How to Manage an Overconfident Newsvendor

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Abstract

Previous experimental work has shown that individuals make suboptimal decisions in newsvendor problems (e.g. Schweitzer and Cachon 2000). We present a theoretical (behavioral) model of overconfident newsvendors that is consistent with these observed results. We show that overconfident newsvendors place suboptimal orders (which can be either higher or lower than optimal quantities) and earn lower profits than well-calibrated newsvendors. We also derive incentive contracts using salvage costs and price adjustments which a well-calibrated manager might offer to an overconfident newsvendor in order to induce optimal orders.

Key words: overconfidence, overprecision, newsvendor, inventory, behavioral operations management

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1. Introduction / Motivation

The newsvendor problem is a staple of Operations Management models, and serves as the basis of many models of inventory management (e.g., Sliver, Pyke and Peterson 1998; Porteus 2002). Although optimal solutions to the newsvendor problem have been known since Arrow, Harris and Marschak (1951), numerous case studies document firms’ difficulties in implementing these solutions (e.g. Fisher and Raman 1996, Katok et al. 2001). Recently, experimental work has identified systematic deviations in ordering behavior from optimal solutions; in particular, individuals under-order in high-profit regimes and over-order in low-profit regimes (e.g., Schweitzer and Cachon 2000, Bolton and Katok 2008, Bostian et al. 2008, Benzion et al. 2007).

We propose a behavioral explanation for these observations, based on overconfidence. When individuals are overconfident, they believe their information or their estimate to be more precise (accurate) than it actually is (see Moore and Healy 2008 for a review). In the newsvendor setting, we model this overconfidence in precision as a biased belief that the distribution of demand has variance lower than its true variance. Previous research has appealed to overconfidence to explain and describe behavior in other settings (e.g. Camerer and Lovallo 1999, Odean 1998, Malmendier and Tate 2005, Hilary and Menzly 2006). We are the first to investigate the impact of overconfidence in a theoretical inventory management setting.

Given our analysis of overconfident newsvendors’ behavior, we then ask the question “How might a well-calibrated manager correct this bias?” We examine two types of corrective incentives. Our modeling follows the style (although not exactly the letter) of Cachon’s (2003) coordinating contracts. In Cachon’s setting, a principal and agent have identical information and beliefs (and perfect rationality), but their differing incentives induce individual behavior inconsistent with joint profit maximization. Coordinating contracts have the potential to (at least partially) resolve these conflicting incentives, typically yielding second-best outcomes. Our setting is different, emphasizing beliefs rather than
incentives. In contrast, we assume that the manager and newsvendor have no conflicting incentives; both wish to maximize the total profits of the newsstand. They differ in their beliefs about demand, however; the manager is well-calibrated and knows the true distribution of demand, whereas the newsvendor is overconfident about the precision of his estimate of the demand distribution. We then derive incentives which the manager can offer the newsvendor in order to induce him to order the optimal inventory quantities, *given the newsvendor’s biased beliefs*.

We thus make two main contributions. First, we present a model of an overconfident newsvendor, and show how his orders deviate from optimal orders; the predictions of this model correspond to established experimental observation. We derive the costs of this overconfidence on profitability. Second, we derive incentives which unbiased managers can offer to overconfident newsvendors to induce optimal ordering behavior. We thus identify a technique for solving the problem of managing an overconfident agent.

The paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes our environment and defines the overconfident newsvendor. Section 4 derives the decisions made by the overconfident newsvendor and demonstrates how they differ from the classical normative solution to the newsvendor problem. Section 5 derives optimal contractual structures for an unbiased manager to offer to an overconfident newsvendor in order to induce optimal ordering. Section 6 concludes.

2. Previous literature

2.1 Overconfidence

While the theoretical operations-management literature generally addresses the actions of a fully rational agent, actual ordering decisions in practice are made by individuals. A substantial body of research from psychology (the field of Judgment and Decision Making) shows that most individuals use heuristics and suffer from a variety of biases in their decision-making. In this paper we examine the impact of one particular bias, *overconfidence*. 
Moore and Healy (2008) present an excellent literature review of over 350 papers demonstrating overconfidence in a variety of settings over a wide range of the population. They distinguish three types of overconfidence demonstrated in the literature, each with its own implications: overestimation, overplacement, and overprecision. In the bias of overestimation, people overestimate their own abilities; they believe these abilities are greater than they indeed are. For example, students robustly overestimate their exam performance (Clayson 2005), and people overestimate the speed with which they can complete a task (Buehler, Griffin and Ross 1994). In the bias of overplacement, people believe they are better than others. For example, in Zenger (1992), 37% of professional engineers in a firm rated themselves among the top 5% of performers at the firm. Although overplacement and overestimation can (and often do) appear together, they are not necessarily causally related. As an example, individuals can simultaneously overestimate their own abilities, while believing that others are even more capable than their true abilities, and thus suffer from overestimation and underplacement simultaneously.

The third bias, which we will invoke in our model, is overprecision. In this bias, individuals believe that their estimates are more accurate than they actually are; they are overconfident about their precision. In a classic test of overprecision, participants are asked to provide estimates of varying types of information and confidence intervals around the estimates. Confidence intervals are on average unbiased (centered around an accurate mean) but too narrow; previous studies have shown that fewer than half of responses include the true answers in the offered 90% confidence intervals (e.g., Alpert and Raiffa 1982, Soll and Klayman 2004). In our model we will capture overprecision as a biased belief that a distribution (of demand) has the same mean but a lower variance than it truly has.

Previous research has appealed to overconfidence in its various forms to explain and describe a variety of behavior. Hilary and Menzly (2006) argue that analysts can become overconfident after a short series of accurate predictions, and that this causes lower accuracy in the future. Camerer and Lovallo (1999) argue that overoptimism about one’s level of ability (overestimation) can explain supraoptimal rates of market entry and entrepreneurship. Odean (1998) shows that investors who overestimate the precision of their information signals (overprecision) will trade more than is optimal and have lower
expected utilities as a result. Malmendier and Tate (2005) show that CEOs who suffer from overplacement are more likely to engage in (unprofitable) acquisitions. As far as we know, we are the first to investigate the impact of overconfidence (or overprecision) in an inventory management setting.

2.2 Newsvendor Problems

The newsvendor problem has a rich history which can be traced back to Edgeworth (1888), who first developed it as a model to study the cash holdings of banks. In the newsvendor model, an agent decides how much of a given item (for example, newspapers) to stock each day. There is a single purchasing opportunity before the start of the selling period; for example, the newsvendor has to decide how many papers he wants to purchase the night before he will sell them. The constant marginal cost of acquiring each newspaper \((c)\) and price at which he can sell them \((p)\) are fixed, with price higher than cost. Actual realized demand for the item is assumed to be random, drawn from a known and stationary distribution.

If the newsvendor under-stocks, he runs out of inventory and loses the expected profits from selling newspapers \((p-c)\). If the newsvendor over-stocks, he has newspapers left over at the end of the day and can recover only their salvage value \((s)\), assumed to be less than the marginal cost of acquiring them. He thus loses the difference \((c-s)\) on each unit unsold.

