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Research Article

Dynamics Analysis of Avian Influenza A(H7N9) Epidemic Model

Yun Li,[1] Peng Qin ,[2] and Juping Zhang[3]

[1]*School of Science, North University of China, Shanxi, Taiyuan 030051, China*
[2]*School of Electrical and Control Engineering, North University of China, Shanxi, Taiyuan 030051, China*
[3]*Complex Systems Research Center, Shanxi University, Taiyuan, Shanxi 030006, China*

Correspondence should be addressed to Peng Qin; qinpeng@nuc.edu.cn

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The avian influenza A(H7N9) virus has certain fatal effects on human. In this paper, a mathematical model describing the transmission dynamics of avian influenza A(H7N9) between human and poultry is investigated. The basic reproduction number of the model is obtained by applying the method of the next generation matrix. Then the local and global stability of the equilibria are proven. At last, we use numerical simulations to verify the theoretical results.

1. Introduction

Infectious diseases are caused by a variety of bacteria, fungi, virus, and other pathogens, which can be transmitted between human and human, human and animals, animals and animals. Infectious diseases have always been the enemy of human; avian influenza is one of the most common diseases. According to the relevant studies, avian influenza was reported early in 1878 in Italy, which is caused by influenza A virus and is popular among animals. Meanwhile, it is transmitted to human by chickens, ducks, and other animals. The overwhelming majority of avian influenza virus cannot infect human; however, some avian influenza viruses are zoonotic [1]. The clinical manifestations of human diseases caused by some avian influenza virus are not very obvious, which are not harmful enough to human health. However, certain virus such as H5N1 and H7N9 can cause serious human diseases. The virus subtype H5N1, a highly pathogenic avian influenza virus, was firstly detected in Hong Kong in 1997 and was transmitted from Asia to Europe and Africa in 2003 and 2004, resulting in the death of a large amount of poultry and humans [2, 3]. The virus subtype H7N9, a low pathogenic avian influenza virus, is transmitted mainly through contact. In March 2013, 3 people were firstly infected, and, by May 31, 132 cases were found, including 37 deaths, and the mortality rate even reached 30%. These cases are distributed in some provinces such as Beijing, Shanghai, Jiangsu, Zhejiang, Anhui, Shandong, Henan, Taiwan, and Fujian [4, 5]. At present, infected humans of avian influenza A(H7N9) are still sporadic, and the ability of the virus to spread among humans has not yet been found. Since 2013, avian influenza A(H7N9) virus has been included in the main content of influenza virus research. The virus is not fatal to poultry, but it can cause severe respiratory diseases and certain lethality to human, which has attracted the attention of WHO [6].

At present, some scholars have studied avian influenza. In 2007, Iwami et al. [7] considered a dynamic model of avian influenza that might be transmitted by infected birds and infected humans with variant avian influenza. Che et al. [8] studied a model of highly pathogenic avian influenza with saturated contact rate. Initially, the health authorities controlled the outbreak of H7N9 by closing live-poultry markets, which can reduce the probability of human infected by reducing the contact between human and poultry. However, due to economic reasons, live-poultry markets cannot be permanently closed. Therefore, humans need to adopt other intervention strategies, such as screening poultry and killing infected poultry. Liu and Fang [9] established a dynamical model of avian influenza A(H7N9) that can spread between poultry and poultry, poultry and human, and human and human to evaluate the impact of these measures on avian influenza A(H7N9) epidemic. Chen and Wen [10] based on the bilinear disease incidence studied a model with mutant avian influenza A(H7N9) virus in 2015. In 2017, Liu et al. [11] proposed two avian influenza models with different growth rates of the avian population, one with logistic growth and the other with Allee effect.

The organization of this paper is as follows. In Section 2, consider that people mainly contact with poultry in wet markets. We construct an avian influenza A(H7N9) model of different groups in specific environment to study the transmission of avian influenza A(H7N9). In Sections 3 and 4, the basic reproduction number and the existence of feasible equilibria are studied. What is more, by using suitable Lyapunov functions, we demonstrate the global stability of equilibria. The numerical simulations used to verify the theoretical results and some conclusions are included in Sections 5 and 6.

2. The Model

The exposure of infected poultry is one of the key factors in the human infection of avian influenza A(H7N9). In this section, we combine poultry with human to establish a mathematical model with the aim of understanding the spread of avian influenza A(H7N9) from poultry to human. Contagion occurs only between poultry and poultry as well as poultry and human; it cannot spread among humans. The human population is classified into three subclasses: susceptible, infected, and recovered, denoted by S_h, I_h, and R_h, respectively. S_{fa} and I_{fa} denote susceptible and infective poultry in farms and S_{ma} and I_{ma} represent susceptible poultry and infective poultry of markets, respectively. The flowchart of avian influenza A(H7N9) transmission between poultry and human is described in Figure 1.

The dynamic model of avian influenza A(H7N9) is described as the following ordinary differential equations:

$$\begin{aligned}
\frac{dS_{fa}}{dt} &= A_a - d_a S_{fa} - aS_{fa} - \beta_a S_{fa} I_{fa},\\
\frac{dI_{fa}}{dt} &= \beta_a S_{fa} I_{fa} - d_a I_{fa} - aI_{fa} - \alpha_a I_{fa},\\
\frac{dS_{ma}}{dt} &= aS_{fa} - \beta_m S_{ma} I_{ma} - d_m S_{ma},\\
\frac{dI_{ma}}{dt} &= aI_{fa} + \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma},\\
\frac{dS_h}{dt} &= A_h - \beta_h S_h I_{ma} - d_h S_h,\\
\frac{dI_h}{dt} &= \beta_h S_h I_{ma} - d_h I_h - rI_h - \alpha_h I_h,\\
\frac{dR_h}{dt} &= rI_h - d_h R_h,
\end{aligned} \tag{1}$$

where A_h and A_a represent the birth rates of human and poultry, respectively. d_h, d_a, and d_m indicate the natural mortality rates of human, poultry of farms, and poultry of markets, respectively. α_h, α_a, and α_m are the disease-related death rates of infected human, infected poultry of farms, and infected poultry of markets, respectively. β_a is the transmission coefficient from infective poultry of farms to susceptible poultry of farms. β_m is the contact rate from infective poultry of markets to susceptible poultry of markets. β_h is the transmission rate from infected poultry of markets to susceptible human. r is the recovery rate of infected human. a is the proportion of poultry from farms to markets. All the parameters are nonnegative.

Let $S_{fa}(t) + I_{fa}(t) = N_{fa}(t)$, $S_{ma}(t) + I_{ma}(t) = N_{ma}(t)$, $S_h(t) + I_h(t) + R_h(t) = N_h(t)$. From system (1), these can find that

$$\begin{aligned}
\frac{dN_{fa}}{dt} &= A_a - (d_a + a) N_{fa} - \alpha_a I_{fa}\\
&\le A_a - (d_a + a) N_{fa},
\end{aligned} \tag{2}$$

$$\begin{aligned}
\frac{dN_{ma}}{dt} &= aN_{fa} - d_m N_{ma} - \alpha_m I_{ma} \le aN_{fa} - d_m N_{ma},\\
\frac{dN_h}{dt} &= A_h - d_a N_h - \alpha_h I_h \le A_h - d_h N_h.
\end{aligned} \tag{3}$$

Then, from (2), it follows that

$$N_{fa}(t) \le \frac{A_a}{a + d_a} + \left(N_{fa}(0) - \frac{A_a}{a + d_a}\right) e^{-(a+d_a)t}, \tag{4}$$

and $e^{-(a+d_a)t} \to 0$ as $t \to \infty$, so $\lim_{t\to\infty} N_{fa}(t) \le A_a/(a + d_a)$.

