preprint hep-ph96xxxxx
Preprint DFPD 96/TH/61

The Paths of Unification In The GUST With The $G \times G$ Gauge Groups of $E(8) \times E(8)$

A. Maslikov [b] **, I. Naumov** [b]**, G. Volkov** [a,b]

[a] INFN Sezione di Padova and Dipartimento di Fisica
Università di Padova,
Via Marzolo 8, 35100 Padua, Italy

[b] Institute for High Energy Physics
142284 Protvino, Moscow Region, Russia

ABSTRACT

In the framework of the four dimensional heterotic superstring with free fermions we discuss the rank eight and/or sixteen Grand Unified String Theories (GUST) which contain the $SU(3)_H$-gauge family symmetry. We explicitly investigate the paths of the unification in the GUST with gauge symmetry $G \times G = [SU(5) \times U(1) \times (SU(3) \times U(1))_H]^{\otimes 2}$. We show that the GUSTs with the $G \times G$ gauge group allow to make the scale of unification to be consistent with the string scale $M_{SU} \sim g_{string} \cdot 5 \cdot 10^{17} GeV$.

1 Introduction

For a couple of years superstring theories, and particularly the heterotic string theory [1, 2], have provided an efficient way to construct the Grand Unified Superstring Theories ($GUST$) of all known interactions, despite the fact that it is still difficult to construct unique and fully realistic low energy models resulting after decoupling of massive string modes.

In the fermionic formulation of the four-dimensional heterotic string theory [3, 4] in addition to the two transverse bosonic coordinates X_μ ,$\bar{X}_\mu$ and their left-moving superpartners ψ_μ, the internal sector $\mathcal{M}_{c_L;c_R}$ contains 44 right-moving ($c_R = 22$) and 18 left-moving ($c_L = 9$) real fermions (each real world-sheet fermion has $c_f = 1/2$). String theories possess infinite dimensional symmetries that place many specific constraints on the theory spectrum. These symmetries origin from 2 dimensional conformal invariance, modular invariance, and Virasoro and Kac-Moody algebras. Because of the presence of the affine Kac-Moody algebra (KMA) $\hat{g}$ (which is a 2-dimensional manifestation of gauge symmetries of the string itself) on the world sheet, string constructions yield definite predictions concerning representation of the symmetry group, especially for the rank 8 and greater, that can be used for low energy models building.

There are not so many GUSTs describing the observable sector of Standard Model. They are well known: the $SU(3) \times SU(2) \times U(1)^n \times G_{hid}$ gauge group, the Pati-Salam ($SU(4) \times SU(2) \times SU(2) \times U(1) \times G_{hid}$) gauge group, the flipped $SU(5) \times U(1) \times G_{hid}$ gauge group and $SO(10) \times G_{hid}$ gauge group [5, 6]. For the heterotic 10-dimensional string the groups $E_8 \otimes E_8$ and $spin(32)/Z_2$ are characteristic. Hence it is interesting to consider GUSTs in four dimension based on its various rank 8 and 16 subgroup [7]. As the GUSTs originating from level one Kac-Moody algebra (KMA) contain only low-dimensional representations, new types of GUSTs with the $G \times G$ gauge groups can naturally appear in consideration [7, 8]. Moreover for the observable gauge symmetry one can consider the diagonal subgroups G'^{sym} of the rank 16 group $G \times G \subset SO(16) \times SO(16)$ or $\subset E(8) \times E(8)$. Early [7] we considered the possible ways of breaking the "string" gauge subgroups $\subset E_8 \otimes E_8$ down to low energy supersymmetric model that includes Standard Model group and horizontal factor $SU(3)$. There are good physical reasons for including the horizontal $SU(3)_H$ group into the unification scheme. Firstly, this group naturally accommodates three fermion families presently observed (explaining their origin) and, secondly, can help to solve the flavour problem in SUSY GUTs and can provide correct and economical description of the fermion mass spectrum and mixing without invoking high dimensional representation of conventional $SU(5)$, $SO(10)$ or $E(6)$ gauge groups[10]. Construction of a string model (GUST) containing the horizontal gauge symmetry provides additional strong motivation to this idea. Moreover, the fact that in GUSTs high dimensional representations are forbidden by the KMA is a very welcome feature in this context. The constraints of horizontal model parameters followed from this approach allow the existence of the interesting flavour-changing physics in the TeV region. Also these models gives rise to a rather natural way of the superweak-like CP-violation[10]. All this leads us naturally to considering possible forms for horizontal symmetry G_H, and

G_H quantum number assignments for quarks (anti-quarks) and leptons (anti-leptons) which can be realized within GUST's framework.

Here we present shortly the string models including Grand Unification group $[SU(5)\times U(1)\times G_H]^{\otimes 2}$, along with horizontal gauge symmetry $G_H = U(3)$. Using the (2,0) world-sheet superconformal symmetry we study the superpotential. The form of obtained superpotential implies that 2 generations remain massless comparing with the M_W scale. Using the condition of $SU(3)_H$ anomaly cancellation the theory predicts the existence of the Standard Model singlet "sterile" particles that participate only in horizontal $SU(3)_H$ and/or $U(1)_H$ interactions. As following from the form of the superpotential some of them could be light (much less than M_W) that will be very interesting in sense of experimental accelerator and astrophysical searches. In this model after anomalous $U(1)$ D-term suppressing the only surviving horizontal gauge group is $SU(3)_H$.

We outline the perspective way of including the symmetric subgroup on the intermediate stage that does not involve higher level of Kac-Moody algebra representations. Starting from the rank 16 grand unified gauge group of the form $G\times G$ [7, 8] and making use of the KMA which select the possible gauge group representations we discuss some ways of breaking of string rank 16 gauge group $[SU(5)\times U(1)\times G_H]^{\otimes 2}$ down to the symmetric diagonal subgroups [7],[9]. This model allows two ways of embedding chiral matter (16 quarks, leptons and right neutrino) in $\underline{1}$, $\underline{\bar{5}}$ and $\underline{10}$ representation of $SU(5)\times U(1)$, which correspond to the flipped and non-flipped $SU(5)\times U(1)$ models respectively [11].

The main goal of our paper is to solve the problem of discrepancy between the unification scales of $SU(3^c)$, $SU(2)_{EW}$, $U(1)_Y$-gauge coupling constants , $M_G \sim 10^{16} GeV$, and string scale in GUSTs, $M_{GUST} = M_{SU} = g_{string}\cdot 5\cdot 10^{17} GeV$.

We consider two possibilities of the breaking of the primordial $[U(5)\times G_H]^{\otimes 2}$ gauge symmetry with two variants of Q_{em} charge quantization correspondingly. For the various chains of gauge symmetry breaking in flipped and non-flipped cases of the $SU(5)$ model we carry out the RGE analysis of the behaviour of the gauge coupling constants taking into account the possible intermediate thresholds (the additional Higgs doublets, color triplets, SUSY threshold, massive fourth generation) and the threshold effects due to the massive string states. We show that only in non-flipped case of $[U(5)\times G_H]^{\otimes 2}$ GUST it is possible to make the unification scale of $g_{1,2,3}$- coupling constants in supersymmetric standard model, M_G, to be consistent with the string scale unification, $M_{GUST} = M_{SU}$ and obtain estimation of the string coupling constant $g_{str} = O(1)$. As an additional benefit, the values of the g_{str} and M_{SU} allow us to estimate the horizontal coupling constant g_{3H} on the scale of $\sim$1 TeV.