Arrow, Harris, and Marschak (1951) calculated the famous critical fractile solution for the newsvendor problem. This optimal solution is characterized by balancing the expected cost of understocking and overstocking, and will be described in more detail below. This simple problem with its intuitively appealing optimal solution has formed the foundation for an enormous literature on stochastic inventory theory more generally. Reviews of recent theoretical work on the newsvendor model can be found in Sliver, Pyke, and Peterson (1998) and Porteus (2002).

A few previous papers have examined the impact of demand variance on the optimal newsvendor solution. For example, Naddor (1978) demonstrates that optimal orders depend only on the mean and variance of the demand distribution, and not on its specific shape. Gerchak and Mossman (1992) demonstrate that expected profits decrease when the variance of the demand distribution increases,
holding the mean constant (see also Song, 1994). Virtually all of the previous work on the newsvendor problem, however, assumes that the newsvendor fully understands the distribution from which market demand is drawn. In contrast, in our model below, the newsvendor will be mistaken about the demand distribution in a way consistent with the overprecision bias.

2.3 Empirical and Experimental Evidence

Although the optimal newsvendor solution is well-known (and, indeed, taught in virtually every MBA core operations-management course), evidence suggests that inventory managers often deviate from its recommendations. For example, Fisher and Raman (1996) model the inventory management decisions of a Skiwear firm, Sport Obermeyer. They conclude, from a sample of 339 inventory decisions, that Sport Obermeyer’s managers consistently ordered too little; had its managers ordered inventory in a way consistent with the newsvendor solution, the firm’s profits would have increased by 60%. In contrast, in another study by Katok et al. (2001), managers at Jeppesen Sanderson (a company that sells maps) systematically ordered too much. The authors derived a periodic-review inventory system which, when implemented, recommended significantly smaller orders and resulted in $800,000 of cost reduction for the firm.

Laboratory studies run under controlled conditions help to reconcile these competing findings from the field. In the classic paper by Schweitzer and Cachon (2000), MBA students who have just learned the optimal solution for the newsvendor problem made suboptimal choices. In experimental conditions where the optimal order was higher than the mean demand (the high-profit condition), participants ordered too little (as at Sport Obermeyer). In experimental conditions where the optimal order was lower than the mean demand (the low-profit condition), participants ordered too much (as at Jeppesen Sanderson). Figure 1 from their paper shows the optimal and average orders in the two conditions.

\[2\] In a slightly different context, Olivares et al. (2008) use a newsvendor framework to explain operating-room reservations at hospitals. They similarly find suboptimal overordering behavior by doctors.
Schweitzer and Cachon (2000) examined and rejected a number of competing explanations for this pattern of results, including risk preferences (risk aversion or risk-seeking), Prospect Theory-based preferences (loss aversion, reflection effects, and probability weighting), waste aversion, stockout aversion, and underestimation of opportunity costs. In the end, they describe two independent and possible biases which could explain their results: anchoring (and insufficient adjustment) and the desire to minimize ex-post inventory error (rather than ex-ante inventory error). In this paper we show analytically that a different bias -- overconfidence (in particular, overprecision) -- is consistent with their results, and thus emerges as a parsimonious unified explanation for their findings.

Four recent papers replicate the Schweitzer and Cachon (2000) result. Bolton and Katok (2008), Benzion et al. (2007), Bostian et al. (2008) and Moritz et al. (2008) examine behavior and learning dynamics in the newsvendor setting. All these papers find that while behavior improves over time, biased orders are still observed. Table 1 contains more detail on results from these previous experimental studies.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Demand distribution</th>
<th>High-profit: Q*&gt; mean of demand</th>
<th>Low-profit: Q*&lt; mean of demand</th>
<th>Total # of subjects</th>
<th># of rounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolton and Katok (2008)</td>
<td>U(0,100) in H condition U(50,150) in L condition;</td>
<td>75</td>
<td>61</td>
<td>75</td>
<td>88</td>
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<tr>
<td>Benzion et al. (2007)</td>
<td>U(1,300)</td>
<td>225</td>
<td>176</td>
<td>75</td>
<td>142</td>
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<tr>
<td></td>
<td>N(150,2500)</td>
<td>184</td>
<td>157</td>
<td>116</td>
<td>150</td>
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<tr>
<td>Bostian et al. (2008)</td>
<td>U(1,100)</td>
<td>75</td>
<td>66</td>
<td>25</td>
<td>39</td>
</tr>
<tr>
<td>Moritz et al. (2008)</td>
<td>N(100,20)</td>
<td>120</td>
<td>112</td>
<td></td>
<td>370</td>
</tr>
</tbody>
</table>

**Table 1.** Results from previous newsvendor experiments

While Schweitzer and Cachon (2000) eliminated theories inconsistent with the observed data, only one previous paper offers a formal model of a behavioral explanation consistent with these observations. Su (2008) provides a logit overlay on optimal decision-making in inventory settings. In particular, he assumes that individuals make decisions with noise; they choose the optimal order with a higher probability than suboptimal orders, but not with certainty. He demonstrates that, assuming a uniform or triangular demand distribution, this trembling-hand model of decision making generates predictions consistent with those observed in these experiments.3

Our study makes several contributions to the current literature. We are the first to propose and demonstrate that overconfidence (in fact, overprecision) is a consistent explanation for suboptimal ordering in the newsvendor problem, both in the high- and low-profit settings. Furthermore, we demonstrate the ability of overconfidence to explain these results under a general demand function. We calculate the extent of bias in the orders, and show how the orders’ deviation from optimal levels change as the level of overconfidence increases. We also calculate the cost of these deviations to the biased newsvendor. In section 5 we go one step further, asking how an unbiased manager might construct incentives to manage such an overconfident newsvendor, seeking to align their ordering decisions with those recommended by the critical fractile solution. Thus, in addition to identifying, analyzing, and assessing the impact of the problem, we also propose a possible solution.

3. **Our Environment and Overconfidence**

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3A few contemporaneous papers take a different approach, experimentally examining the relationship between individual personality characteristics and their performance in the newsvendor problem. Moritz et al. (2008) showed that individuals who performed better on a test of cognition performed better in the newsvendor problem. Bolton et al. (2008) compared the performance of first-year undergraduates, graduate students and experienced managers, and found that all three groups behaved similarly (and suboptimally).
We follow the classical treatment of the newsvendor problem and use the following variables and assumptions about their levels, consistent with Cachon (2003):

\[ p = \text{unit sales price} \ (p>0) \]
\[ c = \text{unit cost} \ (0<c<p) \]
\[ s = \text{unit salvage value} \ (0<s<c<p) \]
\[ Q = \text{inventory to be ordered} \ (Q\geq0) \]
\[ D = \text{quantity demanded} \ (D\geq0), \text{a realization of a continuous random variable} \ D \]
\[ \mu = \text{mean of} \ D \]
\[ \sigma^2 = \text{variance of} \ D \]
\[ F_D = \text{cumulative distribution function of} \ D \]
\[ m = F_D(\mu), \text{the probability that actual demand is less than expected demand.} \]

