Iterative Detectors for Trellis-Code Multiple-Access

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***Abstract*—Trellis-code multiple-access (TCMA) is a narrowband multiple-access scheme based on trellis-coded modulation. There is no bandwidth expansion, so K users occupy the same bandwidth as one single user. The load of the system, in number of bits per channel use, is therefore much higher than the load in, for example, conventional code-division multiple-access systems. Interleavers are introduced as a new feature to separate the users. This implies that the maximum-likelihood sequence detector (MLSD) is now too complex to implement. Iterative detectors are therefore suggested as an alternative to the joint MLSD. The conventional interference cancellation (IC) detector has lower complexity than the MLSD, but its performance is shown to be far from acceptable. Even after a novel improvement of the IC detector, the performance is unsatisfactory. Instead of using IC, another iterative detector is suggested. This detector updates the branch metric for every iteration, and avoids the standard Gaussian approximation. Simulations show that the performance of this detector can be close to single-user performance, even when the interleaver and the phase offset are the only user-specific features in the TCMA system.**

***Index Terms*—Interference cancellation, iterative methods, multiple-access technique, trellis-code multiple-access, trellis-coded modulation.**

I. Introduction

With the explosive growth of mobile radio systems, the efficient use of available spectrum is becoming increasingly important. All conventional multiple-access (MA) techniques rely on bandwidth expansion in some form. Frequency- and time-division MA (FDMA, TDMA) accommodate multiple users by stacking individual users in time and/or frequency. Conventional single-user (SU) detection techniques are then usually applied. It follows that one user is dedicated a small fraction of the available resources for his sole use.

In code-division MA (CDMA), each user is assigned N times the resources normally required for an SU. The same resources are then shared by multiple users, where a total load of N users are considered very good applying conventional techniques, i.e., the same amount of resources that N single users would require. More advanced methods for coded CDMA have been developed based on iterative techniques using *a posteriori* probabilities (APP) [1]–[4]. These schemes have demonstrated SU performance for loads up to $2N$ users [1], thus increasing the bandwidth efficiency. Similar results have been shown in code-spread CDMA with interference cancellation (IC) [5], where low-rate convolutional coding is used for bandwidth expansion rather than traditional spreading code techniques.

In this paper, a narrowband MA scheme is suggested where multiple users effectively share the same bandwidth as required by an SU. This MA scheme is based on bandwidth-efficient trellis-coded modulation (TCM) and is termed trellis-code multiple-access (TCMA) [6]–[9]. Instead of using a unique spreading sequence for each user as in CDMA, the MA is provided entirely by user-specific TCM.

The principle advantage of TCMA is that there is *no bandwidth expansion* ($N = 1$) relative to the SU case, in contrast to other conventional MA systems. The total bandwidth of the potential K users is the same as the bandwidth for one user. For example, if an SU is transmitting one bit in T seconds, using a bandwidth of W Hz, a TCMA system with K users is able to transmit K bits in the same time and still only use a total bandwidth of W Hz. This technique can, of course, be combined with existing MA schemes to increase the spectral efficiency by a factor of K, the number of users in the TCMA system.

The TCMA concept was first presented in [6], where the joint maximum-likelihood sequence detector (MLSD) was derived, together with general union bounds on the bit error performance. The drawback of TCMA is the trellis complexity (the number of states in the detector trellis), which grows exponentially with the number of users when considering joint MLSD for all users. The group of users sees a Gaussian channel (when transmitting over an additive white Gaussian noise (AWGN) channel), having a certain capacity C. As long as the users jointly do not exceed this capacity, error-free communication is, in principle, possible [10], [11] even without a bandwidth expansion relative to the SU case.

In this paper, a series of iterative detector structures for TCMA are suggested and compared at a trellis complexity which does not grow exponentially with the number of users. The structures are based on the forward–backward algorithm [12]. The first structure considered is based on an iterative IC technique similar in principle to the structures suggested in [1], [3], and [13] for coded CDMA. This problem has also been considered in [14], where an iterative detector based on cross entropies is suggested. Further, an improved cancellation algorithm is suggested based on better initialization. The Gaussian approximation (GA) usually used in all iterations is here

Paper approved by C. Schlegel, the Editor for Coding Theory and Techniques of the IEEE Communications Society. Manuscript received August 31, 2000; revised May 23, 2001, August 6, 2001, and January 31, 2002. This work was supported in part by NUTEK under Grant 99-06734, and by the Swedish Research Council for Engineering Sciences. The work of T. M. Aulin was supported by the Swedish Foundation for Strategic Research under an Individual Grant. This paper was presented in part at the IEEE International Conference on Communications (ICC ’01), Helsinki, Finland, June 11–14, 2001.

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Publisher Item Identifier 10.1109/TCOMM.2002.802563.

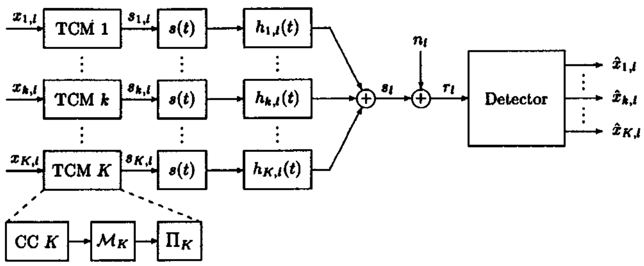

Fig. 1. System model of a general TCMA system with K users.

replaced by another approximation in the first iteration step. Finally, a detector structure with no cancellation is modified to suit TCMA. This approach is based on iterative improvements of the estimated probability density function (pdf) used to generate branch transition metrics in the forward-backward algorithm, and is similar to the structures suggested for CDMA in [2], [3], [15], and [16].

