Abstract—Quickest detection is applied in spectrum sensing in
cognitive radio systems when multiple secondary users collabor-
ate with limited communication time slots. When transmissions
are not coordinated to avoid confliction, random broadcast is
used to exchange information. A necessary condition for optimal
broadcast probability, as a function of log likelihood ratio of local
observation, is obtained using variational analysis. To alleviate the
difficulty of computing optimal broadcast probability, a simple
threshold broadcast scheme is proposed. Simulation shows that
the proposed threshold broadcast scheme can achieve substantial
performance gain (less than 65% in detection delay for the same
false alarm rate) over schemes of random broadcast without
regulation and single-user spectrum sensing.

I. INTRODUCTION

Spectrum sensing is a key issue in cognitive radio systems
[9] which are becoming the focus of study in the commu-
nity of wireless communications. In cognitive radio systems,
secondary users (without license) must detect the emergence
of primary users and then quit the corresponding licensed
frequency band as quickly as possible. An oversensitive sec-
tary user may unnecessarily interrupt its own communi-
ration while a dull spectrum sensor may cause substantial
interference to primary users.

A novel framework of spectrum sensing using the theory
of quickest detection (we coin it quickest spectrum sensing)
[1] [8] [11] has been exploited in recent years [2] [7] [12].
Essentially, quickest detection is used to detect the change
in the distribution of observations, thus being useful in many
areas like financial analysis, econometrics, network security,
et al. Since the spectrum sensing is to detect the change
of spectrum occupancy, it is natural to introduce quickest
detection into the task of spectrum sensing. In [2] and [7],
cumulative sum (CUSUM) test, originally proposed in [10]
in 1954, is applied to detect the emergence of primary users.
Bayesian quickest detection is applied in [12] based on the as-
sumption of available information about primary user activity.
Simulation results in these publications show that the quickest
detection can improve the agility of the spectrum sensing.

However, the above papers are all based on single-user
observations. Due to many random factors in wireless en-
vironments, e.g. fast fading and shadowing, a single-user
spectrum sensing may not be reliable. Therefore, collabora-
tive spectrum sensing has attracted intensive studies, which
allows the collaboration among multiple secondary users to
enhance the robustness of spectrum sensing [4] [5]. Motivated
by the collaborative spectrum sensing, collaborative quickest
detection is also studied in [6], in which communication
delay is considered. However, most studies assume perfect
coordination for the information exchange, e.g., in [6], a
spanning tree is assumed to be generated for information
exchange and the communication is assumed to be noiseless
and conflictionless. Such a perfect coordination may not be
reasonable in practical systems, especially in mobile systems,
since organizing such a coordination may bring substantial
overhead and the negotiation may not achieve consensus when
the network topology is rapidly changing. Therefore, there
is a pressing need to study the cooperation under imperfect
coordination.

In this paper, we study the collaborative quickest spectrum
sensing without communication coordination. We consider
the case in which there are only limited time slots for the
secondary users to exchange information and there is no
coordination to avoid transmission collisions. The cooperation
of spectrum sensing is based on broadcasting the local obser-
vation or statistics by randomly choosing one time slot. The
key issue is to determine whether to broadcast based on the
current observation and population nearby. Using asymptotic
analysis (both the number of secondary users and the number
of available time slots tend to infinity) and variational analysis,
we obtained a necessary condition for the optimal broadcast
probability (Prop. 1). Since it is prohibitively difficult to com-
pute the optimal broadcast probability numerically, we pro-
posed a simple threshold broadcast scheme. Numerical results
show that the threshold broadcast scheme can substantially
improve the performance of spectrum sensing (reducing at
least 40% of the detection delay), compared with the schemes
of random broadcast without regulation (always broadcast) and
single-user spectrum sensing.

The remainder of this paper is organized as follows. The
system model and known results about quickest detection are
introduced in Section II. In Section III, the necessary condition
for optimal broadcast probability in collaborative quickest
detection is derived and a simple threshold broadcast scheme
is proposed. Numerical results and conclusions are provided
in Sections IV and V, respectively.

II. SYSTEM MODEL AND KNOWN RESULTS

In this section, we introduce the system model and known
results about single-user quickest spectrum sensing.