In the same way, from (3), it can be obtained that $\lim_{t\to\infty} N_{ma}(t) \le aA_a/d_m(a + d_a)$, $\lim_{t\to\infty} N_h(t) \le A_h/d_h$. The feasible region of system (1) is

$$\begin{aligned}
\Omega = \Bigg\{ &\left(S_{fa}, I_{fa}, S_{ma}, I_{ma}, S_h, I_h, R_h\right) \in R_+^7 : N_{fa}\\
&\le \frac{A_a}{a + d_a},\ N_{ma} \le \frac{aA_a}{d_m (a + d_a)},\ N_h \le \frac{A_h}{d_h} \Bigg\}.
\end{aligned} \tag{5}$$

3. The Existence of Equilibria

By resolving these equations of system (1), it is easy to see that system (1) always has the disease-free equilibrium $E^0 = (S_{fa}^0, 0, S_{ma}^0, 0, S_h^0, 0, 0)$, where

$$\begin{aligned}
S_{fa}^0 &= \frac{A_a}{d_a + a},\\
S_{ma}^0 &= \frac{aA_a}{d_m (d_a + a)},\\
S_h^0 &= \frac{A_h}{d_h}.
\end{aligned} \tag{6}$$

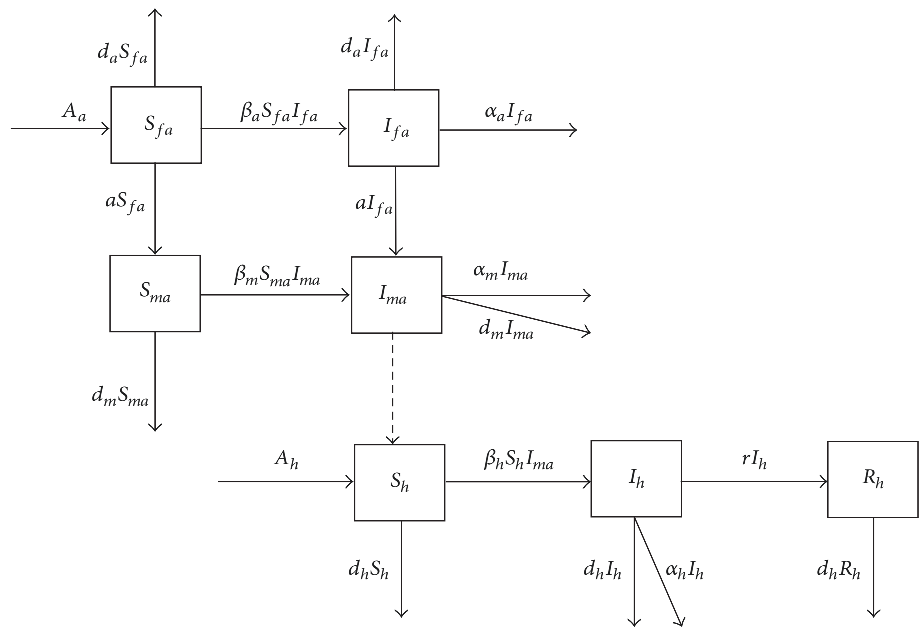

FIGURE 1: Flowchart of avian influenza A(H7N9) transmission.

According to the next generation matrix formulated in van den Driessche and Watmough [12], we can obtain

$$F = \begin{pmatrix} \dfrac{\beta_aA_a}{a+d_a} & 0 & 0 \\ 0 & \dfrac{\beta_m aA_a}{d_m(a+d_a)} & 0 \\ 0 & \dfrac{\beta_hA_h}{d_h} & 0 \end{pmatrix},$$
$$V = \begin{pmatrix} \alpha_a + a + d_a & 0 & 0 \\ -a & d_m+\alpha_m & 0 \\ 0 & 0 & d_h+\alpha_h+r \end{pmatrix}. \tag{7}$$

Then

$$R_{01} = \frac{\beta_aA_a}{(a+d_a+\alpha_a)(a+d_a)},$$
$$R_{02} = \frac{\beta_m aA_a}{d_m(a+d_a)(d_m+\alpha_m)}. \tag{8}$$

Hence, the basic reproduction number of system is as follows:

$$R_0 = \max\{R_{01}, R_{02}\}. \tag{9}$$

Theorem 1. *The disease-free equilibrium E^0 of system (1) always exists. If $R_0 < 1$, E^0 is locally asymptotically stable; if $R_0 > 1$, it is unstable.*

Proof. The Jacobian matrix at the disease-free equilibrium E^0 is

$$J_{|E^0} = \begin{pmatrix} -d_a-a & -\beta_aS_{fa}^0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_aS_{fa}^0-d_a-a-\alpha_a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & -d_m & -\beta_mS_{ma}^0 & 0 & 0 & 0 \\ 0 & a & 0 & \beta_mS_{ma}^0-d_m-\alpha_m & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_hS_h^0 & -d_h & 0 & 0 \\ 0 & 0 & 0 & \beta_hS_h^0 & 0 & -d_h-\alpha_h-r & 0 \\ 0 & 0 & 0 & 0 & 0 & r & -d_h \end{pmatrix}. \tag{10}$$

The characteristic equation of the Jacobian matrix is

$$\begin{aligned}&\left[\left(\lambda+d_a+a\right)\left(\lambda+d_a+a+\alpha_a-\beta_a S_{fa}^0\right)\right]\\&\cdot\left[\left(\lambda+d_m\right)\left(\lambda+\alpha_m+d_m-\beta_m S_{ma}^0\right)\right]\\&\cdot\left[\left(\lambda+d_h\right)\left(\lambda+d_h+r+\alpha_h\right)\left(\lambda+d_h\right)\right]=0.\end{aligned}\tag{11}$$

The eigenvalues of the characteristic equation are

$$\begin{aligned}\lambda_1&=-d_h,\\\lambda_2&=-d_h-r-\alpha_h,\\\lambda_3&=-d_h,\\\lambda_4&=-d_a-a,\\\lambda_5&=-d_m,\\\lambda_6&=-\alpha_a-a-d_a+\frac{\beta_a A_a}{a+d_a},\\\lambda_7&=-d_m-\alpha_m+\frac{\beta_m a A_a}{d_m\left(a+d_a\right)}.\end{aligned}\tag{12}$$

Hence, if $R_{01}<1$ and $R_{02}<1$, all eigenvalues have negative real parts. Namely, if $R_0<1$, the disease-free equilibrium E^0 is locally asymptotically stable.

The discussion of the existence of positive equilibrium is as follows.

(1) Let $I_{fa}\neq 0$; solving the second equation of system (1), one obtains that

$$S_{fa}^{**}=\frac{a+\alpha_a+d_a}{\beta_a}.\tag{13}$$

Equation (13) is replaced with the first equation of system (1), which can be solved as follows:

$$\begin{aligned}I_{fa}^{**}&=\frac{A_a-d_a S_{fa}^{**}-aS_{fa}^{**}}{\beta_a S_{fa}^{**}}\\&=\frac{\beta_a A_a-\left(a+d_a+\alpha_a\right)\left(d_a+a\right)}{\beta_a\left(a+d_a+\alpha_a\right)}\\&=\frac{\left(a+d_a\right)\left(R_{01}-1\right)}{\beta_a}.\end{aligned}\tag{14}$$

Again, let $I_{ma}=0$; the fourth equation of system (1) appears as follows: its left is equal to aI_{fa}^{**}, but the right is 0. Both sides are not equal, so there is no $I_{ma}=0$. Let $I_{ma}\neq 0$, the third equation of system (1) can be solved as follows:

$$S_{ma}^{**}=\frac{aS_{fa}^{**}}{\beta_m I_{ma}^{**}+d_m}.\tag{15}$$

Substituting (15) into the fourth equation of system (1) gives

$$\begin{aligned}&\beta_m S_{ma}^{**}I_{ma}^{**}+aI_{fa}^{**}-d_m I_{ma}^{**}-\alpha_m I_{ma}^{**}\\&\quad=aI_{fa}^{**}\left(\beta_m I_{ma}^{**}+d_m\right)\\&\qquad+\left[a\beta_m S_{fa}^{**}-\left(d_m+\alpha_m\right)\left(d_m+\beta_m I_{ma}^{**}\right)\right]I_{ma}^{**}\\&\quad=0,\end{aligned}\tag{16}$$

and then

$$a_1 I_{ma}^{**2}+b_1 I_{ma}^{**}+c_1=0,\tag{17}$$

where

$$\begin{aligned}a_1&=-\beta_m\left(d_m+\alpha_m\right),\\b_1&=a\beta_m I_{fa}^{**}+a\beta_m S_{fa}^{**}-d_m^2-d_m\alpha_m,\\c_1&=ad_m I_{fa}^{**}.\end{aligned}\tag{18}$$