In the newsvendor model, each morning the agent decides how many newspapers to purchase (Q) for marginal (and constant) cost (c) per unit. He sells the newspapers over the course of the day for a constant price (p) each, where p>c. The number of papers demanded is drawn from the demand distribution D, which is assumed to be stationary. The demand distribution D has mean \(\mu\) and variance \(\sigma^2\), and is described by its cumulative distribution function \(F_D\). When Q>D, excess newspapers are salvaged by the newsvendor with recovery \(s\) per unit, where \(s<c\). We further restrict our attention to \(F_D\) which are continuous and strictly increasing. We denote the well-known critical fractile solution (Arrow et al., 1951) for this single period newsvendor problem as \(\beta\), where

\[ \beta \equiv Pr(D \leq Q) = \frac{p-c}{p-s} \quad (1) \]

Since \(F_D\) is continuous and strictly increasing, we can find a unique optimal solution:

\[ Q^* = F_D^{-1}(\beta), \text{where} \ F_D^{-1} \text{denotes the inverse CDF. We will use this benchmark of optimal orders to compare against the orders placed by an overconfident newsvendor.} \]
For our overconfident newsvendor, his estimate of the mean consumer demand is accurate, but his estimate of the variance of consumer demand is biased. In particular, the true market demand distribution is more variable than the market demand distribution which the newsvendor believes he faces. We adopt the general notion of “more variable” from Ross (1983): if $X_1$ and $X_2$ are nonnegative random variables and $E(X_1) = E(X_2)$, then $X_1$ is said to be more variable than $X_2$ (written $X_1 \geq_v X_2$) if and only if $E[h(X_1)] \geq E[h(X_2)]$ for all convex functions $h$.

For practical purposes, we assume that the demand in the overconfident newsvendor’s mind $D_o$ is a mean-preserving but variance-reducing transformation of the true consumer demand $D$, mixing the true distribution with a zero-variance distribution around $\mu$.4

$$D_o = \gamma D + (1-\gamma)\mu, 0 \leq \gamma \leq 1$$

(2)

Here $\gamma$ represents the extent to which the newsvendor is well-calibrated: $\gamma = 1$ corresponds to a perfectly-calibrated newsvendor who predicts demand based on its actual distribution $D$, whereas $\gamma = 0$ corresponds to an infinitely overconfident newsvendor who believes that $Pr(D = \mu) = 1$. As $\gamma$ increases, overconfidence thus decreases. We will refer below to the level of overconfidence as $(1-\gamma)$. Since $D_o$ is an affine transformation of $D$, $E[D_o] = \mu$ and $Var[D_o] = \gamma^2\sigma^2$, denoted $\sigma^2_o$ for convenience.

**Proposition 3.1 (Basic Properties of $D_o$):** $D \geq_v D_o$.

**Proof:** For any convex function $h$,

$$E[h(D_o)] = E[h(\gamma D + (1-\gamma)\mu)] \leq E[\gamma h(D) + (1-\gamma)h(\mu)]$$

$$E[\gamma h(D) + (1-\gamma)h(\mu)] = \gamma E[h(D)] + (1-\gamma)h(\mu) = E[h(D)] + (1-\gamma)[h(\mu) - E[h(D)]]$$

The right-hand side is less than or equal to $E[h(D)]$ since Jensen’s inequality guarantees that $[h(\mu) - E[h(D)]]$ is never positive.

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4We use this particular formulation for two reasons. First, it accurately captures the well-documented psychological phenomenon of overprecision, dating back to Raiffa and Alpert (1982): on average individual estimates are unbiased, but confidence intervals are too small. Second, we want to isolate the impact of this particular mistake (underestimating the variance) on ordering behavior from other possible biases. A more general formulation of newsvendor mistakes, a topic for future research, might include a misestimation of both the mean and variance of the demand distribution.
4. Overconfident Newsvendor Decisions

In this section, we study the effects of newsvendor overconfidence on the ordering decision. The results are intuitive: Because the overconfident newsvendor underestimates the variance of demand, he places orders closer to the mean of demand than the optimal order would suggest -- exactly the empirically-observed result of Schweitzer and Cachon (2000) and subsequent papers. We show that this order bias has a linear relationship with the level of overconfidence (1-\(\gamma\)).

4.1 Biased Orders of Overconfident Newsvendors

We begin by deriving \(Q^*_o\), the order placed by the overconfident newsvendor, and comparing it with the optimal order \(Q^*\).

**Proposition 4.1.**

(a) When \(\beta > m\), \(Q^*_o = \mu + \gamma \sigma F_S^{-1}(\beta) < Q^*\).

(b) When \(\beta < m\), \(Q^*_o = \mu + \gamma \sigma F_S^{-1}(\beta) > Q^*\).

(c) When \(\beta = m\), \(Q^*_o = Q^* = \mu\).

**Proof.** Appendix A provides a complete proof. The proof's basic steps are as follows: Define \(X\) as a random variable satisfying \(D = \sigma X + \mu\). \(X\) is thus a standardized distribution with mean 0 and variance 1. Substitute \(D = \sigma X + \mu\) into (2) above, yielding \(D_o = \sigma_o X + \mu\). Now define \(F_S(x)\) as the CDF of \(X\) and define \(F_S(b) = \beta\) and thus \(F_S^{-1}(\beta) = b\). Then \(Q^* = F_D^{-1}(\beta) = \mu + \sigma b\) and \(Q^*_o = F_o^{-1}(\beta) = \mu + \sigma_o b\). Since we know \(\sigma_o < \sigma\), the relative order sizes depend only on the sign of \(b\). When \(\beta > m\) then \(b > 0\), and thus \(Q^*_o < Q^*\). When \(\beta < m\), then \(b < 0\) and thus \(Q^*_o > Q^*\). When \(\beta = m\), then \(b=0\) and \(Q^*_o = Q^* = \mu\).

We note that when \(\beta > m\), we are in the high-profit condition as defined by Schweitzer and Cachon (2000) and used in subsequent experiments (op. cit.). In exactly these cases, the optimal order is higher than mean demand and overconfident newsvendors order less than is optimal – moving their order towards the demand mean. In contrast, when \(\beta < m\) we are in the low-profit condition: the optimal order is lower than mean demand, and overconfident newsvendors order more than is optimal – again moving
their order towards the demand mean. Our model of overconfidence is thus consistent with the observed data in the experimental literature.

How much are these orders biased? Since \( \sigma_w = \gamma \sigma \), we know that \( Q^* - Q_o^* = (1 - \gamma) \sigma b \), a linear relationship between the deviation from the optimal order \( Q^* - Q^* \) and the level of overconfidence \((1 - \gamma)\). As \(1 - \gamma\) approaches 0, the newsvendor’s level of overconfidence (bias) goes down, and the deviation from the optimal order decreases linearly in both the high- and low-profit cases.

The effect of overconfidence also has a positive relationship with the (true) variance of demand since \( \frac{\partial (Q^* - Q_o^*)}{\partial \sigma} = (1 - \gamma) b \). When facing a market demand with higher variation, the overconfident newsvendor makes a more biased order, given the same level of overconfidence – in effect making a larger mistake.

