) with the PRF at 2 kHz, the velocity of 100 m/s, and the worst case frequency of 8 GHz, the beamwidth requirement becomes 22° in the along-track. These requirements must all be considered while designing antennas for the snow radar.

4.2 Vivaldi Antenna Array

Vivaldi antenna arrays are popular for their ability to achieve multi-octave bandwidths. They are easy to manufacture using PCBs, resulting in small and lightweight elements. Previous work here at KU [12] has shown the potential for designing an E-plane array of Vivaldi antennas that utilizes mutual coupling to achieve a large bandwidth. This coupling allows the use of elements much smaller than would be required for a single element to achieve the same low frequency.
A published design was utilized [13] that showed a VSWR off less than 2 for a bandwidth of 1 to 5 GHz, which is near the 2 to 8 GHz band desired. A single element of the design is shown below.

![Diagram of Vivaldi antenna element](image)

Figure 4.2. Vivaldi antenna element [13].

The antenna consists of three sections. First, a stripline feed provides access to the antenna and sets the reference impedance. Next, a stub and a cavity are used to provide a transition from the stripline feed to a slotline structure. Finally, the slotline transitions to an exponentially tapered flair. This flair provides a transition into free-space for the signals.

The paper provides a parametric study of the affect on the input impedance when modifying different properties of the design. Some modifications were needed in order to make the designs from the literature match this project’s needs. These included a change in the substrate height so the design could be manufactured, and a change in the stripline width to create a 50 Ω input impedance. Many simulations were performed while modifying the parameters of the elements until eventually a
suitable design was found. A coaxial feed was then included, producing the following.

Table 4.1. Vivaldi element parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>aperture height</td>
<td>H</td>
<td>1.7 cm</td>
</tr>
<tr>
<td>taper length</td>
<td>L</td>
<td>8 cm</td>
</tr>
<tr>
<td>opening rate</td>
<td>R</td>
<td>30 cm</td>
</tr>
<tr>
<td>stripline width</td>
<td>$W_{ST}$</td>
<td>0.27 cm</td>
</tr>
<tr>
<td>slotline width</td>
<td>$W_{SL}$</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>radial stripline stub radius</td>
<td>$R_R$</td>
<td>0.6 cm</td>
</tr>
<tr>
<td>radial stripline stub angle</td>
<td>$A_R$</td>
<td>80°</td>
</tr>
<tr>
<td>circular slotline cavity diameter</td>
<td>$D_{SL}$</td>
<td>0.4 cm</td>
</tr>
<tr>
<td>slotline cavity offset</td>
<td>$L_G$</td>
<td>0.9 cm</td>
</tr>
<tr>
<td>distance from transition to cavity</td>
<td>$L_{TC}$</td>
<td>0.25 cm</td>
</tr>
<tr>
<td>distance from transition to taper</td>
<td>$L_{TA}$</td>
<td>0.25 cm</td>
</tr>
<tr>
<td>element length</td>
<td>d</td>
<td>9.8 cm</td>
</tr>
<tr>
<td>element width</td>
<td>b</td>
<td>2.288 cm</td>
</tr>
</tbody>
</table>
Figure 4.3. Infinite array simulation in HFSS.

Figure 4.4. Infinite array simulation VSWR results.
Figure 4.5. Array beam pattern at 2, 5 and 8 GHz.

The simulation was performed as an infinite array, which should approximate a long array of elements. The final VSWR performance rises above two at the very bottom of the bandwidth, and a couple of other locations as well, but overall the array performs well. The radiation patterns and gains along the bandwidth also look satisfactory.

An array was built from this design and tested using a 12-way power splitter. The elements were fed by utilizing a notch cut into the top substrate, allowing access to the stripline. Coaxial connectors were soldered onto the stripline and stabilized with some additional hardware.
Figure 4.6. Constructed Vivaldi array.

The VSWR of the 12-element array is shown below.

Figure 4.7. Constructed 12 element array VSWR results.
These results are very similar to the simulation, and actually appears to be performing better across the band.

Next, data were collected using the antenna range. First, the gain of the array was found across the bandwidth by using two horn antennas as references, then replacing one of them with the Vivaldi. These values were modified to compensate for the losses through the cable and the power splitter and then compared to the simulation results.

Table 4.2. Measured versus simulated antenna gain.

