

Three-dimensional structures in nonlinear cavities containing left-handed materials

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Abstract: We study the coupling between negative diffraction and direct dispersion in a nonlinear ring cavity containing slabs of Kerr nonlinear right-handed and left-handed materials. Within the mean field approximation, we show that a portion of the homogeneous response curve is affected by a three-dimensional modulational instability. We show numerically that the light distribution evolves through a sequence of three-dimensional dissipative structures with different lattice symmetry. These structures are unstable with respect to the upswitching process, leading to a premature transition to the upper branch in the homogeneous hysteresis cycle.

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1. Introduction

Self-organization in large area two-dimensional optical devices is an active topic in nonlinear optics [1–4]. It has been shown that the competition between nonlinearities, diffraction and dissipation results in the formation of dissipative structures. When diffraction can be neglected (e.g., by using waveguides), the interplay between dispersion and nonlinearity can lead to temporal modulation of continuous wave beams [5]. When diffraction and dispersion have a comparable influence on the system, 3D dissipative structures are formed. These structures consist of self-organized light bullets traveling in the cavity and have been reported in nonlinear Kerr cavities [6], optical parametric oscillators [7], type II second harmonic generation [8], and wide aperture lasers with a saturable absorber [9]. Furthermore, 3D structures in nonlinear resonators have also been predicted from models going beyond the mean field approximation [10].

In the field of material science, experimental and analytical work has shown evidence for materials with negative permittivity and permeability [11]. Such materials, which are often called left-handed materials (LHM), have been considered in Veselago's seminal work [12] and have now been experimentally demonstrated, both in the microwave [13] and in the optical domain [14]. The use of LHMs for the construction of lenses with sub-wavelength resolution was suggested by Pendry [15]. Recently, the inclusion of nonlinear effects in models for LHMs has been addressed. Hysteretic behaviour and unusual propagation was predicted in Refs. [16, 17]. Second harmonic generation in a LHM was treated in [18].

Here, we study the interplay between diffraction, dispersion and nonlinearity in a coherently driven cavity containing slabs of materials with different handedness. In Sect. 2, we construct a model for this resonator and show that the inclusion of a LHM allows for the inversion of the sign of the diffraction coefficient. A linear stability analysis shows that, when the dispersion and diffraction coefficients are both negative, a portion of the bistability curve becomes unstable with respect to spatially and/or temporally modulated perturbations. In Sect. 3, we study numerically the dissipative structures that emerge from the unstable homogeneous steady states (HSS) and show that the light distribution evolves through a number of different 3D patterns. However, these patterns are unstable and the cavity switches finally to the higher homogeneous state. The latter phenomenon leads to a narrower homogeneous hysteresis cycle.

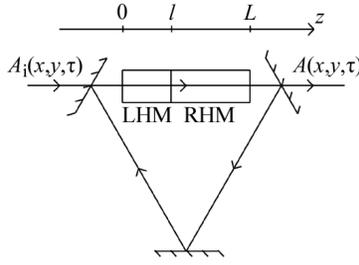


Fig. 1. A driven ring resonator filled with left- (LHM) and right-handed (RHM) materials.

2. Model equations and modulational instability

We consider the optical ring cavity depicted in Fig. 1, which is filled with two adjacent layers containing a right-handed material (RHM) and a left-handed material (LHM), respectively. Both materials are assumed to be local materials with an instantaneous Kerr nonlinearity and with dispersive dielectric and magnetic properties. The evolution of the envelope A of the electric field during propagation in the RHM can be described by the standard nonlinear Schrödinger equation (NLSE) and it has been shown that the NLSE also applies to LHMs [19, 20]:

$$\frac{\partial A}{\partial z} = i\gamma_{R,L}|A|^2A + i\left[\frac{1}{2k_{R,L}}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \beta_{R,L}\frac{\partial^2}{\partial \tau^2}\right]A, \quad (1)$$

where z , x and y are the longitudinal and transverse space coordinates and τ is the time coordinate in the frame moving with the group velocity. Subscripts R and L denote parameters in the RHM and LHM. The strength of the nonlinearity is determined by the coefficients $\gamma_{R,L}$ and the diffraction term is inversely proportional to the wavenumber $k_{R,L}$. This means that the diffraction coefficient is always negative in a LHM [19, 20] and thus that diffraction acts with positive sign in the RHM layer and with negative sign in the LHM layer, leading to partial diffraction compensation between the layers. In Ref. [19], the group velocity dispersion coefficient β has been calculated based on a Drude model describing the dispersive properties of $\epsilon(\omega)$ and $\mu(\omega)$, which has been verified to a good extent for the microwave LHMs. There, it is demonstrated that β can be positive or negative depending on the ratio of the working frequency to the resonance frequency of the LHM. The stability of the NLSE for LHM has been discussed in Ref. [21].