The market’s inherent profitability also affects the amount of deviation from optimal behavior caused by overconfidence. More profitable markets lead to more overstocking relative to expected demand; less profitable markets lead to more understocking relative to expected demand. As the distance \( b \) between the optimal solution \( Q^* \) and the mean market demand \( \mu \) increases, holding the level of overconfidence \(1 - \gamma\) constant, the overconfident newsvendor’s orders move farther away from the optimal order levels, as \( \frac{\partial (Q^* - Q_o^*)}{\partial b} = (1 - \gamma) \sigma > 0 \). We thus expect overconfident newsvendors to make particularly large errors in both highly profitable markets (such as Sport Obermeyer) and very low-profit markets (such as Jeppson-Sanderson).

**4.2 The Costs of Overconfidence**

The previous section demonstrates that overconfident newsvendors order suboptimal amounts. We have thus far focused on the newsvendor’s “mistake” in terms of quantity over- or under-ordered. How much does this mistake cost the newsvendor in terms of forgone profitability?

We can rewrite the general form for the profit of the newsvendor as:

\[
\pi(D, Q) = (p - c)\mu - (p - c)E(D - Q)^* - (c - s)E(Q - D)^* = (p - c)\mu - G(D, Q)
\]  
(3)
where \((X)^+\) denotes \(\max\{X, 0\}\).

Since \((p - c)\mu\) (expected revenue from sales) is fixed, to maximize expected profit the newsvendor simply minimizes the cost term \([G(D, Q)]\) in (3), which (following the classic formulation of Arrow et al., 1951) includes both economic costs of overstocking and opportunity costs of understocking.

For the optimal order quantity, we can rewrite \(G(D, Q)\) as:

\[
G(D, Q^*) = (p - s)\sigma E(X | X \geq b)
\]

(4)

The overconfident newsvendor, however, will face different costs by ordering \(Q_o^*\):

\[
G(D, Q_o^*) = (c - s)\gamma \sigma b - (p - s)(1 - F_s(\gamma b))\sigma \gamma b + (p - s)\sigma E(X | X \geq \gamma b)
\]

(5)

Taking the difference of the two cost functions yields

\[
G(D, Q_o^*) - G(D, Q^*) = (p - s)\sigma [\gamma b(1 - F_s(b)) - (1 - F_s(\gamma b))] + \int_{\gamma b}^{b} x f_s(x) dx
\]

(6)

**PROPOSITION 4.2.** If \(Q^* = \mu + \sigma b\) and \(Q_o^* = \mu + \sigma_o b\), then \(\pi(D, Q^*_o) < \pi(D, Q^*)\).

**Proof:** Appendix B contains a complete proof that whenever \(\beta \neq m\):

\[
G(D, Q^*_o) > G(D, Q^*) \iff \pi(D, Q^*_o) < \pi(D, Q^*)
\]

When \(\beta = m\), then \(G(D, Q^*_o) = G(D, Q^*) \iff \pi(D, Q^*_o) = \pi(D, Q^*)\). We have thus shown that the profits of an overconfident newsvendor never exceed the profits of a well-calibrated one, and are strictly less when \(\beta \neq m\). We can now turn our attention to quantifying the magnitude of these forgone profits.

**PROPOSITION 4.3.** The relationship between the level of overconfidence and expected lost profit is positive and convex.

**Proof.** The lost profit due to overconfidence can be written as:

\[
\pi(D, Q^*) - \pi(D, Q^*_o) = G(D, Q^*_o) - G(D, Q^*)
\]

Taking the first and second derivatives with respect to \(\gamma\) yield:

\[
\frac{\partial}{\partial (1 - \gamma)} (\pi(D, Q^*) - \pi(D, Q^*_o)) = -(p - s)\sigma b[F_s(\gamma b) - F_s(b)] > 0
\]
Appendix B shows that \( b[F_s(\gamma b) - F_i(b)] \) is always negative, while \((p-s)\) and \( \sigma \) are positive by assumption. Thus (7), representing the first derivative of the difference in profits, is negative, demonstrating that as \((1-\gamma)\) approaches 1 (i.e., as overconfidence grows), the magnitude of lost profit increases.

The positive second derivative in (8) tells us that the loss function is not only increasing but convex. Again, \((p-s)\) is assumed to be positive. \( b^2 \), \( \sigma \) and \( f_s(\gamma b) \) are nonnegative by definition. This implies that the downward response of profits to increases in levels of overconfidence is convex: as the bias gets larger, profits fall faster. We illustrate these relationships in Figure 2, using a Gaussian demand distribution with mean 10 and variance 1 as an example. We set \( p=1 \) and \( s=0.4 \), and plot the relationship between the level of overconfidence \((1-\gamma)\) and the lost profits due to overconfidence \( \pi(D, Q^o) - \pi(D, Q_s^o) \).

Figure 2 shows curves for two different levels of \( c=\{0.5, 0.6\} \) which generate two different values of \( \beta=\{5/6, 4/6\} \). Note that, since the normal distribution is symmetric about its mean, these specific cost levels will also determine the value of lost profits for the (different) costs \( c=\{0.9, 0.8\} \) generating \( \beta =\{1/6, 2/6\} \) respectively even though the baseline profits of these four values of \( \beta \) are all distinct. The numerical values of these expected profits for \( \beta=\{1/6, 2/6, 4/6, 5/6\} \) are \{0.85, 2.76, 3.78, 4.85\} respectively. As shown above, these costs of overconfidence clearly increase and are convex in \((1-\gamma)\).
Figure 2: Profits Lost from Overconfidence when \( p=1, s=0.4, c = \{0.5 \text{ or } 0.9, 0.6 \text{ or } 0.8\} \)

5. How to Manage an Overconfident Newsvendor

We have shown that overconfident newsvendors make suboptimal inventory decisions and that these decisions are costly. Now imagine that a well-calibrated manager of newsstands is attempting to manage an overconfident newsvendor. How might she do so?

One possibility, of course, is a hierarchical command-and-control solution; with complete authority over the newsvendor’s actions, the manager could simply force the newsvendor to order \( Q^* \). Given that the newsvendor (and not the manager) must choose the quantity ordered, however, and that the newsvendor has both motive and method to substitute her own judgment for the manager’s, our analysis instead explores incentives or simple contractual arrangements which the manager might use to induce the newsvendor to order the optimal amount.

We are thus invoking the spirit (if not hewing to the letter) of Cachon’s (2003) method of creating coordinating contracts. In contrast, we assume there are no incentive problems between the manager and the newsvendor; both want to maximize the profits of the newsstand to the best of their abilities. We instead assume that the manager and the newsvendor have different levels of understanding of the demand distribution; the manager knows the true demand distribution, whereas the newsvendor is mistaken about it. Our challenge is thus to design an incentive structure to induce the newsvendor to order the optimal amount given his biased beliefs.\(^5\)

The most straightforward way to correct the biased behavior is by changing the cost structure facing the newsvendor, thereby inducing him to choose his order to be the same as the optimal order. The design of this cost structure entails finding \( \beta_o \) to solve the problem: \( Q_o(D_o, \beta_o) = Q^*(D, \beta) \). Under this

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\(^5\)One might object that this is simply a communication problem: that the unbiased manager could simply “tell” the overconfident newsvendor the true variance of demand. However, the overconfident newsvendor has no reason to believe that the manager’s estimate is better than his own; indeed, we can safely assume that this communication has already occurred and has already been factored into the newsvendors’ overconfident beliefs and thus into the \( \gamma \)-weighting from (2).
\(\beta_o\) regime, the newsvendor who orders the optimal amount given his own beliefs will, in fact, order the correct amount given the true demand distribution.

