On the Use of Weighted Linear Combination Method in GIS: Common and Best Practice Approaches

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Abstract
The weighted linear combination (WLC) technique is a decision rule for deriving composite maps using GIS. It is one of the most often used decision models in GIS. The method, however, is frequently applied without full understanding of the assumptions underlying this approach. In many case studies, the WLC model has been applied incorrectly and with dubious results because analysts (decision makers) have ignored or been unaware of the assumptions. This paper provides a critical overview of the current practice with respect to GIS/WLC and suggests the best practice approach.

1 Introduction
The weighted linear combination (WLC) model is one the most widely used GIS-based decision rules (Hopkins 1977, Tomlin 1990, Carver 1991, Eastman et al 1993, Heywood et al 1995, Malczewski 1999). The method is often applied in land use/suitability analysis, site selection, and resource evaluation problems (Hobbs 1980, Han and Kim 1988, Eastman et al 1995, Herzfeld and Merriam 1995, Lowry et al 1995). The primary reason for its popularity is that the method is very easy to implement within the GIS environment using map algebra operations and cartographic modeling (Tomlin 1990, Berry 1993). The method is also easy-to-understand and intuitively appealing to decision makers (Hwang and Yoon 1981, Massam 1988). However, GIS implementations of WLC are often used without full understanding of the assumptions underlying this approach. In addition, the method is frequently applied without full insight into the meanings of two critical elements of WLC: the weights assigned to
attribute maps and the procedures for deriving commensurate attribute maps. In many case studies the WLC model has been applied incorrectly and with dubious results because analysts (decision makers) have ignored or been unaware of these elements. Hobbs (1980), Lai and Hopkins (1989), Heywood et al (1995) and Chrisman (1996) provide discussions on some aspects of the incorrect use of the method. Although the difficulties encountered in using WLC are well-researched by the decision analysts (Hwang and Yoon 1981, Belton 1986, Goodwin and Wright 1998), they have received relatively little attention from the GIS community (Heywood et al 1995). This paper aims to clarify and systematize the most common errors and misconceptions associated with the use of WLC in the raster GIS environment.

WLC can be formalized by means of the multi-attribute decision making (MADM) problem (Massam 1988, Pereira and Duckstein 1993, Malczewski 1996). Let the set of decision alternatives be represented by $X = \{x_i | i = 1, 2, \ldots, m\}$. The alternatives are represented by the set of cells or pixels in a raster GIS database. Thus, the index $i$ indicates the location of the $i$-th alternative. For the sake of simplicity we will use a single subscript to indicate the location of an alternative (the cell designated by $i = 1$ is in the top left-hand corner of a grid-cell map and the cells are numbered left-to-right for each row; the cell $m$ is located in the bottom right-hand corner of the raster map). Each alternative is described by means of its locational attribute (coordinate data) and attribute data (attribute values associated with the location). Since the attributes serve as decision variables we can designate a decision outcome (criterion value) by $x_{ij}$, that represents the level of the $j$-th attribute with respect to alternative $i$. Hence, an alternative $i$ can be characterized by the vector in equation (1), and the levels of attributes across an alternative are represented by the vector in equation (2).

$$x_{is} = (x_{i1}, x_{i2}, \ldots, x_{in}), \quad \text{for } i = 1, 2, \ldots, m, \quad (1)$$

$$x_{sj} = (x_{s1j}, x_{s2j}, \ldots, x_{snj}), \quad \text{for } i = 1, 2, \ldots, n. \quad (2)$$

The input data for equations (1) and (2) can be organized in a tabular form (evaluation matrix or geographical matrix). Accordingly, the data can be stored in a GIS as a set of map layers. The data consists of a set of $n$ data layers and each grid-cell in the data layer contains an attribute value, $x_{ij}$. In a particular decision situation the set of alternatives can be limited by imposing constraints on the attribute values (aspatial constraints) or on the locational attributes (spatial constraints).

Given the input data, the problem is to aggregate the map layers according to the WLC decision rule. Formally, the decision rule evaluates each alternative, $a_i$, by the following value function:

$$V(x_i) = \sum_j w_j v_j(x_i) = \sum_j w_j r_{ij} \quad (3)$$

where $w_j$ is a normalized weight, such that $\sum_j w_j = 1$, $v_j(x_i)$ is the value function for the $j$-th attribute, $x_j = (x_{j1}, x_{j2}, \ldots, x_{jn})$, and $r_{ij}$ is the attribute transformed into the comparable scale. The weights represent the relative importance of the attributes. The most preferred alternative is selected by identifying the maximum value of $V(x_i)$ for $i = 1, 2, \ldots, m$.