2 The features of the GUST spectrum with $[SU(5) \times U(1) \times SU(3) \times U(1)]^{\otimes 2}$. The ways of the gauge symmetry breaking.

Model 1 is defined by 6 basis vectors given in Table 1 which generates the $Z_2 \times Z_4 \times Z_2 \times Z_2 \times Z_8 \times Z_2$ group under addition.

Table 1: **Basis of the boundary conditions for all world-sheet fermions. Model 1.**

Vectors	$\psi_{1,2}$	$\chi_{1,..,6}$	$y_{1,...,6}$	$\omega_{1,...,6}$	$\bar{\varphi}_{1,...,12}$	$\Psi_{1,...,8}$	$\Phi_{1,...,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1/2^8$	0^8
b_3	11	111100	000011	000000	$0^4 1^8$	0^8	1^8
$b_4 = S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	001100	000000	110011	1^{12}	$1/4^5 {-3/4}^3$	$-1/4^5\ 3/4^3$
b_6	11	110000	000011	001100	$1^2 0^4 1^6$	1^8	0^8

The model corresponds to the following chain of the gauge symmetry breaking: $\longrightarrow U(8)^2 \longrightarrow [U(5) \times U(3)]^2$.

Since the matter fields form the chiral multiplets of $SO(10)$, it is possible to write down $U(1)_{Y_5}$–hypercharges of massless states. In order to construct the correct electromagnetic charges for matter fields we must define the hypercharges operators for the observable $U(8)^I$ group as follows

$$Y_5 = \int_0^\pi d\sigma \sum_a \Psi^{*a} \Psi^a, \quad Y_3 = \int_0^\pi d\sigma \sum_i \Psi^{*i} \Psi^i \tag{1}$$

and analogously for the $U(8)^{II}$ group.

Then the orthogonal combinations

$$\tilde{Y}_5 = \frac{1}{4}(Y_5 + 5Y_3), \quad \tilde{Y}_3 = \frac{1}{4}(Y_3 - 3Y_5), \tag{2}$$

play the role of the hypercharge operators of $U(1)_{Y_5}$ and $U(1)_{Y_H}$ groups respectively. In Table 2 we give the hypercharges $\tilde{Y}_5^I, \tilde{Y}_3^I, \tilde{Y}_5^{II}, \tilde{Y}_3^{II}$.

With the chiral matter and "horizontal" Higgs fields available in Model 1 the possible form of the renormalizable (trilinear) part of the superpotential responsible for fermion mass matrices is restricted not only by the gauge symmetry. Another strong constraint comes from the interesting observation that a modular invariant N=1 space-time supersymmetric theory also extends to a global N=2 world sheet superconformal symmetry [18] which now contains two distinct fermionic components of the energy-momentum tensor, T_F^+ and T_F^- and there is also the $U_J(1)$ current J. This conserved $U(1)$ current of the N=2 superalgebra may play a key role in constructing of realistic phenomenology.

Table 2: **The list of quantum numbers of the states. Model 1.**

N^o	$b_1, b_2, b_3, b_4, b_5, b_6$	SO_{hid}	$U(5)^I$	$U(3)^I$	$U(5)^{II}$	$U(3)^{II}$	$\tilde{Y}_5^I$	$\tilde{Y}_3^I$	$\tilde{Y}_5^{II}$	$\tilde{Y}_3^{II}$
1	RNS		5	$\bar{3}$	1	1	–1	–1	0	0
			1	1	5	$\bar{3}$	0	0	–1	–1
	0 2 0 1 2(6) 0		5	1	5	1	–1	0	–1	0
$\hat{\Phi}$			1	3	1	3	0	1	0	1
			5	1	1	3	–1	0	0	1
			1	3	5	1	0	1	–1	0
2	0 1 0 0 0 0		1	3	1	1	5/2	–1/2	0	0
			$\bar{5}$	3	1	1	–3/2	–1/2	0	0
			10	1	1	1	1/2	3/2	0	0
$\hat{\Psi}$	0 3 0 0 0 0		1	1	1	1	5/2	3/2	0	0
			$\bar{5}$	1	1	1	–3/2	3/2	0	0
			10	3	1	1	1/2	–1/2	0	0
3	0 0 1 1 3 0	$-_1\ \pm_2$	1	1	1	3	0	–3/2	0	–1/2
	0 0 1 1 7 0	$-_1\ \pm_2$	1	$\bar{3}$	1	1	0	1/2	0	3/2
$\hat{\Psi}^H$	0 2 1 1 3 0	$+_1\ \pm_2$	1	$\bar{3}$	1	3	0	1/2	0	–1/2
	0 2 1 1 7 0	$+_1\ \pm_2$	1	1	1	1	0	–3/2	0	3/2
4	1 1 1 0 1 1	$\mp_1\ \pm_3$	1	1	1	$\bar{3}$	0	–3/2	0	1/2
	1 1 1 0 5 1	$\mp_1\ \pm_3$	1	$\bar{3}$	1	1	0	1/2	0	–3/2
$\hat{\Phi}^H$	1 3 1 0 1 1	$\pm_1\ \pm_3$	1	$\bar{3}$	1	$\bar{3}$	0	1/2	0	1/2
	1 3 1 0 5 1	$\pm_1\ \pm_3$	1	1	1	1	0	–3/2	0	–3/2
5	0 1(3) 1 0 2(6) 1	$-_1\ \pm_3$	1	$3(\bar{3})$	1	1	$\pm 5/4$	$\pm 1/4$	$\pm 5/4$	$\mp 3/4$
		$+_1\ \pm_3$	$5(\bar{5})$	1	1	1	$\pm 1/4$	$\mp 3/4$	$\pm 5/4$	$\mp 3/4$
$\hat{\phi}$	0 1(3) 1 0 4 1	$-_1\ \pm_3$	1	1	1	$3(\bar{3})$	$\pm 5/4$	$\mp 3/4$	$\pm 5/4$	$\pm 1/4$
		$+_1\ \pm_3$	1	1	$5(\bar{5})$	1	$\pm 5/4$	$\mp 3/4$	$\pm 1/4$	$\mp 3/4$
6	1 2 0 0 3(5) 1	$\pm_1\ -_4$	1	1	1	1	$\pm 5/4$	$\pm 3/4$	$\mp 5/4$	$\mp 3/4$
	1 1(3) 0 1 5(3) 1	$+_1\ \mp_4$	1	1	1	1	$\pm 5/4$	$\pm 3/4$	$\pm 5/4$	$\pm 3/4$
$\hat{\sigma}$	0 0 1 0 2(6) 0	$\mp_3\ +_4$	1	1	1	1	$\pm 5/4$	$\mp 3/4$	$\pm 5/4$	$\mp 3/4$

Thus all vertex operators have the definite $U(1)$ charge. Let us consider the contribution $R^2(NS)$ to the three point fermion-fermion-boson matter superpotential:

$$
\begin{aligned}
W_1 &= g\sqrt{2}\Big[\hat{\Psi}_{(1,3)}\hat{\Psi}_{(\bar{5},1)}\hat{\Phi}_{(5,\bar{3})} + \hat{\Psi}_{(1,1)}\hat{\Psi}_{(\bar{5},3)}\hat{\Phi}_{(5,\bar{3})} + \\
&+ \hat{\Psi}_{(10,3)}\hat{\Psi}_{(\bar{5},3)}\hat{\Phi}_{(\bar{5},3)} + \hat{\Psi}_{(10,3)}\hat{\Psi}_{(10,1)}\hat{\Phi}_{(5,\bar{3})}\Big] \qquad (3)
\end{aligned}
$$

From the above form of the Yukawa couplings it follows that two (chiral) generations have to be very light (comparing to M_W scale).