<table>
<thead>
<tr>
<th>Freq(GHz)</th>
<th>Simulated Gain(dB)</th>
<th>Divider Loss(dB)</th>
<th>Cable Loss(dB)</th>
<th>Total(dB)</th>
<th>Measured Gain(dB)</th>
<th>Error(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.3</td>
<td>-1.0</td>
<td>-0.4</td>
<td>6.9</td>
<td>5.3</td>
<td>-1.6</td>
</tr>
<tr>
<td>3</td>
<td>11.2</td>
<td>-1.3</td>
<td>-0.4</td>
<td>9.5</td>
<td>8.9</td>
<td>-0.6</td>
</tr>
<tr>
<td>4</td>
<td>13.3</td>
<td>-1.8</td>
<td>-0.6</td>
<td>10.9</td>
<td>8.9</td>
<td>-2.0</td>
</tr>
<tr>
<td>5</td>
<td>14.9</td>
<td>-2.3</td>
<td>-0.7</td>
<td>11.9</td>
<td>11.0</td>
<td>-0.9</td>
</tr>
<tr>
<td>6</td>
<td>16.2</td>
<td>-2.8</td>
<td>-0.7</td>
<td>12.7</td>
<td>12.8</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>17.5</td>
<td>-3.3</td>
<td>-0.8</td>
<td>13.4</td>
<td>12.6</td>
<td>-0.8</td>
</tr>
<tr>
<td>8</td>
<td>18.3</td>
<td>-3.8</td>
<td>-0.8</td>
<td>13.7</td>
<td>12.8</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

These results show that the simulated and measured gain match to within 2 dB. Finally, the radiation pattern of the array was tested.
The results show that, once normalized to adjust for the small differences in gain, the simulated and the actual results match well through the first sidelobe.

Once operation of the Vivaldi array has been established, the design must be considered in terms of the goals set earlier. If the linear array is oriented in the along track, then a short array of the closely spaced elements can be used to meet the relatively liberal beamwidth requirements in that direction. It is seen in simulation that a 22° beamwidth at 2 GHz can be nearly achieved by a 12 element array. The next objective is to satisfy the more stringent cross-track requirement. The process of
spacing array elements far apart and balancing the beamwidth versus the grating lobes is discussed later in Section 4.3.

In order to utilize a two-dimensional array, a proper feeding system must be devised. The linear array was tested using a connectorized commercial power splitter. However, these are expensive, large and heavy, so the use of a two-tier system of them is not inviting. If a power splitter were to be integrated within each E-plane array, then only a single connectorized power splitter would be needed externally for feeding the linear arrays. This would greatly simplify the mounting of the system.

A Wilkinson power splitter was designed for use in the antenna array. The design was made in stripline to match the antenna feeds. The Wilkinson divider can provide a match at all 3 ports while isolating the 2 output ports [18]. This only occurs at a single frequency, however, and the bandwidth for a single section Wilkinson divider can be very narrow. A 4-section design is utilized here to provide an adequately large bandwidth. The design was created using Microwave Office to create the layout and provide a preliminary simulation. Then, the layout was imported into HFSS for more accurate simulation results. A 4-section microstrip divider was designed, constructed and tested first, because it could be done so cheaply and quickly.
The results show a down-shifting in the response, as the nulls in the S11 of the simulated design were placed exactly at 2, 4, 6 and 8 GHz.

It was decided to proceed with a stripline design rather than spending time searching for the cause of the shifting, which may only be an issue in the microstrip design. The stripline simulation was conducted similarly to the microstrip with one
modification. Holes are cut in the top substrate to allow for the placement of resistors on the stripline.

Figure 4.11. 4-section stripline Wilkinson power divider model in HFSS.

The simulation provides the following results.
Figure 4.12. 4-section stripline Wilkinson divider results in HFSS. The through to an output port is in red, the input return loss is in blue, the output return loss is in green, and the output port isolation is in purple.

These results show return loss and isolation performance greater than 16 dB across the band.

Unfortunately this design was never tested due to a problem with the manufacturer. It turned out that there was no way to unite the two boards that create the stripline without unintentionally filling in the holes necessary for dropping in the resistors.

4.3 Coordinated Tx and Rx Arrays

Another idea for creating a suitably narrow antenna beamwidth in the cross-track involves the use of two two-element horn arrays. The first array, used for
transmission, is spaced far apart, creating a narrow beamwidth, but resulting in grating lobes located unacceptably close to the main lobe. The second array, used for reception, is spaced closer together with its first null placed to cancel the first grating lobe of the transmit array. The goal is to emulate a large aperture size while keeping the grating lobes as far off center as possible.