A. System Model

Suppose that there are $N$ secondary users in a cognitive
radio system. The observations\(^1\), denoted by $X_n(t)$ for user

\(^1\)We consider a generic observation, which could be received power or
cyclostationary features
n at sample period \( t \), are mutually independent for different secondary users and different sample periods. For simplicity, we assume that the distributions of observations for different secondary users are the same, which is reasonable if all the secondary users are geographically close to each other. We assume that there is no primary user at the beginning and then primary user(s) emerge at an unknown time. Therefore, the distribution of observations satisfies hypothesis \( H_0 \) at the beginning and changes to \( H_1 \) at the unknown time. We further assume that the probability density functions for both hypotheses \( H_0 \) and \( H_1 \) exist and are denoted by \( f_0 \) and \( f_1 \), respectively. The task of these secondary users is to detect the emergence of primary users as quickly as possible, i.e., incurring minimal detection delay while keeping a reasonable false alarm rate.

We suppose that each secondary user is equipped with a wireless transceiver. Therefore, the users can broadcast their current observations (or sufficient statistics) such that the performance can be improved. The timing structure of the spectrum sensing and broadcast is illustrated in Fig. II-A, where there are \( M \) time slots \(^2\) for broadcast following each spectrum sensing period (\( M = 3 \) in the illustration). We define \( \alpha \equiv \frac{N}{\gamma} \), which represents the extent of congestion in the communication time slots. One secondary user can finish broadcasting within one time slot. For simplicity, we assume that the secondary user obtains one observation during the spectrum sensing period. It is easy to extend to the general case of multiple observations per spectrum sensing period.

For simplicity of analysis, the following assumptions are made for the broadcast:

- The secondary users are close to each other such that a broadcast can cover all users. Only one broadcast is allowed at each time slot. If more than one broadcasts are within one time slot, they will collide with each other and thus cannot be decoded.
- The quantization error and possible decoding error are ignored by assuming sufficient communication bandwidth.
- There is no centralized controller or consensus mechanism to coordinate the broadcast of the secondary users. Therefore, the secondary users have to broadcast in a random manner, i.e., choosing a random time slot for broadcast.
- We do not consider the per-frame data communication explicitly. For practical systems, the log likelihood ratios during the communication period are all set to 0 since the secondary user stops the spectrum sensing.

\(^2\)They can also be \( M \) orthogonal channels in the frequency domain.

B. Known Results for Single-user Spectrum Sensing

For a single secondary user, CUSUM test \([10]\) can be used to detect the emergence of primary users. The corresponding stopping time of claiming the change is given by

\[
T^* = \inf \left\{ t \mid m(t) \geq \gamma \right\},
\]

where \( \gamma \) is a predetermined threshold and metric \( m(t) \) is defined as \( m(t) = \max (m(t-1) + l(t), 0) \) and \( l(t) \) is log likelihood ratio of observations, which is defined as \( l(t) \triangleq \log \left( \frac{f_1(X(t))}{f_0(X(t))} \right) \). Intuitively, \( T^* \) is the first time slot in which the metric \( m(t) \) passes the threshold \( \gamma \).

We use two metrics, namely average detection delay, denoted by \( D \), and false alarm rate, denoted by \( F \), which have been used in many studies \([11]\), to measure the performance of quickest spectrum sensing. These metrics are defined as

\[
D = \text{esssup} \left( E [(T^* - T)^+] \mid \mathcal{F}_{T-1} \right),
\]

\[
F = P(T^* < T),
\]

where \( T \) is the change time and \( \mathcal{F}_{T-1} \) is the filtration, namely the smallest \( \sigma \)-field with respect to observation history \( X(0), \ldots, X(t-1) \) (here we drop the subscript of secondary users). The esssup means the worst case of detection delay. Obviously, we desire small \( D \) and \( F \). The details can be found in \([1] [11]\).

When \( \gamma \to \infty \), Brownian motion approximation based asymptotic analysis shows that, for the CUSUM test, the metrics can be approximated by \([1] [11]\)

\[
D \approx \frac{\gamma}{I_1},
\]

\[
\log F \approx \frac{2\gamma I_0}{V_0},
\]

where, \( \forall i = 0, 1 \), \( I_i \) is the expectation (variance) of log likelihood ratio under hypothesis \( i \), i.e.

\[
I_i = E_i \left[ \frac{f_1(X)}{f_0(X)} \right],
\]

\[
V_i = E_i \left[ \left( \frac{f_1(X)}{f_0(X)} \right)^2 \right] - I_i^2.
\]

To be independent of the threshold \( \gamma \), we use the following quantity as the asymptotic performance metric of quickest spectrum sensing:

\[
\mathcal{M} \triangleq \frac{|\log F|}{D} \approx \frac{2I_0I_1}{V_0},
\]

Obviously, larger \( \mathcal{M} \) implies better performance (notice that \( \log F \) is negative).