The $SU(3)_H$ anomalies of the matter fields (row No 2) are naturally canceled by the chiral "horizontal" superfields forming two sets: $\hat{\Psi}^H_{(1,N;1,N)}$ and $\hat{\Phi}^H_{(1,N;1,N)}$, $\Gamma = \underline{1}$, $N = \underline{1}, \underline{3}$, (with both $SO(2)$ chiralities, see Table 2, row No 3, 4 respectively). The superpotential, W_2, consists of the following R^2NS-terms:

$$
W_2 = g\sqrt{2}\Big[\hat{\Phi}^H_{(1,1;1,\bar{3})}\hat{\Phi}^H_{(1,\bar{3};1,1)}\hat{\Phi}_{(1,3;1,3)} + \hat{\Phi}^H_{(1,1;1,1)}\hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(1,3;1,3)} +
$$

$$+ \quad \hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(1,\bar{3};1,\bar{3})} + \hat{\Psi}^H_{(1,\bar{3};1,1)}\hat{\Psi}^H_{(1,\bar{3};1,3)}\hat{\Phi}_{(1,\bar{3};1,\bar{3})} +$$
$$+ \quad \hat{\Psi}^H_{(1,1;1,3)}\hat{\Psi}^H_{(1,\bar{3};1,3)}\hat{\Phi}_{(1,3;1,3)} + \hat{\sigma}_{(-1-4)}\hat{\sigma}_{(+1+4)}\hat{\Psi}_{(1,1;1,1)}\Big] \qquad (4)$$

From (4) it follows that some of the horizontal fields in sectors (No 3, 4) remain massless at the tree-level. This is a remarkable prediction: "horizontal" fields are "sterile", e.g. they interact with the ordinary chiral matter fields only through the $U(1)_H$ and $SU(3)_H$ gauge boson and therefore this "sterile" matter is of an interest in the context of the experimental searches on accelerators or in astrophysics. The Higgs fields could give the following $(NS)^3$ contributions to the renormalizable superpotential:

$$W_3 = \sqrt{2}g\Big\{\hat{\Phi}_{(5,1;1,3)}\hat{\Phi}_{(\bar{5},1;\bar{5},1)}\hat{\Phi}_{(1,1;5,\bar{3})} + \hat{\Phi}_{(5,1;1,3)}\hat{\Phi}_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(\bar{5},3;1,1)}$$
$$+\hat{\Phi}_{(1,3;5,1)}\hat{\Phi}_{(\bar{5},1;\bar{5},1)}\hat{\Phi}_{(5,\bar{3};1,1)} + \hat{\Phi}_{(1,3;5,1)}\hat{\Phi}_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(1,1;\bar{5},3)} + \text{conj.}\Big\} \qquad (5)$$

So, $W_1+W_2+W_3$ is the most general renormalizable superpotential which includes all nonzero three-point (F-type) expectation values of the vertex operators for corresponding 2-dimensional conformal model. The only non-vanishing nonrenormalizable superpotential $W_4 = R^4$ is as follows

$$W_4 \sim \frac{g^2}{M_{Pl}}\hat{\Psi}_{(1,3;1,1)}\hat{\Phi}^H_{(+1,-3)(1,\bar{3};1,1)}\hat{\sigma}_{1\ (-1,-4)}\hat{\sigma}_{3\ (+3,+4)}. \qquad (6)$$

As we can see from the states list for Model 1, the hidden group $U(1)_1$ in this model appears to be anomalous: Tr $U(1)_1 = 12$.

This means that at one-loop string level there exists Fayet-Iliopoulos D-term determined by VEV of the dilaton and it is proportional to Tr $U(1)_1 = 12$. Potentially this term could break SUSY at the high scale and destabilize the vacua [19]. This could be avoided if the potential has D-flat direction on $U(1)_1$-charged fields which have VEVs that break anomalous group (and may be some other groups), compensate D-term, and restore SUSY. Those fields have to have appropriate charges on the remaining groups in order that cause no SUSY breaking via their D-terms. In our case we can avoid the SUSY breaking caused by D-term by using the pairs of fields $\hat{\phi}_1$ and/or $\hat{\phi}_3$ from the sector 5. In addition to anomalous group breaking we also obtain breaking of the groups $U(1)_3^{hid}$ (hidden), $U(1)_H^I \times U(1)_H^{II} \to U(1)'_H$ (in the case of using only one of $\hat{\phi}_1$, $\hat{\phi}_3$, otherwise the group is broken totally) and $U(1)_5^I \times U(1)_5^{II} \to U(1)_5^{sym}$, and also horizontal group $SU(3)_H^I \to SU(2)_H^I$ and/or $SU(3)_H^{II} \to SU(2)_H^{II}$. For complete breaking the initial horizontal symmetry $SU(3)_H^I \times SU(3)_H^{II}$ we also need to use the VEVs of the $\hat{\Phi}_{(1,3;1,3)}$- Higgs fields. Besides that fields potentially the fields $\hat{\sigma}_2$ can also obtain VEVs, that additionally breaks down the hidden group $SO(6)_4$ and also breaks $U(1)'_H$. Note that since the superfields mentioned above besides the $\hat{\sigma}_2$-field do not participate in construction of the renormalizable superpotential $W_1 + W_2 + W_3$ in this scheme we have no problem with the F-flat directions. Finally, note that in non-flipped $SU(5) \times U(1)$ model we can give the VEVs to the fields $\hat{\sigma}_1$ for breaking $U(1)_5^{sym}$. In this case for the choice of the D-flat direction we also need to use VEVs of the fields $\hat{\sigma}_3$. From the form of the $\hat{\sigma}$-depended contribution to the superpotential W_2 (4) it follows that the field of the fourth generation $\hat{\Psi}_{(1,1;1,1)}$ (the fourth neutrino in non-flipped scheme) obtains a heavy mass.

Further we shortly discuss the problem of gauge symmetry breaking in Model 1. The most important point is that the Higgs fields $(\underline{10}_{1/2} + \bar{\underline{10}}_{-1/2})$ do not appear. However there exists some possibilities to break the GUST group $[(U(5) \times U(3))^I]^{\times 2}$ down to the symmetric subgroups using the following VEVs of the Higgs fields $(\underline{5}, \underline{1}; \underline{5}, \underline{1})_{(-1,0;-1,0)}$:

a) $\quad < (\underline{5}, \underline{5}) >= a \cdot \mathrm{diag}(1,1,1,1,1)$

b) $\quad < (\underline{5}, \underline{5}) >_0= \mathrm{diag}(x,x,x,y,y),$

c) $\quad < (\underline{5}, \underline{5}) >_0= a \cdot \mathrm{diag}(0,0,0,1,1),$

d) $\quad < (\underline{5}, \underline{5}) >_0= a \cdot \mathrm{diag}(1,1,1,0,0).$

With the VEV a) the GUST group $[(U(5) \times U(3))^I]^{\otimes 2}$ breaks to symmetric group:

$$\textbf{a)} \qquad U(5)^I \times U(5)^{II} \to U(5)^{sym} \to ...$$

With such a breaking tensor Higgs fields transform under the $(SU(5) \times U(1))^{sym} \times G_H$ group in the following way:

$$(\underline{5}, \underline{1}; \underline{5}, \underline{1})_{(-1,0;-1,0)} \quad \to \quad (\underline{24}, \underline{1})_{(0,0)} \quad + \quad (\underline{1}, \underline{1})_{0,0}. \tag{7}$$