When present in an array, the radiation patterns of the individual elements will be modified by an array factor created by the spacing of the elements. If the elements are to be separated in the cross-track with the main lobe pointed toward the ground, a broadside array with uniform phase excitation is required. The array factor is expressed as [10]:

\[
AF = \sum_{n=1}^{N} e^{j(n-1)\psi}
\]

\[
\psi = kd \cos \theta
\]

with \(N\) as the number of elements in the array, \(k\) as the wave number, and \(\theta\) as the angle, with \(\theta = 90^\circ\) representing broadside.

This array factor is responsible for the creation of the grating lobes and nulls that are to be utilized. The location of the first grating lobe can be calculated as [10]:

\[
\theta_{g1} = \cos^{-1}\left( \pm \frac{\lambda}{d} \right)
\]

with \(d\) as the element spacing and \(\lambda\) as the wavelength of the signal. Similarly, the location of the first null can be found as [10]:

\[
\]
\[ \theta_{r1} = \cos^{-1}\left( \pm \frac{\lambda}{Nd} \right) \]  

(4.6)

with \( N \) as the number of elements in the array. These equations have been implemented, along with the simulated radiation patterns from a single horn element, to test this theory.

The process is explained here with an example. If the effective grating lobes (after accounting for the effects of both the transmit and receive arrays) are desired to be 10° off-nadir, than the transmit array can be designed with worst-case grating lobes at 5°, since the first one will be canceled. The resulting radiation pattern is shown below.

Figure 4.13. Transmit array performance with grating lobes placed 5° off-nadir at 8 GHz.
Next, the receive array is created with nulls located 5° off-nadir at 8 GHz.

Figure 4.14. Receive array performance with nulls placed 5° off-nadir at 8 GHz.

The total effect on the signal is seen by multiplying the radiation patterns from each of the two arrays.
It is evident in Figure 4.15 that the grating lobe 5° off-nadir has been negated, leaving the effective first grating lobe at 10°. The sidelobes created on either side of the eliminated grating lobes are 11 dB down from the main lobe. Additionally, the maximum beamwidth, which occurs at 2 GHz, is 9°. Although this is somewhat larger than the desired 5° bandwidth mentioned earlier, that was not a steadfast limitation. Additionally, by 5 GHz the beamwidth has already reduced below 4° and by 8 GHz it is less than 2.5°, so at least a portion of the sweep will remain clutter free for very shallow snow depths.

The maximum element spacing used to achieve this is 0.43 m, so it is possible that this could even be achieved inside the bomb bay of an aircraft. Since the grating
lobe requirements were met and the beamwidth requirements very nearly met, this should be considered a legitimate option for future snow radar experiments.

As another way to test the validity of the coordinated two-elements arrays, they can be compared to a system using a 3-element array on one end and a single element on the other. If the total length of the 3-element array is set to the previous spacing of the transmit array, then the longest dimension and total number of elements remains unchanged. Also, the addition of an extra element to the 3-element array cuts the spacing in half, resulting in grating lobes for the 3-element array that are at the same distance off-nadir as the cumulative grating lobes from the 2 array system.

A simulation shows that the performance of this new setup is very similar to that shown previously, but slightly worse in a two ways. First, the sidelobes come within 10 dB of the main lobe, which is slightly closer than the 11 dB seen previously. Second, the beamwidths seen at 2, 5 and 8 GHz are about 12°, 5° and 3° respectively. Again, this just slightly lags the performance from the two 2-element arrays.
5.1 Introduction

It was shown in Section 3.10 that the PLL did not function properly in its initial configuration. It was found that by drastically slowing down the chirp rate the radar was able to resolve the surface. It can be shown that when nonlinearities are present on the frequency ramp, problems with the beat frequency become more evident as the beat frequency is increased. Therefore, the source of the improvement seen is likely the reduction in the beat frequency produced.

A simulation is run in MATLAB to illustrate this point. It starts with an ideal chirp covering the bandwidth of 200-300 MHz in 100 µs. A selection of 100 random sinusoidal terms is added to the linear frequency, with a maximum frequency and amplitude set for the terms. A beat frequency is created using two different delays, with one much longer than the other. It is evident in the results that the return is better resolved with the shorter delay. The process is illustrated below, and the code is included in the appendix.
Figure 5.1. Frequency nonlinearities added to an ideal chirp.