The paper is organized as follows. In Section II, the concept and system model of TCMA is explained in detail. In Section III, a brief review of the forward-backward algorithm is included to accommodate the description of the iterative detector structures in Sections IV and V. In Section VI, a series of numerical examples are presented to demonstrate the excellent performance observed for the suggested detectors, and in Section VII, concluding remarks round off the paper.

II. System Model

The TCMA system [6]–[8] in Fig. 1 consists of K users transmitting independent and identically distributed (i.i.d.) binary data. The data from user k is collected in a column vector of size L, and will be referred to as the *data block* of user k, $\mathbf{X}_k = (x_{k,1}, x_{k,2}, \ldots, x_{k,L})^T$. Each component in $\mathbf{X}_k$; $k = 1, 2, \ldots, K$, is a 2^{b_k}-ary symbol $x_{k,l}$, referred to as the *data symbol*. This data symbol is representing the b_k binary digits from user k at time index l. The data-symbol alphabet is selected such that only one data symbol enters the corresponding TCM encoder at each time interval [11], [17].

Each user has a TCM scheme consisting of a convolutional code (CC), a memoryless mapper $\mathcal{M}_k$, and a random symbol interleaver Π_k, as shown in Fig. 1. Since the interleaver is a symbol interleaver, it can swap places with the mapper without influencing the TCM scheme. Each TCM scheme has A_k states, i.e., the number of states in the CC. The size of the symbol interleaver is L, the same size as the data block. The 2^{b_k}-ary data symbol from user k at time index l, $x_{k,l}$, is passed through the convolutional encoder and appropriately mapped onto an M_k-ary two-dimensional (2–D) symbol $s_{k,l}$, referred to as the *user symbol*, according to the corresponding TCM encoder [11], [17]. All users have their own set of user symbols $s_{k,l} \in \{u_{k,1}, u_{k,2}, \ldots, u_{k,M_k}\}$. The user symbols here are complex numbers with a total energy of $E_{s,k} = b_k E_{b,k}$, where $E_{b,k}$ is the energy of one information bit from user k. The code rate of the CC for user k is $R_{c,k} = b_k/m_k$, where $m_k = \log_2(M_k)$. Note that *one* data symbol, $x_{k,l}$, representing b_k information bits from user k, is mapped onto *one* M_k-ary user symbol $s_{k,l}$. The user symbols are then passed through the symbol interleaver Π_k, as shown in Fig. 1, and modulated onto a complex continuous-time waveform $s_k(t-(l-1)T)$; $l = 1, 2, \ldots, L$, every symbol interval of T seconds. This waveform has unit energy and it is equal to zero outside the interval $0 < t < T$.

The crystal oscillator in the receiver has usually some frequency offset from the crystal oscillator in the transmitter. This will cause each user to have a phase offset $\Delta\phi$ [18]. The phase offset during one block of L symbols depends on the accuracy of the oscillators, the carrier frequency, the block length, and the symbol interval, $\Delta\phi = 2\pi \Delta f LT$ radians [19]. For example, if the accuracy of the oscillator (the tolerance value) is 1 ppm, the carrier frequency 1 GHz, the block length $L = 1000$, and the symbol rate 10^6 symbols per second, the phase offset for the whole block will be $\Delta\phi = 2\pi$ radians. If the accuracy of the oscillator is lower or if the carrier frequency is higher, the offset will be larger. The phase offset will also be larger if the block length is larger, or if the symbol rate is lower.

The waveform of user k is affected by the corresponding channel impulse response $h_{k,l}(t)$, shown in Fig. 1. This is a complex-valued function with energy $w_{k,l}^2$. Based on the discussion above, the channel impulse response is here modeled as a delayed Dirac impulse with a constant phase offset during one symbol interval of T seconds

$$h_{k,l}(t) = \delta(t - \tau_k)\, w_{k,l} e^{j\phi_{k,l}}, \quad \left(j = \sqrt{-1}\right). \tag{1}$$

Here, $\delta(\cdot)$ is the Dirac impulse response [20] and τ_k is the transmission delay of user k. The amplitude and the phase offset of user k at time index l is denoted by $w_{k,l}$ and $\phi_{k,l}$, respectively. From now on, the transmission delay is assumed to be equal for all users, i.e., a totally synchronous system. This assumption can be made since TCMA is a narrowband system without any spreading. The synchronization at bit level has successfully been handled in global systems for mobile communications (GSM), using timing advances and guard periods between the time slots. Without loss of generality, the transmission delay is chosen to zero for all users, $\tau_k = 0$.

The signals from all users are superimposed on the AWGN channel, leading to the following complex continuous-time baseband signal observed at the receiver

$$r(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} s_k\left(t - (l-1)T\right) w_{k,l} e^{j\phi_{k,l}} s_{k,l} + n(t). \tag{2}$$

$n(t)$ is here a complex baseband representation of thermal noise with a double-sided power spectral density of $N_0/2$ W/Hz [18]. Note that in (2), $s_{k,l}$ is the symbol of user k transmitted at time index l as shown in Fig. 1. In other words, it is not directly related to the data symbol $x_{k,l}$ anymore due to the interleaver. From now on, this is the definition of $s_{k,l}$. The signal-to-noise ratio (SNR) for user k is defined as $\text{SNR}_k \triangleq (E_{b,k}/N_0)$. The whole data block is transmitted in LT seconds.

