Incentives for Transshipment in a Supply Chain with Decentralized Retailers

(Forthcoming in Manufacturing & Service Operations Management)

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December, 2010

Abstract

We examine transshipment incentives in a decentralized supply chain where a monopolist distributes a product through independent retailers. A key insight is that the transshipment price determines whether the firms benefit from, or are hurt by, transshipment. In particular, we show that the manufacturer prefers to set the transshipment price as high as possible, while retailers prefer a lower transshipment price. Given the important role of the transshipment price in determining the benefits that each firm gets from transshipment, it is useful to consider transshipment in the case where retailers are under joint ownership (a “chain store”) and the transshipment price does not play a role. This comparison yields two surprising results. First, if decentralized retailers control the transshipment price, they will choose a relatively low transshipment price as a way to mitigate the manufacturer’s ability to extract profits by increasing wholesale prices; therefore, the manufacturer may prefer dealing with the chain store which does not have a transshipment price rather than with decentralized retailers. Similarly, the decentralized retailers can use a low transshipment price to achieve higher total profits than a chain store.

Keywords: Supply chain incentives; Transshipment; Decentralized retailers; Chain store retailer

1 Introduction

Many manufacturers distribute their products through a network of independent dealers or retailers. Within these networks, “an increasing number of manufacturers [including] Caterpillar, John Deere, General Motors, etc. [promote] inventory sharing among the dealers in their distribution network” (Zhao et al., 2005). Narus and Anderson (1996) discuss similar strategies followed by Okuma America Corporation (a machine tool manufacturer), Grainger Integrated*

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Supply Operations (an industrial distributor), Dunlop-Enerka (a supplier of conveyor belts for mining and manufacturing operations), and others.

The logic behind these inventory sharing programs is straightforward. By sharing inventories, for example through inventory transshipment, each retailer or dealer can potentially achieve higher service levels at lower costs. However, as Narus and Anderson (1996) note, “significant hurdles” exist in successfully implementing inventory sharing programs. In particular, they note that “channel members are likely to be skeptical about the rewards of participation.”

In this paper, we examine transshipment incentives in a decentralized supply chain where the retailers are independent from the manufacturer and also from each other, i.e., in a vertically and horizontally decentralized supply chain. (As we discuss in the literature review, the existing literature considers either a horizontally or a vertically decentralized supply chain, but not both.)

In practice, transshipment often takes place at a predetermined “transshipment price” (Narus and Anderson, 1996; Zhao et al., 2005; Zhao and Atkins, 2009). A key insight is that the transshipment price determines whether the firms benefit from, or are hurt by, transshipment. In particular, we show that the manufacturer prefers a high transshipment price, while retailers prefer a lower transshipment price from the manufacturer’s perspective.

The intuition (which we analytically support in Section 4) is as follows. Suppose the transshipment price increases. This has two effects: (1) for each retailer, transshipment to another retailer is more profitable; and (2) transshipment from another retailer is more costly. Both these effects will induce each retailer to increase their inventory levels. On the other hand, a decrease in the transshipment price will suppress inventory holdings by reducing the revenue and lowering the cost from transshipment. A higher transshipment price therefore benefits the manufacturer because the retailers will buy more units. Furthermore because a higher transshipment price increases inventory levels, it also makes the retailers less sensitive to higher wholesale prices. Therefore, with a higher transshipment price, the manufacturer can charge a higher wholesale price and extract more profits from the retailers. Conversely, a lower transshipment price limits the manufacturer’s ability to extract profits by raising the wholesale price. If the retailers control the transshipment price, they will prefer to set a lower transshipment price than what the manufacturer would choose.

The conflict between the manufacturer and retailers’ preference for different transshipment prices is hinted at Zhao et al. (2005) who note that in practice:

“...to encourage sharing among dealers, the manufacturer may give a commission equal to a certain percentage of the part’s price to the sharing dealer. ... Alternatively, some manufacturers try to encourage inventory sharing by reducing the cost incurred by the dealers who take part in the inventory sharing system, e.g., by subsidizing the cost to transport a shared part between dealers.”

As the above paragraph indicates, manufacturers encourage transshipment by increasing the payoffs to transshipping retailers. This is consistent with our result that the manufacturer will
seek to increase the transshipment price. In some cases, however, the manufacturer’s ability to control the transshipment price is limited. For example, in the case of Grainger Integrated Supply Operations (GISO), “written contracts with each supplier specify the products it will supply” and “the suppliers set the prices of those products themselves” (Narus and Anderson, 1996). Our analysis will show that when retailers control the transshipment price, they will choose a price that is too low from the manufacturer’s perspective.

Given the important role of the transshipment price in determining the benefits that each firm gets from transshipment, it is useful to compare our fully decentralized supply chain with one where the retailers are under joint ownership (a “chain store”). If the decentralized retailers control the transshipment price, as noted above, they can commit to a low transshipment price. This leads to a counter-intuitive result that the manufacturer may prefer dealing with the chain store, which does not have a transshipment price, rather than with decentralized retailers. Similarly, the decentralized retailers can use a low transshipment price to achieve higher total profits than a chain store.

Figure 1 demonstrates the contribution of our paper relative to the literature. The primary research question of our paper is to investigate the benefits of transshipment in a horizontally and vertically decentralized supply chain (i.e., comparing the left and right panels of Figure 1(a)). The prior literature considers the benefits of transshipment in a vertically decentralized supply chain with a centralized chain store downstream (i.e., comparing the left and right panels of Figure 1(b)). (See a detailed review in Section 2.) Our secondary research question is to study firms’ preference for retail centralization or decentralization under transshipment (i.e., comparing the right panels of Figures 1(a) and 1(b)).

![Figure 1: Transshipment vs. no transshipment under two different supply chain structures](image)

The remainder of the paper is organized as follows. Section 2 reviews the related literature;
and Section 3 presents the modeling framework. In Section 4, we study the manufacturer’s and
decentralized retailers’ incentives for transshipment. In Section 5, we compare decentralized
retailers and the case of a chain store. In Section 6 we examine two extensions to the model. In
Section 7 we conclude the paper and suggest some future research directions.

2 Literature Review

Traditional work on transshipment studied a vertically integrated firm and focused on issues
such as optimal inventory and transshipment policies (Krishnan and Rao (1965), Tagaras (1989),
Robinson (1990), Wee and Dada (2005), Herer et al. (2006), etc.). Incentive conflicts do not exist
in this setting.