Given the decision rule (equation 3) the GIS/WLC method involves the following steps: (1) define the set of attribute (objectives and associated attribute map layers); (2) identify the set of feasible alternatives; (3) derive commensurate attribute maps; (4)
define the criterion weights (that is, a weight of “relative importance” is directly assigned to each attribute); (5) combine the commensurate attribute maps and weights using the multiplication and addition overlay operations to obtain the overall score for each cell (alternative); and (6) rank the alternatives according to the overall performance score (the cell with the highest score is the “best” cell). WLC can be operationalized using any GIS system having overlay capabilities. The overlay techniques allow the attribute map layers (input maps) to be aggregated in order to determine the composite map layer (output map). Some GIS systems (e.g. IDRISI and SPANS) feature WLC modules performing the WLC procedure (Eastman 1997, TYDAC Research Inc 1997). Although the methods can be implemented in both raster and vector GIS environments (ERSI 1995, Chrisman 1996), discussion is limited here to the raster-based implementation. Most of the discussion is, however, of relevance to vector-based approaches as well. The structure of this paper follows the six steps involved in the GIS/WLC procedure (see Table 1). Each section focuses on a particular aspect of the methodology and discusses the common practice and associated problems as well as the best-practice approach to the GIS/WLC analysis.

2 The GIS/WLC Procedure

2.1 Defining the set of attributes

Before proceeding, we need to clarify the terms evaluation criteria, objectives and attributes. In the GIS literature, these terms are often used interchangeably or they are assigned meanings that are inconsistent with the decision analysis literature (Eastman et al 1995; Thill 1999). These terms have specific meanings that decision analysts agree on (Keeney 1980, Hwang and Yoon 1981, Goodwin and Wright 1998). Evaluation criterion is a generic term that includes both the concept of objectives and attributes. An objective is a statement about the desired state of a real-world geographical system (e.g. a land use pattern). It indicates the directions of improvement of one or more attributes of the elements of the system. For any given objective, several different attributes might be necessary to provide a complete assessment of the degree to which the objective might be achieved. An attribute is used to measure performance in relation to an objective. For example, if the objective is “maximize forest habitat preservation”, then attributes associated with this objective might be “the populations of different animal and bird species”, “the quality of water in streams”, and “the acreage of different tree species in the forest”. The specific objectives represent the direction of improvement, such as improving the water quality in streams or increasing the population of certain animal species. In this context, quantification of an objective is the adoption of some quantitative (numerical) scale that provides an indicator for how well the objective would be achieved. Thus, the concept of an objective is made operational by assigning to each objective under consideration, one or more attributes that directly or indirectly measure the level of achievement. The relationship between objectives and attributes has a hierarchical structure (sometimes referred to as a value structure). At the highest level are the most general objectives. They may be defined in terms of more specific objectives, which themselves can be further defined at still lower levels. At the lowest level of the hierarchy are attributes, which are quantifiable indicators of the extent to which associated objectives are realized. In spatial decision
making, the lowest level of the hierarchical structure (i.e. attributes) can be represented on maps and can be stored in a GIS database.

The hierarchical structure of objectives and associated attributes provides us with guidelines for identifying the set of attribute maps that should be included in a particular decision analysis. Furthermore, both an individual attribute and a set of attributes must possess some properties to adequately represent a spatial decision problem in a GIS database. First, each attribute must be comprehensive and measurable. An attribute is comprehensive if its level for a particular decision problem clearly indicates the degree to which the associated objective is achieved. An attribute is measurable if it is practical to: (1) assign a number to the attribute for each alternative, and (2) assess the preferences of the decision maker for various levels of the attribute. Second, a set of attributes should be complete (the attributes should cover all relevant aspects of the decision problem and adequately indicate the degree to which the overall objective is achieved), operational (they can be meaningfully used in the analysis), decomposable (the performance of an alternative on one attribute can be evaluated independently of its performance on other attributes), nonredundant (this is required to avoid problems of double counting), and minimal (the number of attributes should be kept as small as possible) (Keeney 1980).

