Multiple-sensor Fusion Tracking Based on Square-root Cubature Kalman Filtering

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Abstract—Nonlinear state estimation and fusion tracking are always hot research topics for information processing. Compared to linear fusion tracking, nonlinear fusion tracking takes many new problems and challenges. Especially, the performances of fusion tracking, based on different nonlinear filters, are obviously different. The conventional nonlinear filters include extended Kalman filter (EKF), unscented Kalman filter (UKF), particle filter (PF) and cubature Kalman filter (CKF), and the recent square-root cubature Kalman filter (SCKF) has been paid more and more attention by researchers because of its advantages of computation complexity and estimation accuracy over other nonlinear filters. However, the SCKF is mainly designed for single sensor system, and for present nonlinear multi-sensor system, it is not applicable. Based on the current results of fusion tracking algorithms, this paper provides a novel multi-sensor fusion tracking algorithm based on the SCKF. Firstly, a brief introduction of basic SCKF algorithm is given. Then, the centralized fusion tracking frame with augmented measurements and the sequential fusion tracking frame are presented respectively. Finally, two computer simulation examples are demonstrated. Simulations results show that the centralized SCKF can obtain better performances of accuracy, stability and convergence.

Index Terms—Multi-sensor Fusion; Nonlinear System; Cubature Kalman Filter; Unscented Kalman Filter

I. INTRODUCTION

Data fusion or information fusion, which was proposed first by USA in national defense field, is a new information processing technology for multi-sensor systems recently [1]. With the developments of sensor technology, communication technology and computer technology, the study of data fusion is facing many new challenges in both civil and military fields. Promoted by demands from industry application and national defense, the research on data fusion has been developed fleetly and a large number of theoretical achievements and application technologies have been taken [2]. However, as the modern control systems and various networks are becoming more and more complicated, the research on data fusion must be further promoted in order to satisfy more and more practical demands [3].

Contributed by many researchers in the past decades, data fusion theory for linear multi-sensor system is gradually perfected and most basic fusion theories and methods have been reported [4]. But most modern systems are nonlinear and the linear fusion theory cannot be directly extended to deal with nonlinear data fusion. Thus the research of nonlinear fusion is paid more attentions and is a popular topic in data fusion field [5]. It is exactly different from linear fusion theory based on the conventional linear Kalman filter, the performance of nonlinear fusion algorithms depends on the nonlinear filters adopted. Up to now, there are four kinds of common nonlinear filters such as extended Kalman filter (EKF) [6], unscented Kalman filter (UKF) [7], particle filter (PF) [8], cubature Kalman filter (CKF) [9] and its improved square-root form (SCKF) on computation [10]. Although SCKF is similar to the EKF and UKF, but it utilizes different sets of deterministic weighted point, so it has clear advantages on computational complexity and estimation performance, and it is becoming most popular nonlinear filter at present. In addition, it is important for academic research and application foreground to design some novel fusion algorithms on the basis of the cubature Kalman filter.

Data fusion frames include generally centralized and distributed ways, which have different ratios on fusion performance and cost [11]. Under the centralized fusion frame, based on the extended Kalman filter, the centralized fusion algorithm with augmented measurements and the data compression fusion algorithm were presented in [12]. In [13], the centralized fusion algorithms on basis of the extended information filter were also designed for the nonlinear system with correlated noises. For the unscented Kalman filter, some nonlinear fusion algorithms with synchronous sampling and asynchronous systems have been established and the corresponding performance analysis was given in [14]. In fact, the original UKF was first proposed in its augmented form. But the non-augmented UKF has been employed to analyze the practical systems, because it is widely accepted for such a case that the resultant non-augmented UKF yields similar results to the augmented UKF. For the distributed fusion way, some nonlinear data fusion algorithms were also researched by using the EKF, the UKF and the PF [15-16]. After the CKF and SCKF were presented in 2009, many researchers follow it to develop many significant works [17-20].

