Yamaguchi et al. [5] have expressed the three models of Petri nets [4] for modeling and analysis of workflows. There are three models to connect those workflows: chained, nested, and parallel synchronized. Business continuity requires the interworkflow to preserve the behavior of the existing workflows. This requirement is called behavioral inheritance, which has three variations: protocol inheritance, projection inheritance, and life-cycle inheritance. Van der Aalst et al. have proposed workflow nets, WF-nets for short, and have shown that the behavioral inheritance problem is decidable but intractable. In this paper, we first show that all WF-nets of the chained model satisfy life-cycle inheritance, and all WF-nets of the nested model satisfy projection inheritance. Next we show that soundness is a necessary condition of projection inheritance for an acyclic extended free choice WF-net of the parallel synchronized model. Then we prove that the necessary condition can be verified in polynomial time. Finally we show that the necessary condition is a sufficient condition if the WF-net is obtained by connecting state machine WF-nets.

key words: interworkflow, workflow net, behavioral inheritance, soundness, polynomial time verification

1. Introduction

Interoperability technology enables two or more organizations to participate in a workflow. The workflow crossing organizational boundaries is called interorganizational workflow, interworkflow for short [1]. The Workflow Management Coalition, WfMC for short, has given a protocol for interworkflows [2]). In the protocol, an interworkflow is constructed by connecting two or more existing workflows; and there are three models to connect those workflows: chained, nested, and parallel synchronized.

A workflow \( W_X \) is intuitively said to inherit the behavior of another workflow \( W_Y \) iff \( W_X \) can do what \( W_Y \) can do. An interworkflow is constructed by connecting existing workflows, so for business continuity, the interworkflow must inherit the behavior of those workflows. Nevertheless, it may happen that the interworkflow does not inherit the behavior. Therefore, it is very important to verify behavioral inheritance.

Workflow nets [3], WF-nets for short, are a subclass of Petri nets [4] for modeling and analysis of workflows. Yamaguchi et al. [5] have expressed the three models of interworkflows given in the WfMC protocol by means of WF-nets. For convenience’s sake, a WF-net representing an interworkflow of a specific model is abbreviated to a WF-net of that model. They have also analyzed logical correctness, called soundness, for WF-nets of each model.

Van der Aalst et al. [6] have given a formal definition of behavioral inheritance in terms of WF-nets, and have proposed the behavioral inheritance problem: Given two WF-nets \( N_X \) and \( X_Y \), to decide whether \( N_X \) inherits the behavior of \( N_Y \). This problem is decidable but intractable. Van der Aalst et al. have also proposed four rules for attaching a Petri net to a WF-net, and have shown that those rules preserve behavioral inheritance. However, there is no discussion on polynomial time verification of behavioral inheritance.

In this paper, we limit our analysis on the behavioral inheritance problem to interworkflows based on the WfMC protocol, and try to solve the restricted behavioral inheritance problem in polynomial time. The three models of the WfMC protocol are used as the interoperability specification between workflow management systems. To support interoperability, many workflow management systems are being developed on the basis of those models. This implies that many actual interworkflows are based on those models. By analyzing behavioral inheritance of those models, we are aiming to help workflow designers to construct interworkflows guaranteed business continuity. However, there has been no analysis on behavioral inheritance taking account of the WfMC protocol. We first investigate whether all WF-nets of each model satisfy behavioral inheritance. There is a model of which some WF-nets do not satisfy behavioral inheritance. We give a necessary condition of behavioral inheritance for WF-nets of that model. Next we prove that the necessary condition can be verified in polynomial time. Finally we show that the necessary condition is a sufficient condition if a WF-net of that model is obtained by connecting a particular subclass of WF-nets.

2. Preliminary

1. Workflow Net (WF-Net)

A (labeled) WF-net is a labeled Petri net [4] which represents a workflow. Each transition represents an action. Some actions can be observed, others cannot. The former and the latter are called external and internal, respectively. Actions are identified by label. Internal actions are labeled
as a designated label \( \tau \).

**Definition 1** (WF-net [3]): A labeled Petri net \( N=(P, T, A, \ell) \) is a (labeled) WF-net iff (i) \( N \) has a single place \( p_I \) and a single sink place \( p_O \) and \( p \neq p_I \) and \( p \neq p_O \); and (ii) every place or transition is on a path from \( p_I \) to \( p_O \).