These principles of defining a set of attribute map layers for WLC have largely been ignored in GIS-based approaches (Table 1). This is partly due to the fact that it is usually very difficult to meet all these requirements in spatial decision making. The process of identifying the set of attributes is heavily dependent on the availability of georeferenced data in digital form. Simply, even if the analyst is aware that some attributes are important for a particular decision problem, the required data may not be available or may be of poor quality. In addition, the requirement of decomposability and nonredundancy are very difficult to satisfy for spatial decision problems. Given a set of attribute maps to be considered in the context of a decision problem it is likely that some pairs of attributes will be correlated (redundant). The test for interdependence should be performed with every pair of attribute maps. If they prove to be approximately independent, the set of attributes is said to be nonredundant. If, on the other hand, some pairs of attributes turn out to be redundant, some further steps are required. For example, a surrogate attribute may be identified to replace the overlapping pair or subset of attributes. If attributes are highly redundant, the overlapping attributes should be dropped from the analysis. It should be emphasized that if the value structure is incomplete or incorrectly defined the GIS/WLC procedure may lead to misleading results. The procedure may generate “incorrect” ordering of alternatives if an important element of the problem has not been captured by the analysis and/or the interaction between attributes has not been taken into account.

2.2 Defining the set of feasible alternatives

In raster GIS-based applications, it is assumed that an alternative is represented by an individual pixel or combination of pixels (see Section 1). An alternative is feasible if it satisfies all constraints. The feasible alternatives are identified either by exclusionary screening (Boolean constraints) or by imposing target constraints on the set of all alternatives (Eastman et al 1993). For example, in the context of the problem of landfill facility location we may require “the sites must be outside wetlands” or “the sites must be 1 km away from any river”. The two limitations imposed on the set of alternatives
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are examples of Boolean constraints. Another example might be that “the area for establishing the landfill facility must be approximately 200 acres”. This provides an example of a target constraint. The Boolean constraints are typically operationalized in terms of conjunctive and disjunctive screening using GIS overlay operations. Under conjunctive screening, an alternative is accepted if it meets specified standards or thresholds for all attributes. Disjunctive screening accepts alternative scores sufficiently high on at least one of the attributes under consideration. It is important to note that the feasible alternative should be identified before the standardization procedure is undertaken (see Section 2.3).

The way in which spatial decision problems are represented in the GIS database depends on the resolution (scale) of the images from which the attribute maps are to be derived (Goodchild and Quattrochi 1997). Different decision problems require maps of different scales for representing the decision. Scales define the space over which the decision is to be made. One of the main reasons scale is considered important is that it deals with the interactions between various elements of real-world geographical systems. No one element of a geographical system is independent of the surrounding components. A phenomenon may appear homogenous at one spatial scale, but heterogeneous at another. Many geographical systems show a hierarchical organization, with nested patterns and processing occurring over a wide range of characteristic spatial scales. The difficulty encountered in using a set of attributes for evaluating spatial alternatives (e.g. areal units) of different size is that the best alternative at one level does not necessarily hold at another level. It can be argued that every change in scale for which a decision problem is formulated will bring about the statement of a new set of alternatives and a new problem. Therefore it must be recognized that a new set of alternatives and evaluation criteria may emerge as the scale of observation is varied. The mismatch between the scale of the problem (decision alternatives) and the scale within which individuals involved in the decision process operate may be one of the major obstacles of effective and efficient use of GIS-based decision techniques (see Table 1).

The GIS/WLC methods conventionally employ fixed spatial data (prespecified shape and size of decision alternatives) and thus the decision analysis is prone to scale and aggregation effects (or the modifiable area unit problem) (Openshaw 1984). Although, the research on the modifiable area unit problem focuses on its implications for statistical analysis, a similar problem arises in the context of spatial decision making. The scale effect is related to the question of how many zones should be used. In general, using data gathered and reported at different resolutions or aggregation levels can derive significantly different analytical results. Usually data of higher resolution or disaggregated data are less biased. For example, the larger the unit of aggregation, the larger, on average, is the correlation between two variables. Since WLC requires that the attribute maps are independent of each other (a high correlation between attributes is considered an undesirable property of a set of criteria – see Section 2.1), it is suggested that the more aggregated the input data the higher the chance of violating the assumptions underlying WLC. To illustrate this problem consider a situation that requires combination of two attribute maps representing population density (people/km$^2$) and slope (%). Figure 1a shows the distribution of these two attributes on the two maps where grid-cells have an equal size of 1 km$^2$. Assuming that the two attributes have been assigned the weights of 0.6 and 0.4 respectively, and standardized by the score range procedure (see Section 2.3), the WLC
method identifies cell 1 as the best (most suitable) area. Now let us assume that the population density data are available in a finer resolution. Figure 1b shows that each cell of 1 km² has been disaggregated into four cells of 0.25 km². The number of people in each square is identified to obtain the population density attribute for the new resolution level. It is assumed, for the sake of simplicity, that the spatial pattern of the slope attribute remains the same (compare Figure 1a and 1b). Then, the two attribute maps are aggregated using the same weights of importance and the WLC procedure. We can see that the cells 15 and 16 form the most suitable area. Comparing this result with that obtained for the 1 km² cell-data-layers, we see that the best areas do not overlap. Thus, we conclude that changing the resolution levels may significantly influence the results of WLC.