Some papers study transshipment in a “vertically decentralized supply chain,” where a man-
ufacturer sells to a single downstream firm with several locations (a chain store). Assuming a
normal demand distribution, Dong and Rudi (2004) show that, under mild assumptions, the
manufacturer will benefit from transshipment. Zhang (2005) generalizes the results of Dong and
Rudi (2004) to an arbitrary demand distribution.

Other papers study transshipment that occurs between retailers not owned by one firm but
without considering the upstream manufacturer (which we refer to as a “horizontally decentral-
ized supply chain”). There are two approaches. One adopts a non-cooperative game theoretic
framework. In particular, Rudi et al. (2001) and Hu et al. (2007) compare equilibrium inventory
levels under transshipment and under no transshipment. Zhao et al. (2005) and Rong et al.
(2010) consider transshipment games in multiple periods. Jiang and Anupindi (2010) and Zhao
and Atkins (2009) compare the transshipment game with a game where customers search between
retailers.

The other approach uses a cooperative game theoretic framework (Anupindi et al., 2001;
Granot and Sošić, 2003; Sošić, 2006). Specifically, these papers consider retailers’ “coopetition,”
i.e., retailers independently determine their inventories before demand realization, but cooper-
atively determine how to share inventory through transshipment after demand realization. Huang
and Sošić (2010) compare the two approaches and conclude that either approach can be more
efficient than the other in terms of generating extra profit from transshipment depending on
model parameters.

Our work differs from these papers, as we study a vertically and horizontally decentralized
supply chain. As noted in the introduction, our findings show that the transshipment price is
important in determining whether firms benefit or lose from transshipment. However, in much
of the literature, the parameters of the transshipment decision such as transshipment prices are
assumed to be exogenously made. Our work also differs by allowing the firms to determine
whether, and at what transshipment price, transshipment will occur; and we investigate how the
control of the parameters of the transshipment decision affects firms’ transshipment incentives.
3 Model

Consider a single period model where a monopolist produces a single product at production cost \( c \) per unit, and distributes it through two identical retailers. (We extend our model to asymmetric retailers in Section 6.1 and \( n \) retailers in Section 6.2.) The retailers are independent from the manufacturer and from each other.

Denote by \( s \) the per unit “transshipment price” that the retailers pay to each other to obtain the transshipped goods. For simplicity, assume that the cost incurred during transshipment, e.g., the transportation cost, is zero. (Relaxing this assumption does not affect our analysis and results.) To avoid trivial outcomes, we assume that \( s \in [0, p] \), where \( p \) is the fixed retail price.

The supply chain’s decisions occur in three stages as shown in Figure 2. In stage 1, the firms decide whether the retailers should transship, and at what transshipment price. We consider three cases (in Section 4.3) where the manufacturer has an increasing amount of control of the above decisions.

![Figure 2: Timeline of the game](image)

In stage 2, with full knowledge of the decisions made in stage 1, the manufacturer offers a take-it-or-leave-it contract to the retailers, specifying the wholesale price \( w \). If the manufacturer has chosen any transshipment parameter in the first stage, it is also included in the contract. (The above contract, i.e. the wholesale price and potentially the transshipment parameters, cannot in general coordinate the supply chain. See a discussion of coordinating contracts in Shao et al. (2010).) In stage 3, the retailers simultaneously decide order quantities \( y_1 \) and \( y_2 \) before demand is realized. (Note that, as common in the inventory literature, we consider \( y_1 \) and \( y_2 \) to be continuous variables.)

Our assumption that the transshipment price is set prior to the wholesale price is consistent with observed practice. For instance, Narus and Anderson (1996) note that the procedures for transshipment “and the appropriate remuneration are all defined in advance.” In cases of car dealers, dealers normally practice “dealer swap” (i.e., transshipment) based on a transshipment price “that has been existing ex ante” (Zhao and Atkins, 2009). However, we also discuss an alternative sequence of events in Section 7 where the transshipment price is determined after the manufacturer sets the wholesale price.

Retailer \( i \) faces random demand \( \xi_i \), which has an arbitrary distribution on a support \( [a, b] \), where \( 0 \leq a < b \). The distributions of \( \xi_1 \) and \( \xi_2 \) are identical, but \( \xi_1 \) and \( \xi_2 \) are not nec-
essarily independent. Let $F(\cdot)$ and $f(\cdot)$ denote their cumulative distribution function (CDF) and probability distribution function (PDF) respectively, which we assume are continuous and differentiable.

After demands are realized, if one retailer is out of stock, the retailer may transship from the other retailer if the latter has leftover inventory. Note that after the realization of demand, it is in the interest of a stocked out retailer to use transshipment to satisfy as much demand as possible; it is also in the interest of an over-stocked retailer to transship as many units as requested. Thus the number of transshipped units from retailer $i$ to $j$ is given by $T_i = \min((y_i - \xi_i)^+, (\xi_j - y_j)^+)$, which is the minimum of $i$’s excess inventory and $j$’s excess demand.

4 Completely Decentralized Supply Chain

To analyze the benefits of transshipment, we consider two cases: (1) where the retailers do not transship (indicated by superscript “NT” and; (2) where the (decentralized) retailers transship (indicated by superscript “DT”). By backward induction, we first analyze the retailers’ inventory game (stage 3). We then consider the second stage where the manufacturer optimally sets the wholesale price. Finally, we consider the first stage where the firms determine the parameters of the transshipment decision.

4.1 Stage 3: Retailers’ Inventory Game

Under no transshipment, the retailer’s equilibrium inventory, denoted by $y_1^{NT}(w)$, is the newsvendor quantity, given by $F(y_1^{NT}(w)) = (p - w)/p$ (because in this case each retailer’s inventory is unaffected by the other’s inventory decision).

Under transshipment the expected profit of retailer $i$ is given by

$$\pi_i = pE \min(\xi_i, y_i) + sET_i + (p - s)ET_j - wy_i, \ i = 1, 2,$$

(1)

where $E$ denotes expectation.