Let \( N=(P, T, A, \ell) \) be a WF-net. We represent a marking of \( N \) as a bag over \( P \). A marking is denoted by \( M \in \{p^M|p \in P, M(p)>0\} \), where \( M(p) \) denotes the number of tokens in \( p \). Let \( M_X \) and \( M_Y \) be markings. \( M_X=M_Y \) denotes that \( \forall p \in P : M_X(p)=M_Y(p) \). \( M_X \geq M_Y \) denotes that \( \forall p \in P : M_X(p) \geq M_Y(p) \). The restriction of \( M \) to some subset \( P' \) of \( P \), denoted by \( M|P' \), is defined as \( \{p^M|p \in P', M(p)>0\} \). A transition \( t \) is said to be fireable in a marking \( M \) if \( M \geq t \). Firing \( t \) in \( M \) results in a new marking \( M'=(M \circ t \cdot) \). This is denoted by \( M[N, t]M' \). A marking \( M' \) is said to be reachable from a marking \( M \) if there exists a firing sequence \( \sigma \) of transitions transforming \( M \) to \( M' \). This is denoted by \( M[N, \sigma]M' \). \( |\sigma| \) denotes the set of transitions corresponding to nonzero entries in \( \sigma \), called the support of \( \sigma \). \( M[N, \sigma]M' \) means \( \exists \sigma : M[N, \sigma]M' \). The set of all possible markings reachable from \( M \) is denoted by \( R(N, M) \). The Petri net obtained by connecting \( p_I \) with \( p_I \) via an additional transition \( t \) is called the short-circuited net of \( N \), denoted by \( N'=(P, T \cup \{t\}, A' \cup \{(p_I, t'), (t', p_I)\}, \ell \cup \{(t', \tau)\}) \).

We give subclasses of WF-nets as follows: A WF-net is a single path if \( \forall t \in T \cup \{p_I, p_O\} : |\dot{\sigma}|=|\dot{\sigma}|=1 \) and \( |\dot{\sigma}|=|\dot{\sigma}|=1 \). A WF-net is a state machine (SM for short) if \( \forall t \in T : |\dot{\sigma}|=|\dot{\sigma}|=1 \). A WF-net is marked graph (MG for short) if \( \forall p \in P \cup \{p_I, p_O\} : |\dot{\sigma}|=|\dot{\sigma}|=1 \) and \( |\dot{\sigma}|=|\dot{\sigma}|=1 \). A WF-net is extended free choice (ECF for short) if \( \forall p_I, p_O \in P : p_I \circ \{p^p|p \in P\} \Rightarrow p_I \circ \{p^p|p \in P\} \). It is known [3] that most actual workflows can be modeled as EFCWF-nets.

(2) Soundness

Soundness is a criterion of logical correctness.

**Definition 2** (Soundness [3]): A WF-net \( N \) is sound iff (i) \( \forall t \in R(N, [p_I]) : |\dot{\sigma}|=|\dot{\sigma}|=1 \) and \( |\dot{\sigma}|=|\dot{\sigma}|=1 \); (ii) \( \forall t \in R(N, [p_I]) : |\dot{\sigma}|=|\dot{\sigma}|=1 \) and \( |\dot{\sigma}|=|\dot{\sigma}|=1 \); and (iii) there is no dead transition in \( N \).

**Property 1** ([7]): The following problem can be solved in polynomial time: Given an EFCWF-net, to decide whether \( M[N, \ell]M' \) is reachable from \( M \) by following an edge labeled as \( \alpha \). We write \( M[N, \alpha]M' \) if \( M' \) is reachable from \( M \) by following any number of edges labeled as \( \tau \). We write \( M[N, \alpha]M' \) if either \( \alpha=\tau \) and \( M=M' \), or \( M[N, \alpha]M' \).

**Definition 3** (Branching bisimilarity [8]): Let \( G_X \) and \( G_Y \) be the reachability graphs of a WF-net \( (N_X, [p_I]) \) and another WF-net \( (N_Y, [p_I]) \), respectively. A binary relation \( \mathcal{R} \) on \( \{R(N_X, [p_I]) \times R(N_Y, [p_I])\} \) is branching bisimulation iff (i) \( \forall M_X \in R(N_X, [p_I]) \) and \( \forall M_Y \in R(N_Y, [p_I]) : \exists M'_X, M'_Y \in R(N_X, [p_I]) : M_X[N, \tau][M'_X] \) and \( M_Y[N, \tau][M'_Y] \); (ii) \( \forall M_X \in R(N_X, [p_I]) \) and \( \forall M_Y \in R(N_Y, [p_I]) : \exists M'_X, M'_Y \in R(N_X, [p_I]) : M_X[N, \tau][M'_X] \) and \( M_Y[N, \tau][M'_Y] \); and (iii) \( \forall M_X \in R(N_X, [p_I]) \) and \( \forall M_Y \in R(N_Y, [p_I]) : \exists M'_X, M'_Y \in R(N_X, [p_I]) : M_X[N, \tau][M'_X] \) and \( M_Y[N, \tau][M'_Y] \).