The model in (2) is similar to the models given in [14], [16], [21], and [22]. In these cases, there are no phase offset ($\phi_{k,l} = 0$), and the modulation is binary phase-shift keying (BPSK) for all users. With the model in (2), each user can have their own set of user symbols, modulating waveform, amplitude (near-far

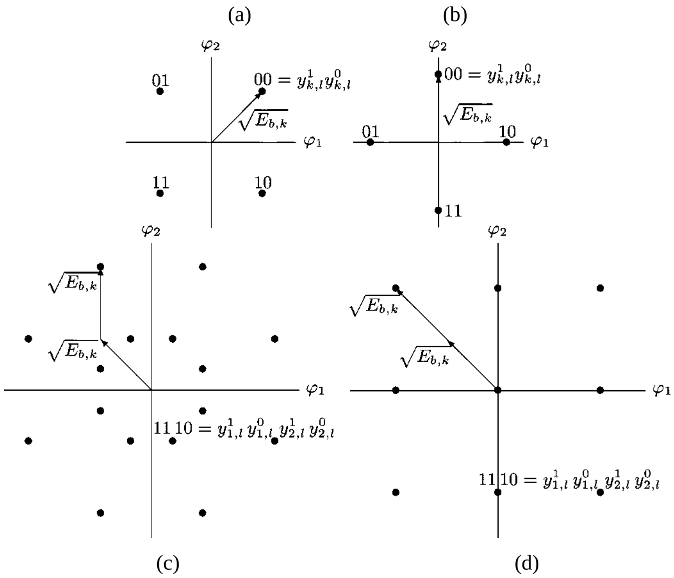

Fig. 2. QPSK constellation with Gray mapping. (a) $\phi_{k,l} = 0$. (b) $\phi_{k,l} = \pi/4$. (c) Super-symbol constellation when $\phi_{1,l} = 0$ and $\phi_{2,l} = \pi/4$. (d) Super-symbol constellation when $\phi_{1,l} = \phi_{2,l} = 0$.

scenario), and phase offset. It is also relatively straightforward to extend the model to a multipath channel. This is done by replacing the channel impulse response in (1) with a sum over delayed Dirac impulses.

In a conventional CDMA system, $s_k(t)$ is the spreading modulating waveform and it is unique for each user. In TCMA, there is no spreading in the frequency domain. The modulating waveform uses the same bandwidth as an SU TCM scheme, depending on the chosen modulation. From now on, this modulating waveform is identical for all users and denoted by $s(t)$. In other words, the modulating waveforms of the users, in one time interval, are now fully correlated, $\rho = 1$. This is in direct contrast to traditional MA systems, e.g., conventional CDMA [1], [5], [15], where $\rho < 1$. Since there is no spreading in the TCMA system, only one complex filter is needed (one real-valued filter for each basis function) in contrast to CDMA systems where K matched filters are used [2], [14]. In the CDMA case, a K-dimensional signal space is created through bandwidth expansion. For TCMA, as many users as possible are stacked within the 2-D signal space, provided by conventional, bandwidth-efficient SU techniques. The received observables can be represented in discrete time, using the vector channel concept [20]

$$r_l = \sum_{q=1}^{K} w_{q,l} e^{j\phi_{q,l}} s_{q,l} + n_l \triangleq s_l + n_l. \tag{3}$$

The sum of the superimposed user symbols $s_{k,l}$, including amplitude and phase offset over all K users in (3), is denoted by s_l and is referred to as the *super symbol*. This super-symbol constellation is the joint constellation of all user-specific constellations. It follows that the maximum number of signal points in the super-symbol constellation in (3) is the product of the number of signal points for the users, $M = \prod_{k=1}^{K} M_k$. The set of super symbols is therefore $s_l \in \{y_1, y_2, \ldots, y_M\}$.

Figs. 2(a) and (b) show examples of Gray-encoded quaternary phase-shift keying (QPSK) constellations ($M_k = 4$) with different phase offset (their rotation differs with $\pi/4$ radians). $y_{k,l}^1$ and $y_{k,l}^0$ are the two coded bits delivered from, in this case, the rate–1/2 CC of user k at time index l.

Fig. 2(c) illustrates the super-symbol constellation for a TCMA system with two users, where the users have equal amplitude, $w_{1,l} = w_{2,l}$, but different phase offset. The super-symbol constellation in Fig. 2(c) has only distinct (nonambiguous) signal points. On the other hand, if both users have equal amplitude and phase offset, e.g., the constellation in Fig. 2(a), the super-symbol constellation has only nine distinct points, as shown in Fig. 2(d). Only the four corner points are the result of a unique combination of user symbols. The other signal points are the result of four (the signal point in the middle) and two (the signal points at the edges) different combinations of symbols from the two users [7]. Because of the ambiguous points, the super-symbol constellation in Fig. 2(d) gives the lowest constellation-constrained capacity [7], [9] and also worst performance in bit error rate (BER) among all combinations of two QPSK users with equal energy. The super-symbol constellation in Fig. 2(c) gives the best performance in BER, due to the maximum possible separation of the signal points. The four constellations in Figs. 2(a)–(d) are just examples. In a real system, these constellations will appear with a probability of zero, since the phase offset can be any value between 0 and 2π.