The diagonal VEVs of the Higgs fields break the GUST with $G \times G$ down to the "skew"-symmetric group with the generators $\triangle^{sym}$ of the form:

$$\triangle^{sym}(t) = -t^* \times 1 + 1 \times t, \tag{8}$$

The corresponding hypercharge of the symmetric group reads:

$$\bar{Y} = \tilde{Y}^{II} - \tilde{Y}^I. \tag{9}$$

Adjoint representations which appear on the rhs of (7) can be used for further breaking of the symmetric group. This can lead to the final physical symmetry

$$SU(3^c) \times SU(2_{EW}) \times U(1)_5 \times U(1)^{sym} \times G_H^{'}$$

with the low-energy gauge symmetry of the quark–lepton generations with an additional $U(1)^{sym}$–factor. As we already discussed above the form of the $G_H^{'}$ depends on the way of the U(1) anomaly group cancellation and the complete breaking of this group is realized by the VEVs of the $\hat{\Phi}_{(1,3;1,3)}$ Higgs fields and/or $\hat{\Phi}_{(1,1;5,\bar{3})}$, $\hat{\Phi}_{(5,\bar{3};1,1)}$ Higgs fields.

Note, that when we use the VEVs b),c),d) there exist also the others interesting ways of breaking the $G^I \times G^{II}$ gauge symmetry down to

b) $\quad SU(3^c) \times SU(2)_{EW} \times U(1)_5 \times U(1)^{sym} \times G_H^{'} \to \ldots,$

c) $\quad SU(3^c) \times SU(2)^I_{EW} \times SU(2)^{II}_{EW} \times U(1)_5 \times U(1)^{sym} \times G_H^{'} \to \ldots\ ,$

d) $SU(3^c)_I \times SU(3^c)_{II} \times SU(2)_{EW} \times U(1)_5 \times U(1)^{sym} \times G_H^{'} \to \ldots$.

We could consider the GUST construction involving $[SO(10) \times G_H]^{\otimes 2} \times G_{hidden}$ as the gauge group [7]. In that model the only Higgs fields appeared are $(10, 10)$ of $SO(10) \times SO(10)$ and the hidden group $G_{hidden} = U(1) \times SO(6)$ is anomaly free. As an illustration we would like to remark that for the $SO(10) \times SO(10) \times G_H \times G_H$ GUST we can consider similarly the following VEVs of the Higgs fields $(10, 10)$:

a') $< (10, 10) >_0 = a \cdot \mathrm{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$,

b') $< (10, 10) >_0 = a \cdot \mathrm{diag}(1, 1, 1, 1, 1, 1, 0, 0, 0, 0)$,

c') $< (10, 10) >_0 = a \cdot \mathrm{diag}(1, 1, 1, 1, 1, 1, x, x, x, x)$.

These cases lead correspondingly to the following chains of $[SO(10)]^{\otimes 2}$ breaking ways:

a') $[SO(10)]^{sym}$,

b') $SU(4) \times SU(2)_{I1} \times SU(2)_{I2} \times SU(2)_{II1} \times SU(2)_{II2}$,

c') $SU(4) \times SU(2)_I \times SU(2)_{II}$.

3 The GUT and string unification scale. The paths of unification in flipped and non-flipped GUST models.

Indeed the estimates on the M_{H_0} scale depend on the value of the family gauge coupling. String theories imply a natural unification of the gauge and gravitational couplings, g_i and G_N respectively. For example, it turns out that these couplings unify at tree level to form one coupling constant g_{string} [12]:

$$8\pi \frac{G_N}{\alpha'} = g_i k_i = g_{str}^2. \tag{10}$$

Here α' is the Regge slope, the coupling constants g_i correspond to the gauge group G_i with the Kac-Moody levels k_i. In string theory the scale of unification is fixed by the Planck scale $M_{Pl} \approx 10^{19}$. In one-loop string calculations the value of the unification scale could be divided into moduli independent part and a part that depends on the VEVs of the moduli fields. The latter part considered as the correction to the former and obviously it is different for the various models. Like in the gauge fields theory this correction is called the string threshold correction of the massive string states. Moduli independent contribution depends on the renormalizing scheme used, so in $\overline{DR}$ renormalization scheme the scale of string unification is shifted to [13]a :

$$M_{SU} = \frac{e^{(1-\gamma)/2} 3^{-3/4}}{4\pi} g_{str} M_{Pl} \approx 5\, g_{str} \times 10^{17} GeV. \tag{11}$$

There exists the most important difference between the unification scales of gauge coupling unification in string theory and in field theory. In field theory this scale is determined via extrapolation of data within the Supersymmetric $SU(3) \times SU(2) \times U(1)$ Standard Model using the renormalization group equations (RGE) for the gauge couplings

$$\alpha_3^{-1}(M_Z) = 8.93 \ ,$$

$$\alpha_2^{-1} = (\alpha_{em}/\sin^2\theta_W)^{-1}|_{M_Z} = 29.609 \ ,$$
$$\alpha_1^{-1} = (5\alpha_{em}/3\cos^2\theta_W)^{-1}|_{M_Z} = 58.975 \ .$$

The factor 5/3 in the definition of α_1 has been included for the normalization at the unification scale M_G.

The one loop renormalization group equations for these gauge couplings are given by

$$\frac{d\alpha_i}{d\ln\mu} = \frac{\alpha_i^2}{2\pi} b_i \tag{12}$$

Beta functions coefficients for the $SU(3) \times SU(2) \times U(1)$ coupling constants in SUSY models are given by the following:

$$b_3 = -9 + 2N_g + 0 \cdot N_h + \frac{1}{2} \cdot N_3, \tag{13}$$
$$b_2 = -6 + 2N_g + \frac{1}{2} \cdot N_h + 0 \cdot N_3, \tag{14}$$
$$b_1 = 0 + 2N_g + \frac{3}{10} \cdot N_h + \frac{1}{5} \cdot N_3, \tag{15}$$

where N_g is the number of generation and N_h is the number of Higgs doublets and we also include some possible intermediate thresholds for heavy Higgs doublets $(N_h - 2)$ and color triplets, N_3, which exist in the spectrum of Model 1 and can be take into account for RGE.

The RGE are integrated from M_Z-mass to the unification scale M_G. In the presence of various intermediate scale, M_I, $I = 1, 2, 3, ...$, the RGE are given by:

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_G) + \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} - \frac{b_{i1}}{2\pi} \ln \frac{M_1}{M_Z} - \frac{b_{i2}}{2\pi} \ln \frac{M_2}{M_Z} - ... \tag{16}$$

where b_{iI} are the additional corresponding contributions of the new thresholds to the beta functions. At the Z-mass scale we have:

$$\sin^2\theta_W(M_Z) \approx 0.2315 \pm 0.001 \ ,$$
$$M_Z = 91.161 \pm 0.031 GeV, \tag{17}$$

Note, that for flipped models

$$\sin^2\theta_W(M_G) = \frac{15k^2}{16k^2+24} \tag{18}$$

where $k^2 = g_1^2/g_5^2$. In the SO(10) limit (or for non-flipped case) we have $k^2 = 1$ and $\sin^2\theta_W = 3/8$.