The cavity is driven by a coherent plane wave of frequency $\nu = \omega_0/2\pi$. The intracavity field A undergoes a coherent superposition with this input field at the input mirror:

$$A^{m+1}(0, x, y, \tau) = \rho A^m(L, x, y, \tau) \exp(i\Phi) + \theta A_i. \quad (2)$$

ρ is the product of the amplitude reflection coefficients of the input and output mirrors, θ is the amplitude transmission coefficient at the input mirror.

In order to simplify the propagation model, which consists of Eqs. (1) and (2), we have reduced the infinite dimensional map A^m to a single partial differential equation using the well-known mean field approach. This method is based on three assumptions: (1) reflections at the interfaces of the layers can be neglected; (2) the dissipative Fresnel number is large; (3) the cavity is shorter than the diffraction, dispersion and nonlinearity space scales. The first of these approximations needs some more explanation as such reflections and the associated counter-propagating beam could alter the dynamics considerably. Having two independent material parameters (ϵ and μ), the structure of the LHM can be tuned to have an impedance $\eta = \sqrt{\epsilon/\mu}$ equal to that of the RHM, i.e., the RHM and the LHM are impedance matched. Note that this,

in general, does not mean that the LHM will have equal but opposite index. In another possible realization, an optical isolator can be used to inhibit the counterpropagating signal. A slow dimensionless time scale T is introduced, where $T = \pi t / (\mathcal{F} t_r)$, with t_r the cavity roundtrip time and \mathcal{F} its finesse. The slow time evolution of the envelope $A(x, y, \tau, T)$ is then governed by

$$\frac{\partial A}{\partial T} = -(1 + i\Delta)A + \mathcal{E} + i\Gamma|A|^2A + i \left[\mathcal{D} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \beta \frac{\partial^2}{\partial \tau^2} \right] A. \quad (3)$$

The detuning Δ is related to the linear phase accumulated by the light during one roundtrip, $\Phi = \pi\Delta/\mathcal{F}$. The input field amplitude has been scaled by $A_i = \pi\mathcal{E}/\mathcal{F}$ and the coefficient of the nonlinear term describes the combined effect of the Kerr nonlinearities in both layers: $\Gamma = 3\mathcal{F}(\gamma_R l_R + \gamma_L l_L)/\pi$. A self-focusing nonlinearity ($\Gamma > 0$) is assumed in the remainder of this text. Finally, the diffraction and dispersion coefficients are given by

$$\mathcal{D} = \frac{\mathcal{F}}{\pi} \left(\frac{l_R}{2k_R} + \frac{l_L}{2k_L} \right), \quad \beta = \frac{\mathcal{F}}{\pi} (\beta_R l_R + \beta_L l_L). \quad (4)$$

Equation (3) has the same mathematical structure as the generalized Lugiato-Lefever (LL) model [6]. However, we see from Eqs. (4) that the diffraction and dispersion coefficients \mathcal{D} and β are an arithmetic average of the respective coefficients in the NLSE weighted by the lengths of the layers and, consequently, that they can be tuned between the values that they would have if the cavity were filled with either of both materials. The inclusion of the left-handed element in the ring cavity thus allows to engineer the strength of the diffraction coefficient \mathcal{D} and even to change its sign when $l_L > -l_R k_L/k_R$. Furthermore, it is possible to obtain both signs for β , since both the RHM and the LHM can come with normal or anomalous dispersion. As we will see, these changes of sign have an important consequence in the 3D pattern formation process. We will consider the case of $\mathcal{D} < 0$ and $\beta < 0$ in the numerical simulations presented below.