The received observations (3) can be used in different ways to make a decision, $\hat{x}_{k,l}$, on the data symbol of user k. To be able to make decisions, something has to be unique to each individual user. This will be referred to as the unique user feature (UUF). There are three possible UUFs for each user; the CC, the constellation (the mapper including the amplitude and the phase offset), and the interleaver as shown in Fig. 1. They can be combined in different ways as long as all users have a unique combination of the three UUFs. For example, all users can have the same CC, but different constellations and different interleavers. Another example is when either the constellation or the interleaver is the only UUF [7].

The TCMA model in [6] has no interleaver. In that case, the system can be identified as a single TCM scheme driven by the data blocks $\mathbf{X}_k$ in parallel. This TCM is referred to as the *super TCM* and associated with the *super-data block*; $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_k, \ldots, \mathbf{X}_K) \triangleq (x_1, x_2, \ldots, x_L)^T$, where each component in the column vector $\mathbf{X}$ is a 2^b-ary symbol, referred to as the *super-data symbol*. This super-data symbol, x_l, is obtained as the collection of the K data symbols from all users at time index l, $x_l \triangleq x_{1,l}\, x_{2,l} \ldots x_{K,l}$, and it is representing a total of b binary digits. It follows that b is the sum of all b_k, $k = 1, 2, \ldots, K$. The 2^b-ary super-data symbol at time index l is mapped, with help from all the K TCM schemes, onto an M-ary super-symbol constellation as defined in (3). It is now possible to perform an MLSD on the received channel observations and extract the super-data sequence $\widehat{\mathbf{X}}$ as described in [6].

The trellis complexity of the joint MLSD is proportional to the number of states in the corresponding super TCM. The number of states in the super TCM is $A = \prod_{k=1}^{K} A_k$, where A_k is the number of states in the TCM of user k. The trellis complexity of this joint MLSD is, therefore, exponential with respect to the number of users. If interleavers are inserted in the system as in Fig. 1, they will extend the memory of the individual TCM schemes and thus, also the super TCM scheme. For even moderately sized interleavers, the corresponding super TCM scheme is prohibitively complex for MLSD [23]. The number of states in the super TCM is now growing exponentially with both the number of users and the size of the interleaver. Less complex iterative detector techniques are, therefore, considered here as potential alternatives.

III. The APP Algorithm

First, a brief review of the APP algorithm, known as the forward-backward algorithm [12], is given in order to introduce the necessary notation for the iterative treatment in Sections IV and V. It has been shown (e.g., [1]–[3]) that iterative detectors based on the APP of the user symbols provide excellent performance.

If detection of user k is considered, the channel observation at time index l can, according to (3), be written as

$$\begin{aligned} r_l =& w_{k,l} e^{j\phi_{k,l}} s_{k,l} + \sum_{\substack{q=1\\q\neq k}}^{K} w_{q,l} e^{j\phi_{q,l}} s_{q,l} + n_l \\ \triangleq& w_{k,l} e^{j\phi_{k,l}} s_{k,l} + z_{k,l}. \end{aligned} \tag{4}$$

Here, $z_{k,l}$ contains both the additive noise and the multiple-access interference (MAI) from all users except user k.

The algorithm calculates the APP for both the data symbols, $\Pr(x_{k,l}|\mathbf{r})$, and for the user symbols, $\Pr(s_{k,l}|\mathbf{r})$. Here, $\mathbf{r} = (r_1, r_2, \ldots, r_L)^T$ is a column vector of the received observables in (4). These APPs are recursively calculated from the branch metric (BM) used in the algorithm [7], [12]

$$\gamma_{k,l}(v_k) \triangleq p_r(r_l | s_{k,l} = u_{k,v_k}) = p_{z_k}\left(r_l - w_{k,l} e^{j\phi_{k,l}} u_{k,v_k}\right) \tag{5}$$

where $p_{z_k}(\cdot)$ is the joint pdf of the additive noise and the MAI in (4). This pdf will be referred to as the branch metric function (BMF).

In an SU system, $K = 1$, the BMF is equal to the pdf of the additive noise. In a multiuser system, $K > 1$, where the amplitude, $w_{k,l}$, and the phase offset, $\phi_{k,l}$, are known for all users and time index, the conditional BMF can be expressed as

$$\begin{aligned} &p_{z_k}\left(\alpha | s_{q,l} = u_{q,v_q};\ \forall q \neq k\right) \\ &= p_n\left(\alpha - \sum_{\substack{q=1\\q\neq k}}^{K} w_{q,l} e^{j\phi_{q,l}} u_{q,v_q}\right). \end{aligned} \tag{6}$$

Here, $p_n(\cdot)$ is the 2-D (or complex) Gaussian pdf of the additive noise and the conditional BMF is just a biased version of this pdf.