III. Random Broadcast

In this section, we discuss random broadcast for collaborative quickest spectrum sensing, where the randomness is from both the decision of whether to broadcast and the time slot selection. We first derive a necessary condition for the optimal broadcast probability and then propose a simple threshold broadcast scheme.
A. Random Broadcast Scheme

Since there is no centralized controller or consensus mechanism for coordination, the only approach for the secondary users is to broadcast their information by selecting a random time slot.

The procedure of random broadcast is given below.

1) At the end of the spectrum sensing period, secondary user \( n \) first determines whether to broadcast, based on the log likelihood ratio of \( X_n(t) \). We denote by \( P_B(l) \), called broadcast probability, the probability to broadcast when the log likelihood ratio is \( l \).

2) If a secondary user decides to broadcast, it broadcasts \( l_n(t) \) in a randomly selected time slot. If there is no other secondary user broadcasting in the same time slot, all other secondary users will receive the information; if there is a collision, the colliding packets will be dropped.

3) At the end of time slot \( M \), each secondary user collects the information received during this period and update the metric of CUSUM test in (1)\(^3\).

Once \( N, M, f_1 \) and \( f_0 \) are all known, we can optimize \( P_B(l) \) to maximize the asymptotic performance metric \( M \) in (8).

B. Asymptotic Analysis

We assume that the broadcast probability \( P_B(l) \) has been fixed and then analyze the asymptotic performance metric \( M \), for which we need to obtain expressions of the quantities \( I_1, I_0 \) and \( V_0 \). Then, we perturb \( P_B(l) \) and evaluate the corresponding change in \( M \).

First, we analyze \( I_1 \). On defining probability (intuitively, it means the probability that a secondary user broadcasts when the true distribution is \( H_1 \))

\[
p_{i} = \int_{-\infty}^{\infty} P_B(l)f_i(l)dl, \quad i = 0, 1, \tag{9}
\]

and events \( S_n \triangleq \{ \text{success in receiving } l_n \} \) and \( B_n \triangleq \{ \text{user } n \text{ broadcasts} \} \), the average of log likelihood ratios that can be successfully received when the distribution is \( H_1 \) is given by \((I(S_n)) \) is the characteristic function of event \( S_n \)\(^4\)

\[
\frac{1}{N}E \left[ \sum_{n=1}^{N} I_n(S_n) \right] = \frac{1}{N} \sum_{n=1}^{N} E[I_n(S_n)] \leq \frac{1}{N} \sum_{n=1}^{N} \int_{-\infty}^{\infty} P_B(l)P(S_n|B_n)f_i(l)dl \\
= \frac{1}{N} \sum_{n=1}^{N} \int_{-\infty}^{\infty} P_B(l) \prod_{k=1,k\neq n}^{N} \left( 1 - \frac{P_1}{M} \right) f_i(l)dl \\
\to \exp(-\alpha p_0) \int_{-\infty}^{\infty} lP_B(l)f_0(l)dl. \tag{10}
\]

\(^{3}\)Note that the secondary user transmits in a later time slot may obtain more samples and thus can update its broadcast. This issue has been addressed in [3] but is out of the scope of this paper.

\(^{4}\)Note that secondary user \( n \) can always use its own observation; however, in the asymptotic case, the single observation is negligible since the received information increases in \( N \); therefore, we can assume that all observations of a secondary user are from received broadcasts.

Fig. 2: Performance metric \( M \) versus \( N \) when \( \alpha = 0.5 \) and 1.

As \( M, N \to \infty \) and \( \frac{N}{M} = \alpha \), we have \((1 - \frac{P_1}{M})^N \to \exp(-\alpha p_0) \). Note that \( P(S_n) \) is equal to the probability that all other secondary users do not broadcast in the same time slot, thus being equal to \( \prod_{k=1,k\neq n}^{N} (1 - \frac{P_1}{M}) \). Therefore, \( I_1 \), normalized by \( N \), converges to (10).