(3) Behavioral Inheritance

Behavioral inheritance is a relaxation of a behavioral equivalence, called branching bisimilarity. Branching bisimilarity equates WF-nets which have the same external behavior. The behavior of a WF-net \( N=(P, T, A, \ell) \) is captured by the reachability graph of \( (N, [p_I]) \). It is denoted by \( G=(V, E) \), where \( V=R(N, [p_I]), E=(M, (t), M')[M, M' \in V; t \in T; M[N, t]M'] \). Let \( M, M' \in V, a \in \ell(T) \). We write \( M[N, a]M' \) if \( M' \) is reachable from \( M \) by following an edge labeled as \( a \). We write \( M[N, \tau]M' \) if \( M' \) is reachable from \( M \) by following any number of edges labeled as \( \tau \). We write \( M[N, \alpha]M' \) if either \( \alpha=\tau \) and \( M=M' \), or \( M[N, \alpha]M' \).

**Definition 4** (Behavioral inheritance [6]): Let \( N_X, N_Y \) be WF-nets.

(i) **Protocol inheritance**: \( N_X \) is a subclass under protocol inheritance of \( N_Y \) if \( \forall t \in R(N_X, [p_I]) : (\bar{\sigma}(N_X, [p_I])) \) and \( (\bar{\sigma}(N_Y, [p_I])) \), respectively. Let \( x \) be a place or a transition. \( \bar{\sigma}(x) \) and \( \bar{\sigma}(x) \) denote \( (x, y, A) \) and \( (y, x, A) \), respectively. \( \ell : T \to \{A \} \) is a labeling function of transitions, where \( A \) denotes the set of all possible external labels. For a subset \( T' \) of \( T \), \( (T') \) denotes \( (t, t') \in T' \).

(ii) **Projection inheritance**: \( N_X \) is a subclass under projection inheritance of \( N_Y \) if \( \forall t \in R(N_X, [p_I]) : (\bar{\sigma}(N_X, [p_I])) \) and \( (\bar{\sigma}(N_Y, [p_I])) \), respectively.

(iii) **Life-cycle inheritance**: \( N_X \) is a subclass under life-cycle inheritance of \( N_Y \) if \( \forall t \in R(N_X, [p_I]) : (\bar{\sigma}(N_X, [p_I])) \) and \( (\bar{\sigma}(N_Y, [p_I])) \), respectively.

**Property 2** ([6]): Protocol, projection, and life-cycle inheritance relations are partial-order.

In this section, we investigate whether WF-nets of each model given in the WfMC protocol satisfy behavioral inheritance. We assume that the WF-net representing an interworkflow is constructed from sound EFC WF-nets because most actual workflows can be modeled as sound EFC WF-nets, and that any transition represents a unique external action because of rationalization.

3.1 The Chained Model

In the chained model, a workflow \( W_X \) deputes the later part after a specified action \( \alpha \) to another workflow \( W_Y \) and \( W_Y \) terminates. Let \( N_X = (P_X, T_X, A_X, \ell_X) \) and \( N_Y = (P_Y, T_Y, A_Y, \ell_Y) \) be the sound EFC WF-nets representing \( W_X \) and \( W_Y \), respectively, and \( \tau_X \in \tau_X \) the transition representing \( \alpha = (\ell_X(t_X)) \). The interworkflow of the chained model, obtained by connecting \( N_Y \) with \( N_X \) after \( \alpha \), can be modeled as the following WF-net \( N_Z = (P_Z, T_Z, A_Z, \ell_Z) \).

\[
\begin{align*}
P_Z &= P_X \cup P_Y \cup \{p_1^X\} \\
T_Z &= T_X \cup T_Y \cup \{t_1^X, t_2^Y, t_3^Y\} \\
A_Z &= A_X \cup A_Y \cup \{(t_2^X, t_1^X), (t_2^X, t_2^Y), (t_2^X, t_3^Y), (t_2^X, p_1^Y),
\quad (t_1^Y, p_1^Y), (t_2^Y, p_1^Y),
\quad (t_2^Y, \chi_X)\} \\
\ell_Z : t(\in T_Z) &\mapsto
\begin{cases}
\ell_X(t) & \text{if } t \in T_X \\
\ell_Y(t) & \text{if } t \in T_Y \\
\beta(t) & \text{if } t = t_1^X \\
\gamma(t) & \text{if } t = t_2^Y \\
\epsilon(t) & \text{if } t = t_3^Y
\end{cases}
\end{align*}
\]

Example 1: Connecting WF-net \( N_2 \) shown in Fig. 1(b) with WF-net \( N_1 \) shown in Fig. 1(a) after transition \( t_1^X \), we can obtain a WF-net \( N_3 \) of the chained model, which is shown in Fig. 2.