**Proposition 5.1.** Define \(\alpha = \beta_o - \beta\) and \(\alpha^*\) as the value of \(\alpha\) which induces the agent to choose \(Q^* = Q^o\). Then

\[
\alpha^* = \int_{F_s^{-1}(\beta)}^\beta f_s(x)dx = \int_{\beta}^\infty f_s(x)dx
\]

(9)

**Proof:** if we want the agent to choose \(Q^* = Q^o\), we need to set \(F_s^{-1}(\beta_o) = \frac{b}{\gamma}\), since

\[
Q^* = Q^o \iff \mu + \gamma\sigma F_s^{-1}(\beta_o) = \mu + \sigma F_s^{-1}(\beta) \iff \gamma F_s^{-1}(\beta_o) = F_s^{-1}(\beta) \iff F_s^{-1}(\beta_o) = \frac{b}{\gamma}
\]

Since \(\alpha^* = \int_{\beta}^\infty f_s(x)dx\), we can find \(\int_{F_s^{-1}(\beta)}^\beta f_s(x)dx = \beta_o - \beta = \int_{\beta}^\infty f_s(x)dx \iff F_s^{-1}(\beta_o) = \frac{b}{\gamma}\).

From the definition of \(\alpha^*\) as the quantity adjustment required to achieve the optimal order, we can derive the following relationships from Proposition 4.1:

- If \(\beta > m\), then \(\alpha^* > 0\). If \(\beta < m\), then \(\alpha^* < 0\), and if \(\beta = m\), \(\alpha^* = 0\).

When the optimal order is larger than the mean of the demand distribution, the manager wants the overconfident newsvendor to order more than he normally would. She thus needs to add \(\alpha^*\) to the \(\beta_o\) he would otherwise face. The reverse holds when the optimal order is less than the mean of the demand distribution.

The manager has numerous tools available to alter the \(\beta_o\) that goes into the newsvendors’ decision. Here we discuss two common and easily-implemented tools frequently investigated in the operations management literature: (a) altering the salvage cost facing the newsvendor using a buyback contract (as examined in Song et al., 2008) or (b) altering the basic unit cost schedule facing the newsvendor (as in Lariviere and Porteus, 2001).

**5.1 Using Buyback Contracts**
By using a buyback contract, the manager could augment the true salvage value \( s \) with a bonus (or penalty) \( I_s \) for each unit left at the end of the period. Thus the effective salvage value facing the newsvendor, which we call the *return value*, would be \((s+I_s)\). Intuitively, if the overconfident newsvendor would order less than optimal, the manager should choose a positive \( I_s \) (making the return value higher than the true salvage value – a classic buyback contract), whereas if he would order more than optimal, the manager should choose a negative \( I_s \) (making the return value lower than the true salvage value, e.g. demanding a fee to return, presumably coupled with monitoring to ensure that the inventory is not surreptitiously sold to avoid the penalty). Recall that the optimal order quantity given the true salvage value is \( Q^* = F^{-1}_d(\beta) = F^{-1}_d\left(\frac{p-c}{p-s}\right) \).

**Proposition 5.2** The unique \( I_s \) which induces \( Q^* = Q^*_s \) is \( I^*_s = \frac{(p-s)\alpha^*}{(p-c+(p-s)\alpha^*)} \).

*Proof.* When facing a return value of \((s+I_s)\), the newsvendor will order

\[
Q^*_o = F^{-1}_o(\beta^*_o) = F^{-1}_o\left(\frac{p-c}{p-s-I_s}\right).
\]  

(10)

We can rewrite \( \beta^*_o = \frac{p-c}{p-s-I_s} = \frac{p-c}{p-s} + \frac{p-c}{p-s} \frac{I_s}{p-s-I_s} = \beta + \alpha^* \),

Therefore,

\[
\alpha^* = \frac{p-c}{p-s} \frac{I_s}{p-s-I_s} \Leftrightarrow (p-c)I_s = \alpha^* (p-s)(p-s-I_s) \Leftrightarrow (p-c+(p-s)\alpha^*)I_s
\]

\[
= (p-s)^2 \alpha^* \Leftrightarrow I^*_s = \frac{(p-s)^2 \alpha^*}{(p-c+(p-s)\alpha^*)}.
\]

We can see that the denominator is strictly positive for any value of \( \alpha \).

\( p-c+(p-s)\alpha > 0 \Leftrightarrow \frac{p-c}{p-s} > -\alpha \Leftrightarrow \beta > -\alpha \Leftrightarrow \beta + \alpha > 0 \Leftrightarrow \beta^*_o > 0 \).

Since \( p>c \) and \((p-s)^2\) is always positive, \( I^*_s \) has the same sign as \( \alpha^* \).
PROPOSITION 5.3. In the high-profit condition, \( I'_s > 0 \); in the low-profit condition \( I'_s < 0 \). When \( \beta = m \), \( I'_s = 0 \).

In the high-profit regime (\( \beta > m \)), the manager should offer to supplement the existing salvage value by \( I'_s \) to encourage the overconfident newsvendor to order more. This implies that the manager will (probabilistically) buy back unsold goods at the end of the period at a price above the true salvage value. On the other hand, in the low-profit regime (\( \beta < m \)), the manager should demand a salvage penalty (a negative \( I'_s \)) to encourage the overconfident newsvendor to order less. This implies that the manager will buy back unsold goods below their true salvage value.

Note, however, that given an overconfidence level \( \gamma \), the extra salvage value and the extra salvage penalty are not simply negatives of each other, which would require a linear relationship between \( I'_s \) and \( \alpha^+ \). In particular, \( \frac{\partial I'_s}{\partial \alpha^+} = \frac{(p-s)^2(p-c)}{(p-c+(p-s)\alpha^+)^2} \) which is not constant given that \( p \neq s \). \( I'_s \) thus does not have a linear relationship with \( \alpha^+ \) and the optimal adjustments to salvage value are asymmetric.

5.2 Using Price Contracts

Instead of altering the effective salvage value facing the overconfident newsvendor, the well-calibrated manager can subsidize (or increase) his unit cost by adding an additional cost or subsidy \( I_c \) for each unit ordered. The marginal cost facing the newsvendor, which we call the apparent cost, would thus be \( (c + I_c) \). In the high-profit regime, the overconfident newsvendor would order less than is optimal and thus the manager should choose a negative \( I_c \) (making the apparent cost lower than the true marginal cost to induce larger orders). Conversely, in the low-profit regime, the overconfident newsvendor would order more than optimal and the manager should thus choose a positive \( I_c \) (making the apparent cost higher than the true marginal cost). We can solve for the unique \( I_c \) for a given level of overconfidence and market structure.