Then, we calculate \( I_0 \) and \( V_0 \). Similarly to (10), \( I_0 \), normalized by \( N \), is asymptotically equal to

\[
\frac{1}{N}E_0 \left[ \sum_{n=1}^{N} I_n(S_n) \right] \to \exp(-\alpha p_0) \int_{-\infty}^{\infty} l^2 P_B(l)f_0(l)dl. \tag{11}
\]

With more complicated calculation (the details are omitted due to limited space), we obtain \( V_0 \) which is given by

\[
\frac{1}{N}V_0 \to \exp(-\alpha p_0) \int_{-\infty}^{\infty} l^2 P_B(l)f_0(l)dl. \tag{12}
\]

Therefore, the metric \( M \), normalized by \( N \), is asymptotically given by

\[
\lim_{N \to \infty} \frac{M}{N} = \frac{2P_1E_0}{V_0}, \tag{13}
\]

where

\[
\begin{align*}
P_1 &= \exp(-\alpha \int_{-\infty}^{\infty} P_B(l)f_1(l)dl) \\
E_1 &= \int_{-\infty}^{\infty} lP_B(l)f_1(l)dl \\
E_0 &= -\int_{-\infty}^{\infty} l^2 P_B(l)f_0(l)dl \\
V_0 &= \int_{-\infty}^{\infty} l^2 P_B(l)f_0(l)dl
\end{align*}
\]

Note that \( M \) increases linearly with respect to \( N \), as \( N \to \infty \), and the slope represents the performance gain from collaboration. This can be validated by simulation with finite \( M \) and \( N \), whose results are shown in Fig. 2 (\( M \) is shown in the vertical axis and we set \( P_B(l) = 1 \)). Then, we can optimize the broadcast probability by maximizing the increasing slope of \( M \).

C. Optimization on Broadcast Probability

The broadcast probability is optimized to maximize the increasing slope of performance metric \( M \), defined as \( S = \)
\[
\lim_{N \to \infty} \frac{M}{N}, \quad \text{i.e.} \\
\max_{P_B} S(P_B) \\
s.t. \quad 0 \leq P_B(l) \leq 1, \quad \forall l \in \mathbb{R}, \\
\int_{-\infty}^{\infty} lP_B(l)f_1(l)dl > 0, \\
\int_{-\infty}^{\infty} lP_B(l)f_0(l)dl < 0. \\
\]  

(15)

For an arbitrary fixed broadcast probability \( P_B \), we perturb it by a sufficiently small \( \delta P_B \). Then the perturbation on \( S \) is given by (the detailed calculation is omitted due to limited space)

\[
\delta M = \frac{\int_{-\infty}^{\infty} g(l)\delta P_B(l)dl}{V_B^2}, \\
\]  

(16)

where

\[
g(l) = P_1E_0V_0f_1(l) - P_1E_1V_0f_0(l) - \alpha P_1E_0E_1V_0f_1(l) - P_1E_0E_1V_0f_0(l)^2 + \alpha P_1E_0E_1V_1f_0(l) \\
\]  

(17)

Then, a necessary condition for an optimal \( P_B \) satisfies

\[
\int_{-\infty}^{\infty} g(l)\delta P_B(l)dl \leq 0, \quad \forall \delta P_B. \\
\]  

(18)

For a fixed \( P_B(l) \), we can set \( \delta P_B(l) \) in the following situations to make the left hand side of (18) larger than zero if applicable (note that all the changes \( \delta P_B(l) \) are sufficiently small).

- When \( P_B(l) \neq 1 \) or 0, if \( g(l) > 0 \), we can set \( \delta P_B(l) \) to be positive; if \( g(l) < 0 \), we can set \( \delta P_B(l) \) to be negative.
- When \( P_B(l) = 1 \), if \( g(l) < 0 \), we can set \( \delta P_B(l) \) to be negative.
- When \( P_B(l) = 0 \), if \( g(l) > 0 \), we can set \( \delta P_B(l) \) to be positive.

Therefore, we obtain the following proposition which states a necessary condition for the optimal broadcast probability (the rigorous proof via variational calculus is omitted due to limited space).

**Proposition 1:** The optimal broadcast probability \( P_B(l) \) should satisfy\(^5\)

\[
P_B(l) = \begin{cases} 
0, & \text{if } g(l) < 0 \\
1, & \text{if } g(l) > 0 
\end{cases} \\
\]  

(19)

\(^5\)Here we ignore the case \( g(l) = 0 \) which is of zero measure.