The BMF for user k can now be expressed as a sum of the conditional BMF over all possible user symbols for the other $K-1$ users

$$\begin{aligned} p_{z_k}(\alpha) = &\sum_{v_1=1}^{M_1} \cdots \sum_{v_{k-1}=1}^{M_{k-1}} \sum_{v_{k+1}=1}^{M_{k+1}} \cdots \sum_{v_K=1}^{M_K} \\ &p_n\left(\alpha - \sum_{\substack{q=1\\q\neq k}}^{K} w_{q,l} e^{j\phi_{q,l}} u_{q,v_q}\right) \prod_{\substack{d=1\\d\neq k}}^{K} \Pr(s_{d,l} = u_{d,v_d}). \end{aligned} \tag{7}$$

$\Pr(s_{d,l} = u_{d,v_d})$ is the *a priori* probability that user k has transmitted user symbol u_{d,v_d}. The general expression of the BMF in (7) is similar to expressions in [2] and [3], but in this case, every user can have their own symbol alphabet, amplitude, and phase offset. For CDMA, this strategy is usually too complex due to a large number of users, typically in the order of N (the bandwidth expansion factor or processing gain). In these cases, the concept of IC has been successfully applied. The IC technique is developed for TCMA in Section IV together with a simple modification which leads to noticeable improvements.

IV. The IC Detector

To reduce the complexity of the detector, IC can be used as an alternative to MLSD. IC detectors have successfully been used to detect the users in other MA systems [1], [3], [13]. The main idea for detecting user k is that soft tentative decisions of all the other users except user k are subtracted from the channel observation

$$\tilde{r}_{k,l}^{i} = r_l - \sum_{\substack{q=1\\q\neq k}}^{K} w_{q,l} e^{j\phi_{q,l}} \tilde{s}_{q,l}^{i}. \tag{8}$$

$\tilde{s}_{q,l}^{i}$ denotes the soft tentative decisions of user q and time index l at iteration i.[1] $\tilde{r}_{k,l}^{i}$ is first passed through a deinterleaver, according to the interleaver in the corresponding TCM scheme of user k, before an SU detector is applied on the resulting statistics.

The APPs for the user symbols are used to calculate the soft tentative decisions in (8). This is done by taking the sum of the M_k different symbols of user k weighted by the matching APP [7]. This approach has also been used for BPSK symbols in CDMA [1], [3]. After calculation, the soft tentative decision is delivered to the detectors for the other $K-1$ users.

The most commonly used way, if not the only way, is to approximate the BMF with a GA [1], [13]. It makes the assumption that $z_{k,l}$ in (4) is a biased complex Gaussian random variable with variance $\sigma_{k,i}^2 + \sigma_n^2$. Here, $\sigma_{k,i}^2$ is an approximation of the remaining variance of the MAI at iteration i [7] and $\sigma_n^2 = N_0$. After some iterations, $\tilde{s}_{q,l}^{i}$ is hopefully getting closer to the correct user symbol and then $\sigma_{k,i}^2$ will be close to zero. In that case, the BMF is approaching the conditional BMF (6), which is exactly the main object [7].

The GA of the BMF for the first, second, and third iteration is shown in Fig. 3(a)–(c). This example is for a two-user QPSK system [7] at an SNR of 5 dB. It can be seen in Fig. 3(a)–(c) that the peak of the approximation of the BMF is getting closer and closer to one of the four possible user symbols of the other user, i.e., the conditional BMF (6).

The conventional IC detector can be improved by introducing a different approximation of the BMF in the first iteration, before any cancellation is made. Instead of using the GA for all iterations, it will be used for every iteration except the first one. With perfect knowledge of the other user, the BMF would be a scaled version of one of the four bell-shaped peaks in Fig. 3(d),

[1] Note that in (8) and everywhere else, the data from the ith iteration is used. If this data is not available, it is replaced by the data from the previous iteration; $i-1$.

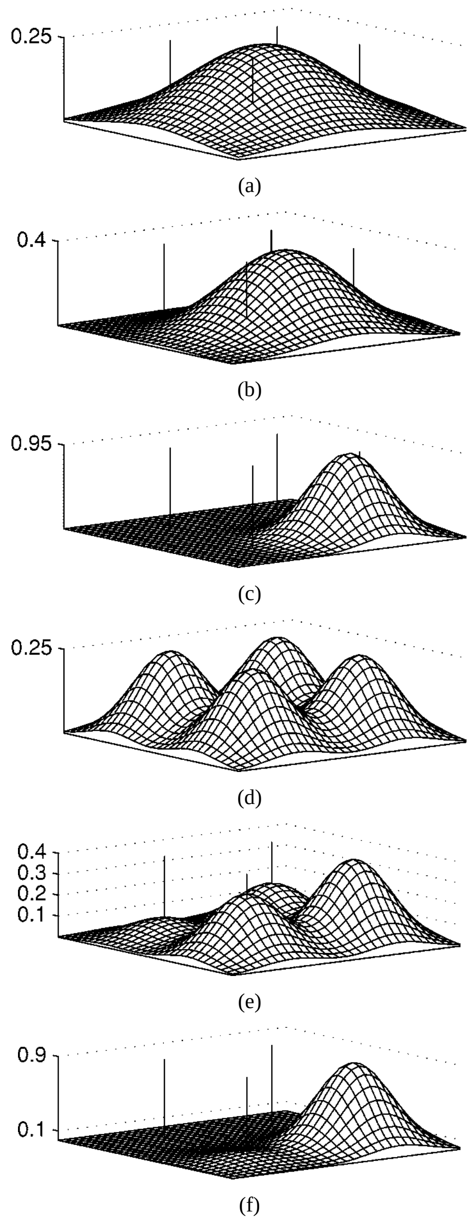

Fig. 3. Approximations of the BMF at an SNR of 5 dB. (a)–(c) GA for the first, second, and third iteration in the IC detector. (d)–(f) First, second, and third iteration in the BMU detector.

i.e., the conditional BMF, and the performance would be equal to the performance in the corresponding SU system. It is obvious that the GA for the first iteration shown in Fig. 3(a) is not a very good approximation, especially at the point in the middle of the four peaks in Fig. 3(d). It is assumed that all transmitted symbols are equally likely. The approach suggested here is, therefore, to approximate the BMF in the first iteration with the weighted sum of all possible conditional BMFs, as shown in Fig. 3(d).