Figure 5.2. Resulting beat frequency with the shorter delay.
In the experiment in which this PLL was used previously [8], the system was assumed to be working satisfactorily after viewing the settling of the tuning voltage of the VCO. This is not a satisfactory means of testing for this application. The error shown in Figure 5.1, for example, results in a ramp linearity of $5 \times 10^{-4}$ (maximum nonlinearity divided by bandwidth), which is enough to seriously effect the results. This would relate to a voltage swing of only 10 mV on the 20 V ramp and would be very difficult to see. Additionally, the previous experiment was run with a range of around 2 m and a slower chirp time, resulting in much lower beat frequencies. This explains why the nonlinearities were not noticed previously.

If there are nonlinearities causing problems in the PLL, then the issue at hand is to locate their source and attempt to eliminate them. To this end, and attempt was made to capture the output chirp up to 6 GHz using an oscilloscope so it could be

![Figure 5.3. Resulting beat frequency with the longer delay.](image)
examined. Unfortunately, the sampling process of the oscilloscope created a number of large spurs that made the data unusable. Efforts to filter out the spurs in a number of ways proved unsuccessful. It could be seen that the data were still corrupted because the results in MATLAB for an ideal target were coming back worse than the results from actual data collected over the ocean.

Without the ability to directly capture the phase of the chirp, another direction was taken. An ADS model was created to explore some of the potential issues that could be causing problems in the PLL. An overview of phase-locked loop operation will be given first, followed by a presentation of the simulation and results.

5.2 Phase-Locked Loop Overview

With the inclusion of a frequency divider in the feedback path, a phase-locked loop (PLL) will essentially act as a frequency multiplier. A simple PLL circuit is shown below.

![Phase-Locked Loop Diagram](image)

Figure 5.4. Simple phase-locked loop.
The frequency output of the voltage-controlled oscillator (VCO) is divided by a value, N, before reaching the phase detector (PD). This value will create the multiplication of the reference. At the PD, the phases of the reference input and the divided VCO output are compared, with the output current proportional to the difference. The loop filter is used to condition this output, dictating the speed and stability of the system. Finally, the voltage present at the input of the VCO produces its frequency output, as the loop continues around through the feedback.

When properly designed, the phases of the reference signal and the divided VCO output will become locked together, resulting in a frequency output that is a multiple of the reference. When the reference is linearly swept, the output of the PLL will be a linear sweep of a multiplied frequency band.

The PD employed is a phase/frequency detector with a charge pump. This has become the most common phase detector in use due to a variety of benefits that it provides [21]. It is complimented by a 3rd order passive loop filter that has been shown to strike a good balance between stability and frequency rejection. The VCO has a nonlinear transfer function typical of most VCOs. An external divide-by-8 counter is used in conjunction with the one internal to the PLL chip to achieve greater multiplication.

5.3 Simulation Overview

The ADS schematic is shown below.
Figure 5.5. ADS PLL simulation schematic.
The simulation is set up to work with variable frequencies for the reference and VCO output. This was done, rather than using the actual values from the experiment, because it is impossible to simulate the 6 GHz bandwidth due to the amount of data produced.

The reference chirp is created with a voltage ramp incident on a VCO component. The center comparison frequency is set as a variable and the bandwidth of the reference sweep depends on the VCO output center frequency and bandwidth chosen. A phase/frequency detector component is used next, with an adjustable output current. This is followed by the 3rd order loop filter, which has been designed so that all of the component values will update themselves to match an adjustable cutoff frequency. The filter output feeds into another VCO component, which acts as the VCO of the system. There are two of these components that can be switched in and out. One has an ideally linear tuning curve, and one has the actual tuning curve of the VCO used in the experiment, which was imported previously [8]. These VCOs have been designed to work with an adjustable center frequency and bandwidth as mentioned previously. Additionally, a summer has been placed at the input of the VCO with the additional input provided by either a ground or a linear voltage sweep. When the voltage sweep is used, it allows the loop filter to be tightened further because it does not have to pass the transient of the ramp. Finally, the feedback is run through a frequency divider whose value is set by the specified center frequencies of the reference and the VCO.
The output of the VCO is delayed and multiplied to produce a beat frequency. The value of the beat frequency is another variable set by the user. When run, the simulation creates a number of plots showing the operation of the loop, concluding with a plot of the beat frequency, which illustrates the overall effectiveness of the PLL in linearizing the ramp.