Due to $a_1<0$, $c_1>0$, $\Delta=b_1^2-4a_1c_1>0$ is always constant and there is a unique positive root. From (17), we obtain

$$I_{ma}^{**}=\frac{-aI_{fa}^{**}}{\beta_m S_{ma}^{**}-d_m-\alpha_m}=\frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1}.\tag{19}$$

Substituting (19) into the fifth, sixth, and seventh equations of system (1) gives

$$\begin{aligned}S_h^{**}&=\frac{A_h}{\beta_h I_{ma}^{**}+d_h},\\I_h^{**}&=\frac{\beta_h S_h^{**}I_{ma}^{**}}{d_h+r+\alpha_h},\\R_h^{**}&=\frac{rI_h^{**}}{d_h}.\end{aligned}\tag{20}$$

(2) Let $I_{fa}=0$, from the first equation of (1), we can obtain

$$S_{fa}^{*}=\frac{A_a}{d_a+a}.\tag{21}$$

Again, let $I_{ma}\neq 0$. Substituting $I_{fa}=0$ into the fourth equation in (1) gives

$$S_{ma}^{*}=\frac{d_m+\alpha_m}{\beta_m}.\tag{22}$$

Combining (21) and (22), from the third equation of system (1), it is known that

$$\begin{aligned}I_{ma}^{*}&=\frac{aS_{fa}^{*}-d_m S_{ma}^{*}}{\beta_m S_{ma}^{*}}=\frac{aA_a}{\left(d_m+\alpha_m\right)\left(a+d_a\right)}-\frac{d_m}{\beta_m}\\&=\frac{d_m\left(R_{02}-1\right)}{\beta_m}.\end{aligned}\tag{23}$$

Substituting (23) into the fifth, sixth, and seventh equations of system (1) gives

$$S_h^* = \frac{A_h}{\beta_h I_{ma}^* + d_h},$$
$$I_h^* = \frac{\beta_h S_h^* I_{ma}^*}{d_h + r + \alpha_h}, \tag{24}$$
$$R_h^* = \frac{r I_h^*}{d_h}.$$

Hence,

(i) when $R_{01} < 1$, there are two cases:

when $R_{02} < 1$, the disease-free equilibrium $E^0 = (S_{fa}^0, 0, S_{ma}^0, 0, S_h^0, 0, 0)$ is obtained,

when $R_{02} > 1$, the boundary equilibrium $E^* = (S_{fa}^*, 0, S_{ma}^*, I_{ma}^*, S_h^*, I_h^*, R_h^*)$ is obtained;

(ii) when $R_{01} > 1$, there are two cases:

when $R_{02} < 1$, the positive equilibrium does not exist,

when $R_{02} > 1$, the endemic equilibrium $E^{**} = (S_{fa}^{**}, I_{fa}^{**}, S_{ma}^{**}, I_{ma}^{**}, S_h^{**}, I_h^{**}, R_h^{**})$ is obtained.

□

In conclusion, we can obtain the following theorem.

Theorem 2. *For system (1), if $R_{01} < 1$, $R_{02} > 1$, there is the unique boundary equilibrium E^*; if $R_{01} > 1$, $R_{02} > 1$, there is the unique endemic equilibrium E^{**}.*

4. Stability of Equilibria

We note that the variable R_h does not appear in the first six equations of system (1). The last equation is independent of the first six equations; we can only consider the following subsystem of system (1):

$$\begin{aligned}
\frac{dS_{fa}}{dt} &= A_a - d_a S_{fa} - a S_{fa} - \beta_a S_{fa} I_{fa},\\
\frac{dI_{fa}}{dt} &= \beta_a S_{fa} I_{fa} - d_a I_{fa} - a I_{fa} - \alpha_a I_{fa},\\
\frac{dS_{ma}}{dt} &= a S_{fa} - \beta_m S_{ma} I_{ma} - d_m S_{ma},\\
\frac{dI_{ma}}{dt} &= a I_{fa} + \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma},\\
\frac{dS_h}{dt} &= A_h - \beta_h S_h I_{ma} - d_h S_h,\\
\frac{dI_h}{dt} &= \beta_h S_h I_{ma} - d_h I_h - r I_h - \alpha_h I_h.
\end{aligned} \tag{25}$$

4.1. Stability of the Disease-Free Equilibrium. The discussion of global stability of the disease-free equilibrium E^0 is as follows.

The poultry subsystem of farms, the poultry subsystem of markets, and the human subsystem are independent of each other. We firstly consider the poultry subsystem in farms and define a Lyapunov function

$$V_{11} = S_{fa} - S_{fa}^0 - S_{fa}^0 \ln \frac{S_{fa}}{S_{fa}^0} + I_{fa}, \tag{26}$$

and then the derivative of V_{11} along solutions of system (25) is

$$\begin{aligned}
\frac{dV_{11}}{dt} &= \frac{S_{fa} - S_{fa}^0}{S_{fa}} \frac{dS_{fa}}{dt} + \frac{dI_{fa}}{dt}\\
&= \frac{S_{fa} - S_{fa}^0}{S_{fa}} \left(A_a - d_a S_{fa} - a S_{fa} - \beta_a S_{fa} I_{fa}\right)\\
&\quad + \beta_a S_{fa} I_{fa} - d_a I_{fa} - a I_{fa} - \alpha_a I_{fa}\\
&= \frac{S_{fa} - S_{fa}^0}{S_{fa}} \left(d_a \left(S_{fa}^0 - S_{fa}\right) + a \left(S_{fa}^0 - S_{fa}\right)\right)\\
&\quad + \beta_a S_{fa}^0 I_{fa} - I_{fa} \left(d_a + a + \alpha_a\right)\\
&= -\frac{a + d_a}{S_{fa}} \left(S_{fa} - S_{fa}^0\right)^2\\
&\quad + \left(a + d_a + \alpha_a\right) \left(R_{01} - 1\right) I_{fa},
\end{aligned} \tag{27}$$

and if $R_0 < 1$, we get $dV_{11}/dt \le 0$. Thus,

$\Omega_1 = \{(S_{fa}, I_{fa}) \in R_+^2 : dV_{11}/dt = 0\} = \{(S_{fa}, I_{fa}) \in R_+^2 : S_{fa} = S_{fa}^0,\ I_{fa} = 0\} = \{E_{fa}^0\}$. According to Lasalle's invariance principle [13, 14], E_{fa}^0 is globally asymptotically stable.