In a general TCMA system with K users, this approximation is the same as putting the *a priori* probability in (7) equally likely $\Pr(s_{d,l} = u_{d,v_d}) = 1/M_d$, for all users $d = 1, 2, \ldots, K$, time index $l = 1, 2, \ldots, L$, and symbols u_{d,v_d}; $v_d = 1, 2, \ldots, M_d$. This improvement can only be used in the first iteration of the IC, when the soft tentative decisions of the other users are still zero, i.e., before any cancellation is made. It will be shown with an example in Section VI that this improved IC (IIC) detector is better than the conventional IC detector using the GA. The only difference is the approximation of the BMF in the first iteration, but it is enough to give the detection a push in the right direction, resulting in a better final performance for the IIC, compared to the conventional IC. This improvement could probably also improve the performance for the first iteration in a CDMA system using IC, e.g., [1], and [3].

The trellis complexity of the IC grows linearly with the number of users (K SU detectors), in contrast to exponentially as in the MLSD case. On the other hand, the soft tentative decision has to be calculated and the variance of the remaining MAI must be approximated in some way.

V. BM Update Detector

Instead of using the GA and IC, another method to approximate the BMF and to detect the users in a TCMA system is proposed here. It is known that the APP algorithm delivers the APPs of the user symbols at iteration i for every time index l, $\Pr^i(s_{d,l} = u_{d,v_d}|\mathbf{r})$, $v_d = 1, 2, \ldots, M_d$. These can now be used to approximate the *a priori* probabilities in (7) by simply replacing $\Pr(s_{d,l} = u_{d,v_d})$ with $\Pr^i(s_{d,l} = u_{d,v_d}|\mathbf{r})$, [2], [3], [7]. The BMF is now changing for every iteration, and it is hopefully approaching the conditional BMF (6). This detector approach is here called the branch metric update (BMU) detector.

The BM in the BMU detector for user k, time index l, and iteration i can now, as shown in Section III, be expressed as $\gamma_{k,l}^i(v_k) = p_{z_k}^i(r_l - w_{k,l}e^{j\phi_{k,l}}u_{k,v_k})$. In contrast to similar detectors in CDMA systems [2], [3], the BM is in this case only depending on the received observable, not on the output of a user-specific matched filter. Since there is no bandwidth expansion ($N = 1$, $\rho = 1$) in a TCMA system, the additive noise is still white after the filter (3). As soon as there is another correlation between the waveforms ($\rho \neq 1$), the noise is no longer white, and the pdf in (7) is a multidimensional Gaussian pdf with correlated variables [2], [3], [21], [22].

The main difference between the well known IC detector [1], [3], [13] and the BMU detector, is that the observations fed into the BMU detector are exactly the same for every user. Here, no cancellation is made and there is no reason to approximate the variance of the MAI, $\sigma_{k,i}^2$. As in the IIC detector, the symbols are initialized to be equally probable for the first iteration; $\Pr^1(s_{d,l} = u_{d,v_d}|\mathbf{r}) = (1/M_d)$.

An example of the approximation of the BMF used in the BMU detector for the first, second, and third iteration in a two-user QPSK system is shown in Figs. 3(d)–(f). The different strategies between the IC detector and the BMU detector of approaching the conditional BMF can be seen by comparing Figs. 3(a)–(c) with Figs. 3(d)–(f). The IC detector has only one moving Gaussian pdf with decreasing variance. The BMU detector has a sum of several weighted time-varying Gaussian pdfs with constant variance in fixed positions.

The trellis complexity of the BMU detector is, as for the IC detector, linear with respect to the number of users. The drawback is that the metric (7) is now a sum of M/M_k terms, where M grows exponentially with the number of users. However, the number of users in a TCMA system is considerably smaller than in a conventional CDMA system. This is because a TCMA system only uses the same time and frequency resources as required by one user. Fundamental capacity limits and convergence analysis then dictate that only a small number of users can be accommodated within these resources [7], [9]. On the other hand, the trellis complexity reduction is big compared to the MLSD trellis complexity. The BMU detector can therefore be a potential alternative to IC in TCMA systems.

VI. Simulation Results

The performances of the different detectors described earlier are examined and compared in this section. All users, in the examples here, are exclusively using CC(5,7) in octal notation ($R_{c,k} = 1/2$ and $A_k = 4$ states) together with a QPSK mapper, see Figs. 2(a)–(b). Different CCs, together with other symbol constellations (e.g., 8PSK and 16QAM), are further evaluated in [7]. The performance is measured in BER versus SNR_k. In all examples following, the SNR is equal for all users. Therefore, $\text{SNR} \triangleq \text{SNR}_k$ for all $k = 1, 2, \ldots, K$. The size of the data block and the symbol interleaver is chosen to $L = 1000$ in all examples. Results in [7] show that increasing the size of the interleaver only makes the slope of the performance curve steeper over a certain threshold, which has also been observed in systems using conventional turbo codes [23]. For reasons of simplicity, it is assumed that all users have the same received energy, $w_{k,l} = 1$. Near–far scenarios where $w_{k,l} \neq 1$ have been investigated in [7].