Theorem 1: Let \( N_Z \) be a WF-net of the chained model, which is obtained by connecting a sound EFC WF-net \( N_Y \) with another sound EFC WF-net \( N_X \) after an action \( \alpha = (\ell_X(t_X)) \) of \( N_X \). \( N_Z \) is a subclass under life-cycle inheritance of \( N_X \).

Proof: This proof consists of the following three steps: In the first step, let \( N_X \) be a WF-net obtained by sequentially inserting \( p_1^X \) and \( t_1^X \) after \( \tau_X \) into \( N_X \), we show that \( N_X \) is a subclass under projection inheritance of \( N_X \). In the second step, we show that \( N_Z \) is a subclass under protocol inheritance of \( N_X \). In the third step, we show that \( N_Z \) is a subclass under life-cycle inheritance of \( N_X \).

First step: The formal definition of \( N_X \) is given as \( (P_X, T_X, A_X, \ell_X) \), where \( P_X = P_X \cup \{p_1^X\} \), \( T_X = T_X \cup \{t_1^X\} \), \( A_X = A_X \cup \{(t_2^X, p_1^X), (t_1^Y, p_1^Y), (t_2^Y, p_1^Y)\} \cup \{(t_2^Y, \chi_X)\} \). In order to show that \( N_X \) is a subclass under projection inheritance of \( N_X \), we must prove that the following relation \( R \) between \( R(N_X, [p_1^X]) \) and \( R(t_1^Y, N_X, [p_1^Y]) \) with \( t = (t_1^X, t_2^Y) \) satisfies Conditions (i)–(iii) of branching bisimilarity:

\[
R = \{([M_X, M_Y]) : R(N_X, [p_1^X]) \times R(t_1^Y, N_X, [p_1^Y])[p_1^X]
\]

It is obvious that \( N_X \) is a sound EFC WF-net because \( N_X \) is a sound EFC WF-net. Thus \( (t_1^Y, N_X, [p_1^Y]) \) are safe.

Condition (i): Let us first consider the case of \( M_X = M_K \).

If \( \tau_X \) fires, i.e., \( M_X[N_X, \ell_X(t_X)]M_X' = M_X \cup \chi_X \bullet \leftarrow \bullet \times \bullet \chi_X \), then \( M_X[t_1^Y(N_X, \ell_X(t_X))]M_X' = M_X \cup \chi_X \bullet \leftarrow \bullet \chi_X \) holds. Since \( t_1^Y(N_X, [p_1^X]) \) is safe, the number of tokens of \( p_1^X \) is at most one. Therefore \( M_X' \cup \chi_X \bullet \leftarrow \bullet \chi_X \) holds. And \( M_X' \cup \chi_X \bullet \leftarrow \bullet \chi_X \) holds. Next let us consider the case of \( M_Y = M_K' \).

If \( \tau_Y \) is the first transition \( t_1^X \) after \( \tau_X \) fires, i.e., \( M_Y[N_Y, \ell_Y(t_Y)]M_Y' = M_Y \cup \chi_Y \bullet \leftarrow \bullet \chi_Y \), then \( M_Y[t_1^Y(N_Y, \ell_Y(t_Y))]M_Y' = M_Y \cup \chi_Y \bullet \leftarrow \bullet \chi_Y \) holds. Since \( t_1^Y(N_Y, [p_1^Y]) \) is safe, the number of tokens of \( p_1^Y \) is at most one. Therefore \( M_Y' \cup \chi_Y \bullet \leftarrow \bullet \chi_Y \) holds.

Condition (ii): Let us first consider the case of \( M_X = M_K \cup \chi_X \bullet \leftarrow \bullet \chi_X \) holds.
\[ M_X = M_K. \]

If \( r^k \) fires, i.e. \( M_K[\tau_I(N_K), \ell_K(t^k)] \), \( M'_K = M_K[\{p^i\}] \), then \( M_X[N_X, \ell_X(t^k)] = M'_X \). 

If \( r^k \) holds. Next let us consider the case of \( M_X = M_K \cup \{p^i\} \). If \( r^k \) holds. Thus \( M_X = M_K \). 