Next, considering the poultry subsystem of markets with the avian components of farms already at the disease-free steady state

$$\begin{aligned}
\frac{dS_{ma}}{dt} &= a S_{fa}^0 - \beta_m S_{ma} I_{ma} - d_m S_{ma},\\
\frac{dI_{ma}}{dt} &= \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma}.
\end{aligned} \tag{28}$$

We define a Lyapunov function

$$V_{12} = S_{ma} - S_{ma}^0 - S_{ma}^0 \ln \frac{S_{ma}}{S_{ma}^0} + I_{ma}. \tag{29}$$

Calculating the derivative of V_{12} along solutions of system (28), it follows that

$$\begin{aligned}
\frac{dV_{12}}{dt} &= \frac{S_{ma} - S_{ma}^0}{S_{ma}} \frac{dS_{ma}}{dt} + \frac{dI_{ma}}{dt}\\
&= \frac{S_{ma} - S_{ma}^0}{S_{ma}} \left(a S_{fa}^0 - d_m S_{ma} - \beta_m S_{ma} I_{ma}\right)\\
&\quad + \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma}
\end{aligned}$$

$$\begin{aligned}
&= \frac{S_{ma}-S_{ma}^0}{S_{ma}}\left(d_m\left(S_{ma}^0-S_{ma}\right)\right)+\beta_m S_{ma}^0 I_{ma} \\
&\quad - d_m I_{ma} - \alpha_m I_{ma} \\
&= -\frac{d_m}{S_{ma}}\left(S_{ma}-S_{ma}^0\right)^2 \\
&\quad + \left(d_m+\alpha_m\right)\left(R_{02}-1\right) I_{ma},
\end{aligned} \tag{30}$$

and if $R_0 < 1$, $dV_{12}/dt_{|(28)} \le 0$. Thus, $\Omega_2 = \{(S_{ma}, I_{ma}) \in R_+^2 : dV_{12}/dt = 0\} = \{(S_{ma}, I_{ma}) \in R_+^2 : S_{ma} = S_{ma}^0,\ I_{ma} = 0\} = \{E_{ma}^0\}$. According to Lasalle's invariance principle [13, 14], E_{ma}^0 is globally asymptotically stable.

Finally, considering the human subsystem with the avian components already at the disease-free steady state

$$\begin{aligned}
\frac{dS_h}{dt} &= A_h - d_h S_h, \\
\frac{dI_h}{dt} &= -d_h I_h - r I_h - \alpha_h I_h.
\end{aligned} \tag{31}$$

We define a Lyapunov function

$$V_{13} = S_h - S_h^0 - S_h^0 \ln\frac{S_h}{S_h^0} + I_h, \tag{32}$$

and then the derivative of V_{13} along solutions of system (31) is

$$\begin{aligned}
\frac{dV_{13}}{dt} &= \frac{S_h - S_h^0}{S_h}\frac{dS_h}{dt} + \frac{dI_h}{dt} \\
&= \frac{S_h - S_h^0}{S_h}\left(A_h - d_h S_h\right) - d_h I_h - r I_h - \alpha_h I_h \\
&= -\frac{d_h}{S_h}\left(S_h - S_h^0\right)^2 - \left(r + d_h + \alpha_h\right) I_h \le 0,
\end{aligned} \tag{33}$$

and, thus, $\Omega_3 = \{(S_h, I_h) \in R_+^2 : dV_{13}/dt = 0\} = \{(S_h, I_h) \in R_+^2 : S_h = S_h^0,\ I_h = 0\} = \{E_h^0\}$. According to Lasalle's invariance principle [13, 14], E_h^0 is globally asymptotically stable. In summary, the following theorem can be obtained.

Theorem 3. *For system (1), if $R_0 < 1$, the disease-free equilibrium E^0 is globally asymptotically stable.*

4.2. Stability of the Boundary Equilibrium and the Endemic Equilibrium. After calculation, the Jacobian matrix of system (1) is given as

$$J = \begin{pmatrix} G & 0 \\ H & K \end{pmatrix}, \tag{34}$$

where

$$\begin{aligned}
G &= \begin{pmatrix}
-d_a - a - \beta_a I_{fa} & -\beta_a S_{fa} & 0 & 0 \\
\beta_a I_{fa} & \beta_a S_{fa} - d_a - a - \alpha_a & 0 & 0 \\
a & 0 & -\beta_m I_{ma} - d_m & -\beta_m S_{ma} \\
0 & a & \beta_m I_{ma} & \beta_m S_{ma} - d_m - \alpha_m
\end{pmatrix}, \\
H &= \begin{pmatrix}
0 & 0 & 0 & -\beta_h S_h \\
0 & 0 & 0 & \beta_h S_h \\
0 & 0 & 0 & 0
\end{pmatrix}, \\
K &= \begin{pmatrix}
-\beta_h I_{ma} - d_h & 0 & 0 \\
\beta_h I_{ma} & -d_h - r - \alpha_h & 0 \\
0 & r & -d_h
\end{pmatrix}.
\end{aligned} \tag{35}$$

The characteristic equation of the Jacobian matrix is

$$\begin{aligned}
&\left[\left(\lambda + d_a + a + \beta_a I_{fa}\right)\left(\lambda + d_a + a + \alpha_a - \beta_a S_{fa}\right)\right. \\
&\quad \left. + \beta_a^2 S_{fa} I_{fa}\right] \\
&\quad \cdot \left[\left(\lambda + d_m + \beta_m I_{ma}\right)\left(\lambda + \alpha_m + d_m - \beta_m S_{ma}\right)\right. \\
&\quad \left. + \beta_m^2 S_{ma} I_{ma}\right] \\
&\quad \cdot \left[\left(\lambda + d_h + \beta_h I_{ma}\right)\left(\lambda + d_h + r + \alpha_h\right)\left(\lambda + d_h\right)\right] \\
&\quad = 0.
\end{aligned} \tag{36}$$

(1) For the boundary equilibrium $E^* = (S_{fa}^*, 0, S_{ma}^*, I_{ma}^*, S_h^*, I_h^*, R_h^*)$, five eigenvalues are

$$\begin{aligned}
\lambda_1 &= -d_h, \\
\lambda_2 &= -d_h - r - \alpha_h, \\
\lambda_3 &= -d_h - \beta_h I_{ma}^*, \\
\lambda_4 &= -a - d_a, \\
\lambda_5 &= -\alpha_a - a - d_a + \frac{\beta_a A_a}{a + d_a}.
\end{aligned} \tag{37}$$

The remaining two eigenvalues λ_6, λ_7 depend on

$$\lambda^2 + \lambda d_m (R_{02} - 1) + d_m (\alpha_m + d_m)(R_{02} - 1) = 0. \quad (38)$$

Hence, if $R_{01} < 1$, $R_{02} > 1$, all eigenvalues have negative real parts.

(2) For the endemic equilibrium $E^{**} = (S_{fa}^{**}, I_{fa}^{**}, S_{ma}^{**}, I_{ma}^{**}, S_h^{**}, I_h^{**}, R_h^{**})$, three eigenvalues are

$$\begin{aligned}\lambda_1 &= -d_h,\\ \lambda_2 &= -d_h - r - \alpha_h,\\ \lambda_3 &= -d_h - \beta_h I_{ma}^{**}.\end{aligned} \quad (39)$$

In addition, λ_4, λ_5 are satisfied:

$$\lambda^2 + \lambda R_{01}(d_a + a) + (d_a + a)(\alpha + a + d_a)(R_{01} - 1) = 0. \quad (40)$$

The remaining two eigenvalues λ_6, λ_7 are satisfied:

$$\begin{aligned}&\lambda^2 + \lambda(2d_m + \alpha_m - \beta_m S_{ma}^{**} + \beta_m I_{ma}^{**}) + \beta_m d_m I_{ma}^{**}\\ &\quad + \beta_m \alpha_m I_{ma}^{**} + d_m (d_m + \alpha_m - \beta_m S_{ma}^{**}) = 0.\end{aligned} \quad (41)$$

It can be obtained that $d_m + \alpha_m - \beta_m S_{ma}^{**} > 0$ by $I_{ma}^{**} > 0$. Hence, if $R_{01} > 1$, $R_{02} > 1$, all eigenvalues have negative real parts. According to the above discussion, we can derive the following theorem.

Theorem 4. *For system (1), if $R_{01} < 1$, $R_{02} > 1$, the boundary equilibrium E^* is locally asymptotically stable; if $R_{01} > 1$, $R_{02} > 1$, the endemic equilibrium E^{**} is locally asymptotically stable.*

Consider the global stability of the boundary equilibrium and the endemic equilibrium.

(1) If $R_{01} < 1$, $R_{02} > 1$, there is the boundary equilibrium E^*.