The string unification scale could be contrasted with MSSM, $SU(3^c)\times SU(2)\times U(1)$ naive unification scale,

$$M_G \approx 2\times 10^{16} GeV \tag{19}$$

obtained by running the SM particles and their SUSY-partners to high energies.

One of the first way to explain the difference between these two mass scales, M_{SU} and $M_{MSSM} = M_G$, was the attempts to take into account the string thresholds corrections of the massive string states [13]:

$$\frac{1}{g_i^2(\mu)} = k_i \frac{1}{g^2_{string}} + 2b_i \ln(\frac{\mu}{M_{SU}}) + \tilde{\Delta}_{G_i}. \tag{20}$$

where the index i runs over gauge coupling and μ is some phenomenological scale such as M_Z or M_G. The coefficients k_i are the Kac-Moody levels (e.g. for SU(5) $k_2 = k_3 = 1, k_1 = 5/3$). The quantities $\tilde{\Delta}_{G_i}$ represent the heavy thresholds corrections, which are the corrections arising from the infinite towers of massive string states. Although these states have the Planck mass scale, there are infinite number of them, so they together could have the considerable effect. In general the full string thresholds corrections are of the form

$$\tilde{\Delta}_i = \Delta_i + k_i Y, \tag{21}$$

where Y is independent of the gauge group factor. Moreover, the low energy predictions for $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ depend only on the differences $(\tilde{\Delta}_i - \tilde{\Delta}_j) = (\Delta_i - \Delta_j)$ for the different gauge groups.

However the Y factor makes influence on the estimation of the value of g_{str} if we base on low energy gauge constants and use RGE. Note that in general g_{str} is defined by VEV of the dilaton moduli field but because of degeneracy of the classic potential of the moduli fields we do not know the g_{str} a priori. If we could have the value of the g_{str} then the its coincidence with our estimates will show the correctness of our model.

It is supposed that the value of the Y factor is small enough [20] so we neglect it in our calculations of g_{str} via RGE.

In the GUSTs examples considered the threshold corrections of the massive string states are not large enough to compensate the difference between the scales of unification of the string and the MSSM (GUT) models. Later we will discuss the possible effects of them in $G\times G$ models, for example in Model 1.

In our calculations for Model 1 we will follow the way suggested in [13]c,[14]. According to it we have to calculate an integral of the modified partition function over the fundamental domain Γ of modular group.

$$\delta\Delta = \int_\Gamma \frac{\mathrm{d}^2\tau}{\tau_2}\left(|\eta(\tau)|^{-4}\hat{Z}(\tau) - b_G\right)$$

where b_G is the beta function coefficient, $\hat{Z}(\tau)$ is modified partition function. Modified in this context means that the charge operators Q are inserted in the trace in partition function in the following way:

$$\hat{Z}(\tau) = -\mathrm{Tr} Q_s^2 Q_I^2 q^H q^{\bar{H}},$$

where Q_s is helicity operator and Q_i is a generator of a gauge group. We are interesting in particular in the difference between groups $SU(5)_I$ and $U(1)_I$ in Model 1. For this case charge polynomial is $5Q_1Q_2$ (see [13]c). Rewritten via well known theta function, the modified function $\hat{Z}(\tau)$ for our case reads:

$$\hat{Z}(\tau) = -\frac{5}{512}\sum_{\mathbf{a},\mathbf{b}} c\binom{\mathbf{a}}{\mathbf{b}}\eta^{-1}\theta'\binom{a_\Psi}{b_\Psi}\bar{\eta}^{-2}\hat{\bar{\theta}}^2\binom{a_I}{b_I}\prod\eta^{-1}\theta\binom{a_j}{b_j}\prod\bar{\eta}^{-1}\bar{\theta}\binom{a_k}{b_k}$$

where 512 is normalizing factor and products calculated over all fermions excluding fermions which Q operators apply to. Namely θ' and $\hat{\bar{\theta}}$ denote action of helicity and gauge group operators respectively. The sum is taken over all pairs of boundary condition vectors that appear on the model. According to [14] we expand the resulting expression with θ-functions via q and $\bar{q}$ in order to achieve appropriate precision.

The final results for Model 1 are as follows (δ denotes the difference between corresponding quantities for $U(1)_I$ and $SU(5)_I$). $\delta b = 26.875$; $\delta\Delta = 5.97$ Given this relative threshold corrections we can compute its effect on string unification scale M_{SU}. We find that the correction unification scale is:

$$M_{SU}^{corr} = M_{SU}\,\exp\frac{(\Delta_5 - \Delta_1)}{2(b_5 - b_1)} \approx g_{str.}\cdot 5.6\cdot 10^{17} GeV\ . \tag{22}$$

However there are some ways to explain the difference between scales of string (M_{SU}) and ordinary ($M_G = M_{SU}$) unifications (without additional intermediate exotic vector matter fields that does not fit into 5 or $\bar{5}$ representations of SU(5). [14]) Perhaps the most natural way is related to the $G^I \times G^{II}$ String GUT. If one uses the breaking scheme $G^I \times G^{II} \to G^{sym}$ (where $G^{I,II} = U(5) \times U(3)_H \subset E_8$) on the M_{sym}-scale, then unification scale $M_{MSSM} = M_G \sim 2\cdot 10^{16}$ GeV is the scale of breaking the G_{sym} group, and string unification do supply the equality of coupling constant $G \times G$ on the string scale $M_{SU} \sim g \cdot 5 \cdot 10^{17}$ GeV. Otherwise, we can have an addition scale of the symmetry breaking $M_{sym} > M_G$. In any case on the scale of breaking $U(5)^I \times U(5)^{II} \to U(5)^{sym}$ the gauge coupling constants satisfy the equation

$$(\alpha^{Sym})^{-1} = (\alpha^I)^{-1} + (\alpha^{II})^{-1}\ . \tag{23}$$

Thus in this scheme the knowledge of scales M_{SU} and M_{Sym} gives us a principal possibility to trace the evolution of coupling constant of the original group $SU(5)^I \times U(1)^I \times SU(5)^{II} \times U(1)^{II}$ through the $SU(5)^{Sym} \times U(1)^{Sym}$ to the low energies and estimate the values of all coupling constants including the horizontal gauge constant g_{3H}.

The coincidence of $\sin^2\theta_W$ and α_3 with experiment will show how realistic this model is. The evolution of the gauge constant from the string constant g_{str} to the scale of M_G is described by the equation:

$$(\alpha_{5,1}^{Sym})^{-1}(M_G) = 2(\alpha_{5,1}^{Str})^{-1}(M_{SU}) + \frac{(b_{5,1}^I + b_{5,1}^{II})}{2\pi}\ln(M_{SU}/M_{Sym}) + \frac{b_{5,1}^{Sym}}{2\pi}\ln(M_{Sym}/M_G) \tag{24}$$

where $\alpha_5^{Str}(M_{SU}) = g_{str}^2/4\pi$ and g_{str} is the string coupling. Now we can get the relation between $g_{str} = g$ and M_{Sym} from RGE's for gauge running constants $g_5^{Sym} = g_5$, g_5^I and , g_5^{II} on the $M_G - M_{SU}$ scale. For example in Model 1 for the breaking scheme a) one can get:

$$b_5^{Sym} = 12, \;\; b_5^I = 5, \;\; b_5^{II} = -3 \tag{25}$$

and

$$b_1^{Sym} = 27, \;\; b_1^I = \frac{105}{4}, \;\; b_1^{II} = \frac{73}{4}. \tag{26}$$