Rudi et al. (2001) show that a unique Nash equilibrium exists in this game. (They consider two asymmetric retailers; our model considers symmetric retailers, so the equilibrium in our case is also symmetric.) From the best response functions of (1), we obtain $y_1^{DT}(w(s))$, the equilibrium inventory of a retailer under transshipment, which is given by

$$F(y_1^{DT}(w(s))) = \frac{p - w}{p} + \frac{s}{p} \frac{\partial ET_i}{\partial y_i} + \frac{p - s}{p} \frac{\partial ET_j}{\partial y_i} = 0.$$

(2)

For a fixed wholesale price, the retailers’ inventory choice under transshipment, $y_1^{DT}$ is different from that under no transshipment, $y_1^{NT}$ (we drop the arguments for expositional convenience). Under transshipment, there are two forces that cause retailer $i$’s inventory choice to deviate from $y_1^{NT}$. First (for any inventory choice of retailer $j$), when retailer $i$ increases inventory from $y_1^{NT}$,
it will transship more to retailer $j$ at the end of the period. Therefore, retailer $i$ profits by collecting the transshipment price $s$ from retailer $j$ on the extra units. Second, when retailer $i$ decreases inventory from $y^{NT}$, it will transship more from retailer $j$ at the end of the period. It pays retailer $j$ the transshipment price $s$ on these units and then sells to consumers at $p$, collecting a margin $(p - s)$ on these units.

These two forces pull retailer $i$’s optimal inventory choice under transshipment in opposite directions. The net effect depends on the magnitude of the two margins, i.e., $s$ and $p - s$, as well as the demand distribution; therefore, it is hard to determine in general. However, it becomes clear in the two extreme cases where $s = 0$ and $s = p$. In the first case, the second force becomes zero and it is optimal for the retailer to decrease inventory from $y^{NT}$. In the second case, the first force becomes zero and it is optimal to increase inventory from $y^{NT}$. (Rudi et al. (2001) prove the two cases.) Furthermore,

**Lemma 1** Each retailer’s order quantity under transshipment, $y^{DT}$, is increasing in the transshipment price $s$.

(All proofs are in the Appendix. Lemma 1 is similar to Proposition 2 of Rudi et al. (2001) with asymmetric retailers, but we provide a new approach to the proof.)

### 4.2 Stage 2: Manufacturer Sets Wholesale Price

In this stage, the manufacturer’s problem is to choose a wholesale price that maximizes $\pi_M = 2(w - c)y^{NT}(w)$ under $NT$, and $\pi_M = 2(w(s) - c)y^{DT}(w(s))$ under $DT$, where $y^{NT}(w)$ and $y^{DT}(w(s))$ are the (symmetric) equilibrium inventories derived from stage 3. (The analysis in this section does not require the manufacturer’s profit function to be unimodal.)

From Section 4.1, for a fixed wholesale price, the manufacturer is better off with transshipment at $s = p$ and worse off at $s = 0$. We show that this continues to hold when the manufacturer optimally chooses the wholesale price:

**Lemma 2** When the manufacturer optimally sets the wholesale price, the manufacturer’s profit is increasing in the transshipment price $s$. It makes a higher profit under transshipment at $s = p$, and a lower profit under transshipment at $s = 0$.

Lemmas 1 and 2 provide a basis for the intuition, outlined in the introduction, for why the manufacturer prefers a high transshipment price. Furthermore, from Lemma 2 we have:

**Proposition 1** When the manufacturer optimally sets the wholesale price, there exists a unique transshipment price $\hat{s}_M$, such that the manufacturer prefers transshipment when $s > \hat{s}_M$, no transshipment when $s < \hat{s}_M$, and is indifferent when $s = \hat{s}_M$.

(See the top panels of Figure 3 for an illustration.)
We next look at the retailers’ profits. When the wholesale price is fixed, the retailers always make higher profits under transshipment, since even with the newsvendor inventory the retailers will earn extra profit through transshipment. However,

**Observation 1** When the manufacturer optimally sets the wholesale price, the retailers’ profit can be lower under transshipment.

For an illustration of Observation 1, assume that the demands at the two retailers have independent uniform distribution on the support \([0, b]\). When the retail price is relatively small \((p/c \leq 1.952)\), the retailers are better off from transshipment for all transshipment prices (see, e.g., the bottom panel of Figure 3(a)). However, when the retail price is sufficiently large \((p/c > 1.952)\), the retailers are worse off under transshipment at a large transshipment price (e.g., the bottom panel of Figure 3(b); proofs for independent uniform distribution on \([0, b]\) are in Shao et al. (2010).)

The intuition is that when the retail price is high, the upper limit of the transshipment price increases. At a large transshipment price, the retailers have stronger incentives to increase inventory under transshipment. Therefore, the manufacturer can raise the wholesale price to take advantage of retailers’ over-stocking and extract more profits from the retailers. As a result, the retailers are worse off from transshipment.
4.3 Stage 1: Firms Determine Parameters of Transshipment Decision

Now consider the first stage where the firms determine the parameters of the transshipment decision, i.e., whether to transship and at what transshipment price. In reality, both the manufacturer and the retailers may have different levels of control over these parameters. We will show that the allocation of decision rights regarding transshipment will affect the optimal choices related to transshipment and the benefits to the firms. Here we provide three cases where the manufacturer has an increasing amount of control over transshipment. This is not an exhaustive list of possible allocations of decision rights; but using these representative cases, we present an approach to analyze how transshipment decisions are made and who benefits from transshipment.

Case 1: Manufacturer has no control.

This is the case of a weak manufacturer (whose only decision is to set the wholesale price) and powerful retailers who jointly decide whether to transship and the transshipment price. Denote by \( s^*_R \) the transshipment price at which the retailers obtain the highest profits under transshipment. (Depending on demand distributions, the retailers may or may not benefit from transshipment at \( s^*_R \). For example, Shao et al. (2010) show that for independent uniform demand on \([0, b]\), the retailers always make more profit under transshipment at \( s^*_R \); however, for uniform distribution on \([a, b]\) \((a > 0)\) and truncated normal, the retailers may make less profit under transshipment for all transshipment prices.) From Proposition 1, we have:

**Proposition 2** Suppose the retailers benefit from transshipment at \( s^*_R \) and the manufacturer has no control of the parameters of the transshipment decision. The manufacturer is better off from retailers’ transshipment if \( s^*_R > \hat{s}_M \), worse off if \( s^*_R < \hat{s}_M \), and no worse off if \( s^*_R = \hat{s}_M \).