We firstly consider the poultry subsystem in farms and define a Lyapunov function

$$V_{21} = S_{fa} - S_{fa}^* - S_{fa}^* \ln \frac{S_{fa}}{S_{fa}^*} + I_{fa}, \quad (42)$$

and then the derivative of V_{21} along solutions of system (25) is

$$\begin{aligned}\frac{dV_{21}}{dt} &= \frac{S_{fa} - S_{fa}^*}{S_{fa}} \frac{dS_{fa}}{dt} + \frac{dI_{fa}}{dt}\\ &= \frac{S_{fa} - S_{fa}^*}{S_{fa}} \left(A_a - d_a S_{fa} - a S_{fa} - \beta_a S_{fa} I_{fa}\right)\\ &\quad + \beta_a S_{fa} I_{fa} - d_a I_{fa} - a I_{fa} - \alpha_a I_{fa}\\ &= \frac{S_{fa} - S_{fa}^*}{S_{fa}} \left(d_a \left(S_{fa}^* - S_{fa}\right) + a \left(S_{fa}^* - S_{fa}\right)\right)\\ &\quad + \beta_a S_{fa}^* I_{fa} - I_{fa}(d_a + a + \alpha_a)\\ &= -\frac{a + d_a}{S_{fa}} \left(S_{fa} - S_{fa}^*\right)^2\\ &\quad + (a + d_a + \alpha_a)(R_{01} - 1) I_{fa},\end{aligned} \quad (43)$$

and if $R_{01} < 1$, $dV_{21}/dt \le 0$, thus $\Omega_4 = \{(S_{fa}, I_{fa}) \in R_+^2 : dV_{21}/dt = 0\} = \{(S_{fa}, I_{fa}) \in R_+^2 : S_{fa} = S_{fa}^*, I_{fa} = 0\} = \{E_{fa}^*\}$. According to Lasalle's invariance principle [13, 14], E_{fa}^* is globally asymptotically stable.

Next, considering the poultry subsystem of markets with the avian components of farms already at the disease-free steady state

$$\begin{aligned}\frac{dS_{ma}}{dt} &= a S_{fa}^* - \beta_m S_{ma} I_{ma} - d_m S_{ma},\\ \frac{dI_{ma}}{dt} &= \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma}.\end{aligned} \quad (44)$$

We define a Lyapunov function

$$\begin{aligned}V_{22} &= S_{ma} - S_{ma}^* - S_{ma}^* \ln \frac{S_{ma}}{S_{ma}^*} + I_{ma} - I_{ma}^*\\ &\quad - I_{ma}^* \ln \frac{I_{ma}}{I_{ma}^*},\end{aligned} \quad (45)$$

and then the derivative of V_{22} along solutions of system (44) is

$$\begin{aligned}\frac{dV_{22}}{dt} &= \frac{S_{ma} - S_{ma}^*}{S_{ma}} \frac{dS_{ma}}{dt} + \frac{I_{ma} - I_{ma}^*}{I_{ma}} \frac{dI_{ma}}{dt}\\ &= \frac{S_{ma} - S_{ma}^*}{S_{ma}} \left(a S_{fa}^* - d_m S_{ma} - \beta_m S_{ma} I_{ma}\right)\\ &\quad + \frac{I_{ma} - I_{ma}^*}{I_{ma}} (\beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma})\\ &= \frac{S_{ma} - S_{ma}^*}{S_{ma}} (d_m (S_{ma}^* - S_{ma}) + \beta_m S_{ma}^* I_{ma}^*\\ &\quad - \beta_m S_{ma} I_{ma}) + \frac{I_{ma} - I_{ma}^*}{I_{ma}} (\beta_m S_{ma} I_{ma}\\ &\quad - \beta_m S_{ma}^* I_{ma}) = \left(2 - \frac{S_{ma}^*}{S_{ma}} - \frac{S_{ma}}{S_{ma}^*}\right)(d_m S_{ma}^*\\ &\quad + \beta_m S_{ma}^* I_{ma}^*) = \left(2 - \frac{S_{ma}^*}{S_{ma}} - \frac{S_{ma}}{S_{ma}^*}\right)(d_m S_{ma}^*\\ &\quad + S_{ma}^* d_m (R_{02} - 1)).\end{aligned} \quad (46)$$

Since $2 - S_{ma}^*/S_{ma} - S_{ma}/S_{ma}^* \le 0$, if $R_{02} > 1$, $dV_{22}/dt_{|(44)} \le 0$, thus, $\Omega_5 = \{(S_{ma}, I_{ma}) \in R_+^2 : dV_{22}/dt = 0\} = \{(S_{ma}, I_{ma}) \in R_+^2 : S_{ma} = S_{ma}^*, I_{ma} = I_{ma}^*\} = \{E_{ma}^*\}$. According to Lasalle's invariance principle [13, 14], E_{ma}^* is globally asymptotically stable.

Finally, considering the human subsystem with the avian components of markets already at the endemic steady state

$$\begin{aligned} \frac{dS_h}{dt} &= A_h - \beta_h S_h I^*_{ma} - d_h S_h, \\ \frac{dI_h}{dt} &= \beta_h S_h I^*_{ma} - d_h I_h - r I_h - \alpha_h I_h. \end{aligned} \tag{47}$$

We define a Lyapunov function

$$V_{23} = S_h - S^*_h - S^*_h \ln \frac{S_h}{S^*_h} + I_h - I^*_h - I^*_h \ln \frac{I_h}{I^*_h}, \tag{48}$$

and then the derivative of V_{23} along solutions of system (47) is

$$\begin{aligned} \frac{dV_{23}}{dt} &= \frac{S_h - S^*_h}{S_h}\frac{dS_h}{dt} + \frac{I_h - I^*_h}{I_h}\frac{dI_h}{dt} \\ &= \frac{S_h - S^*_h}{S_h}\left(A_h - \beta_h S_h I^*_{ma} - d_h S_h\right) \\ &\quad + \frac{I_h - I^*_h}{I_h}\left(\beta_h S_h I^*_{ma} - d_h I_h - r I_h - \alpha_h I_h\right) \\ &= \frac{S_h - S^*_h}{S_h}\left(d_h S^*_h - d_h S_h + \beta_h S^*_h I^*_{ma} - \beta_h S_h I^*_{ma}\right) \\ &\quad + \frac{I_h - I^*_h}{I_h}\left(\beta_h S_h I^*_{ma} - \beta_h S^*_h I^*_{ma}\frac{I_h}{I^*_h}\right) \\ &= d_h S^*_h\left(2 - \frac{S^*_h}{S_h} - \frac{S_h}{S^*_h}\right) \\ &\quad + \beta_h S^*_h I^*_{ma}\left(3 - \frac{S^*_h}{S_h} - \frac{I_h}{I^*_h} - \frac{S_h I^*_h}{S^*_h I_h}\right) \\ &= d_h S^*_h\left(2 - \frac{S^*_h}{S_h} - \frac{S_h}{S^*_h}\right) \\ &\quad + \beta_h S^*_h \frac{d_m\left(R_{02} - 1\right)}{\beta_m}\left(3 - \frac{S^*_h}{S_h} - \frac{I_h}{I^*_h} - \frac{S_h I^*_h}{S^*_h I_h}\right). \end{aligned} \tag{49}$$

Since $2 - S^*_h/S_h - S_h/S^*_h \le 0$, $3 - S^*_h/S_h - I_h/I^*_h - S_h I^*_h/S^*_h I_h \le 0$, if $R_{02} > 1$, $dV_{23}/dt_{|(47)} \le 0$, thus, $\Omega_6 = \{(S_h, I_h) \in R^2_+ : dV_{23}/dt = 0\} = \{(S_h, I_h) \in R^2_+ : S_h = S^*_h,\ I_h = I^*_h\} = \{E^*_h\}$. According to Lasalle's invariance principle [13, 14], E^*_h is globally asymptotically stable. In conclusion, if $R_{01} < 1$, $R_{02} > 1$, the boundary equilibrium E^* is globally asymptotically stable.

(2) If $R_{01} > 1$, $R_{02} > 1$, there is the endemic equilibrium E^{**}.