However, these works mentioned above, whether they are under the centralized frame or the distributed one, mostly aim at single sensor system. To our best knowledge, the single sensor data fusion tracking algorithms are hardly applicable at present because the real motion system is generally on the basis of multi-
sensor measurements. Thus, that is the motivation of this paper. On the basis of the current results on data fusion algorithms, this paper devotes itself to designing centralized fusion tracking algorithm with augmented measurements and the sequential structure fusion tracking algorithm, and both of them are based on the SCKF for multi-sensor system.

The rest of this paper is as follows. Problem formulation is given in section II. Section III summarizes the square-root cubature Kalman filter and a centralized fusion estimator is presented. In section IV, a novel multi-sensor sequential square-root cubature Kalman filter is proposed. A brief algorithm analysis is done in section V and section VI is a simple numerical simulation. We conclude in section VII.

II. PROBLEM FORMULATION

In this paper, the following dynamic system equation is considered, namely,

$$x(k) = f(x(k-1), x(k-1)) + v(k, k-1)$$

(1)

where $x(k) \in \mathbb{R}^{n_t}$ is state variable interested. $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a known differential function. $v(k, k-1)$ is a Gaussian white noise with zero mean.

The nonlinear measurement equations of multi-sensor synchronous systems are given as follows:

$$z_j(k) = h(x(k)) + w_j(k) \quad (j = 1, 2, \ldots, J)$$

(2)

where $z(k) \in \mathbb{R}^{n_m}$ is measurement vector and $h$ is a known and differential function with $\mathbb{R}^n \rightarrow \mathbb{R}^{n_m}$. $w_j(k)$ is also a zero mean Gaussian white noise.

The steps of the SCKF are as follows:

1. Step 1: Firstly, $x(0)$ and $P(0 \mid 0)$, and it is uncorrelated with $v(k, k-1)$ and $w_j(k)$.

2. Step 2: Compute $x_0 = f(x(0), x(0))$ and $P_0(0 \mid 0)$.

3. Step 3: Compute the state prediction and its covariance $\hat{x}(k | k-1) = f(x(k-1), x(k-1))$ and $P(k | k-1)$.

4. Step 4: Compute the square-root factor of the predicted measurement equation $X_0(k | k-1)$.

5. Step 5: Compute the measurement prediction $z(k | k-1)$ and $P(k | k-1)$.

6. Step 6: Compute the cross-covariance matrix $P_{xz}(k | k-1)$.

7. Step 7: Compute measurement prediction $\tilde{z}(k | k-1) = f(x(k), x(k | k-1))$

A. Notations

- $x(k | k-1)$ and $P(k | k-1)$: estimate and its covariance of estimate error at $k-1$
- $S(k-1 | k-1)$: a square-root of $P(k-1 | k-1)$
- $X_0(k | k-1)$: the propagated cubature points from the state equation
- $\hat{x}(k | k-1)$ and $P(k | k-1)$: one step state predict estimate and its associated covariance from $k-1$ to $k$
- $S(k | k-1)$: a square-root of $P(k | k-1)$
- $Z_i(k | k-1)$: the propagated cubature points
- $\tilde{z}(k | k-1)$ and $P_{xz}(k | k-1)$: the measurement prediction and the associated covariance matrix from $k-1$ to $k$
- $P_{xz}(k | k-1)$: the cross-covariance matrix

When $j = 1$ and neglect the sensor index, we have $z_1(k) = z(k)$, $h_1(x(k)) \rightarrow h(x(k))$, and $w_1(k) \rightarrow w(k)$. Then, the SCKF algorithm based on single sensor measurement can be summarized in next subsection.

B. Square-Root Cubature Kalman filter

The steps of the SCKF are as follows:

1. Step 1: Firstly, $S(k-1 | k-1)$ can be taken estimated on-line in the SCKF algorithm.

2. Step 2: Compute

$$X_0'(k | k-1) = f(k-1, S(k-1 | k-1) - 1)$$

(3)

3. Step 3: Compute the state prediction and its covariance

$$\hat{x}(k | k-1) = \frac{1}{2n} \sum_{i=1}^{2n} X_0'(k | k-1)$$

(4)