### 5.4 Ramp Tracking

The first issue explored in the simulation is the possibility that the loop filter is too tight and is unable to properly follow the frequency ramp. It is estimated that in order to track an incident frequency ramp, the following condition must be met [21]:

\[
\omega_n > \sqrt{2\Delta \omega_{\text{max}}}
\]  

(5.1)

where \( \omega_n \) is the natural frequency of the loop filter (which is closely tied to the cutoff frequency), and \( \Delta \omega_{\text{max}} \) is the maximum slope of the reference frequency. Problems can arise when this condition is not met, as described here.

The loop is first run with a very wide bandwidth, assuring that, after a brief settling time, the ramp will pass through unhindered.
Figure 5.6. VCO output frequency and its 1st derivative versus time for a wide loop bandwidth.

Next, the loop is tightened until it becomes evident that the ramp is not passing through properly. This can be seen in the existence of a slope in the derivative of the output frequency ramp.
Figure 5.7 VCO output frequency and its 1st derivative versus time for a narrow loop bandwidth.

It can be seen in the figures above that by tightening the loop, the noise on the sweep has reduced dramatically. This noise is produced at the output of the charge pump, and does not appear to be large enough in either case to adversely affect the results. The effect of the loop’s inability to track the reference ramp can be seen in the beat frequencies produced by the above two chirps.
The effect this has on the beat frequency is unique to this problem. The return in Figure 5.8 looks very symmetrical with sidelobes approaching the 32 dB level expected for the Hanning window used. The beat frequency in Figure 5.9, on the
other hand, has been shifted slightly down in frequency with additional noise on the lower end, while the upper end actually has improved sidelobe performance.

5.5 Additive Noise

Any additive white Gaussian noise appearing within the loop will be limited in bandwidth by the loop filter before appearing on the tuning voltage. There was no way found to produce white noise in ADS while running a transient simulation, so a simulation was run in MATLAB instead. Noise was created for the length of the chirp time, and was then filtered to limit it in frequency to below the loop filters cutoff frequency. The cutoff frequency was set according to (5.1). A linear chirp frequency versus time curve was created and this noise was added onto it. The chirp curve was then converted back into voltage versus time, and then delayed so that a beat frequency could be formed. This is illustrated in the following plots.
Figure 5.10. Frequency noise.

Figure 5.11. Beat frequency from an ideal chirp.
Figure 5.12. Beat frequency after adding the noise from Figure 5.10.

Note that the amplitude of noise needed to affect the quality of the beat frequency here is significantly greater than that encountered in Figure 5.6. In the event that the noise in a system is large enough to be causing an issue, it can be reduced if the bandwidth of the loop filter is reduced.

5.6 Phase Noise

The final problem addressed here is the existence of phase noise, most likely originating from the VCO [23]. The first plot shows the results of a PLL with no phase noise.
Next, a large amount of phase noise is added to the output of the VCO.

Figure 5.13. Beat frequency resulting from a VCO without phase noise and a 50 kHz loop bandwidth.

Figure 5.14. Beat frequency from a VCO with phase noise and a 50 kHz loop bandwidth.
It is suggested that opening up the loop bandwidth allows the loop to quickly compensate for the phase noise, improving the performance of the loop [23]. This works because, if the reference is free of noise, the nonlinearities created by the VCO will be fed back to the phase detector where they will be adjusted for. This can be seen from the simulation results below, where the loop bandwidth has been increased by a factor of 10.

![Figure 5.15. Beat frequency from a VCO with phase noise and a 500 kHz loop bandwidth.](image)

**5.7 Analysis**

Of the three problems addressed, the ramp tracking issue is the easiest to understand. The condition of Equation (5.1) was met both before and after the sweep was slowed down during the experiment. There is no reason to believe that this was a problem for the system.
The problems of additive and phase noise are more complex. First of all, both produce results which look relatively similar on the beat frequency, which is currently the only way available to test the linearity of the chirp. Second, if both are found to be an issue, there can be a serious problem because the means of correcting them are opposites [23]. While the effect of additive noise is reduced with a small loop bandwidth, the effect of phase noise is reduced by using a very large loop bandwidth in coordination with a large comparison frequency.