Before we will discuss this model, we want to remark that we have neglected propagation losses, which seems unacceptable due to the high losses in current metamaterials. However, such losses can be straightforwardly included in the model by adding a linear loss term to the NLSE and averaging this term under the mean-field approximations. Finally, the latter losses will add up to the mirror losses and they will only change the scaling of the field envelope A . Additionally, the combination of the RHM and the LHM allows to satisfy the resonance condition with cavities that are much thinner than $\lambda = c/v$ [22]. In such thin cavities, the propagation losses can be decreased significantly until the mirrors are again the major loss mechanism.

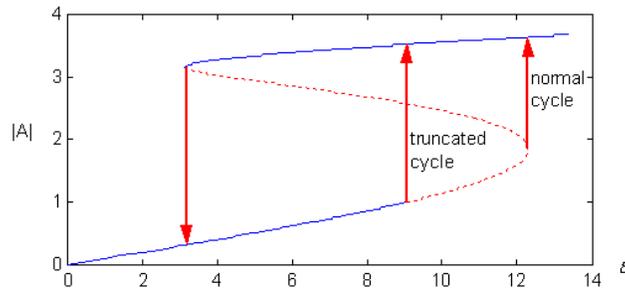


Fig. 2. Bistability curve of the resonator. Modulationally stable regions are denoted with blue solid lines, unstable regions with red dashed lines. Parameters are $\mathcal{D} = \beta = -2$ and $\Delta = 10$.

The homogeneous steady state (HSS) solutions A_s of Eq. (3) are $\mathcal{E} = [1 + i(\Delta - \Gamma|A_s|^2)]A_s$ (Fig. 2). $|A_s|^2$ as a function of $|\mathcal{E}|^2$ is single-valued for $\Delta < \sqrt{3}$ and multiple-valued for $\Delta > \sqrt{3}$.

We have performed the stability analysis of these HSS. With periodic boundaries, the deviation from the steady state is taken proportional to $\exp(i\mathbf{k} \cdot \mathbf{r} + \lambda T)$ with $\mathbf{k} = (k_x, k_y, k_\tau)$ and $\mathbf{r} = (x, y, \tau)$. The threshold associated with modulational instability is $|A_c| = 1/\sqrt{\Gamma}$ and $\mathcal{E}_c = \sqrt{\Delta^2 - 2\Delta + 2}$. At that bifurcation point, the HSS becomes unstable with respect to modes satisfying the relation, $\mathcal{D}(k_x^2 + k_y^2) + \beta k_\tau^2 = 2 - \Delta$. These modes form an ellipsoid in Fourier space (k_x, k_y, k_τ) if \mathcal{D} and β have equal sign, and an hyperboloid for opposite signs (see Fig. 3).

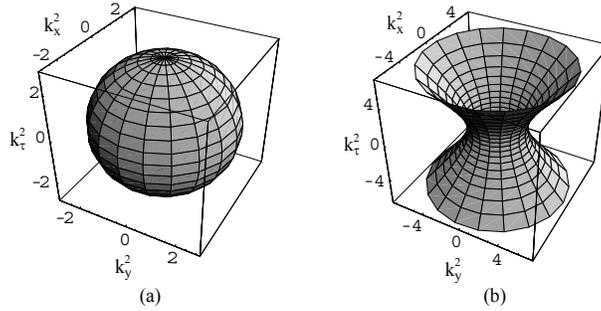


Fig. 3. At the bifurcation point, the unstable modes form (a) an ellipsoid or (b) a hyperboloid in the Fourier space (k_x, k_y, k_τ) . Parameters are $\mathcal{D} = \beta = -2$ and $\Delta = 10$.

3. Three-dimensional structures and up-switching process

In the absence of diffraction and dispersion, the lower homogeneous branch is stable and the transition to the upper branch in the homogeneous hysteresis cycle occurs at the limit point (as indicated in Fig. 2). When diffraction and dispersion are taken into account, a critical input intensity exists above which the lower branch is unstable. Above this threshold, an infinite number of linearly unstable Fourier modes exist. The wavevectors of these modes lie on an ellipsoid in the Fourier space (k_x, k_y, k_τ) , as given by the linear stability analysis, which means that they are of constant magnitude, but arbitrarily directed in the Fourier space. Therefore, the modulational instability will tend to impose periodicity in the Euclidean space (x, y, τ) . Due to the nonlinearity, the unstable modes will interact, resulting in the formation of dissipative structures. However, in the same input intensity range, stable homogeneous states still exist on the upper branch. A second possible dynamical process is thus the up-switching phenomenon, which will tend to restore uniformity in the transverse plane of the cavity.