For example, in Figure 3, the retailers will choose to transship and set the transshipment price at \( s^*_R \); the manufacturer is better off in (a) but worse off in (b). (Note that if the retailers are worse off at \( s^*_R \), then transshipment will not occur.)

The implication here is that if the manufacturer has little power compared with downstream retailers, the manufacturer may suffer from the negative impact of transshipment: if the transshipment price that the retailers optimally choose is low, the manufacturer does not get enough orders from the retailers as they share their inventories.

Case 2: Manufacturer has partial control.

This case is similar to case 1 except that the manufacturer has the power to disallow retailers’ transshipment. (But the manufacturer cannot control the transshipment price if the retailers are allowed to transship. Neither can the manufacturer force transshipment when the retailers do not voluntarily transship.) The manufacturer knows that if the retailers transship they will set the transshipment price at \( s^*_R \). So the manufacturer needs only to determine whether it will be worse off by transshipment at \( s^*_R \). Since the manufacturer (weakly) benefits from transshipment only if \( s^*_R \geq \hat{s}_M \), it allows transshipment when this condition is satisfied. (In the case where the
retailers are always hurt by transshipment, as shown in Shao et al. (2010), the retailers would simply avoid transshipment even if the manufacturer allows it.) For example, in Figure 3(a), the manufacturer will allow the retailers to transship; but not in Figure 3(b).

Thus, when the manufacturer has partial control of transshipment parameters, it can use its control to protect itself from being hurt by transshipment.

**Case 3: Manufacturer has complete control.**

In this case the (powerful) manufacturer can enforce or prohibit retailers’ transshipment and also choose the transshipment price. From Proposition 1, we get

**Proposition 3** When the manufacturer has complete control, it always forces the retailers to transship and sets \( s = p \). The manufacturer is always better off as a result of transshipment.

Notice that it is not adequate for a manufacturer to encourage transshipment, e.g., by providing information systems and other resources; it also needs to dictate and monitor the retailers’ transshipment price. And while the manufacturer always benefits, the retailers may be worse off. Figure 3(a) is an example where the retailers also benefit from the transshipment decisions enforced by the manufacturer; and Figure 3(b) is an example where the retailers are hurt by the manufacturer’s decisions. This leads to:

**Observation 2** When the manufacturer has complete control of the parameters of the transshipment decision, the retailers may be worse off as a result of transshipment.

5 Decentralized Retailers vs. a Chain Store Retailer

Through transshipment, decentralized retailers “share” their inventories based on the transshipment price. In contrast, when a chain store retailer transships, the inventories at different locations are “pooled;” the chain store does not have a transshipment price, since it maximizes the total profit of the chain. Given the importance of the transshipment price in determining whether firms benefit from transshipment, how do the benefits of transshipment in the decentralized retailer case compare with the benefits of transshipment in the chain store case (comparing the right panels of Figures 1(a) and 1(b))? (Anupindi and Bassok (1999) also study whether a manufacturer prefers retail centralization, but in the context of customer search instead of transshipment.)

Our approach is to compare a decentralized retailer’s profit with the profit of a single location of the chain store, which, by symmetry, equals one half of the total profit of the chain store. Recall that superscript DT indicates the case where decentralized retailers transship; now let superscript CT represent the case where the chain store transships between its locations.

In this section, we assume that both the decentralized retailers and the chain store always transship so that “whether to transship” is not a decision. This assumption allows us to focus on the main question, i.e., whether firms prefer retail centralization or decentralization. Under
this assumption, in the decentralized retailer supply chain, either the manufacturer or the decentralized retailers determine the transshipment price; but no one needs to decide whether the retailers should transship.

5.1 Manufacturer’s Preference for Retail Centralization

Will decentralized retailers hold more or less inventory than a chain store? Consider, first, the inventory levels for a fixed wholesale price in the two cases. Denote by $y^{CT}(w)$ the optimal inventory of a location of the chain store. Note that $y^{CT}$ (with the argument dropped) is independent of $s$ because the transshipment price is irrelevant for the chain store. Compare a decentralized retailer’s inventory choice with $y^{CT}$. Supposing retailer $j$’s inventory is fixed at $y^{CT}$, then if retailer $i$ increases inventory from $y^{CT}$ it profits by transshipping to retailer $j$; if retailer $i$ decreases inventory from $y^{CT}$ it profits by transshipping from retailer $j$. It is therefore not clear, in equilibrium, whether the retailer’s optimal inventory is higher or lower than $y^{CT}$. However, this becomes clear in the two extreme cases:

Lemma 3 For a given wholesale price, when $s = 0$ the decentralized retailers order less than the chain store; and when $s = p$ the decentralized retailers order more than the chain store.

(Propositions 3 and 4 of Rudi et al. (2001) show similar results as our Lemma 3 in the case of asymmetric retailers. However, Hu et al. (2007) provide a counterexample to Propositions 3 and 4 of Rudi et al. (2001). Here we use a different proof and show that they hold for symmetric retailers. We analyze the case of asymmetric retailers in Section 6.1.)

The intuition is that at $s = 0$, the margin of transshipping one unit to the other retailer is zero; therefore, retailer $i$ will decrease inventory from $y^{CT}$. Retailer $j$ hence will have less opportunity to transship from retailer $i$, and its profit declines; but retailer $i$ ignores this externality. Similarly, at $s = p$, the margin of transshipping becomes zero; so the argument is reversed and retailer $i$ increases inventory from $y^{CT}$. In a word, the decentralized retailers ignore the impact of their inventory decisions on the other retailers, while the chain store internalizes the impact. (This is a similar argument to the one preceding Lemma 1 where the comparison was between no transshipment and transshipment among decentralized retailers.)

The inventory comparison for a fixed wholesale price in Lemma 3, combined with the monotonicity of inventory (Lemma 1), gives us the manufacturer’s preference for a chain store or decentralized retailers when it optimally sets the wholesale price:

Proposition 4 When the manufacturer optimally sets the wholesale price, there exists a unique transshipment price $s_{CT}^M$ such that the manufacturer prefers dealing with the chain store when $s < s_{CT}^M$, the decentralized retailers when $s > s_{CT}^M$, and is indifferent when $s = s_{CT}^M$.