Let us try to make the behaviours of these coupling constants consistent above and below M_G scale. To do this we have to remember that there are two possibilities to embed quark-lepton matter in $SU(5)\times U(1)$ of SO(10) multiplets, $\underline{1}_{5/2}$, $\underline{\bar{5}}_{-3/2}$, $\underline{10}_{1/2}$. For the electromagnetic charge we get:

$$\begin{aligned} Q_{em} &= Q^{II} - Q^I = \bar{T}_3 + \frac{1}{2}\bar{y}, \\ \bar{T}_3 &= T^{II} - T^I; \qquad \bar{y} = y^{II} - y^I. \end{aligned} \tag{27}$$

where

$$\frac{1}{2}y^{I,II} = \alpha T^{I,II}{}_{5z} + \beta Y^{I,II}{}_5 \;, \quad T^{I,II}{}_{5z} = \mathrm{diag}(-1/3, -1/3, -1/3, 1/2, 1/2). \tag{28}$$

In usual non-flipped Georgi-Glashow SU(5) model the $Q_{e.m.}$ is expressed via SU(5) generators only: $\alpha = 1$; $\beta = 0$. For flipped $SU(5)\times U(1)$ we have $\alpha = -\frac{1}{5}$; $\beta = \frac{2}{5}$. Note, that this charge quantization does not lead to exotic states with fractional electromagnetic charges (e.g. $Q_{em} = \pm 1/2, \pm 1/6$)[16, 7]. Also in non-flipped $SU(5)\times U(1)$ gauge symmetry breaking scheme there are no SU(3) color triplets and SU(2) doublets with exotic hypercharges.

For example in flipped (non-flipped) case of Model 1 we can use the Higgs doublets from $(5,1;1,1)+(\bar{5},1;1,1)$, (the fields $\hat{\phi}_2 + \bar{\hat{\phi}}_2$ from sector 5) for breaking the SM symmetry and low energy $U(1)_5^{sym}$-symmetry. Below the M_G scale in non-horizontal sector the evolution of gauge coupling constants is described by equations

$$\alpha_S^{-1}(\mu) = \alpha_5^{-1}(M_G) + \frac{b_3}{2\pi}\ln(M_G/\mu) \tag{29}$$

$$\alpha^{-1}(\mu)\sin^2\theta_W = \alpha_5^{-1}(M_G) + \frac{b_2}{2\pi}\ln(M_G/\mu) \tag{30}$$

$$\frac{15k^2}{k^2+24}\alpha^{-1}(\mu)\cos^2\theta_W = \alpha_5^{-1}(M_G) + \frac{\bar{b}_1}{2\pi}\ln(M_G/\mu), \tag{31}$$

where for $N_g = 4$ generations and for the minimal set of Higgs fields we have:

$$b_3 = -1\ , \quad b_2 = 3\ , \quad \bar{b}_1 = \frac{25k^2}{k^2+24}\cdot b_1|_{4\ gen.} = \frac{15k^2}{(k^2+24)}\frac{43}{3}.$$

From these equations and from the experimental data we can find for $N_g = 3, 4$, respectively:

$$\begin{aligned} M_G &= 1.2\cdot 10^{16}\text{GeV}\ , \qquad \alpha_5^{-1} = 24.4, \qquad k^2 = 0.98\ , \\ M_G &= 1.17\cdot 10^{16}\text{GeV}\ , \qquad \alpha_5^{-1} = 14.1, \qquad k^2 = 0.97. \end{aligned} \tag{32}$$

Here we assume that additional Higgs doublets and triplets appeared in the theory are heavy ($> M_G$).

From the other hand for the Model 1 and with the mass of the fourth generation sufficiently heavy to be invisible but less than M_G the equations for $\alpha_{5,1}^{I.II.Sym}$ for all M_{sym}-scale in the range, $M_G < M_{sym} < M_{SU}$, give the contradicting value for k^2 that is considerably less than 1. For example, for $M_{Sym} = 1.6\cdot 10^{17} GeV$, we get:

$$M_{SU} = 9.6\cdot 10^{17} GeV,\ \ g_{str} = 1.7\ \rightarrow k^2 = 0.44. \tag{33}$$

In the non-flipped case in Model 1 we have an additional neutral singlet $\hat{\sigma}^1$ field, which could be used for breaking $U(1)^{Sym}$ group (of $U(1)_5^I \times U(1)_5^{II}$) at any high scale, independently on the M_{Sym} scale, where the

$$SU(5)^I \times SU(5)^{II} \longrightarrow SU(5)^{Sym}$$

Therefore we have no constraints on k^2 parameter in the range from string scale down to M_G. As a result in non-flipped case of the $G^I \times G^{II}$ ($G = SU(5)\times U(1)\times G_H$) GUST the string unification scale, M_{SU}, can be consistent with the M_G (M_{MSSM}) scale, i.e using low energy values of the $g_{1,2,3}$- coupling constants and their RGE (12, 15, 23,24) we get for the GUST scale M_{SU} the expression (22) with the corresponding value of the string coupling constant.

In this case while M_{sym} changes in the range from $M_G = 1.17\cdot 10^{16}$ GeV to 10^{18} GeV we could expect the string constant and string scale (22) to be in the range

$$g \sim (1.40 \div 2.13)\ , \quad M_{SU} \sim (0.79 \div 1.20)\times 10^{18} GeV\ .$$

It is interesting to estimate the value of horizontal coupling constant. The analysis of RG-equations allows to state that the horizontal coupling constant g_{3H} does not exceed the electro-weak one g_2.

In Model 1 after cancellation of the U(1) anomaly by VEVs of the fields $\hat{\phi}_1$ or $\hat{\phi}_3$ corresponding $SU(3)^{II}$ or $SU(3)^{I}$ horizontal gauge symmetry group survives.

Using RG equations for the running constant $g_{3H}^{I,II}$ and the value of the string coupling constant g_{str} at M_{SU} we can estimate a value of the horizontal coupling constant at low energies. For Model 1 we have

$$b_{3H}^{I} = 21 \ , \qquad b_{3H}^{II} = 13 \ ,$$

and we find from RGE for g_{3H} that

$$g_H^{I} \sim 0.3 \ , \qquad g_H^{II} \sim 0.4 \ .$$

Below we consider in details the gauge symmetry breaking by VEV's b), c), d) applied to Model 1 both in flipped and non-flipped cases. We assume that some additional Higgs doublets and triplets originating from the group G^{II} (see sector 1 in table 2) could be lighter than M_G. The fact that this Higgs fields were initially (i.e. before breaking $G^I \times G^{II} \to G$) related to the group G^{II} excludes their dangerous interaction with matter that could lead to proton decay.

Also we investigate the dependency on the fourth generation mass M_4 and take into account the SUSY threshold.

In general the RGE with thresholds between M_Z and M_G are as follows ($k^2 \equiv 1$ for non-flipped $SU(5)$):

$$\begin{aligned}
\alpha_3^{-1} &= \alpha_5^{-1} + \frac{1}{2\pi}\left(z + b_3 \ln M_G - b_3^0 \ln M_Z\right) \\
\alpha_2^{-1} &= \alpha_5^{-1} + \frac{1}{2\pi}\left(y + b_2 \ln M_G - b_2^0 \ln M_Z\right) \\
\frac{25k^2}{k^2+24}\,\alpha_1^{-1} &= \alpha_5^{-1} + \frac{1}{2\pi}\left(x + \bar{b}_1 \ln M_G - \bar{b}_1^0 \ln M_Z\right)
\end{aligned} \qquad (34)$$

$\bar{b}_1,\ b_2,\ b_3$ denote beta function coefficients for the corresponding coupling constants that take into account all fields below M_G scale. Similarly $\bar{b}_1^0,\ b_2^0,\ b_3^0$ are beta function coefficients that take all fields of the MSSM excluding superpartners. We also introduce thresholds factors $x,\ y,\ z$ that depend on various thresholds M_I:

$$x, y, z = -\sum_I \Delta b_{1,2,3}^I \ \ln M_I$$

In particular we are going to consider SUSY threshold M_{SUSY}, 4th generation masses M_4 and effective masses of addition doublets and triplets $M_{2,3}$.