We firstly consider the poultry subsystem in farms and define a Lyapunov function

$$\begin{aligned} V_{31} &= S_{fa} - S^{**}_{fa} - S^{**}_{fa} \ln \frac{S_{fa}}{S^{**}_{fa}} + I_{fa} - I^{**}_{fa} \\ &\quad - I^{**}_{fa} \ln \frac{I_{fa}}{I^{**}_{fa}}, \end{aligned} \tag{50}$$

and then the derivative of V_{31} along solutions of system (25) is

$$\begin{aligned} \frac{dV_{31}}{dt} &= \frac{S_{fa} - S^{**}_{fa}}{S_{fa}}\frac{dS_{fa}}{dt} + \frac{I_{fa} - I^{**}_{fa}}{I_{fa}}\frac{dI_{fa}}{dt} \\ &= \frac{S_{fa} - S^{**}_{fa}}{S_{fa}}\left(A_a - d_a S_{fa} - a S_{fa} - \beta_a S_{fa} I_{fa}\right) \\ &\quad + \frac{I_{fa} - I^{**}_{fa}}{I_{fa}}\left(\beta_a S_{fa} I_{fa} - d_a I_{fa} - a I_{fa} - \alpha_a I_{fa}\right) \\ &= \frac{S_{fa} - S^{**}_{fa}}{S_{fa}}\left[\left(d_a + a\right)\left(S^{**}_{fa} - S_{fa}\right)\right] + \beta_a S^{**}_{fa} I^{**}_{fa} \\ &\quad - \beta_a S_{fa} I_{fa} - \beta_a S^{**}_{fa} I^{**}_{fa}\frac{S^{**}_{fa}}{S_{fa}} + \beta_a S^{**}_{fa} I_{fa} \\ &\quad + \beta_a S_{fa} I_{fa} - \beta_a S^{**}_{fa} I_{fa} - \beta_a S_{fa} I^{**}_{fa} + \beta_a S^{**}_{fa} I^{**}_{fa} \\ &= -\frac{a + d_a}{S_{fa}}\left(S_{fa} - S^{**}_{fa}\right)^2 \\ &\quad + \beta_a S^{**}_{fa} I^{**}_{fa}\left(2 - \frac{S^{**}_{fa}}{S_{fa}} - \frac{S_{fa}}{S^{**}_{fa}}\right) \\ &= -\frac{a + d_a}{S_{fa}}\left(S_{fa} - S^{**}_{fa}\right)^2 \\ &\quad + S^{**}_{fa}\left(a + d_a\right)\left(R_{01} - 1\right)\left(2 - \frac{S^{**}_{fa}}{S_{fa}} - \frac{S_{fa}}{S^{**}_{fa}}\right). \end{aligned} \tag{51}$$

Since $2 - S^{**}_{fa}/S_{fa} - S_{fa}/S^{**}_{fa} \le 0$, if $R_{01} > 1$, $dV_{31}/dt \le 0$, thus, $\Omega_7 = \{(S_{fa}, I_{fa}) \in R^2_+ : dV_{31}/dt = 0\} = \{(S_{fa}, I_{fa}) \in R^2_+ : S_{fa} = S^{**}_{fa},\ I_{fa} = I^{**}_{fa}\} = \{E^{**}_{fa}\}$. According to Lasalle's invariance principle [13, 14], E^{**}_{fa} is globally asymptotically stable.

Next, considering the poultry subsystem of markets with the avian components of farms already at the endemic steady state

$$\begin{aligned} \frac{dS_{ma}}{dt} &= a S^{**}_{fa} - \beta_m S_{ma} I_{ma} - d_m S_{ma}, \\ \frac{dI_{ma}}{dt} &= a I^{**}_{fa} + \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma}. \end{aligned} \tag{52}$$

We define a Lyapunov function

$$\begin{aligned} V_{32} &= S_{ma} - S^{**}_{ma} - S^{**}_{ma} \ln \frac{S_{ma}}{S^{**}_{ma}} + I_{ma} - I^{**}_{ma} \\ &\quad - I^{**}_{ma} \ln \frac{I_{ma}}{I^{**}_{ma}}, \end{aligned} \tag{53}$$

and then the derivative of V_{32} along solutions of system (52) is

$$\begin{aligned}
\frac{dV_{32}}{dt} &= \frac{S_{ma}-S_{ma}^{**}}{S_{ma}}\frac{dS_{ma}}{dt} + \frac{I_{ma}-I_{ma}^{**}}{I_{ma}}\frac{dI_{ma}}{dt} \\
&= \frac{S_{ma}-S_{ma}^{**}}{S_{ma}}\left(aS_{fa}^{**} - d_m S_{ma} - \beta_m S_{ma} I_{ma}\right) \\
&+ \frac{I_{ma}-I_{ma}^{**}}{I_{ma}}\left(aI_{fa}^{**} + \beta_m S_{ma} I_{ma} - d_m I_{ma} - \alpha_m I_{ma}\right) \\
&= \frac{S_{ma}-S_{ma}^{**}}{S_{ma}}\left[d_m\left(S_{ma}^{**} - S_{ma}\right) + \beta_m S_{ma}^{**} I_{ma}^{**}\right. \\
&\left. - \beta_m S_{ma} I_{ma}\right] + \frac{I_{ma}-I_{ma}^{**}}{I_{ma}}\left(aI_{fa}^{**} + \beta_m S_{ma} I_{ma}\right. \\
&\left. - \left(aI_{fa}^{**} + \beta_m S_{ma}^{**} I_{ma}^{**}\right)\frac{I_{ma}}{I_{ma}^{**}}\right) \\
&= \frac{a\left(a+d_a\right)\left(R_{01}-1\right)}{\beta_a}\left(2 - \frac{I_{ma}^{**}}{I_{ma}} - \frac{I_{ma}}{I_{ma}^{**}}\right) \\
&+ d_m S_{ma}^{**}\left(2 - \frac{S_{ma}^{**}}{S_{ma}} - \frac{S_{ma}}{S_{ma}^{**}}\right) + \beta_m S_{ma}^{**} I_{ma}^{**}\left(2\right. \\
&\left. - \frac{S_{ma}^{**}}{S_{ma}} - \frac{S_{ma}}{S_{ma}^{**}}\right).
\end{aligned} \tag{54}$$

Since $2 - S_{ma}^{**}/S_{ma} - S_{ma}/S_{ma}^{**} \le 0, 2 - I_{ma}^{**}/I_{ma} - I_{ma}/I_{ma}^{**} \le 0$, if $R_{01} > 1, R_{02} > 1$, $I_{ma}^{**} > 0$, $dV_{32}/dt_{|(52)} \le 0$, thus, $\Omega_8 = \{(S_{ma}, I_{ma}) \in R_+^2 : dV_{32}/dt = 0\} = \{(S_{ma}, I_{ma}) \in R_+^2 : S_{ma} = S_{ma}^{**}, I_{ma} = I_{ma}^{**}\} = \{E_{ma}^{**}\}$. According to Lasalle's invariance principle [13, 14], E_{ma}^{**} is globally asymptotically stable.