Fortunately, there is reason to believe that additive noise should not be a factor in the snow radar system. Most studies on noise in PLLs have been regarding reference signals coming from a noisy environment. This is not an issue here, where the reference is created digitally. The signal to noise ratio required for stable operation of a PLL is approximately 4 [22]. This value should be far exceeded by the PLL of the snow radar because there are no significant contributors of additive noise power in the loop.

It is believed, then, that the problems in the PLL are caused by the phase noise of the VCO. Although the VCO employed has what would traditionally be considered good phase noise performance, it is unknown exactly how this performance translates to this unique application. A full simulation in ADS could potentially address this, but is impossible because of the large data rate required.
Chapter 6

CONCLUSION AND RECOMMENDATIONS

6.1 Radar System

A radar was designed and built to measure snow depth from an aircraft. The system was to provide data with nearly a 3 cm resolution over very large areas. This large quantity of new data is to be used in validating snow depth algorithms created for AMSR-E. When operated in the field, a problem was discovered involving the linearity of the chirp created by the PLL. Data were collected after an adjustment was made to the length of the chirp.

The system could benefit from improvements in both the antennas and the generation of the chirp. Additionally, as new components become available, it may soon be possible to create the RF section of the radar using surface mount parts, drastically reducing the size. Finally, depending on the future implementation of the chirp generator, a redesign of the digital system may be necessary for providing the required signals.

6.2 Antennas

Two potential alternatives were explored for the antennas in future iterations. First, a 12 element Vivaldi array was created to act as a single element, producing the desired along track beamwidth. However, in order to use these elements, the design of an integrated Wilkinson power divider must be completed. Both the Vivaldi stick and
a commercial horn can be used in a cross-track array to produce the small beamwidth required. It has been found that, due to the liberal grating lobe requirement, suitable beamwidth performance can be achieved using only 4 elements. Both a 2x2 and a 3x1 configuration have been shown to produce similar results. The benefits of pursuing the more complicated Vivaldi design are seen in the ability to reduce the along-track beamwidth, as well as a considerably lighter size.

6.3 Chirp Generation

Two reasons were found as to why the PLL did not work as expected on the experiment. First, the additional delay of the airborne application was seen to have a large effect, and second, it was shown that the amount of nonlinearity needed to affect the performance was not evident when viewing the tuning voltage. Simulations were run to explore three possible reasons for the performance issues. Both additive and phase noise were seen to create similar problems. It was concluded that, due to the high SNR of the system, phase noise was likely the source of the errors.

If phase error is the issue, then the PLL must be redesigned with a much higher loop bandwidth and comparison frequency. Designs using a constant, high comparison frequency, along with a fractional divider for creating the chirp, have seen success in similar applications [24]. Another paper introduced a two-loop system where, essentially, the second loop was run by a chirping reference signal created by the first loop [25]. The difference between this system and the one implemented for
the snow radar is that the reference chirp in the paper had a minimum value of 150 MHz, resulting in a much higher comparison frequency.

There are also alternatives that would avoid the use of a PLL. One idea is to use pre-compensation to adjust the tuning voltage into the VCO, resulting in a linear output. The concern with this has been trying to achieve enough stability so that the VCO will react the same way to the input sweep every time. It is believed that with enough temperature stabilization this could be a suitable solution. Another idea involves the use of a series of frequency multipliers, rather than a PLL, to create a wideband chirp from a small digital one. This would avoid the complicated nonlinearities that can appear in the loop, as well as eliminate the need for a VCO, that may have phase noise issues. This would be complicated, however, because a large number of frequency multiplications would likely be needed to achieve a 6 GHz bandwidth. In between each multiplication there would have to be filtering and mixing to keep the signal clean, and the circuit could end up rather complex.
REFERENCES


APPENDIX

IF Board PCB Schematic and Layout
Timing Board PCB Schematic and Layout
2x2 Array Code