In the context of Model 1 we have

$$\alpha_5^I = \alpha_5(1+q^2), \quad q = g_5^I / g_5^{II}, \quad g_5^{I2} = \frac{16\pi^2 g_{str}^2}{16\pi^2 - g_{str}^2 b_5^I \ln\left(\frac{M_G^2}{M_{str}^2}\right)}$$

and the similar formulae for $g_5^{II},\ g_1^{I},\ g_1^{II}$.

In this scheme the representations like $(\mathbf{5},\mathbf{3})$ of the Model 1 break into the equal number of vector-like triplets and doublets ($(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{2})$ under the $SU(3)\times SU(2)$ group). We consider the case when the fields in one $(\mathbf{5},\mathbf{3})$ representation with the masses below M_G are from the second group $U(5)\times U(3)$. (In this case we have no problems with the dimension four, five, six operators of proton decay in Higgs sector.) Hence in addition to the MSSM Higgs vector-like doublet we have 2 doublets and 3 triplets.

Below are the values of the b coefficients for our case:

$$\bar{b}_1=\frac{275k^2}{24+k^2}\ ,\ b_2=5\ ,\ b_3=2\ ,\ \bar{b}_1^0=\frac{105k^2}{24+k^2}\ ,\ b_2^0=-3\ ,\ b_3^0=-7\ .$$

Considering flipped $SU(5)$ case we have to pay attention to the consistency of the value of k^2 derived from (34) and from RGE of the string coupling g_{str} above the M_G scale. We use b coefficients from (25, 26).

From the system (34) we can get a set of appropriate masses in the range of M_Z-M_G and values of $\alpha_5,\ k^2$ and q^2 as well. But then we should apply RGE between the string scale and M_G scale to check out whether this values are consistent. This equations give us $k^2<1$. Our calculations show that with $k^2\leq 1$ for flipped $SU(5)$ in the Model 1 one cannot get appropriate values for M_{SUSY}, $M_{2,3,4}$, M_G (i.e. that are within range $M_{SUSY}-M_G$) that are consistent with string RGE.

For non-flipped case we apparently obtain the demand that constants $\alpha_{1,2,3}$ converge to one point (that is equivalent to $k^2\equiv 1$) which is consistent with RGE in the framework of the MSSM-like models.

For this case we consider the b) breaking way of $[SU(5)\times U(1)]^{\otimes 2}$. We can consider the cases c) and d) as a limits ($x\ll y$ and $x\gg y$). As it follows from our analysis there exists a range of parameters values (threshold masses) that make system (34) consistent and we have an appropriate hierarchy of the scales.

The maximum value of M_G one can obtain in this case is $M_G\sim 1.3\cdot 10^{16}$ GeV. The mutual dependencies of the threshold masses are shown on Fig. 1 where $M_{2,3}$ are effective masses because the equation (34) depends on them only ($M_2=\sqrt{M_2^{(1)}M_2^{(2)}}$; $M_3=\sqrt[3]{M_2^{(1)}M_2^{(2)}M_2^{(3)}}$).

Note that the Higgs triplets and doublets considered obtain their masses via F^2-term of the field $\hat{\Phi}_{(5,1;1,3)}$ (see the first term in the W_3 (5)). Hence $M_3^2\sim|x|^2$; $M_2^2\sim|y|^2$. At the same time squared masses of the vector bosons of the broken groups $SU(3)$ and $SU(2)$ are proportional to $(g_I^2+g_{II}^2)|x|^2$; $(g_I^2+g_{II}^2)|y|^2$. This means that in general above the thresholds $M_{2,3}$ we should take into account the restoration of the symmetry $SU(n)\longrightarrow SU(n)_I\times SU(n)_{II},\quad n=2,3$. I.e. our plots are correct only in the region with M_2 close to M_3. The other cases demand more careful accounting of the symmetry restoration thresholds. This question is currently under consideration and we will present the results in the future.

The horizontal gauge constant on the scale of 1 TeV for first or second $SU(3)_H$ group (depending on which of them will survive after suppression of the $U(1)$ anomaly) appears to be of the order

$$ {g^I_{3H}}^2\Big(O(1\mathrm{TeV})\Big) \approx 0.10 \div 0.11 \ , \quad {g^{II}_{3H}}^2\Big(O(1\mathrm{TeV})\Big) \approx 0.15 \div 0.17 \ . $$

The calculations for our model for different breaking chains show that for evaluation of intensity of a processes with a gauge horizontal bosons at low energies we can use inequality $\alpha_{3H}(\mu) \le \alpha_2(\mu)$.

This work was supported in part by Russian Foundation for Basic Researches, grant No 95-02-06121a. One of us (G.G.V.) would like to thank INFN for financial support, especially Prof. G. Puglierin and Prof. C. Voci, for hospitality during his stay in Padova. It is a great pleasure for him to thank Profs. G. Costa, and A. Masiero for useful discussions, for help and support. Many helpful advises and interesting discussions with R. Barbieri, J. Ellis, H. Dahmen, K. Dienes, J. Gasser, L. Ibanez, H. Leutwyler, P. Minkowsky, N.Paver, G.Ross and F. Zwirner during this work are also acknowledged.

References

[1] M.B.Green and J.H.Schwarz *Phys.Lett.B* **149** (1984) 117.

[2] D.J.Gross, J.A.Harvey, E.Martinec and R.Rohm, *Nucl.Phys.* **B256** (1985) 253.

[3] H.Kawai, D. Lewellen and S.Tye, *Phys.Rev.Lett.* **57** (1986) 1832; *Nucl.Phys.* **288B** (1987) 1.

[4] I.Antoniadis, C.Bachas, C.Kounnas, *Nucl.Phys.* **B289** (1988) 87; I.Antoniadis, C.Bachas, *Nucl.Phys.* **B298** (1988) 586.

[5] I.Antoniadis, J.Ellis, J.S.Hagelin and D.V.Nanopoulos, *Phys.Lett.* **B194** (1987) 231,**B208** (1988) 209, **B231** (1989) 65; J.Ellis, J.I. Lopez, D.V. Nanopoulos, *Phys.Lett.* **B245** (1990) 375; S.Kalara, J.I. Lopez, D.V. Nanopoulos, *Nucl.Phys.* **B353** (1991) 650.

[6] I. Antoniadis, G. Leontaris and J. Rizos, *Phys. Lett.* **B245** (1990) 161; A. Faraggi, *Phys.Lett.* **B274** (1992) 47, **B278** (1992) 131, **B302** (1993) 202, **B326** (1994) 62.