4. Step 4: Compute the square-root factor of the predicted error covariance $P(k | k-1)$

$$S(k | k-1) = \text{Trial}(|z(k | k-1) - 1 S_{0}(k | k-1))$$

(5)

5. Step 5: Compute the cubature points from the measurement equation

$$X_j(k | k-1) = S(k | k-1) \tilde{x}(k | k-1)$$

(6)

6. Step 6: Compute the propagated cubature points from the measurement equation

$$Z_i(k | k-1) = h(x(k), x(k | k-1))$$

(7)

7. Step 7: Compute measurement prediction

$$\tilde{z}(k | k-1) = \frac{1}{2n} \sum_{i=1}^{2n} Z_i(k | k-1)$$

(8)
Step 8: Compute the square-root of estimate error covariance of $\tilde{x}(k\mid k-1)$

$$S_{zz}(k\mid k-1) = \text{Tria}([\zeta(k\mid k-1) \quad S_{\rho}(k)])$$ (11)

where

$$\zeta(k\mid k-1) = \frac{1}{2n}$$

$$[Z_1(k\mid k-1) - \tilde{z}(k\mid k-1) \cdots Z_m(k\mid k-1) - \tilde{z}(k\mid k-1)]$$

Step 9: The cross-covariance is

$$P_{xz}(k\mid k-1) = \eta(k\mid k-1)\zeta^T(k\mid k-1)$$ (13)

where

$$\eta(k\mid k-1) = \frac{1}{2n}$$

$$[X_1(k\mid k-1) - \tilde{x}(k\mid k-1) \cdots X_m(k\mid k-1) - \tilde{x}(k\mid k-1)]$$

Step 10: Gain matrix

$$K(k) = \frac{P_{xz}(k\mid k-1)}{S_{zz}(k)}$$ (15)

Step 11: Final state estimate

$$\hat{x}(k) = \tilde{x}(k\mid k-1) + K(k)\zeta(k) - \zeta(k\mid k-1)$$

$$P(k\mid k) = S(k)S^T(k\mid k)$$

where

$$S(k\mid k) =$$

$$\text{Tria}([\eta(k\mid k-1 - K(k)\zeta(k) - \zeta(k\mid k-1)) \quad K(k)S_{\rho}(k)])$$ (17)

C. Centralized Square-Root Cubature Kalman Fusion

For nonlinear measurement equations (2), denote

$$z(k) = [z_1(k); z_2(k); \cdots; z_l(k)]$$ (18)

$$h(k, x(k)) = [h_1(k, x(k)); h_2(k, x(k)); \cdots; h_l(k, x(k)))]$$ (19)

$$w(k) = [w_1(k); w_2(k); \cdots; w_l(k)]$$ (20)

where the covariance of augmented noise $w(k)$

$$R(k) = \text{diag}[R_1(k), R_2(k), \cdots, R_l(k)]$$ (21)

Then the centralized measurement equation becomes

$$z(k) = h(k, x(k)) + w(k)$$ (22)

So, for the multi-sensor system given by Eq.(1) and (22), we can get the centralized square-root cubature Kalman fusion estimate with augmented measurement by use of the SCKF formulas given by Eq.(4)-(17).

But, as it is known to all, the centralized fusion has some shortcomings such as high computational complexity and communication requirement, especially the matrix inversion is easy to fail. So, in the next section we propose a sequential square-root cubature Kalman filtering fusion algorithm, which is better than the centralized one.

IV. SEQUENTIAL SQUARE-ROOT CUBATURE KALMAN FUSION

The meaning of sequential fusion tracking structure is to sequentially use the new measurements to update the previous global fusion estimate step by step. Obviously, it is different from the centralized one with augmented measurements, which deals with all of measurements at the same time. The basic principle of sequential filtering fusion for multi-sensor synchronous sampling systems is shown by Fig. 1.