The aggregation effect focuses on the question of which zoning scheme should be used at a given level of aggregation. In general, using images at different resolutions (or aggregation levels) for analysis can derive significantly different analytical results. For many raster GIS-based decision problems there is an unimaginably large number of combinations of pixels that can be considered as decision alternatives. When WLC is

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**Figure 1** The effect of scale on WLC: (a) the result of WLC for 1 km² resolution map layers; (b) the results of WLC for 0.25 km² resolution map layers.

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used for ordering the set of feasible alternatives, virtually any target ordering can be obtained purely by engineering an appropriate zoning system (configuration of cells). Suppose, for example, that in our sample decision problem (Figure 1b) an alternative is defined as any land parcel of 1 km\textsuperscript{2}. Given a single cell size of 0.25 km\textsuperscript{2} a combination of four cells that meets the requirement of continuity can be considered as a feasible alternative. It is obvious that the different combinations are characterized by different values of attributes. Consequently, one can expect that the different types of aggregation can produce wholly different outcomes, and consequently can lead to different orderings of the alternatives. Figure 2 illustrates this problem for two aggregation schemes. Not surprisingly the two schemes produce different best alternatives.

One may argue that it would be better to process the data at the 0.25 km\textsuperscript{2} resolution using WLC, and in the last step of the procedure one can identify the four highest ranking cells to form the best alternative of 1 km\textsuperscript{2} (Eastman et al 1993). There are, however, two problems with this approach. First, it is likely the alternative will not meet the continuity requirement; that is, the highest ranked cells may not be clustered

Figure 2  The effect of scale on WLC: (a) the result of WLC for an aggregation of the 1 km\textsuperscript{2} resolution map layers; (b) the results of WLC for an aggregation of the 0.25 km\textsuperscript{2} resolution map layers.
(Brookes 1997). Second, there is an interdependency problem involved; the problem is that the selection of one alternative may depend on the choice of another alternative (Rajabi et al 1997). For example, nearby sites for waste disposal facilities are often interdependent with respect to environmental impact (see Section 2.6 for a detailed discussion).

Although GIS can offer some assistance in helping to reduce the scale and aggregation effects, there is no simple and satisfactory solution to the modifiable areal units problem for WLC (Goodchild and Quattrochi 1997). GIS systems allow the decision analyst to access data at various levels of geographical aggregation so that the aggregation level considered best for a particular decision problem can be determined. The systems facilitate the concept of user defined flexible geographic representation. The same set of geographical data can be given a large number of broadly equivalent but different spatial representations. As suggested earlier there are many different ways of aggregating 1 m data to a 1 km grid, for example. This can be achieved by moving the grid-origin and by rotating the grid. Given the GIS capabilities of data manipulation, it is no longer justifiable to assume fixed spatial data for WLC. The attribute data map layers can be processed according to WLC, analyzed, and displayed at the lowest possible level of aggregation, and the scale and aggregation effects can be explored by reaggregating and reanalyzing. While this approach cannot remove the effects, and can be only done at levels of aggregation above the lowest available, the results will be of great importance for letting the decision maker realize the degree of aggregation error in the aggregated data. It can help the decision maker to make a rational decision and build in uncertainty in the decision making process.

2.3 Generating commensurate attribute maps

Given the variety of scales on which attributes can be measured, WLC requires that the values contained in the different attribute map layers be transformed to comparable units. There are a number of approaches that can be used to make the attribute map layers comparable. Linear scale transformation is the most frequently used GIS-based method for transforming input (raw) data into commensurate attribute maps (Table 1). The linear scale transformation methods convert the raw data into standardized attribute scores. A number of linear scale transformations exist (Voogd 1983, Massam 1988). The most often used method is the score range procedure (Eastman et al 1993, Heywood et al 1995). For the maximization criteria, the procedure first subtracts the minimum attribute value from the attribute values, and then rescales them by dividing the results by the range. Values of standardized attributes range from 0 to 1. The higher the value of the score, the more attractive the standardized attribute value. This approach to standardization is based on a strong assumption of linearity. Although it may be argued that the linear transformation is an approximation of the “true” value function, this approximation becomes more inaccurate as the curvature of the “true” value function increases.