(See Figure 4 (top) for an illustration.) The above result is surprising because, intuitively, one might expect that the manufacturer always prefers dealing with decentralized retailers rather than
a centralized chain store that can coordinate inventory decisions. However, if the decentralized retailers control the transshipment price, as discussed in Section 4.3, they may optimally choose a transshipment price that is low. This will not only suppress inventories but also will inhibit the manufacturer’s ability to extract profits by increasing the wholesale price. For instance, in Figure 4, the retailers will choose $s^*_R$ that is lower than the manufacturer’s threshold $\hat{s}_M^{CT}$; so the manufacturer will prefer dealing with the chain store.

Figure 4: Firms’ profits when manufacturer sets wholesale price ($p = 6; c = 1; \xi_i \sim$ i.i.d. Uniform[0,1]; here $\pi_M^{CT}$ and $\pi_R^{CT}$ represent the equilibrium profit of the manufacturer and that of one location of the chain store under $CT$ respectively.)

If the manufacturer controls the transshipment parameters, then from Proposition 4, the manufacturer will always set $s = p$ so that the decentralized retailers have strong incentives to stock. Proposition 5 summarizes the manufacturer’s preference when the retailers or the manufacturer controls the transshipment price:

**Proposition 5** (a) When the manufacturer controls the transshipment price, it always prefers dealing with decentralized retailers; (b) When the retailers control the transshipment price, the manufacturer prefers dealing with the chain store if $s^*_R < \hat{s}_M$, the decentralized retailers if $s^*_R > \hat{s}_M$, and is indifferent if $s^*_R = \hat{s}_M$.

5.2 Retailers’ Preference for Retail Centralization

For a fixed wholesale price, both the chain store and decentralized retailers benefit from transshipment. However, since the chain store makes the inventory decisions for the two locations, its
feasible set includes the decentralized retailers’ equilibrium inventories. Therefore, for a given wholesale price, the chain store always makes a higher profit than the decentralized retailers.

When the manufacturer optimally sets the wholesale price, does the chain store still make more profits than the decentralized retailers? From Figure 4 we have the following observation:

**Observation 3** When the manufacturer optimally sets the wholesale price, the decentralized retailers can make more profits than the chain store.

Recall that when the decentralized retailers control the transshipment price, they will set a low transshipment price, which reduces inventories and also limits the manufacturer’s pricing ability. In this case, the retailers may prefer decentralization to centralization, as the decentralized retailers can use the transshipment price to their advantage. However, when the manufacturer controls the transshipment price, it will set \( s = p \) and the chain store can be better off, as in Figure 4. For the i.i.d. uniform distribution on \([0, b]\), Shao et al. (2010) prove that when the manufacturer has control of the transshipment price, the chain store is always better off than decentralized retailers; and when the retailers have control, they always prefer decentralization.

### 6 Extensions

#### 6.1 Asymmetric Retailers

Our basic model considers identical retailers, since often the retailers who transship to each other, e.g., car dealers, etc., are similar in size. However, transshipping retailers need not be identical in every respect. In this section we consider the case where the retailers are asymmetric. Suppose that the retailers’ demands \( \xi_i \)'s are not necessarily identical (and can be generally correlated as before). Let \( p_i \) be retailer \( i \)'s price. Without loss of generality, suppose \( p_1 \geq p_2 \). We continue to assume that the retailers transship to each other at a transshipment price \( s \) and \( s \in [0, p_2] \). The other notations and the timeline of the game are the same as in Section 3.

First consider the retailers’ total inventory for a fixed wholesale price. The monotonicity result from the symmetric case continues to hold:

**Lemma 4** Suppose that the retailers are asymmetric. For a fixed wholesale price, the retailers’ total inventory is increasing in the transshipment price.

Recall that in the symmetric case, individual retailers’ inventories also increase in \( s \). However, in the asymmetric case, the individual inventories may decrease in the transshipment price. Consider the impact on \( y_1 \) when \( s \) changes. For retailer 1, when \( s \) increases, transshipping to retailer 2 becomes more profitable and transshipping from retailer 2 becomes more costly. Therefore, holding \( y_2 \) fixed, the direct effect is that retailer 1 has an incentive to increase \( y_1 \).

However, an increases in \( s \) also induces retailer 2 to increase \( y_2 \). An increase in \( y_2 \) will in turn induce retailer 1 to reduce \( y_1 \). This is because an increase in retailer 2’s inventory reduces
the marginal benefit of additional inventory at retailer 1, i.e., $y_1$ and $y_2$ are strategic substitutes. In the completely symmetric case, this indirect effect is always dominated by the direct effect; and hence the individual inventories are always increasing in $s$. But in the asymmetric case, the indirect effect may in some cases dominate and non-monotonicity can occur.

Another property that characterizes the manufacturer’s profit in the symmetric case is the inventory comparison at the bounds of the transshipment price. Recall that in the symmetric case, $y_{DT}^i < y_{NT}^i$ at $s = 0$, and $y_{DT}^i > y_{NT}^i$ at $s = p$. For asymmetric retailers, at the lower bound of $s$, i.e., $s = 0$, we still have that $y_{DT}^i < y_{NT}^i$. However, at the upper bound of $s$, i.e., $s = p_2$ ($p_2 \leq p_1$), we only have $y_{DT}^2 > y_{NT}^2$. (These can be easily proved by comparing the retailer’s best response functions under $DT$ and $NT$ at $s = 0$ and $s = p_2$.) But we cannot conclude that $y_{DT}^1 > y_{NT}^1$ at $s = p_2$, especially if $p_2$ is much smaller than $p_1$.

Nevertheless, in reality, when retailers sell the same product, the retail prices are often close. In fact, if the retailers’ prices are equal, then at $s = p_1 = p_2$, we have that $y_{DT}^i > y_{NT}^i$, $i = 1, 2$; and hence, the aggregate inventory under $DT$ and $NT$ has the same relationship at the bounds of $s$.

From the above analysis, for a fixed wholesale price (stage 3), most of our results from the symmetric case continue to hold, especially the ones that we need to establish the manufacturer’s profit (e.g., Lemma 4). Therefore, in stages 1 and 2 we can derive similar results as the symmetric case. Also, the insights that we have obtained for decentralization versus centralization remain true. For detailed analyses of the asymmetric model, see Shao et al. (2010).