Finally, considering the human subsystem with the avian components already at the endemic steady state

$$\begin{aligned}
\frac{dS_h}{dt} &= A_h - \beta_h S_h I_{ma}^{**} - d_h S_h, \\
\frac{dI_h}{dt} &= \beta_h S_h I_{ma}^{**} - d_h I_h - rI_h - \alpha_h I_h.
\end{aligned} \tag{55}$$

We define a Lyapunov function

$$V_{33} = S_h - S_h^{**} - S_h^{**}\ln\frac{S_h}{S_h^{**}} + I_h - I_h^{**} - I_h^{**}\ln\frac{I_h}{I_h^{**}}, \tag{56}$$

and then the derivative of V_{33} along solutions of system (55) is

$$\begin{aligned}
\frac{dV_{33}}{dt} &= \frac{S_h - S_h^{**}}{S_h}\frac{dS_h}{dt} + \frac{I_h - I_h^{**}}{I_h}\frac{dI_h}{dt} \\
&= \frac{S_h - S_h^{**}}{S_h}\left(A_h - \beta_h S_h I_{ma}^{**} - d_h S_h\right) \\
&+ \frac{I_h - I_h^{**}}{I_h}\left(\beta_h S_h I_{ma}^{**} - d_h I_h - rI_h - \alpha_h I_h\right) \\
&= d_h\left(S_h^{**} - S_h\right) + \beta_h S_h^{**} I_{ma}^{**} - \beta_h S_h I_{ma}^{**} \\
&- \beta_h S_h^{**} I_{ma}^{**}\frac{S_h^{**}}{S_h} - d_h S_h^{**}\frac{S_h^{**}}{S_h} + \beta_h S_h^{**} I_{ma}^{**} \\
&+ d_h S_h^{**} + \beta_h S_h I_{ma}^{**} - \beta_h S_h^{**} I_{ma}^{**}\frac{I_h}{I_h^{**}} \\
&+ \beta_h S_h^{**} I_{ma}^{**} - \beta_h S_h I_{ma}^{**}\frac{I_h^{**}}{I_h} \\
&= d_h S_h^{**}\left(2 - \frac{S_h^{**}}{S_h} - \frac{S_h}{S_h^{**}}\right) \\
&+ \beta_h S_h^{**} I_{ma}^{**}\left(3 - \frac{S_h^{**}}{S_h} - \frac{I_h}{I_h^{**}} - \frac{S_h I_h^{**}}{S_h^{**} I_h}\right).
\end{aligned} \tag{57}$$

Since $2 - S_h^{**}/S_h - S_h/S_h^{**} \le 0$, $3 - S_h^{**}/S_h - I_h/I_h^{**} - S_h I_h^{**}/S_h^{**} I_h \le 0$, if $R_{02} > 1$, $I_{ma}^{**} > 0$, $dV_{33}/dt_{|(55)} \le 0$, thus, $\Omega_9 = \{(S_h, I_h) \in R_+^2 : dV_{33}/dt = 0\} = \{(S_h, I_h) \in R_+^2 : S_h = S_h^{**}, I_h = I_h^{**}\} = \{E_h^{**}\}$. According to Lasalle's invariance principle [13, 14], E_h^{**} is globally asymptotically stable. Hence, the following theorem can be obtained.

Theorem 5. *For system (1), if $R_{01} < 1$, $R_{02} > 1$, the boundary equilibrium E^* is globally asymptotically stable; if $R_{01} > 1$, $R_{02} > 1$, the endemic equilibrium E^{**} is globally asymptotically stable.*

Remark 6. Stability of the equilibrium depends on the Lyapunov functions. The quadratic form of Lyapunov functions is usually used in most references [15, 16]. Let us take the function in this paper

$$f(x) = 1 - x + \ln x, \quad \forall x > 0, \tag{58}$$

we have

$$\begin{aligned}
f(1) &= 0, \\
f'(x) &= \frac{1}{x} - 1;
\end{aligned} \tag{59}$$

thus, $f(x) = 1 - x + \ln x \le 0$, and the equality holds only when $x = 1$. Let $x = S_{fa}/S_{fa}^0 > 0$; then $S_{fa} - S_{fa}^0 - S_{fa}^0 \ln(S_{fa}/S_{fa}^0) \ge 0$. Hence, V_{11} is a Lyapunov function. In a similar way, $V_{12}, V_{13}, V_{21}, V_{22}, V_{23}, V_{31}, V_{32}, V_{33}$ are also Lyapunov functions.

5. Numerical Simulations

In this section, the following parameter values are taken as some examples to simulate the stability of the disease-free equilibrium, the boundary equilibrium, and the endemic equilibrium of system (1), and the time-series diagram is given. At last, when β_h has different parameter values, the time-variation diagram of I_h is given.

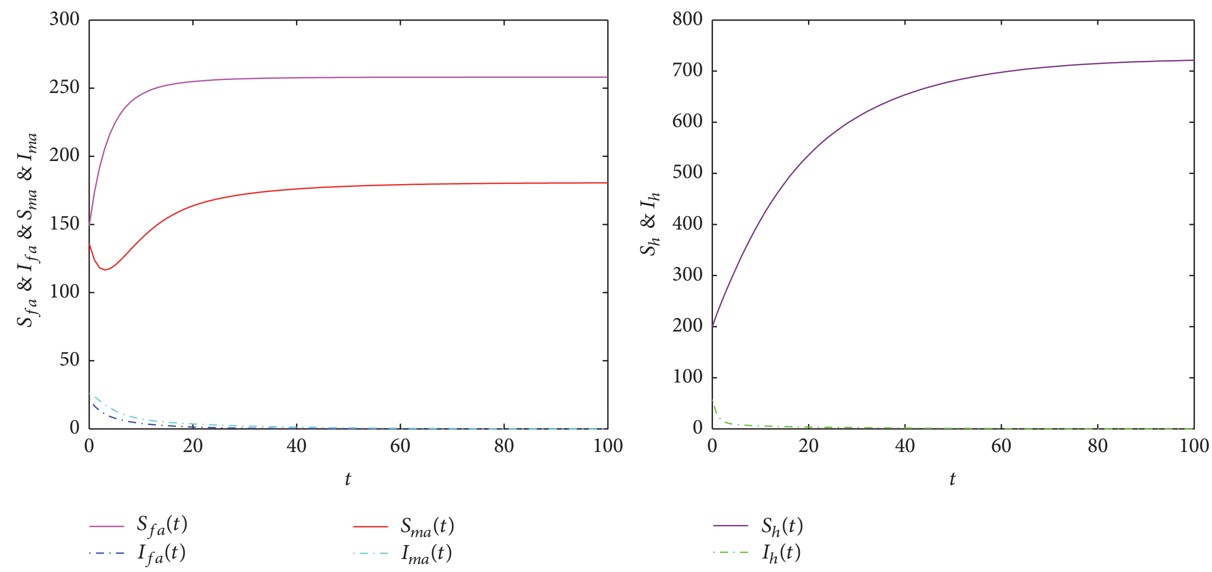

FIGURE 2: If $R_0 = 0.88680353 < 1$, the time-variation diagram of system (25) state variables is shown.

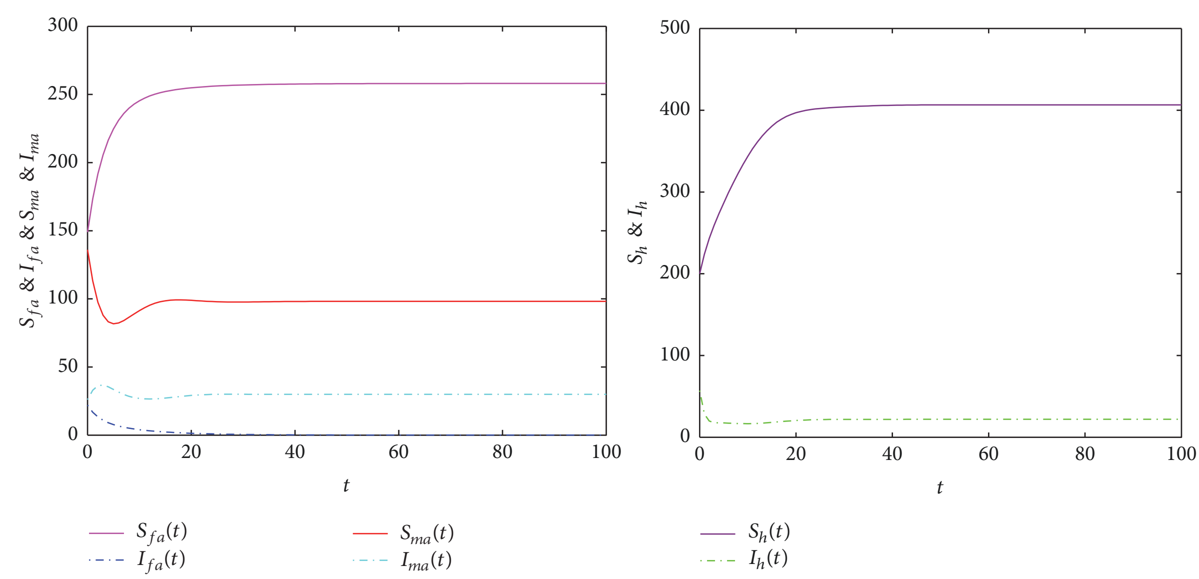

FIGURE 3: If $R_{01} = 0.82949309 < 1$ and $R_{02} = 1.83929618 > 1$, the time-variation diagram of system (25) state variables is shown.