In spatial decision problems, the preferences with respect to the levels of attributes are usually nonlinear. One example of such a function is the “distance decay” phenomenon describing the inversely exponential relationship between the “value of distance” and the distance. Since the proximity operations (widely used in deriving attribute maps for raster GIS applications) are based on the straight line or cell measurements of distance, there is an implicit assumption of linearity between distance
and the proximity attributes. Also note that once a transformation function has been chosen, the transformation of the attribute levels to scores is purely a map algebra operation, without any preference inputs from the decision maker. Finally, since the weights assigned to attribute maps imply the trade-offs among attributes with respect to the transformed attribute levels and they vary with attribute level if the “true” value function is nonlinear, the standardized scores make the procedure for deriving weights more difficult (Lai and Hopkins 1989). The transformed attribute levels to which the weights apply are too abstract for a decision maker to consider. This is particularly true when the standardized attributes are assigned values in the interval 0 to 255 to meet color display standards of GIS supporting SuperVGA (see Section 2.4). Heywood et al (1995) have obtained significantly different results processing the same set of data using IDRISI and SPANS. They demonstrated that the WCL analysis is sensitive *inter alia* to the standardization procedure.

Given the limitations of the linear transformation techniques, some authors advocate the use of the value function approach for transforming the attribute maps (Hepner 1984, Hobbs 1980, Lai and Hopkins 1989, Keisler and Sundell 1997). The value function converts different levels for an attribute into value scores. It relates possible decision outcomes to a scale which reflects the decision maker’s relative preferences. There are a number of techniques for assessing a value function (or curve). The *midvalue method* is one of the most popular techniques for deriving a value curve (Keeney 1980, Lai and Hopkins 1989). First, the technique identifies the minimum and maximum values on an attribute map. The decision maker then estimates the midpoint of the value interval between the minimum and maximum attribute values so that the value difference between the minimum and the midpoint is equal to the difference between the midpoint and the maximum. The midvalue point corresponds to the value of 0.5. This implies that an increase in an attribute value from the minimum to the midpoint is just as attractive as an increase from the midvalue point to the maximum. Having identified the midpoint value, the “quarter points” are identified. The first of these is the point which has a value halfway between the minimum and the midpoint. The second “quarter point” has a value halfway between the midpoint and the maximum. These two points correspond to the values of 0.25 and 0.75. The procedure can be repeated to assign subsequent values of 0.125, 0.375, 0.625, 0.875, etc to the corresponding “midvalue” points; the more points, the greater accuracy level of the curve. The author is not aware of a commercial GIS system that supports this procedure. There are, however, spatial decision support systems that integrate GIS and decision analysis techniques which incorporate a module for deriving value functions (e.g. Jankowski 1995). In addition, value function procedures can be implemented in a spreadsheet environment (Kirkwood 1997), and subsequently the results can be imported to GIS (Keisler and Sundell 1997). It should be emphasized that once the form of the value function is identified, the transformation of the raw attributes maps into the value maps is a matter of simple map algebra manipulation.

Finally, it is important to note that the transformation procedure should be performed on the set of feasible alternatives (see Section 2.2). Since the alternatives are defined by the set of constraints imposed on the values of the attributes, it is incorrect to perform the transformation procedure on the attribute maps and then combine the attribute maps with the constraint maps (see Section 2.3). This is a widespread practice in the GIS/WLC studies. For example, Eastman et al (1993, 1995) suggest that the WLC model can be modified for the GIS application by incorporating the constraint maps.
into equation (3); that is,

\[ V(x_i) = \sum_j w_j r_{ij} (\Pi_j r_{ik}^*) \]  

(4)

where \( r_{ik}^* \) is a value assigned to the \( i \)-th cell on the \( k \)-th constraint map layer; it takes a value of 1 if a cell is a feasible alternative and a value of 0 is assigned to infeasible cells. This essentially means that the standardization procedure is performed first and then the standardized attribute maps are multiplied by the binary constraint maps. It is argued here that this is incorrect. The feasible cells (constraints maps) should be generated first, and then a procedure for deriving commensurate criterion maps should be performed.