PROPOSITION 5.4. The unique \( I_c \) which cause \( Q^* = Q^*_o \) is \( I'_c = -(p-s)\alpha^+ \).
Proof. Recall that the overconfident newsvendor’s optimal order is $Q^* = F^{-1}_a(\beta_c)$ and that

$$Q^* = F^{-1}_a(\beta) = F^{-1}_d\left(\frac{p-c-L_t}{p-s}\right)$$

is the optimal order from the manager’s perspective. Under the additional cost (subsidy) condition, $\beta_n = \frac{p-c-L_t}{p-s} = \frac{p-c}{p-s} - \frac{L_t}{p-s} = \frac{p-c}{p-s} + \left(-\frac{L_t}{p-s}\right)$. Therefore,

$$\alpha^* = -\frac{L_t}{p-s} = \int_b^2 f_s(x)dx \Leftrightarrow L_t = -(p-s)\int_b^2 f_s(x)dx \Leftrightarrow L_t = -(p-s)\alpha^*.$$ 

Unlike the analogous salvage value, we can see that $I_c$ has a simple negative linear relationship with $\alpha^*$. We can therefore conclude:

**Proposition 5.5.** In the high-profit condition, $I_c^* < 0$; in the low-profit condition, $I_c^* > 0$. When $\beta = m$, $I_c^* = 0$.

Thus the optimal cost adjustment for high-profit market is a subsidy (encouraging the newsvendor to order more), while the optimal cost adjustment for low-profit markets is a surcharge (encouraging the newsvendor to order less).

5.3 Relationship between Overconfidence Level and Size of Incentive

Here we describe the relationship between the newsvendor’s level of overconfidence and the size of the incentive that the unbiased manager must offer.

**Proposition 5.6.** The size of the incentive is positively related to the extent of the bias.

Proof. Taking the derivative of $\alpha^*$ (the required adjustment) with respect to the level of overconfidence yields

$$\frac{\partial \alpha^*}{\partial(1-\gamma)} = \frac{\partial}{\partial(1-\gamma)} \int_b^2 f_s(x)dx = \frac{b}{\gamma^2} f_s\left(\frac{b}{\gamma}\right).$$

(11)

Therefore, $\frac{\partial \alpha^*}{\partial(1-\gamma)}$ has the same sign as $b$. If $\beta > m$, then $\frac{\partial \alpha^*}{\partial(1-\gamma)} > 0$. If $\beta < m$, then $\frac{\partial \alpha^*}{\partial(1-\gamma)} < 0$. If $\beta = m$, then $\frac{\partial \alpha^*}{\partial(1-\gamma)} = 0$. 

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When the manager uses an augmented salvage value to align the newsvendor’s actions, the derivate of $I_s$ with respect to $(1-\gamma)$ yields:

$$\frac{dl_s}{d(1-\gamma)} = \frac{\partial I_s}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial (1-\gamma)} = \frac{(p-s)(p-c)}{(p-c+(p-s)\alpha^2)} \frac{\partial \alpha^*}{\partial (1-\gamma)}$$

The numerator being unambiguously nonnegative, $\frac{dl_s}{d(1-\gamma)}$ will have the same sign as $\frac{\partial \alpha^*}{\partial (1-\gamma)}$. If $\beta > m$, then $I_s > 0$, $\frac{\partial \alpha^*}{\partial (1-\gamma)} > 0$ and thus $\frac{dl_s}{d(1-\gamma)} > 0$. Similarly, when $\beta < m$, $\frac{dl_s}{d(1-\gamma)} < 0$. The absolute value of $I_s$, representing the size of the bonus or penalty, thus has a positive relationship with the level of overconfidence. As the newsvendor becomes more overconfident, the manager needs to offer a larger salvage bonus or penalty.

We can derive a similar result for adjusting marginal cost. Taking the first derivative of $I_c$ with respect to $(1-\gamma)$ yields

$$\frac{dl_c}{d(1-\gamma)} = \frac{\partial I_c}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial (1-\gamma)} = -(p-s) \frac{\partial \alpha^*}{\partial (1-\gamma)}$$

Since $p > s$, $(p-s)$ is always positive and thus $\frac{dl_c}{d(1-\gamma)}$ has the opposite sign from $\frac{\partial \alpha^*}{\partial (1-\gamma)}$ and thus of $b$, given that price reductions increase quantity ordered (and vice versa for price increases). Using similar reasoning, we thus find that the absolute size of the incentive (positive or negative) that the manager needs use to adjust the marginal cost is positively related to the level of overconfidence.

Figure 3a and 3b illustrate the relationship between the required subsidies (or bonuses) and the newsvendor’s level of overconfidence using the same normal distribution and parameters as in Figure 2. In Figure 3a we show the optimal price adjustment $I'_c$ as a function of overconfidence level $(1-\gamma)$. We again depict $I'_c$ for different levels of $c$ (ranging from 0.5 to 0.9), yielding $\beta$ ranging from $1/6$ to $5/6$.

When $\beta > \frac{1}{2}$ (the high-profit condition) $I'_c$ is negative and decreases even further as overconfidence
increases. When \( \beta < \frac{1}{2} \) (the low-profit condition) \( I_c^* \) is positive and increases further as overconfidence increases.

In Figure 3b we use the same parameters to show the optimal salvage-value adjustment \( I_s^* \) as a function of \( (1-\gamma) \). When \( \beta > \frac{1}{2} \) (the high-profit condition) \( I_s^* \) is positive and increases with overconfidence. When \( \beta < \frac{1}{2} \) (the low-profit condition) \( I_s^* \) is negative and decreases with overconfidence. Note also that, in contrast to \( I_c^* \), \( I_s^* \) is asymmetric, as demonstrated in section 5.3 above.

**Figure 3a.** \( I_c^* \) under demand \( D \sim N(10, 1) \), with parameters \( p=1, s=0.4, c = \{0.5, 0.6, 0.8, 0.9\} \)

**Figure 3b.** \( I_s^* \) under demand \( D \sim N(10, 1) \), with parameters \( p=1, s=0.4, c = \{0.5, 0.6, 0.8, 0.9\} \)

### 5.5 Constraints on the Tools
Our previous analysis assumed the manager could choose any $I_s^*$ and $I_c^*$ she wanted. However, the manager faces constraints which may make these tools less effective for inventory management.

First, consider changing the salvage value facing the overconfident newsvendor by adding (or subtracting) $I_s^*$ to form the return value. When $\beta > m$, $I_s^* > 0$, and thus the return value promised by the manager will be higher than the true salvage value. The manager faces a basic constraint, however, in using this tool; the return value cannot exceed the marginal cost $c$, or else the newsvendor will have an incentive to buy an infinite amount of the newspaper and return any extra for a profit. Similarly, when $\beta < m$, $I_s^* < 0$, thus the return value will be less than the true salvage value. Again the manager faces a constraint, the return value cannot be negative -- otherwise the newsvendor will have an incentive to simply dispose of the papers or give them away for free rather than to pay to return them. These constraints imply:

\begin{align*}
\text{when } \beta &> m, \ c - s \geq I_s^* > 0 \text{ and } \\
\text{when } \beta &< m, \ 0 > I_s^* \geq -s
\end{align*}

We substitute $I_s^*$ from Proposition 5.2 and find that the first constraint will never bind: it implies that $c - s \geq I_s^* > 0 \iff 1 \geq \beta_s > \beta$, which is always the case when $\beta > m$. Thus, no matter how overconfident the newsvendor, when $\beta > m$ the amount of additional salvage value the manager needs to offer the newsvendor is not sufficient to put the return value above the marginal cost, making this tactic effective.