To resolve which process dominates the dynamics of the resonator, we have performed numerical simulations of the generalized LL-model [Eq. (3)], with both the two-dimensional diffraction coefficient and the dispersion coefficient being negative, i.e., $\mathcal{D} < 0$ and $\beta < 0$. The input is chosen in the unstable part of the lower steady state branch, i.e., between the threshold for modulational instability and the normal up-switching point (red dotted branch in Fig. 2). We have discretized Eq. (3) by applying a 6-point Crank-Nicholson scheme with periodic boundary conditions in all directions (number of mesh points is $40 \times 40 \times 40$ in the cube $0 < x, y, \tau < 16$) and as initial condition the homogeneous field A_s to which a small noise term (10^{-5}) is added.

From Fig. 4, we observe that the intracavity field goes through a sequence of dissipative structures with different lattice symmetry. First, a structure with light bullets ordered on the lattice points of a face centered cubic lattice is formed [Fig. 4(a)]. The lattice constant of this structure ($\Gamma_{\text{num}} = 5.3$) compares well to the value obtained from the linear stability analysis ($\Gamma_c = 2\pi/k_c = \pi$); the small difference is due to the periodic boundary conditions. Subsequently, the bullets on diagonal planes of the cubic structure merge to form plates of interconnected bullets [Fig. 4(b)]. The formation of dissipative structures thus seems to dominate the

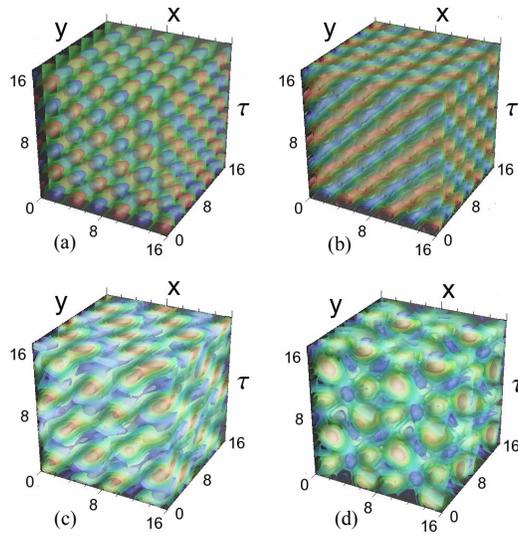


Fig. 4. (2.2 MB) Movie of the 3D structures observed in the cavity (8.1 MB version). (a)-(d) Representative frames of the movie at times $T = 100$, $T = 215$, $T = 236$ and 245 . Parameters are $\mathcal{D} = \beta = -2$ and $\Delta = 10$.

dynamics at small times. However, the patterns further evolve to a homogeneous state (note the contracting scale in the movie of Fig. 4). We conclude therefore that all dissipative structures are unstable and that the intracavity field switches to the upper homogeneous state. Note the dramatic difference with the positive diffraction case, where stable light bullets are formed [6].

We have repeated the simulation described above for several values of the input field in the unstable region and we found each time that the up-switching process is more robust than the formation of stable three-dimensional structures. This means that up-switching in the homogeneous hysteresis cycle of a Kerr cavity with negative diffraction and direct dispersion will occur at the bifurcation point $|A_c|^2 = 1/\Gamma$ rather than at the limit point $|A_-|^2 = (2\Delta \pm \sqrt{\Delta^2 - 3})/(3\Gamma)$, effectively reducing the width of the cycle (see Fig. 2).

4. Conclusions

We have studied the propagation of light in a ring cavity containing materials having opposite handedness. Using the mean field approach, we have derived a generalized LL-model taking into account both two-dimensional diffraction and chromatic dispersion. The linear stability analysis of that model reveals the existence of a 3D instability affecting a portion of the lower homogeneous steady state curve. Numerical simulations show that the modulational instability due to the coupling of negative diffraction and direct dispersion leads to an up-switching process, during which the field evolves through a number of 3D structures having different symmetry. This switching behavior turns out to be more robust than the 3D pattern formation scenario, leading to the truncation of the homogeneous hysteresis cycle.

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