Figure 3 demonstrates the effect of applying the WLC procedure before and after a set of feasible alternatives is identified. Figure 3a shows the WLC procedure according to equation (4). The two attribute maps are first standardized using the score range

![Figure 3](image-url)

**Figure 3** The effect of attribute standardization where: (a) standardization is performed on all cells in attribute map layers; (b) standardization is performed on the set of feasible cells.

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procedure and then they are weighted and added to obtain the overall score map layer. The constraint map is generated by conjunctive screening using 400 and 8 as cut-offs for the population density and slope map layers, respectively. Finally, the constraint maps and the overall score map layer are combined using the multiplication overlay operation to obtain the set of feasible alternatives along with the overall scores assigned to them. Figure 3b shows the WCL procedure that follows the steps listed in Section 1. Thus, the procedure starts with identifying the feasible alternatives, followed by standardization, and the combination of the standardized map layers according to equation (3). Although the same cells are identified as the best ones for both procedures, the results differ substantially in terms of the values of overall scores assigned to corresponding cells. Thus, the procedures result in different orderings. This is of particular importance when more than one cell is to be identified as the best area on the basis of the cell ranking (Eastman et al. 1993).

2.4 Assigning weights to attribute maps

Spatial decision problems typically involve criteria of different importance to the decision makers. Consequently, information about the relative importance of the criteria is required. The derivation of weights is a central step in eliciting the decision maker’s preferences. To this end, one should realize that assigning weights to attribute maps accounts for: (1) the changes in the range of variation for each attribute map, and (2) the different degrees of importance being attached to these ranges of variation.

Incorrect specification of weights is an especially common error in the application of WCL to spatial decision problems (for an overview see Hobbs 1980, Lai and Hopkins 1989). In many GIS-based studies the weights are directly assigned to the attribute maps without full understanding of their meaning (e.g. Williams 1985, IGISE 1991). The weights are obtained simply be asking the decision maker to directly assign numbers between 0 and 1 or 0 and 100 for each thematic data layer. This approach can be criticized for its ignorance of the definition of the units of measurement. Since the weights are multiplied by the ratings for attributes to obtain a score for an alternative, they must depend on the units used for each attribute (Table 1).

It is misleading to interpret the weights as general measures of the importance. An attribute weight is dependent on the range of the attribute values; that is, the difference between the minimum and maximum value for a given attribute. An attribute weight can be made arbitrarily large or small by increasing or decreasing the range. For example, if all alternatives to be evaluated are characterized by the proximity to water of 1000 and 1010 m, the attribute would be less important than in the case where the attribute values range from 1 to 1000 m. The general rule is that we are concerned with the perceived advantage of changing from the maximum level to the minimum level of each attribute, relative to the advantages of changing from the worst to the best level for the other attributes under consideration. Thus, the weights assigned to attribute maps should be derived by asking the decision maker to compare a change from the least-preferred to the most-preferred value on one attribute map to a similar change in another attribute. To achieve this the swing weights technique can be used (see Bodily 1985 for details). This technique is probably the most appropriate for use in conjunction with GIS. We illustrate the approach using a site suitability problem (Banai-Kashani 1989). The problem involves assigning weights of importance to the following attributes: price, slope, and views. The swing weights method derives the weights by
asking the decision maker to compare a change from the least-preferred to the mostpreferred value on one attribute to a similar change in another attribute. The decision maker is asked to imagine a hypothetical grid-cell with the three attributes at their least-preferred levels. Then, he/she is asked, if just one of the attributes could be moved to its best level, which would he/she choose? Suppose that the decision maker selects “price”. After this change has been made, he/she is asked which attribute he would next choose to move to its best level, and so on until all attributes have been ranked. Let us assume that the decision maker’s rankings are: “price”, “slope”, and “view”.

Given the rankings, we can now assign a weight of 100 to the price criterion. Then, the decision maker is asked to compare a swing from the least-preferred slope to the most-preferred slope, with a swing from the least-cost site to the highest-cost site. Suppose that the decision maker indicates that the swing in “slope” is 60% as important as the swing in “price”. Consequently, a weight of 60 is given to the slope criterion. Finally, a swing from the worst “view” to the best is considered 30% as important as a change from the least-cost to the highest-cost site. Accordingly, “view” is given a weight of 30. Finally, the three weights are normalized as follows:

\[ w_1 = \frac{100}{(100 + 60 + 30)} = 0.526 \]
\[ w_2 = \frac{60}{(100 + 60 + 30)} = 0.316 \]
\[ w_3 = \frac{30}{(100 + 60 + 30)} = 0.158 \]