6.2 $n$ Retailers

Transshipment with more than two decentralized retailers is generally a hard problem to model and analyze. The main difficulty is that a transshipment pattern needs to be determined, as there could be multiple over-stocked retailers as well as multiple under-stocked retailers. Our goal in this section is to test the robustness of our main model in the case of more than two retailers. For simplicity, we focus our attention on symmetric retailers. In order to determine the transshipment pattern, we adopt the following proportional rule proposed by Huang and Sošić (2010):

**Proportional rule of transshipment** (Huang and Sošić, 2010): *If the total excess demand of all retailers exceeds their total excess inventory, an under-stocked retailer obtains transshipment proportional to its excess demand; if the total excess inventory of the retailers exceeds their total excess demand, an over-stocked retailer transships in proportion to its excess inventory.*

Denote by $TO_i$ the quantity that retailer $i$ transships out, and $TI_i$ the quantity it transships
in. Under the proportional rule, $TO_i$ and $TI_i$ are given by

$$TO_i = (y_i - \xi_i)^+ \frac{\min[\sum_{i=1}^{n}(y_i - \xi_i)^+, \sum_{i=1}^{n}(\xi_i - y_i)^+]}{\sum_{i=1}^{n}(y_i - \xi_i)^+},$$

(3)

$$TI_i = (\xi_i - y_i)^+ \frac{\min[\sum_{i=1}^{n}(\xi_i - y_i)^+, \sum_{i=1}^{n}(y_i - \xi_i)^+]}{\sum_{i=1}^{n}(\xi_i - y_i)^+},$$

(4)

and a retailer’s expected profit is

$$\pi_i = pE \min(\xi_i, y_i) + sE TO_i + (p - s)E TI_i - wy_i.$$  

(5)

Through numerical simulations, we find that the results from the two-retailer case continue to hold. In particular, the manufacturer’s profit is still lower under transshipment at a low $s$, and higher under transshipment at a high $s$ (see Figure 5(left)). However, the slope of the manufacturer’s profit under transshipment becomes steeper as $n$ increases. For instance, as $n$ increases, at $s = 0$ the retailers can further cut their safety stocks; and at $s = p$ the retailers have more incentives to stock higher to pursue higher profits.

A retailer’s profit decreases with an increase in $n$ for very low or very high transshipment prices (see Figure 5(right)) as the inventory sharing effect becomes stronger. At an intermediate transshipment price, however, the retailers’ profits increase as $n$ increases.

Therefore, when the manufacturer controls the transshipment price, it still sets $s = p$ and obtains an even higher profit when there are more retailers in the channel. However, when the retailers control the transshipment price, they may prefer a higher transshipment price as $n$ increases, which leads to an increase in the manufacturer’s profit (Figure 5 shows an example).

For the comparison of decentralized retailers versus a chain store, the results from the two retailer case also continue to hold. For details see Shao et al. (2010).
7 Discussion and Conclusions

In a fully decentralized supply chain both the manufacturer and retailers can be harmed by transshipment. Whether they benefit from or are hurt by transshipment depends on who controls the parameters of the transshipment decision.

We have shown that if the manufacturer has the control, it will set a high transshipment price to stimulate retailers’ incentive to stock inventory and also reduce retailers’ sensitivity to a high wholesale price; this leads to a higher manufacturer profit under transshipment. But if the retailers have the control, they may choose a low transshipment price to suppress inventory and limit the manufacturer’s ability to extract profit; the manufacturer may end up being worse off from transshipment.

Given the important role of the transshipment price in determining firms’ benefits from transshipment, we also compare the supply chain with decentralized retailers and that with a chain store. When the manufacturer controls the transshipment price, it prefers dealing with decentralized retailers, since it can set a high transshipment price and extract more profits from decentralized retailers. When the decentralized retailers control the transshipment price, they set a low transshipment price to reduce inventory and mitigate the manufacturer’s pricing power. As a result, the decentralized retailers may make more profits than the chain store; and the manufacturer may prefer dealing with the chain store.

Our work also suggests some future research directions. First, in our model we assume that the retail price is fixed. In the real world, however, the retailers often make price decisions as well. Future research should incorporate retailers’ pricing decisions and examine whether the players’ incentives for transshipment will change. (Jiang and Anupindi (2010) and Zhao and Atkins (2009) consider retailers’ price decisions when comparing transshipment and the case of customer search. Dong and Durbin (2005) also incorporate firms’ pricing in their study of secondary markets.)

As we note in Section 3, the sequence of events that we currently assume is consistent with practice. However, it may be worthwhile to consider an alternative sequence of events where the manufacturer sets the wholesale price before the firms determine the parameters of the transshipment decision. Under this alternative sequence of events, if the manufacturer has full control of the transshipment parameters, then the current results remain unchanged, because the first two stages are indeed one stage (the manufacturer’s decision stage).

If the decentralized retailers determine transshipment parameters, for a given wholesale price \( w \), the decentralized retailers will always set the transshipment price such that they obtain the profit of a chain store. Hence this case reduces to a problem where the manufacturer deals with a chain store. Whether firms benefit from transshipment depends on the critical fractile and demand distribution (Dong and Rudi, 2004; Zhang, 2005). Also, the comparison between centralized and decentralized retailers is no longer relevant.

Another interesting extension is to utilize a multi-period model to examine the incentive
issues in a dynamic setting. The single period results of this paper can serve as a building block to understand transshipment incentives in these more complex settings.

References


Dong, L. and Durbin, E. (2005), ‘Markets for surplus components with a strategic supplier’, Naval Research Logistics 52(8), 734–753.


Proof of Lemma A.1: For some of the proofs, we will use the following lemma:

**Lemma A.1** At equilibrium, $\partial ET_i/\partial y_i > 0$, and $\partial ET_i/\partial y_j < 0, i, j = 1, 2$.

**Proof of Lemma A.1:** Since $T_i = \min((y_i - \xi_i)^+, (\xi_j - y_j)^+)$, by the assumption of continuity of demand distribution functions, we have

$$\frac{\partial ET_i}{\partial y_i} = Pr(\xi_i < y_i, \xi_i + \xi_j > y_i + y_j) \geq 0, \quad (6)$$

$$\frac{\partial ET_i}{\partial y_j} = -Pr(\xi_j > y_j, \xi_i + \xi_j < y_i + y_j) \leq 0. \quad (7)$$

At equilibrium (6) and (7) are strict.