Example 1. Taking parameters $A_a = 80$, $\beta_a = 0.0018$, $d_a = 0.17$, $\alpha_a = 0.25$, $a = 0.14$, $\beta_m = 0.0027$, $d_m = 0.2$, $\alpha_m = 0.35$, $A_h = 50$, $\beta_h = 0.0018$, $d_h = 0.069$, $\alpha_h = 0.63$, and $r = 0.301$, Figure 2 shows the time-variation diagram of system (25) state variables. It is found that if $R_0 < 1$, the disease-free equilibrium E^0 is globally asymptotically stable.

Example 2. Taking parameters $A_a = 80$, $\beta_a = 0.0018$, $d_a = 0.17$, $\alpha_a = 0.25$, $a = 0.14$, $\beta_m = 0.0056$, $d_m = 0.2$, $\alpha_m = 0.35$, $A_h = 50$, $\beta_h = 0.0018$, $d_h = 0.069$, $\alpha_h = 0.63$, and $r = 0.301$, as shown in Figure 3, it is found that if $R_{01} < 1$, $R_{02} > 1$, the boundary equilibrium E^* is globally asymptotically stable.

Example 3. Taking parameters $A_a = 80$, $\beta_a = 0.0045$, $d_a = 0.17$, $\alpha_a = 0.25$, $a = 0.14$, $\beta_m = 0.0056$, $d_m = 0.2$, $\alpha_m = 0.35$, $A_h = 50$, $\beta_h = 0.0018$, $d_h = 0.069$, $\alpha_h = 0.63$, and $r = 0.301$, Figure 4 shows the time-variation diagram of system (25) state variables. It is found that if $R_{01} > 1$, $R_{02} > 1$, the endemic equilibrium E^{**} is globally asymptotically stable.

Example 4. Taking parameters $A_a = 80$, $\beta_a = 0.0045$, $d_a = 0.17$, $\alpha_a = 0.25$, $a = 0.14$, $\beta_m = 0.0056$, $d_m = 0.2$, $\alpha_m = 0.35$, $A_h = 50$, $d_h = 0.069$, $\alpha_h = 0.63$, and $r = 0.301$ and letting $\beta_h = 0.00005, 0.0018, 0.02$, Figure 5 shows the curve-trend diagram of I_h with time. It is found that I_h will increase with the increase of β_h.

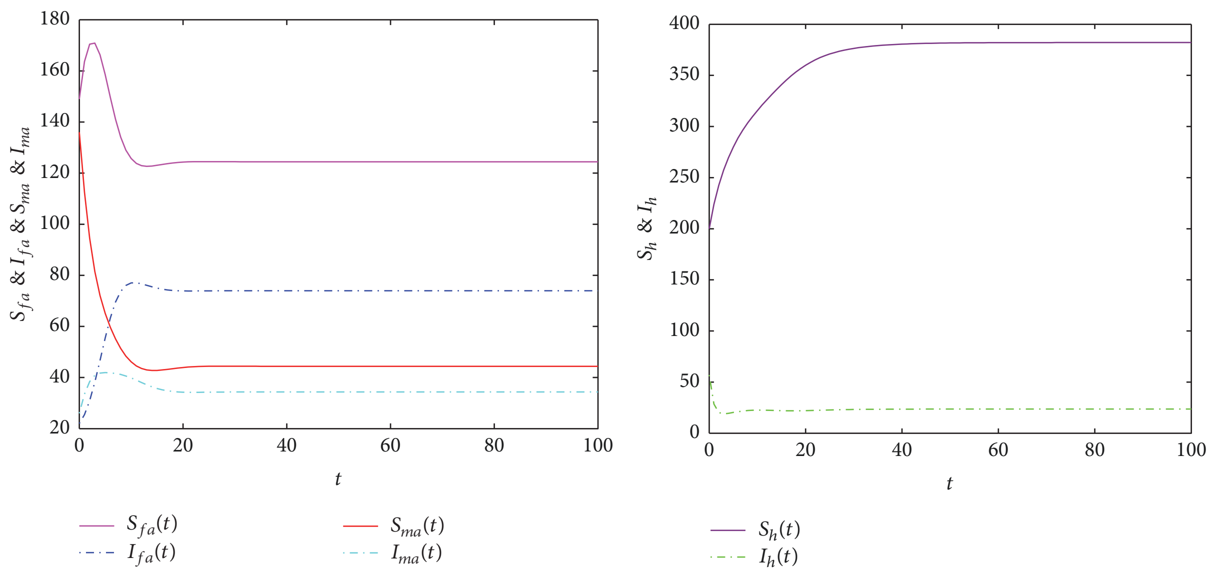

FIGURE 4: If $R_{01} = 2.07373271 > 1$ and $R_{02} = 1.83929618 > 1$, the time-variation diagram of system (25) state variables is shown.

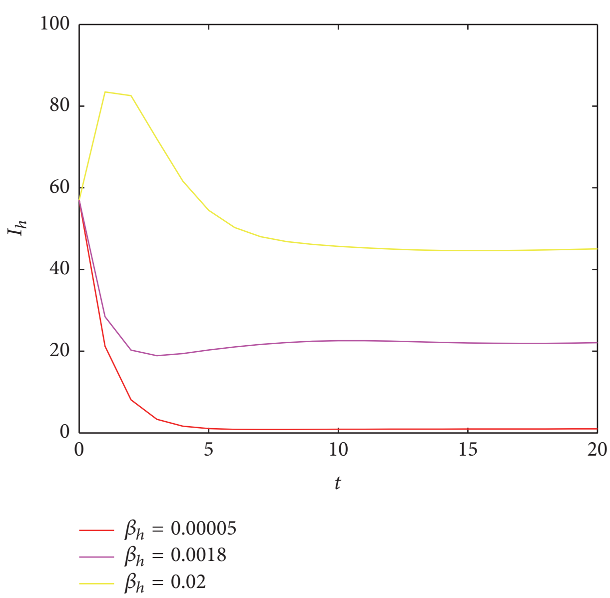

FIGURE 5: The curve-trend diagram of I_h with time, when β_h has different parameter values.

6. Discussion

Avian influenza infectious diseases caused by influenza virus can quickly spread in areas such as farms and markets. In this paper, an SI-SI-SIR dynamic model of avian influenza A(H7N9) is established by combining human and poultry. We get the basic reproduction number R_0; it is the threshold which is endemic or not. If $R_0 < 1$, there is only the disease-free equilibrium E^0, and it is globally asymptotically stable, which implies that the disease dies out. If $R_{01} < 1$, $R_{02} > 1$, at this time $R_0 > 1$, there is the unique boundary equilibrium E^*, which is globally asymptotically stable; namely, the disease will be sustained and lead to epidemic disease eventually. If $R_{01} > 1$, $R_{02} < 1$, at this time $R_0 > 1$, there is no positive equilibrium; that is, the disease spreads in farms and it does not spread in markets, people will not be infected by the virus, and the disease will not be popular. If $R_{01} > 1$, $R_{02} > 1$, at this time $R_0 > 1$, there is the endemic equilibrium E^{**}, which is globally asymptotically stable; that is, the disease will spread. From the above analysis, we can see that if artificial measures are taken to reduce the basic reproduction number to small enough value in the transmission system of avian influenza A(H7N9), the global stability point of the propagation dynamics process exists. Raise α_a and α_m by killing infected poultry or reduce β_a, β_m, and β_h by closing farms and markets. Both of the two methods can reduce R_0 or I_h, so as to control the occurrence and development of diseases.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no competing financial interests.

Acknowledgments

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