![Sequential filtering fusion frame](image)

Figure 1. Sequential filtering fusion frame

Exactly, the sequential filtering fusion is to solve

$$\begin{align*}
\hat{x}(k\mid k) &= E\{x(k) \mid \hat{x}_1(k), \hat{x}_2(k), \cdots, \hat{x}_l(k)\}
P(k\mid k) &= E\{|x(k) - \hat{x}(k\mid k)|^2\}
\end{align*}$$ (23)

where $j = 1,2,\ldots,l$ and

$$\begin{align*}
\hat{x}_j(k\mid k) &= E\{x(k) \mid \hat{x}(k-1\mid k-1), z_1(k), \cdots, z_j(k)\}
P_j(k\mid k) &= E\{|x(k) - \hat{x}_j(k\mid k)|^2\}
\end{align*}$$ (24)

the sequential filtering fusion is to describe the multi-sensor samples at the same time to a series of sequential pseudo-measurements. Then, the total multi-sensor measurements can be taken as a measurement sequence with the following additive state relations [20]:

$$x_i(k) = x_{i1}(k) = \cdots = x_{il}(k) = x(k)$$ (25)

Accordingly, a new state estimate system can be established with corresponding pseudo-state and pseudo-measurement sequences and the standard square-root cubature Kalman filter can be used to estimate the systemic state.

The main steps of the sequential square-root cubature Kalman filter are as follows.

Step 1: Time update process

$$X_{s\rightarrow f}^+(k\mid k) = f(k-1, X_{s\rightarrow f}^-(k\mid k))$$ (26)

where $j = 1,2,\ldots,l$ and $i = 1,2,\ldots,2n$, and

$$\begin{align*}
\hat{x}_i(k\mid k) &= \hat{x}(k\mid k-1)
P_i(k\mid k) &= P(k\mid k-1)
S_i(k\mid k) &= S(k\mid k-1)
\end{align*}$$ (27)
\[ \tilde{z}_{j,i-1}(k | k) = \frac{1}{2n} \sum_{i=1}^{2n} X^*_{j,i-1}(k | k) \] (28)

\[ S_{j,i-1}(k | k) = \text{Tri}a(\zeta_{j,i-1}(k | k) \ S_{j,i-1}(k | k)) \] (29)

where \( S_{j,i-1} \) is the square-root factor of \( Q_{j,i-1} \) which is

\[ Q_{j,i-1} = \begin{cases} Q(k,k-1) & \text{if } j = 0 \\ 0 & \text{else} \end{cases} \] (30)

Step 2: Measurement update process

\[ \begin{bmatrix} X_{j,i}(k | k) = S_{j,i-1}(k | k) \zeta_{j,i-1}(k | k) \\ Z_{j,i}(k | k) = h_j(k, X_{j,i}(k | k)) \end{bmatrix} \] (31)

\[ \begin{align*}
\tilde{z}_{j,i-1}(k | k) &= \frac{1}{2n} \sum_{i=1}^{2n} Z_{j,i}(k | k) \\
\tilde{z}_{j,i-1}(k | k) &= \frac{1}{2n} \sum_{i=1}^{2n} Z_{j,i}(k | k) Z_{j,i}(k | k) \\
&= \tilde{z}_{j,i-1}(k | k) + R_f(k)
\end{align*} \] (32)

The square-root of estimation error covariance of \( \tilde{z}_{j,i-1}(k | k) \)

\[ S_{\tilde{z}_{j,i-1}}(k | k) = \text{Tri}a(\zeta_{j,i-1}(k | k) \ S_{\tilde{z}_{j,i-1}}(k | k)) \] (33)

where

\[ \zeta_{j,i-1}(k | k) = \frac{1}{2n} \sum_{i=1}^{2n} Z_{j,i}(k | k) - \tilde{z}_{j,i-1}(k | k) \] (34)

Then, the cross-covariance matrix is computed according to

\[ P_{\tilde{z}_{j,i-1}}(k | k) = \eta_{j,i-1}(k | k) \tilde{z}_{j,i-1}(k | k) \] (35)

The SCKF estimate after updated by use of \( z_j(k) \)

\[ \begin{align*}
\tilde{x}_j(k) &= \tilde{x}_{j,i-1}(k) + K_j(k) (z_j(k) - \tilde{z}_{j,i-1}(k)) \\
P_j(k | k) &= S_j(k | k) S_j(k | k) \] (36)

where

\[ S_j(k | k) = \text{Tri}a(\eta_{j,i-1}(k | k) \ K_j(k) S_{\tilde{z}_{j,i-1}}(k | k)) \] (37)