However, the second constraint may bind: it implies that $\beta > \beta_o \geq \frac{(P - s)}{p} \beta$, which may or may not hold when $\beta < m$. Thus, when the manager is facing a particularly overconfident newsvendor in the low-profit condition, the salvage penalty she would have to impose might put the return value below zero, making it an inappropriate (and ineffective) tool to use.

Now let us consider the limitations of changing the marginal cost facing the overconfident newsvendor. When $\beta > m$, $I_c^* < 0$, purchases are subsidized, and thus apparent cost is lower than the true...
marginal cost. The manager again faces a constraint on how large this subsidy can be: the apparent cost cannot be less than the salvage value, because otherwise the newsvendor would buy an infinite number of papers and sell any remaining ones back to the manager. Similarly, when $\beta < m$, $I^*_c > 0$, purchases are “taxed,” and thus the apparent cost will be higher than the marginal cost. Again the manager faces a constraint: the apparent cost cannot be higher than the price at which the papers are sold to customers -- otherwise the newsvendor will not buy any papers, as there is no profit to be made.

These constraints imply

$$\text{when } \beta > m, \ 0 > I^*_c \geq s - c \text{ and}$$

$$\text{when } \beta < m, \ p - c \geq I^*_c > 0$$

We substitute $I^*_c$ from Proposition 5.4 and find that neither constraint binds the manager. For (14) we see that $0 > I^*_c \geq s - c \iff 1 \geq \beta, > \beta$, which is always true when $\beta > m$ (as with the salvage value subsidy, above). Thus, even when the manager is facing a very overconfident newsvendor in the high-profit condition, the subsidy she must offer to induce $Q^*$ is never so large as to drive the apparent cost below the salvage value. The second constraint also does not bind. It implies $p - c \geq I^*_c > 0 \iff \beta \geq \beta_0 > 0$ which is always the case when $\beta < m$. Thus, even when the manager is facing a very overconfident newsvendor in the low-profit condition, the cost penalty necessary to adjust the overconfident newsvendor’s order downwards to $Q^*$ will never yield a marginal cost higher than the price.

While both tools we have identified could, in theory, be used to induce an overconfident newsvendor to order an optimal amount, the well-calibrated manager is limited in the size and type of incentives she can offer. These results indicate that in both the high-profit and low-profit regimes cost adjustments can be used to manage an overconfident newsvendor, but that salvage-value adjustments (buyback contracts) may not be practical in low-profit regimes, especially when the newsvendor’s level of overconfidence is high.

5.6 Managing Overconfident Newsvendors In the Field
In this section, we have proposed two techniques to correct biased decisions made by an overconfident newsvendor. In addition to being theoretically effective, these techniques are often employed in business practice. Cachon (2003) describes multiple types of contracts observed in inventory-ordering settings, including wholesale price contracts (which alter the marginal cost faced by the newsvendor) and buyback contracts (which alter the salvage value faced by the newsvendor). Our model provides an additional behavioral explanation for why these contracts might be observed in the field: to correct biased decision making caused by overconfidence. Additionally, our model yields some insight into a reason why wholesale price contracts are common whereas buyback contracts are less universal.

Our model also goes one step further to predict which form of supply contract we expect to observe in various types of industries. In high-profit industries (where $\beta > m$), overconfident agents are likely to order less inventory than is optimal. Managers will be able to correct this biased behavior by either increasing the salvage value or decreasing the marginal cost. In this case, either type of contract might be observed in practice.

In low-profit industries (where $\beta < m$), overconfident agents are likely to order more inventory than is optimal. This biased behavior can be corrected by decreasing the salvage value or increasing the marginal costs. As we saw, however, marginal cost adjustments can always implement optimal orders, whereas the salvage-value adjustments required to implement optimal orders might involve return values below zero, rendering them ineffective as a corrective tool. We would thus predict that in these low-profit industries we would see wholesale-price contracts but not buyback contracts. If wholesale-price contracts that discourage overordering are not practical (or can be easily evaded, e.g., by subdividing orders), we expect that poor stocking decisions will cause channel profitability to decline even further than would be expected from the low-margin regime.

Our results can also explain the conflicting empirical results from Fisher and Raman (1996) and Katok et al. (2001). In both studies, decision-makers placed orders closer to the mean of the demand
distribution than the profit/loss incentives would indicate was optimal. In the former we saw under-ordering of inventory, consistent with a market where $\beta > m$. In the latter, we saw over-ordering of inventory, consistent with a market where $\beta < m$. This model can explain and organize not only the experimental results from newsvendor ordering, but also a series of observational results from the field. It also makes new predictions between industry characteristics and the incidence of different types of contracts, which can be empirically tested in future research.

**Extension: How to Manage an Underconfident Newsvendor**

Although our results have all been presented in terms of overconfident (overprecise) newsvendors, the model could similarly be applied to the problem of underconfident newsvendors. Results from the preliminary investigation of this extension (presented in Appendix C) are quite intuitive. Underconfident newsvendors’ orders also deviate from optimal levels: they order too much in the high-profit condition (to better exploit their perception of a relatively high probability of extremely high demand, and partially protected by their ability to put the inventory back to the manager in case of extreme overstocking). In the low-profit condition, newsvendors may not participate at all, as they fear that they may very often lose c-s of a large portion of their inventory that remains unsold due to extremely low demand, with only meager compensation when they sell their entire stock. Appendix C presents some preliminary mathematical analysis of the underconfident newsvendor.

### 6. Conclusion and Discussion

This paper makes a number of contributions to the growing field of Behavioral Operations Management. We introduce the idea of overconfidence into the traditional newsvendor model, wherein overconfident newsvendors underestimate the variance of the demand distribution they are facing and thus place suboptimal orders. We suggest that overconfidence (and in particular, overprecision) explain systematic deviations from optimal ordering and directly compute the orders that would be placed by such a biased newsvendor, showing that they deviate from optimality in ways consistent with experimental and
field evidence. We characterize the profits lost from these suboptimal orders and show that these forgone profits are convex in the level of overconfidence. Finally, we propose two tools which unbiased managers might use to incentivize overconfident newsvendors: adjusting the salvage value facing the newsvendor (to yield a return value which might be higher or lower than the true salvage value) and adjusting the marginal cost facing the newsvendor (to yield an apparent cost which might be higher or lower than the true marginal cost). We also show that these tools cannot be uniformly applied: salvage-value adjustments can be used in high-profit industries but not always in low-profit industries, whereas marginal cost adjustments can be used in either.

Our model shows that the observed behavior is consistent with the overconfidence bias. Of course, other biases may also occur in inventory settings. Schweitzer and Cachon (2000) suggest that individuals may be subject to an anchoring bias, and that anchoring is consistent with observed behavior. Su (2007) shows that adding noise to an individual’s decisions also yields predictions consistent with observed behavior. Our paper makes a contribution to this literature both by identifying overconfidence as possible culprit in biased orders and specifically modeling the decision process implemented by the overconfident newsvendor.