It should be noted that this form of trade-off analysis is rarely used in GIS-based studies (Hobbs 1980; Keisler and Sundell 1997). Some analysts argue that the pairwise comparison method is a more appropriate method for use in conjunction with GIS (Banai 1993, Siddiqui et al 1996). The popularity of this method is in part due to the fact that the IDRISI GIS incorporates a module for deriving attribute weights using the pairwise comparison technique (Eastman 1997, Eastman et al 1993). In short, this method involves pairwise comparisons to create a ratio matrix. It takes as an input the pairwise comparisons and produces the relative weights as output. Specifically, the weights are determined by normalizing the eigenvector associated with the maximum eigenvalue of the (reciprocal) ratio matrix. However, this approach can be criticized for its meaningfulness of responses to the underlying questions (Goodwin and Wright 1998). The questions simply ask for the relative importance of attributes without reference to the units of measurement and scales on which the attribute are measured. This fuzziness may mean that the questions are interpreted in different, and possibly erroneous ways, by decision makers. As suggested earlier, to have a consistent meaning the weights must depend on the units used for each attribute.

2.5 Combining attribute maps and weights

As mentioned earlier there are two strong assumptions implicit in the WLC method: the linearity and additivity of attributes. The former assumption means that the desirability of an additional unit of an attribute is constant for any level of that attribute. For example, this assumption implies that an additional 10 ha in a parcel of land is valued the same regardless of whether it is added to a land parcel of 100 or 1000 ha. The additivity assumption implies attributes under consideration are mutually preference independent of each other. In many spatial decision situations these two assumptions are very difficult to apply (Table 1). Because of the interaction between
different attributes the WLC method may lead to “false” results. To see what happens when the WLC is applied to a problem where mutual preference independence does not exist, consider the following problem. Suppose that four parcels of land (cells) are evaluated with respect to land suitability for a housing development (Figure 4). Two mutually dependent attributes are considered: cost of land acquisition and slope. Assume the scores assigned to each parcel of land are derived from value functions. In addition, the decision maker assigned weights of 0.6 and 0.4 to the attributes, respectively. Using the WLC model, parcel (cell) 4 is the most preferred one. It is characterized by the lowest cost (highest value) and steepest slope (lowest value). However, it may be that the decision maker considers low cost only to be of “high value” if the parcel is suitable in terms of its slope (notice that a relatively high weight was assigned to the slope attribute). It can be argued that the high value of the cost attribute is virtually “worthless” because it is characterized as steep (low value). Thus, the “best” alternative identified by WLC might not, in fact, be the most preferred one. If the cost attribute is not preferentially independent of the slope attribute, WLC will not correctly represent the decision maker preference structure.

How can the absence of mutual preference independence be identified? The most obvious way in which this will reveal itself is in the use of phrases like “this depends on…” when the decision maker responds to questions. For example, when asked to assign a value to the cost attribute, our decision maker might well have said “that depends on how steep the parcel of land is”. If mutual preference independence does not exist it is usually possible to return to the hierarchical structure of evaluation criteria (see Section 2.1) and redefine the attributes so that a set of attributes which is mutually preference independent can be identified.

Even if the WLC method meets all the underlying assumptions, there still remains a problem of interpreting the overall scores assigned to each cell of the output map. Note that if we multiplied weights of 0.3 and 0.1 by attribute values 0.36 and 1.0, respectively, then the results are about the same. If we interpret 1.0 as being an exceptional attribute value and 0.36 as being a below average attribute, then this identity implies that an exceptional value and a somewhat below average value make the same contribution to the weighted average. Thus, there exist some difficulties in interpreting the output of the WLC method (Hwang and Yoon 1981).

Although it is argued that the value function methods are superior to the \textit{ad hoc}
techniques commonly used in GIS, one should point out the fact that both approaches neglect the existence of spatial relationships among alternatives (ReVelle et al. 1981). For example, in the case of the coal-based power station siting problem, the level of emissions allowed for a given power plant depends on the emissions from other facilities located in a given region. Furthermore, it can be argued that location choice or site selection is a Gestalt process in which alternatives are considered holistically. This means that the overall score obtained by WLC is something more than the sum of its elements. This point is missed in WLC, because its components are determined separately, and then combined.