Intuitively, the above conclusions mean that when the belief of change is large (small), each secondary user should broadcast with large (small) probability. Therefore, we propose the following rule of broadcast:

\[
P_B(l) = \begin{cases} 
1, & \text{if } l > L_{cut} \\
0, & \text{if } l \leq L_{cut} 
\end{cases} \\
\]  

(20)

where \( L_{cut} \) is a predetermined cutoff threshold and we coin the rule as **Threshold Broadcast**.

### IV. Numerical Results

In this section, we use numerical simulation to demonstrate the performance gain of threshold broadcast over the cases of single-user spectrum sensing \( (L_{cut} = \infty) \) and collaborative spectrum sensing without broadcast regulation \( (L_{cut} = -\infty) \). Note that we omit the performance result for \( M \) and \( S \) and display only false alarm rates and detection delays, due to limited space\(^6\).

We assume that the two distribution hypotheses \( H_0 \) and \( H_1 \) are Gaussian distributions \( N(1, 1) \) and \( N(-1, 1) \) (this is reasonable if the observations are sensed power in dB scale). Each statistic is obtained using 5000 realizations of the random time of primary user emergence.

Fig. 3 shows the receiver operation characteristic (ROC) curves when \( N = 20 \) and \( M = 5 \). The performance measures are false alarm rate versus average detection delay. We tested the cases of \( L_{cut} = -1000, 0, 4, 10, 1000 \). It is easy to observe that, without broadcast regulation, the collaborative spectrum sensing achieves only marginally better performance than the single-user spectrum sensing. When the cutoff threshold is properly chosen (e.g. \( L_{cut} = 4 \)), the performance can be substantially improved since collision can be considerably avoided and plenty of information can be exchanged. We also observe that the performance could be seriously impaired if the threshold is not well chosen (e.g. \( L_{cut} = 0 \)).

Fig. 4 shows the ROC curves in which we replace the average detection delay with the 90% percentile of detection delays (sorted in an ascending order) since the tail of detection delay could incur much more damage (sufficiently small detection delay can be completely tolerated by primary users). Although good average detection delay in (2) does not necessarily imply good performance of the tail of detection delay, we observe that the threshold broadcast can also substantially reduce the tail of detection delay.

Figures 5 and 6 show the performance gains (both difference and ratio) of the threshold broadcast with different \( M \). In both figures, \( D_L \) and \( d_L \) denote the average detection delay and the 90% percentile of detection delay, respectively, when the threshold of broadcast, \( L \), is used. The false alarm rate is fixed at 5% by choosing a proper detection threshold \( \gamma \). The optimal thresholds for different \( M \) are obtained from exhaustive search and the corresponding quantities are labeled with subscript ‘opt’. We observe that, averagely, the detection delay (both the average and tail) of threshold broadcast is less than 70% (or 60%) of that of broadcast without regulation (or  

\(^6\)Moreover, the false alarm rates and detection delays are our direct concern for the spectrum sensing.
information exchange). (more communication time slots mean more capability of M gain over the single-user spectrum sensing increases in cast confliction becomes less frequent) while the performance 90% percentile detection delays.

Fig. 5: Difference of average detection delays, as well as the 90% percentile detection delays.

![Figure 5](image1)

Fig. 3: ROC (average delay vs. false alarm rate) curves when N = 20 and M = 5.

Fig. 4: ROC (90% percentile of delay vs. false alarm rate) curves when N = 20 and M = 5.

![Figure 4](image2)

single-user spectrum sensing). Moreover, we observe that the performance gain of threshold broadcast over the broadcast without regulation decreases as M increases (since the broadcast confliction becomes less frequent) while the performance gain over the single-user spectrum sensing increases in M (more communication time slots mean more capability of information exchange).

V. CONCLUSIONS

We have discussed collaborative quickest spectrum sensing via random broadcast in multiple communication channels. We assume that there is no controller or mechanism to coordinate the broadcasts, thus making broadcast collisions possible. Therefore, we consider the strategy that the probability of broadcast is a function of observed log likelihood ratio and have derived a necessary condition for the optimal function of broadcast probability. To alleviate the difficulty of computing the optimal broadcast probability, we proposed a simple scheme of threshold broadcast, in which a secondary user broadcasts only when the log likelihood ratio is larger than a certain cutoff threshold. Numerical results show that this simple strategy can substantially improve the performance of spectrum sensing, compared with the strategies of random broadcast without regulation and single-user spectrum sensing.

REFERENCES