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Setup
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ArraySize = 2; % Number of Elements in Each of 2 Linear Arrays
f = 2E9:3E9:8E9; % Frequencies used in Simulation
Lenf = length(f); % Length of Freq Array
% Center Frequency used for array spacing calculations
f0 = 8E9;
lambda0 = 3E8/f0;
% theta - angle used to plot beam patterns
theta = 0:pi/10000:pi;
theta_deg = theta*180/pi;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Importing Single Element Data from HFSS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% A single element TEM horn was simulated in HFSS. The E-field beam
% pattern is imported here at each of the relevant frequencies
Data5G = csvread('5GHzElem.csv',1,0);
Patt5G = Data5G(:,2);
theta_imp = Data5G(:,1);
Data2G = csvread('2GHzElem.csv',1,0);
Patt2G = Data2G(:,2);
Data8G = csvread('8GHzElem.csv',1,0);
Patt8G = Data8G(:,2);
% Interpolation is used to even up the number of points
Elem(1,:) = interp1(theta_imp,Patt2G,theta_deg,'spline');
Elem(2,:) = interp1(theta_imp,Patt5G,theta_deg,'spline');
Elem(3,:) = interp1(theta_imp,Patt8G,theta_deg,'spline');
Elem = sqrt(Elem);
ElemdB = dB(abs(Elem));

%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Tx Array
%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This creates the transmission array by setting the element spacing
theta_g_on = 5 % grating lobe degrees off nadir
theta_g = (90-theta_g_on)*(pi/180);
d1 = lambda0/cos(theta_g);
k = 2*pi*f/3E8; % wave number

for i = 1:Lenf % for each frequency
    psi1 = k(i)*d1*cos(theta);
    for q = 1:ArraySize % for each element
        AF1(i,:) = AF1(i,:)+exp(j*(q-1)*psi1);
    end
    AF1(i,:) = (1/ArraySize)*AF1(i,:); % array factor
    AP1(i,:) = AF1(i,:).*Elem(i,:); % array pattern
end
end

AF1dB = dB(abs(AF1));
AP1dB = dB(abs(AP1));

figure(1)
for i = 1:Lenf
    subplot(3,1,i)
    hold on
    plot(theta_deg,ElemdB(i,:),'b')
    plot(theta_deg,AF1dB(i,:),'k')
    plot(theta_deg,AP1dB(i,:),'r')
    hold off
    title(['Tx Array Beam Pattern at ' num2str(f(i)/1E9) ' GHz'])
    xlabel('theta (degrees)')
    ylabel('Radiation Intensity (dB)')
    axis([0 180 -10 10])
    grid on
    legend('Single Element Pattern', 'Array Factor', 'Array Pattern')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Rx Array
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Place Nulls
theta_n1 = theta_g_on; % sets location of null to previous location
of grating lobes
theta_n = (90-theta_n1)*pi/180;
d2 = lambda0/cos(theta_n)/ArraySize;

for i = 1:Lenf
    psi2 = k(i)*d2*cos(theta);
    AF2(i,:) = zeros(size(theta));
    for q = 1:ArraySize
        AF2(i,:) = AF2(i,:)+exp(j*(q-1)*psi2);
    end
    AF2(i,:) = (1/ArraySize)*AF2(i,:);
    AP2(i,:) = AF2(i,:).*Elem(i,:);
end

AF2dB = dB(abs(AF2));
AP2dB = dB(abs(AP2));

figure(2)
for i = 1:Lenf
    subplot(3,1,i)
    hold on
    plot(theta_deg,ElemdB(i,:),'b')
    plot(theta_deg,AF2dB(i,:),'k')
    plot(theta_deg,AP2dB(i,:),'r')
hold off
title(['Rx Array Beam Pattern at ' num2str(f(i)/1E9) ' GHz'])
xlabel('theta (degrees)')
ylabel('Radiation Intensity (dB)')
axis([0 180 -30 10])
grid on
legend('Single Element Pattern', 'Array Factor', 'Array Pattern')
end

%%%%%%%%%%%%%%%%
% Together
%%%%%%%%%%%%%%%%
APT = AP1.*AP2;
APTdB = dB(abs(APT));
figure(3)
for i = 1:Lenf
    subplot(3,1,i)
    plot(theta_deg,APTdB(i,:),'r',theta_deg,AP1dB(i,:),'b',theta_deg,AP2dB(i,:),'k')
    title(['Cumulative Radiation Pattern at ' num2str(f(i)/1E9) ' GHz'])
    xlabel('theta (degrees)')
    ylabel('Radiation Intensity (dB)')
    axis([60 120 -20 20])
grid on
end
break