**Proof of Lemma 1:** By the Implicit Function Theorem,

$$\frac{\partial y^{DT}}{\partial s} = \frac{-\frac{\partial^2 \pi_i}{\partial y_i \partial s} \frac{\partial^2 \pi_j}{\partial y_j^2} + \frac{\partial^2 \pi_i}{\partial y_i \partial y_j} \frac{\partial^2 \pi_j}{\partial y_j \partial s}}{|H|} = \frac{\frac{\partial^2 \pi_i}{\partial y_i \partial s} \left( \frac{\partial^2 \pi_i}{\partial y_i \partial y_j} - \frac{\partial^2 \pi_i}{\partial y_j^2} \right)}{|H|}, \quad (8)$$

where $|H|$ is the positive determinant of the Hessian matrix, and the second equality follows from the symmetry between the retailers. Consider the numerator of (8). First, by Lemma A.1 we have $\partial^2 \pi_i/\partial y_i \partial s = \partial ET_i/\partial y_i - \partial ET_j/\partial y_i > 0$.

Define the following marginal probabilities (following the notation of Rudi et al. (2001)):

$$b^1_{ij} = Pr(\xi_i < y_i)f_{D|\xi_i<y_i}(y_i + y_j), \quad b^2_{ij} = Pr(D > y_i + y_j)f_{\xi_i|D>y_i+y_j}(y_i), \quad (9)$$

$$g^1_{ij} = Pr(\xi_i > y_i)f_{D|\xi_i>y_i}(y_i + y_j), \quad g^2_{ij} = Pr(D < y_i + y_j)f_{\xi_i|D<y_i+y_j}(y_i). \quad (10)$$

Then we have

$$\frac{\partial^2 \pi_i}{\partial y_i \partial y_j} = -[sb^1_{ij} + (p-s)g^1_{ij}], \quad \frac{\partial^2 \pi_i}{\partial y_i^2} = -[s(b^1_{ij} - b^2_{ij}) + (p-s)(g^1_{ij} - g^2_{ij}) + pf(y_i)]. \quad (11)$$

So $\partial^2 \pi_i/\partial y_i \partial y_j - \partial^2 \pi_i/\partial y_i^2 = pf(y_i) - sb^2_{ij} - (p-s)g^2_{ij}$. Since $f(y_i) > b^2_{ij}$ and $f(y_i) > g^2_{ij}$, we have $\partial^2 \pi_i/\partial y_i \partial y_j - \partial^2 \pi_i/\partial y_i^2 > 0$. It follows that $\partial y^{DT}/\partial s > 0$ at equilibrium.  

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Proof of Lemma 2: The manufacturer’s profit in stage 2 is given by

\[ \pi_{M}^{DT} = \max_{w} 2(w - c)y^{DT}(w), \]  

where \( y^{DT}(w) \) is the equilibrium inventory of a retailer for a given wholesale price \( w \) (by symmetry, the two retailers’ inventories are equal at equilibrium).

In order to show monotonicity of the manufacturer profit in \( s \), we calculate the total derivative of \( \pi_{M}^{DT} \) with respective to \( s \) as follows:

\[
\frac{d\pi_{M}^{DT}}{ds} = \frac{\partial \pi_{M}^{DT}}{\partial s} + \frac{\partial \pi_{M}^{DT}}{\partial w} \frac{\partial w}{\partial s} + \frac{\partial \pi_{M}^{DT}}{\partial y^{DT}} \frac{\partial y^{DT}}{\partial s}. \tag{13}
\]

On the right hand side, the first term equals zero since \( s \) does not show up (explicitly) in (12); the second term also equals zero due to the optimality of the manufacturer’s problem. Therefore,

\[
\frac{d\pi_{M}^{DT}}{ds} = \frac{\partial \pi_{M}^{DT}}{\partial y^{DT}} \frac{\partial y^{DT}}{\partial s} = 2(w - c)\frac{\partial y^{DT}}{\partial s}. \tag{14}
\]

From Lemma 1 (a retailer’s inventory is increasing in \( s \)) and since \( w > c \), we have \( d\pi_{M}^{DT} / ds > 0 \), which indicates that the manufacturer’s profit is increasing in \( s \).

Next consider the manufacturer’s profit at \( s = 0 \) and \( s = p \). For any given wholesale price \( w \), a retailer’s optimal inventory under \( NT \) is given by \( F(y^{NT}(w)) = (p - w)/p \). From (2), a retailer’s equilibrium inventory under \( DT \) at \( s = 0 \) is given by \( F(y^{DT}(w)) = (p - w)/p + \partial ET_{j}/\partial y_{i} \).

Since \( \partial ET_{j}/\partial y_{i} < 0 \) by Lemma A.1, and from the monotonicity of the function \( F(\cdot) \), we have that for any \( w \), \( y^{NT}(w) > y^{DT}(w) \) at \( s = 0 \). It certainly holds for the manufacturer’s optimal wholesale price under \( DT \), denoted by \( w^{DT} \). Suppose under \( NT \) the manufacturer sets \( w = w^{DT} \); then the manufacturer’s profit is \( 2(w^{DT} - c)y^{NT}(w^{DT}) \), which is greater than \( 2(w^{DT} - c)y^{DT}(w^{DT}) \), i.e., the manufacturer’s optimal profit under \( DT \). When the manufacturer chooses the optimal wholesale price under \( NT \), it will make an even higher profit. The argument is similar but reversed for \( s = p \).