The phase offsets for two received adjacent symbols from the same user are probably not independent, but the example in Section II showed that the total phase offset for a whole block of L symbols can easily be 2π radians. The deinterleaver will, in some sense, make the phase offsets between two adjacent symbols entering the APP block independent. Instead of using system parameters to model the change of the phase offset between adjacent symbols on the channel, the phase offsets in (3) are here assumed to be independent, since that is the case after the deinterleaver anyway. In the simulations in this section, the phase offset is modeled as a uniform random variable between 0 and 2π. Here, it is assumed that the phase offset can be estimated and therefore, it is known at the receiver. Successive methods are used for the iterative detection of the users. This means that the APPs calculated in the first iteration of user one are used in the first iteration of user two, and so on.

The SU performance is used as a reference in the examples. The SU performance is the performance when one user is alone in the system, and it is conjectured to be the lower bound on the performance of the same user in a multiuser TCMA system. In the performance figures below, the SU performance is marked with a dashed line.

A. Identical Interleavers

In the first two-user example, both users have identical interleavers, and that is the same as removing the interleavers, since the AWGN channel is memoryless [7]. The difference between the users, the UUF, is, in this system, the constellation (phase offset) alone. The super trellis has $A = A_1 \cdot A_2 = 16$ states, and the super-symbol constellation $M = M_1 \cdot M_2 = 16$ points.

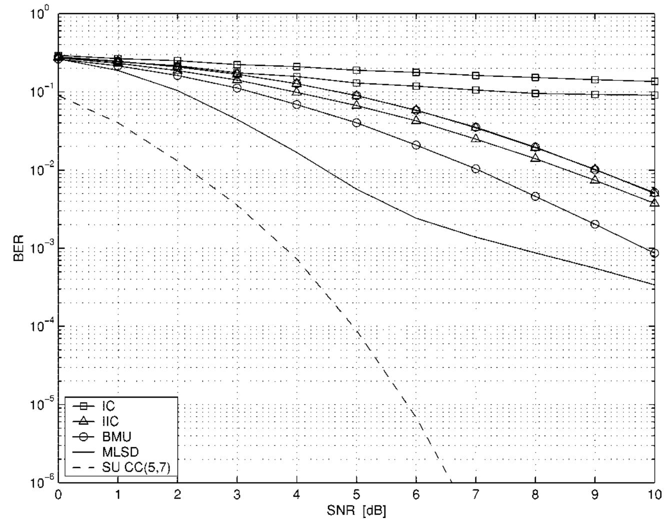

Fig. 4. The performance of user one after one and two iterations in a two-user QPSK system with CC(5,7), identical interleavers, and random phase offset.

Here, the optimal MLSD can be used because there are no interleavers, and only a few number of states in the super TCM.

Fig. 4 shows the performance of the IC detector, the IIC detector, the BMU detector and the MLSD. Iterations one and two are shown, but only for user one, since user two has similar performance. No further improvement of the performance is observed after the second iteration. The difference between the SU performance and the MLSD performance is significant.

The performance of the IC detector is very poor in this case; a BER of 9% at an SNR of 10 dB. The conclusion is that the conventional IC detector is not appropriate for this system. The simple cancellation strategy is not sufficiently powerful to resolve the severe MAI. The improvement in the performance when the IIC detector is used is obvious, but the performance is still very poor. The performance can be further improved by using the BMU detector. Fig. 4 shows that the performance of the BMU detector after the first iteration is equal to the performance of the IIC detector. This is so, since exactly the same BMF is used in both these detectors for the first iteration as described earlier, see Fig. 3(d). The performance of the BMU detector after the second iteration is around 2 dB better than the IIC performance, but there is still a gap down to the MLSD performance.

B. User-Unique Interleavers

If the users have different interleavers, the MLSD performance would most certainly be improved, but too complex to implement [23]. Fig. 5 shows the performance of user one for iterations one and five with the three iterative detectors. The conditions are the same as in the previous example, but the users now have user-unique interleavers. The UUF is now both the interleaver and the constellation. The gain in the performance introduced by the interleavers can be noticed by comparing the performance curves in Fig. 5 with the ones in Fig. 4.