%%%%%%%%%%%%%%%%
% Analysis
%%%%%%%%%%%%%%%%

% Beamwidth
% Max
track = find(theta==pi/2);
theta(track);
APT = abs(APT);
D1 = APT(1,track);
while D1 > 0.5*max(APT(1,:))
    track = track+1;
    D1 = APT(1,track);
end
Beta1 = 2*(theta(track)-pi/2)*180/pi;
% Min
track = find(theta==pi/2);
D2 = APT(end,track);
while D2 > 0.5*max(APT(end,:))
    track = track+1;
    D2 = APT(end,track);
end
Beta2 = 2*(theta(track)-pi/2)*180/pi;
disp(['Max Beamwidth = ' int2str(Beta1) ' degrees'])
disp(['Min Beamwidth = ' int2str(Beta2) ' degrees'])
%
Grating Lobes
track = find(theta==pi/2)+1;
D3 = APT(end,track);
while abs(APT(end,track)) < abs(APT(end,track-1))
    track = track+1;
end
while abs(APT(end,track)) > abs(APT(end,track-1))
    track = track+1;
end
Grat1 = (theta(track)-pi/2)*180/pi;
GratDown = dB(abs(max(APT(end,:))/APT(end,track)),'p');
disp(['Closest Grating Lobes is ' int2str(Grat1) ' degrees from
center'])
disp(['And ' int2str(GratDown) ' dB down'])

figure(4)
for i = 2
    subplot(3,1,1)
    hold on
    plot(theta_deg,ElemdB(i,:),'b')
    plot(theta_deg,AP1dB(i,:),'k')
    plot(theta_deg,AP1dB(i,:),'r')
    hold off
    title('Array 1 Beam Pattern')
    axis([0 180 -20 10])
    grid on
end
for i = 2
    subplot(3,1,2)
    hold on
    plot(theta_deg,ElemdB(i,:),'b')
    plot(theta_deg,AP2dB(i,:),'k')
    plot(theta_deg,AP2dB(i,:),'r')
    hold off
    title('Array 2 Beam Pattern')
    axis([0 180 -20 10])
    grid on
end
for i = 2
    subplot(3,1,3)
    plot(theta_deg,APTdB(i,:),'r',theta_deg,AP1dB(i,:),'b',theta_deg,AP2
dB(i,:),'k')
    title('Cummulative Beam Pattern')
    axis([0 180 -20 15])
    grid on
end
Chirp Nonlinearity Code

% clear
clc

T = 100e-6;      % Sweep Time [s]
fstart = 200e6;   % Total sweep bandwidth
fstop = 300e6;
B = fstop-fstart;
fs = 700e6;      % Sampling Frequency [Hz]
fc = (2*2*pi*(B/T))^(1/2);
lin = 1E-4;
tau = 20E-6;

t = 0:1/fs:T;
N = length(t);
M = 2^(nextpow2(N)+2);
f = ((1:M)/M)*fs;

freq = fstart+(B/T)*t;
phase = 2*pi*cumtrapz(freq)/fs;
a1 = cos(phase);
A1 = fft(a1,M);
A1dB = db(abs(A1));

figure(1)
plot(f,A1dB)

% Adding Crap
Go = 1; % if you want error
if Go == 1;
    Num = 200; % Number of error terms
    dev_BW = 50E3 % maximum frequency (in Hz) of freq deviations
    dev_max = 2000
    Cf = dev_BW.*randn(1,Num); % create random array of values for frequencies
    Ca = dev_max*randn(1,Num); % create random array of values for amplitudes
    error = 0;
    for i = 1:Num
        error = error+Ca(i)*cos(2*pi*Cf(i)*t+randn(1));
    end
    figure(2)
    plot(t*10^6,error/10^3)
    axis([0 100 -1E2 1E2])
    xlabel('Time (us)')
    ylabel('Chirp Nonlinearity (kHz)')
    title('Chirp Nonlinearity')
else
    error = 0;
end
freqc = freq+error; % add error to frequency sweep
figure(3)
plot(t,freqc)
phasec = 2*pi*cumtrapz(freqc)/fs;

ac = cos(phasec);
Ac = fft(ac,M);
AcdB = db(abs(Ac));
figure(4)
plot(f,AcdB)

% Creating Beat
fb = (B/T)*tau;
acd = circshift(ac',floor(N*tau/T))';
beat = ac.*acd;
win = hann(N)';
beatw = beat.*win;
Beat = fft(beatw,M);
BeatdB = db(abs(Beat))-max(db(abs(Beat)));
figure(5)
plot(f/10^6,BeatdB)
offset = 0.01;
axis([offset]*fb/10^6 (1+offset)*fb/10^6 -60 0])
xlabel('Frequency (MHz)')
ylabel('Amplitude (dB)')
title('Beat Frequency')