Proof of Lemma 3: We first derive the optimal inventories of the chain store for a given wholesale price. Denote by \( \Pi^{CT} \) the expected profit of the chain store (both locations) under transshipment. Then

\[
\Pi^{CT} = \sum_{i=1,2} [pE \min(\xi_{i}, y_{i}) + pET_{i} - wy_{i}]. \tag{15}
\]

Recall that \( D = \xi_{1} + \xi_{2} \). Use the equality

\[
E \min(\xi_{i}, y_{i}) + E \min(\xi_{j}, y_{j}) + ET_{i} + ET_{j} \equiv E \min(D, y_{i} + y_{j}) \tag{16}
\]

to rewrite the chain store’s profit as

\[
\Pi^{CT} = pE \min(D, y_{i} + y_{j}) - w(y_{i} + y_{j}). \tag{17}
\]

Denote by \( f_{D}(\cdot) \) and \( F_{D}(\cdot) \) the PDF and CDF of \( D \); and the support of \( D \) is \([2a, 2b] \). Let \( Y \equiv y_{i} + y_{j} \) denote the total inventory of the two locations. From the first order conditions of
(17), the chain store’s optimal total inventory $Y^{CT}$ is given by

$$F_D(Y^{CT}) = \frac{p - w}{p}. \quad (18)$$

Next use (16) again to rewrite a decentralized retailer’s expected profit at $s = 0$ and $s = p$:

$$s = 0: \quad \pi_i^{DT} = pE\min(\xi_i, y_i) + pE\bar{\xi}_j - wy_i,$$

$$s = p: \quad \pi_i^{DT} = pE\min(\xi_i, y_i) + pE\bar{\xi}_j - wy_i,$$

$$= pE\min(D, y_i + y_j) - pE\min(\xi_j, y_j) - pET_i - wy_i, \quad (19)$$

$$s = p: \quad \pi_i^{DT} = pE\min(\xi_i, y_i) + pE\bar{\xi}_j - wy_i,$$

$$= pE\min(D, y_i + y_j) - pE\min(\xi_j, y_j) - pET_j - wy_i. \quad (20)$$

Denote by $Y^{DT} \equiv y_i^{DT} + y_j^{DT}$ the sum of the retailers’ equilibrium inventories under $DT$.

Take the first derivative of (19) and (20) with respect to $y_i$, rearrange and get

$$s = 0: \quad F_D(Y^{DT}) = \frac{p - w}{p} - \frac{\partial ET_i}{\partial y_i} < \frac{p - w}{p}, \quad (21)$$

$$s = p: \quad F_D(Y^{DT}) = \frac{p - w}{p} - \frac{\partial ET_j}{\partial y_i} > \frac{p - w}{p}, \quad (22)$$

where the inequalities follow from Lemma A.1.

Compare (21) and (22) with (18), we have $Y^{DT} < Y^{CT}$ at $s = 0$, and $Y^{DT} > Y^{CT}$ at $s = p$. By symmetry, the individual inventories $y^{DT}$ and $y^{CT}$ have the same relationships.

Proof of Proposition 4: Following Lemma 3 and by a similar argument as that for Lemma 2, we have that the manufacturer gets a higher profit dealing with the chain store at $s = 0$, and it gets a higher profit dealing with the decentralized retailers at $s = p$. Combining with the monotonicity of the manufacturer’s profit (Lemma 2), the threshold result follows.

Proof of Proposition 5: (a) follows from Lemma 3 and the monotonicity of the manufacturer’s profit (Lemma 2); and (b) follows from Proposition 4.

Proof of Lemma 4: By the Implicit Function Theorem, we have

$$\frac{\partial y_i}{\partial s} = \frac{-\frac{\partial^2 \pi_i}{\partial y_i \partial s} \frac{\partial^2 \pi_i}{\partial y_j \partial s} + \frac{\partial^2 \pi_i}{\partial y_i \partial y_j} \frac{\partial^2 \pi_j}{\partial y_j \partial s}}{|H|}$$

$$= \frac{(u_ii - u_ii) p_j f_j(y_j) - s v_{jjj} - (p_j - s)v_{ijj} + (u_ii - u_ii) [sv_{iji} + (p_i - s)v_{jji}]}{|H|},$$

where

$$u_ii = \frac{\partial ET_i}{\partial y_i} \geq 0, \quad u_ii \equiv \frac{\partial ET_j}{\partial y_i} \leq 0, \quad u_ii \equiv \frac{\partial ET_j}{\partial y_j} \geq 0, \quad u_ii \equiv \frac{\partial ET_i}{\partial y_j} \leq 0,$$

$$v_{jjj} \equiv \frac{\partial^2 ET_j}{\partial y_j^3}, \quad v_{ijj} \equiv \frac{\partial^2 ET_i}{\partial y_j^2}, \quad v_{iji} \equiv \frac{\partial^2 ET_i}{\partial y_j \partial y_i}, \quad v_{jji} \equiv \frac{\partial^2 ET_j}{\partial y_j \partial y_i},$$

and $f_j(\cdot)$ is the PDF of $\xi_j$. \( \quad (26) \)
For the total inventory, we have \( \frac{\partial Y}{\partial s} = \frac{\partial y_1}{\partial s} + \frac{\partial y_2}{\partial s} \). Let \([a_i, b_i]\) denote the support of \( \xi_i \), where \( 0 \leq a_i < b_i \), and \( f(\cdot, \cdot) \) the joint PDF of \( \xi_i \) and \( \xi_j \). Since the denominator of \( \frac{\partial Y}{\partial s} \) is positive, we only consider the numerator of \( \frac{\partial Y}{\partial s} \) which equals

\[
(u_{11} - u_{21})[p_2 f_2(y_2) - s(v_{222} - v_{221}) - (p_2 - s)(v_{122} - v_{121})] \\
+ (u_{22} - u_{12})[p_1 f_1(y_1) - s(v_{111} - v_{112}) - (p_1 - s)(v_{211} - v_{212})]
\]

\[= (u_{11} - u_{21})[p_2 f_2(y_2) - s \int_{y_1}^{b_1} f(x_1, y_2) dx_1 - (p_2 - s) \int_{a_1}^{y_1} f(x_1, y_2) dx_1] \\
+ (u_{22} - u_{12})[p_1 f_1(y_1) - s \int_{y_2}^{b_2} f(y_1, x_2) dx_2 - (p_1 - s) \int_{a_2}^{y_2} f(y_1, x_2) dx_2],
\]

(27)

Since \( f_2(y_2) = \int_{y_1}^{b_1} f(x_1, y_2) dx_1 + \int_{a_1}^{y_1} f(x_1, y_2) dx_1 \) and \( f_1(y_1) = \int_{y_2}^{b_2} f(y_1, x_2) dx_2 + \int_{a_2}^{y_2} f(y_1, x_2) dx_2 \), we have that \( \frac{\partial Y}{\partial s} \geq 0 \). 

\[\blacksquare\]