Step 3: Final fusion estimate (when \( j = 1 \))

\[ \begin{align*}
\tilde{x}(k | k) &= \tilde{x}_{j,1}(k | k) \\
P(k | k) &= P_j(k | k)
\end{align*} \] (38)

V. BRIEF DISCUSSION

Aiming at the nonlinear fusion, two kinds of fusion tracking algorithms based on the square-root cubature Kalman filter are proposed in this paper, which are the centralized fusion estimator with augmented measurements (Estimator 1) and the sequential filtering fusion estimator (Estimator 2). Compared to the corresponding centralized fusion estimators based on the EKF and the UKF, the proposed centralized fusion estimators have obvious advantages on fusion estimation accuracy, stability and convergence. This is because the basic nonlinear-used SCKF in the centralized fusion is better than the EKF and UKF [10, 11]. It also shows that performance of the nonlinear fusion algorithms depends on the nonlinear filter. Similarly, the fusion estimators should exceed the centralized fusion algorithms based on the cubature Kalman filter because that computational performance of the SCKF is better than that of the CKF as stated in [14].

Generally, the centralized fusion estimator has the best fusion accuracy despite linear and nonlinear systems because all of initial measurements are processed in the fusion center at the same time. But this fusion way has high requirements on communication network and the ability of processing center, so it will lead to expensive systemic cost. Clearly, in the practical applications, it will be limited strictly due to high investment. The computational complexity of the centralized fusion with augmented measurements must increase when the number of measurements increases. An important problem is that the Estimator 1 can be run after all of measurements getting to the fusion center. Exactly, it is unrealistic because the data transmission often delays across network. This problem requires the fusion estimator has the ability to recursively fuse measurements which arrives sequentially. Naturally, the proposed sequential square-root cubature Kalman filtering fusion estimator can satisfy this. More importantly, this sequential fusion algorithm can adapt to the networked system with short delay which is small than one sample period. The previous work in linear systems shows that the centralized fusion estimator with augmented measurements has the same estimation accuracy with the sequential fusion estimator [4, 5]. However, it is not sure that whether this conclusion should still hold between Estimator 1 and Estimator 2 based on the SCKF in nonlinear systems or not, and we will use a simple computer simulation example to study this.

VI. SIMULATIONS

In this section, two kinds of simulation examples are demonstrated to show efficiency of the proposed square-root cubature Kalman fusion estimators and all of results are the mean of 100 Monte-Carlo runs.

A. Simulation Setup

Consider a target with a constant-velocity motion, whose dynamic model is described in Eq. (1). The state is denoted by \( \mathbf{x}(k) = [x(k) \ \dot{x}(k) \ y(k) \ \dot{y}(k)]^T \), where \( x(k) \) and \( y(k) \) are the position components of east and north, respectively, \( \dot{x}(k) \) and \( \dot{y}(k) \) are the corresponding velocity components respectively. We choose the parameters as follows:
where sampling interval $T = 1s$. The parameters of the target is given by initial state: $[1000 \ 150 \ 1500 \ 200]^T$. Here, the covariance of initial state and process noise are chosen

$$P_0 = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 196 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 255 \end{bmatrix}$$

The variance of process noise is

$$Q(k) = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \times 0.25(m^3/s^2)$$

The measurement system consists of three phased array radars that measure the range and the direction cosines of the target. The measurement matrix are given by

$$h_j(x(k)) = \begin{bmatrix} \sqrt{x^2(k) + y^2(k)} \\ \arccos \left( \frac{x(k)}{\sqrt{x^2(k) + y^2(k)}} \right) \end{bmatrix}, \quad j = 1, 2, 3$$

The covariance matrices of measurement noises are chosen

$$R_1(k) = \begin{bmatrix} 25m & 0 \\ 0 & 0.2^2 \end{bmatrix}, \quad R_2(k) = \begin{bmatrix} 64m & 0 \\ 0 & 0.3^2 \end{bmatrix}$$