[7] S.M. Sergeev and G.G. Volkov, *preprint DFPD/TH/51* (1992); *Rus. Nucl. Phys.* **v.57** (1994) No 1; A.A.Maslikov, S.M. Sergeev, G.G.Volkov. *Phys.Lett.* **B328** (1994) 319; A.A.Maslikov, S.M. Sergeev, G.G.Volkov. *Phys.Rev.D15* **v.50** (1994) 7740; A. Maslikov, I. Naumov, G. Volkov *IJMP* **v.11A** 1117-1168, *hep-ph/ 9505318, hep-ph/9512429.*

[8] A.Font, L.E.Ibanez, F.Quevedo, *Nucl.Phys.* **B345** (1990) 389.

[9] R. Barbieri, G. Dvali, and S. Strumia, *Phys.Lett.* **B333** (1994) 79; R.N. Mohapatra, *hep-ph/9601203*; E.Ma, *Phys.Lett.***B344** (1995),164.

[10] A.N.Amaglobeli, A.R.Kereselidze, A.G.Liparteliani, G.G.Volkov, *Phys. Lett.* **B237** (1990) 417; A.A.Maslikov and G.G.Volkov, *Towards a Low Energy Quark-Lepton Family with Non-Abelian Gauge Symmetry*, *ENSLAPP-A-382/92*, (1992) Z. Berezhiani, *Problem of flavour in SUSY GUT and Horizontal Symmetry INFN-FE 03/96* (1996); V.A.Monich, B.V.Struminsky, G.G.Volkov, *Phys. Lett.* **B104** (1981) 382.

[11] S.M. Barr, *Phys. Lett.* **B112** (1982) 219; J.P. Derendinger, J.E. Kim and D.V. Nanopoulos, *Phys. Lett.* **B139**, (1984), 170.

[12] P. Ginsparg, *Phys.Lett..* **B197** (1987) 139.

[13] **a)** V.S. Kaplunovsky, *Nucl.Phys.* **B307** (1988) 145; Erratum: *Nucl.Phys* **B382** (1992) 436;
b) L.J. Dixon, V.S. Kaplunovsky, J. Lois, *Nucl.Phys.* **B355** (1991)649;
c) I. Antoniadis, J. Ellis, R. Lacaze and D.V. Nanopoulos, *Phys.Lett.***B268** (1991) 188.

[14] K.R. Dienes, A.E. Faraggi, *hep-th/9602045* (1996); *Nucl. Phys* **B457** (1995) 409.

[15] U. Amaldi et al, *Phys.Rev.* **D36** (1987)1385; G. Costa et al., *Nucl. Phys.* **B297** (1988) 244, J. Ellis and G.L. Fogli, *Phys. Lett.* **B213** (1988) 526; P. Langacker, *Phys.Rev.Lett.* **63** (1989) 1920; J. Ellis, S. Kelley and D.V. Nanopoulos *Phys. Lett.* **B249** (1990) 441, **260** (1991) 131; U. Amaldi, W. de Boer, H. Furstenau, *Phys. Lett* **260** (1991) 447.

[16] A. Schellekens, *Phys.Lett.* **B237** (1990) 363.

[17] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, *Phys.Lett.* **B115** (1982) 380.

[18] C.M.Hull, E.Witten, *Phys.Lett.* **B160** (1985) 398; T.Banks, L.J. Dixon, D. Friedan, E. Martinec, *Nucl.Phys.* **B299** (1988) 613.

[19] M.Dine, N.Seiberg and E.Witten, *Nucl.Phys.* **B289** (1987) 585.

[20] S. Kalara, J.L. Lopez, D.V. Nanopoulos, *Phys.Lett.* **B269** (1991), 84

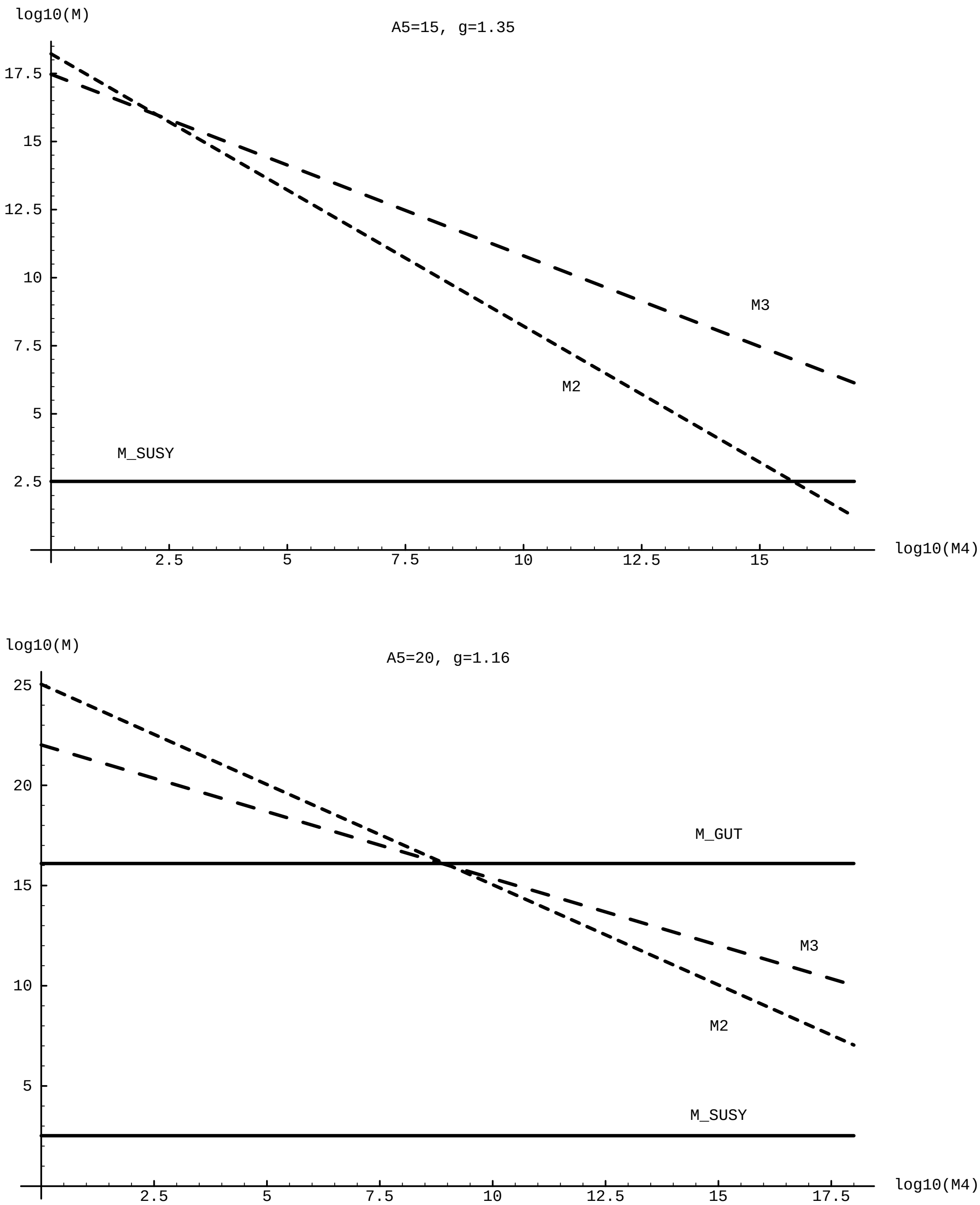

Figure 1: $M_{2,3}$ behavior. `A5`$= \alpha_5^{-1}$, $M_G = 1.26 \cdot 10^{16}$, `g` means g_{str}, $g_3^I = 0.32$, $g_3^{II} = 0.40$.