The significant difference in the performance for all three detectors is only due to the introduced interleavers. The performance of the IIC detector is still better than the performance of the IC detector, but there seems to be a floor for high SNR. Why

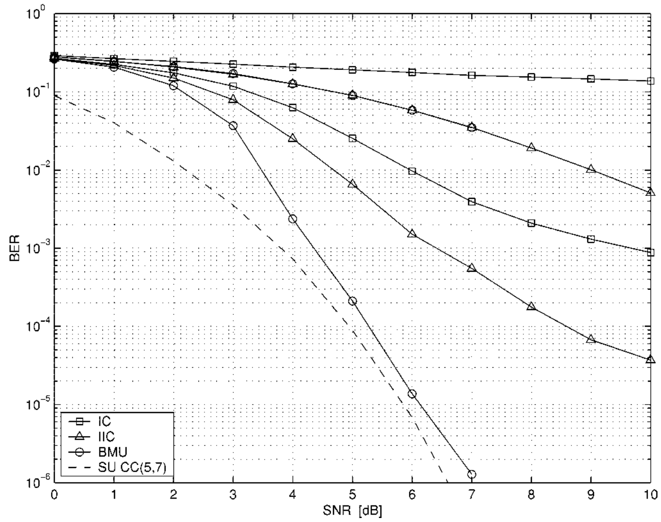

Fig. 5. The performance of user one after one and five iterations in a two-user QPSK system with CC(5,7), user-unique interleavers and random phase offset.

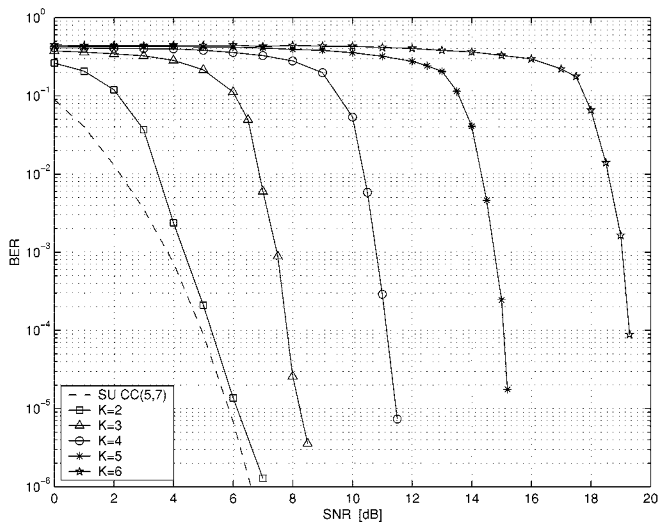

Fig. 6. The performance of one–six users after iteration five in a QPSK system with CC(5,7), user-unique interleavers, and random phase offset.

this is so has not been investigated, since the performance of the IC detector and the IIC detector is so poor anyway. An educated guess could be that the floor originates from the GA used in the BMF. For a small number of users, as in TCMA systems, the approximation that the MAI is Gaussian is not sufficiently accurate. Attempts have been made to try to lower the floor in the performance of the IC detector and IIC detector by increasing the size of the interleaver. Only a minor improvement in the performance was observed in these attempts.

The performance of the BMU detector is almost as good as the SU performance at an SNR of 5 dB. This means that two users can have almost the same performance and occupy the same bandwidth as if there was only one user in the system.

Fig. 6 shows the performance of the BMU detector for one–six users under the same conditions. For $K > 2$, the performance is not close to the SU performance, but the slope is steeper than the slope of the SU performance. By inspection of Fig. 6, it can be concluded that if another user is added to the system, it will require approximately 4 dB higher SNR to get to the same BER as before. Six users is probably not the limit in this case, but more than six users has not been considered here because of computational complexity reasons.

The interleavers make the MAI from the other users uncorrelated in time. The MAI is then merely another additive noise with a limited number of levels, which are known, and the APP of them are calculated with the forward–backward algorithm. The MAI can, in the two-user example, almost totally be removed if the BMU detector is used, and the performance is then almost as good as in the SU case.

VII. Conclusions

A TCMA system has no bandwidth expansion, i.e., $N = 1$. This means that K users are together using a total bandwidth equal to the bandwidth of one user. The load in a TCMA system, i.e., the total number of bits in each symbol interval normalized with the used bandwidth, is therefore much higher than the load in, e.g., a CDMA system.

Here, different iterative detectors for TCMA systems have been considered. Instead of using the optimal, but very complex, MLSD [6] to extract the users' information, certain iterative detectors are proposed. The first iterative detector considered is the well known IC detector [1], [3], [13]. It is shown here that this detector is not suited for TCMA systems. That is because the MAI is too prominent, due to the high load in the TCMA system. An improved IC detector has, therefore, been proposed. This detector uses another approximation of the BMF in the first iteration, instead of the most commonly used GA. The performance is now improved, because the detection gets a push in the right direction from the beginning. The drawback in both IC and IIC is that the variance of the remaining MAI must be approximated and that the soft tentative decision must be calculated.

Random interleavers are introduced in the TCMA system [7]. These interleavers introduce a gain in the performance of the iterative detectors considered. Unfortunately, simulation results show that the performance of both the IC and IIC detector still reaches a floor for high SNR.

Another iterative detector, here called the BMU detector, is also suggested for detecting the users in the TCMA system. This detector does not make any IC. It uses the same channel observations all the time, for every user and for every iteration. The difference is that it uses the APPs delivered from the forward-backward algorithm to approximate and update the *a priori* probabilities in the BMF. The performance of the two-user example system is shown to be as good as if one user is alone in the system. The floor in the detection performance has also vanished. This means that two independent users can transmit data with the same performance as one user, without using any more bandwidth. This performance (just a few fractions of a dB from the SU performance) is achieved when the interleaver and the symbol constellation (phase offset) are the only features separating the users. It is also shown that six users can successfully share the same bandwidth as one user. The load in the system is then six information bits per channel use. This can be compared to a conventional CDMA system, where loads of 1.5–2 information bits per channel use are currently considered high.

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