Integrating any social-science theory with the rich mathematical foundations of the operations management literature must be done carefully to ensure that the integrated theory offers firm, testable predictions, which can be tested (and their causes teased apart) in experimental and empirical work. In general, we believe that this new direction of integrating heuristics and biases into operations management is a fruitful one. The formal analysis from operations management forces consistency in modeling technique and assumptions, highlights the cases which must be analyzed, and provides rigor in supporting quite general conclusions. The psychological research explains anomalies observed in operations management settings, and its predictions generate new testable hypotheses which deepen our understanding of inventory decisions specifically and operational decisions more generally. Given the centrality of inventory management to operations and judgment and decision making to psychology, an
improvement in understanding how biased individuals make inventory decisions can undoubtedly strengthen both fields.
Appendix A: Proof of Proposition 4.1

Recall that \( F_s(b) = \beta \) and \( F_s^{-1}(\beta) = b \), and \( F_s() \) is the CDF of the standardized distribution \( X \).

Then using \( D = \sigma X + \mu \) we can write:

\[
F_D(\mu + \sigma b) = \Pr(D \leq \mu + \sigma b) = \Pr((D - \mu) / \sigma \leq b) = \Pr(X \leq b) = F_s(b) = \beta
\]

The optimal solution can be written as:

\[
Q^* = F_D^{-1}(\beta) = F_D^{-1}(\Pr(D \leq \mu + \sigma b)) = F_D^{-1}(F_D(\mu + \sigma b)) = \mu + \sigma b
\]

Using similar reasoning:

\[
F_d(\mu + \sigma b) = \Pr(D_d \leq \mu + \sigma b) = \Pr((D_d - \mu) / \sigma \leq b) = \Pr(X \leq b) = F_s(b) = \beta
\]

\[
Q_o^* = F_d^{-1}(\beta) = F_d^{-1}(\Pr(D_d \leq \mu + \sigma b)) = F_d^{-1}(F_d(\mu + \sigma b)) = \mu + \sigma b
\]

Appendix B: Proof of Proposition 5.1

The newsvend or’s profit can be written as:

\[
\pi(Q) = pE[\min(Q, D)] + sE(Q - D)^+ - cQ, \quad \text{from which we can directly get equation (3)}.
\]

It can be shown that \( E(D - Q^*)^+ = \sigma E(X - \gamma b)^+ = \sigma [E(X | X \geq b) - b(1 - \beta)] \). As a consequence,

\[
G(D, Q^*) = (c - s)(Q^* - \mu) + (p - s)E(D - Q^*)^+ = (c - s)(\mu + \sigma b - \mu) + (p - s)\sigma [E(X | X \geq b) - b(1 - \beta)]
\]

\[
= (p - s)\sigma E(X | X \geq b)
\]

Using the same calculation, the overconfident newsvend or’s cost is:

\[
E(D - Q_o^*)^+ = \sigma E(X - \gamma b)^+ = \sigma \int_{\gamma b}^{\infty} (X - \gamma b)f_s(x)dx = \sigma [E(X | X \geq \gamma b) - \gamma b(1 - F_s(\gamma b))]
\]

\[
G(D, Q_o^*) = (c - s)(Q_o^* - \mu) + (p - s)E(D - Q_o^*)^+ = (c - s)(\mu + \sigma b - \mu) + (p - s)\sigma [E(X | X \geq \gamma b) - (1 - F_s(\gamma b))\gamma b]
\]

\[
= (c - s)\gamma \sigma b - (p - s)(1 - F_s(\gamma b))\gamma b + (p - s)\sigma E(X | X \geq \gamma b)
\]

Subtracting (16) from (17) yields the difference between the costs:

\[
G(D, Q_o^*) - G(D, Q^*) = \sigma \gamma [c - p + (p - s)(1 - F_s(\gamma b)) + (p - s)\sigma [E(X | X \geq \gamma b) - E(X | X \geq b)]]
\]

\[
= \sigma \gamma [c - p + (p - s)F_s(b) - (p - s)F_s(\gamma b)] + (p - s)\sigma \left[ \int_{\gamma b}^{\infty} x f_s(x)dx - \int_{b}^{\infty} x f_s(x)dx \right]
\]
\[
= \sigma b(p - s)[F_s(\gamma b) - F_s(b)] + (p - s)\sigma \int_{-b}^{b} x f_s(x) dx
\]
\[
= (p - s)\sigma \{\gamma b[(1 - F_s(b)) - (1 - F_s(\gamma b))] + \int_{-b}^{b} x f_s(x) dx\}
\]  
(18)

Since \( b \) can be positive or negative, to identify the sign of equation (18) we need to examine three cases.

Case 1: When \( \beta > m \), then \( b > \gamma b > 0 \cdot 0 < \gamma < 1 \)

The first part of equation (18) can then be signed to be negative:

\[
\gamma b[(1 - F_s(b)) - (1 - F_s(\gamma b))] = \gamma b\int_{-b}^{b} f_s(x) dx - \int_{-b}^{b} f_s(x) dx
\]
\[
= \gamma b\int_{-b}^{b} f_s(x) dx - \int_{-b}^{b} f_s(x) dx - \int_{-b}^{b} f_s(x) dx = -\gamma b\int_{-b}^{b} f_s(x) dx < 0
\]

The second part of equation (18) is greater than or equal to zero by assumption. In addition, we can prove:

\[
\gamma b\int_{-b}^{b} f_s(x) dx < \gamma b\int_{-b}^{b} x f_s(x) dx = \int_{-b}^{b} x f_s(x) dx
\]

Therefore: \( (18) = (p - s)\sigma \{\int_{-b}^{b} x f_s(x) dx - \gamma b\int_{-b}^{b} f_s(x) dx\} > 0 \iff G(D, Q_o) > G(D, Q^*) \iff \pi(D, Q_o) < \pi(D, Q^*) \)

Case 2: When \( \beta > m \), then \( b < \gamma b < 0 \cdot 0 < \gamma < 1 \)

The first part of equation (18) can still be signed to be negative:

\[
\gamma b[(1 - F_s(b)) - (1 - F_s(\gamma b))] = \gamma b\int_{-b}^{b} f_s(x) dx - \int_{-b}^{b} f_s(x) dx
\]
\[
= \gamma b\int_{-b}^{b} f_s(x) dx + \int_{-b}^{b} f_s(x) dx - \int_{-b}^{b} f_s(x) dx = \gamma b\int_{-b}^{b} f_s(x) dx < 0 \cdot b < 0
\]

Now: \( \gamma b\int_{-b}^{b} f_s(x) dx > \gamma b\int_{-b}^{b} x f_s(x) dx = \int_{-b}^{b} x f_s(x) dx = -\int_{-b}^{b} x f_s(x) dx \)

Therefore: \( (18) = (p - s)\sigma \{\int_{-b}^{b} x f_s(x) dx + \gamma b\int_{-b}^{b} f_s(x) dx\} > 0 \iff G(D, Q_o) > G(D, Q^*) \iff \pi(D, Q_o) < \pi(D, Q^*) \)

Case 3: When \( \beta = m \), then \( b = \gamma b = 0 \cdot 0 < \gamma < 1 \). Now both the first part and second parts of (18) are equal to zero. Therefore, \( G(D, Q_o) = G(D, Q^*) \iff \pi(D, Q_o) = \pi(D, Q^*) \)

Since Cases 1 and 2 generate strictly lower profits and Case 3 generates the same profits, we thus conclude that overconfident agents earn the same or less profit as their unbiased counterparts.
References