2.6 Ranking the alternatives

The final stage of GIS/WLC involves the ranking procedure to order all the cells on the output layer according to their overall score value. The cell assigned the rank of 1 is the best alternative. This may look a simple operation, especially when a GIS system featuring the rank procedure is used (e.g. IDRISI). However, in some applications using the rank operation to identify the best alternative may be a problem. For example, consider a situation in which 10 acres of land are to be allocated to a particular use. If each cell has a size of 1 acre, then the WLC procedure is performed and the first 10 cells are recommended as the most suitable land (Eastman et al. 1993, 1995). However, this approach assumes strict independence of alternatives (grid cells), a situation which is very difficult to meet in a spatial decision problem due to the spatial interaction phenomenon (Table 1). Specifically, an evaluation of one alternative with respect to a particular attribute may depend on whether another alternative is selected (Keeney 1980, Rajabi et al. 1997).

To illustrate this problem suppose that four grid cells are evaluated with respect to three attributes: population density, slope, and cost of land (Figure 5a). It is assumed that the weights of 0.4, 0.2 and 0.4 have been assigned to each of the three attributes, respectively. All evaluation criteria are to be minimized. In addition, let us assume that each cell (parcel of land) has a size of 1 ha and the aim is to identify 2 ha of most suitable land. The WLC procedure is performed to obtain a ranking of the four cells. From this cells 2 and 1 are found to be the best and the second best parcels of land, respectively. Therefore, assuming independence, the most suitable area consists of those two parcels of land. However, it can be expected that the value of an attribute in one cell depends on the value of an attribute in the other cells. If interdependency is involved, then the highest ranked parcel (cell 2) should be eliminated and the remaining three parcels should be re-evaluated (Figure 5b). Observe that the value of parcel 4 is now 0.64 and is greater than for parcel 1 (previously ranked as the second best). Accordingly, the most suitable area consists of cells 2 and 4. This simple example demonstrates that in the presence of interdependency, one may need to examine all combinations of grid cells in order to find the best subset; and that selection according to the ranked list of individual cells may not result in the best solution. The difficulties with using ranking for identifying the most suitable land are aggravated as the highest ranked cells may not be clustered into one or more regions. Therefore, depending on the particular situation some further processing of the ranking output is required to identify a continuous region. The reader is referred to Brookes (1997) for a description of this problem and a suggestion for its solution using parameterized region-growing programming.
Figure 5 The effect of spatial dependence on the ordering of alternatives (cells) using WLC: (a) spatial independence of alternatives is assumed; (b) spatial interdependence of alternatives is assumed. Note: $R$ = ranking; $S$ = standardization; the standardized scores are obtained using the following formula:

$$
\hat{r}_{ij} = \frac{r_{ij} \cdot (x_{ij}^{\text{max}} - x_{ij}^{\text{min}})}{x_{ij}^{\text{max}} - x_{ij}^{\text{min}}},
$$

where $x_{ij}^{\text{min}}$ and $x_{ij}^{\text{max}}$ are the minimum and maximum scores for the $j$-th attribute, respectively, and the remaining terms have been defined previously.

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3 Conclusions

This paper has provided an evaluation of commonly used GIS/WLC procedures. It has also offered some advice on best practice approaches to GIS/WLC (Table 1). In each step of the procedure there are problems that may significantly affect the “true” outcome of WLC as applied to spatial decision making. The greatest disadvantage of the current practice with respect to the GIS/WLC methods is that the approaches tend to be ad hoc procedures with little theoretical foundation to support them. It is suggested that incorporating the value function approach and trade-off analysis into the GIS/WLC procedures can substantially improve the decision making process.

One should emphasize that the purpose of any GIS-based decision analysis is to provide insights and understanding, rather than to prescribe a “correct” solution. Often the process of attempting to structure the decision problem is more useful in achieving these aims than the numeric output of the GIS-based modeling. Nevertheless, this process is still best served when the analytical method, such as WLC, bases its suggested solutions on testable axioms and an accurate translation of the decision maker’s judgments. Whether the WLC is the best technique to support this process is a question which is bound to continue to attract debate and controversy.

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References

Eastman J R 1997 *IDRISI for Windows, Version 2.0: Tutorial Exercises*. Worcester, Graduate School of Geography, Clark University

Goodwin P and Wright G 1998 Decision Analysis for Management Judgment. Chichester, John Wiley and Sons


Malczewski J 1999 GIS and Multicriteria Decision Analysis. New York, John Wiley and Sons

Malczewski J 1999 GIS and Multicriteria Decision Analysis. New York, John Wiley and Sons

Massam B H 1988 Multi-criteria decision making (MCDM) techniques in planning. Progress in Planning 30: 1–84

Openshaw S 1984 The Modifiable Areal Unit Problem. Norwich, GeoAbstracts


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