B. Example 1

In this example, we compare the estimation accuracies among the CSCKF, the CEKF and the CUKF. The results are shown by Fig. 2 to Fig. 5 and Table I. From these simulation results, we can easily know that the CSCKF has the best estimation accuracy among three nonlinear fusion algorithms and the CEKF is the worst. In especial, the CEKF diverges based on the parameters given in this section. It is obvious because the estimation performance of nonlinear fusion algorithms depend on the performance of the used basic nonlinear filter. Clearly, the EKF is the worst among them and the SCKF is the best. Thereby, the performance orders of three nonlinear fusion algorithms correspond to the associated nonlinear filters.
B. Example 2

This simulation compares the CSCKF and the SSCKF on tracking performance and the results are shown by Fig. 6 to Fig. 9 and Table II. The results show that the CSCKF is slightly better than the SSCKF on estimation accuracy and it accords to the research on EKF fusion in [10]. Namely, for nonlinear multi-sensor systems, the centralized fusion frame with augmented measurements has better estimation accuracy than the sequential fusion frame. However, due to the inversion operation of a big matrix induced by augmenting measurements, computational complexity of the CSCKF is higher than the SSCKF, and the result is that real-time performance of the CSCKF is worse than the SSCKF. This conclusion which is taken in nonlinear system also accords with linear case. In addition, the sequential structure is more popular than the centralized structure with augmented measurements in practical networked systems with time delays. As a result, selecting the CSCKF or the SSCKF depends on the detailed requirements from practical systems.

Table I. Absolute Estimation Errors of Three Fusion Estimators

<table>
<thead>
<tr>
<th>Fusion Estimators</th>
<th>CSCKF</th>
<th>CEKF</th>
<th>CUKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>East Displacement (m)</td>
<td>36.2277</td>
<td>286.4090</td>
<td>44.5580</td>
</tr>
<tr>
<td>East Velocity (m/s)</td>
<td>1.2696</td>
<td>14.1466</td>
<td>1.7947</td>
</tr>
<tr>
<td>North Displacement (m)</td>
<td>43.8054</td>
<td>315.6695</td>
<td>47.5249</td>
</tr>
<tr>
<td>North Velocity (m/s)</td>
<td>2.0260</td>
<td>16.0007</td>
<td>4.7720</td>
</tr>
</tbody>
</table>

Figure 6. Absolute estimation error of east displacement (x axis)

Figure 7. Absolute estimation error of north displacement (y axis)

Figure 8. Absolute estimation error of east velocity (x axis)

Figure 9. Absolute estimation error of north velocity (y axis)

Table II. Absolute Estimation Errors of Two Fusion Estimators

<table>
<thead>
<tr>
<th>Fusion Estimators</th>
<th>CSCKF</th>
<th>SSCKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>East Displacement (m)</td>
<td>18.4220</td>
<td>25.0037</td>
</tr>
<tr>
<td>East Velocity (m/s)</td>
<td>1.8652</td>
<td>1.9295</td>
</tr>
<tr>
<td>North Displacement (m)</td>
<td>13.1296</td>
<td>17.4185</td>
</tr>
<tr>
<td>North Velocity (m/s)</td>
<td>1.1342</td>
<td>1.3327</td>
</tr>
</tbody>
</table>

VII. Conclusions

The fusion estimation problem is studied by using the square-root cubature Kalman filter for a kind of multi-sensor nonlinear target tracking system, and two nonlinear fusion estimators are proposed. Compared to the fusion estimators based on the EKF and UKF, the fusion estimators by using the SCKF have better estimation performance. For the nonlinear systems, the centralized fusion estimator based on augmenting measurements is slightly better than the sequential structure on the estimation accuracy, but the sequential structure possesses of better real-time performance than the augmented measurement case. Some open problems should be further studied, for examples, to design the corresponding distributed fusion estimator and compare with the centralized estimators on the estimation performance, to study centralized and distributed square-root cubature Kalman fusion with correlated noises and to design the corresponding networked nonlinear fusion algorithms with time delay and limited bandwidth and so on.
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