Condition (iii): If \( r^k \in R \) then \( p^i \). Otherwise \( \{p^i\} \). Thus \( N_Z \) holds. Since \( N_Z \) holds. Thus \( N_Z \) is a subclass under life-cycle inheritance of \( N_K \).

Third step: The former two steps and Property 2 lead that \( N_Z \) is a subclass under life-cycle inheritance of \( N_K \).

Q.E.D.

This theorem means that every WF-net of the nested model, which is obtained by connecting a sound EFC WF-net \( N_Z \) with another sound EFC WF-net \( N_X \) after an action of \( N_Z \), is a subclass under life-cycle inheritance of \( N_X \). Applying it to Example 1, we can know that \( N_3 \) is a subclass under life-cycle inheritance of \( N_1 \).

Let us consider the computation complexity of the following problem: Given a WF-net \( N_Z \) of the chained model, which is obtained by connecting a sound EFC WF-net \( N_Y \) with another sound EFC WF-net \( N_X \) after an action of \( N_Y \), to decide whether \( N_Z \) is a subclass under life-cycle inheritance of \( N_X \). We know from Theorem 1 that the answer of the problem is always yes. Thus the problem can be solved in \( O(1) \) time.

3.2 The Nested Model

In the nested model, a workflow \( W_X \) invokes another workflow \( W_Y \) as a subprocess at a specified action \( \alpha \), and \( W_X \) waits for \( W_Y \) to return a result. Let \( N_X = (P_X, T_X, A_X, \ell_X) \) and \( N_Y = (P_Y, T_Y, A_Y, \ell_Y) \) be sound EFC WF-nets representing \( W_X \) and \( W_Y \), respectively, and \( r^k \) (\( e^k \)) the transition representing \( \alpha \) (\( \ell_k \)). The interworkflow of the nested model, obtained by connecting \( N_Y \) with \( N_X \) at \( \alpha \), can be modeled as the following WF-net \( N_Z \) (\( P_Z, T_Z, A_Z, \ell_Z \)).

\[
\begin{align*}
P_Z &= P_X \cup P_Y \cup \{p^i\} \\
T_Z &= T_X \cup T_Y \cup \{r^k\} \\
A_Z &= A_X \cup A_Y \cup \{(r^k, p^i), (p^i, r^k), (p^i, r^j), (r^j, p^i)\} \\
&\cup \{(r^k, p)|prv^{X_k} = \{(r^k, p)|prv^{X_k} = \}
\end{align*}
\]

\[ N_Z \]

\[ \tau_I(N_K), \ell_I(t^k)[\tau_I(N_K), \ell_I(t^k)] \]

\[ M'_Z = M'_Z \cup \{p^i\} \]

\[ \mathcal{R} = ((M_X, M_Z), M_Z = M_Z \cup \{p^i\}) \]

\[ M_X = M_Z \cup \{p^i\} \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ \mathcal{R} = (M_X, M_Z) \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ \mathcal{R} = (M_X, M_Z) \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]

\[ \tau_I(N_Z), \ell_I(t^k)[\tau_I(N_Z), \ell_I(t^k)] \]

\[ M_X = M_Z \]
3.3 The Parallel Synchronized Model

In the parallel synchronized model, a workflow \( W_X \) runs in parallel with another workflow \( W_Y \), but \( W_X \) and \( W_Y \) synchronize at some points. Let \( N_X = (P_X, T_X, A_X, \Delta_X) \) and \( N_Y = (P_Y, T_Y, A_Y, \Delta_Y) \) be acyclic EFC WF-nets representing \( W_X \) and \( W_Y \), respectively. A synchronization between \( W_X \) and \( W_Y \) is modeled as a single path Petri net whose source and sink nodes are a transition of \( N_X \) and that of \( N_Y \). The whole of synchronizations between \( N_X \) and \( N_Y \) is represented as an acyclic conflict-free labeled Petri net, called synchronization net [5]. The interworkflow of the parallel synchronized model, obtained by connecting \( N_Y \) with \( N_X \) via a synchronization net \( N_S = (P_S, T_S, A_S, \Delta_S) \), can be modeled as the following WF-net \( N_Z = (P_Z, T_Z, A_Z, \Delta_Z) \).

\[
\begin{align*}
P_Z &= P_X \cup P_Y \cup P_S \cup \{p^Y_1, p^O_0\} \\
T_Z &= T_X \cup T_Y \cup T_S \cup \{t^Y_1, t^O_2\} \\
A_Z &= A_X \cup A_Y \cup A_S \cup \{(p^Y_1, t^Y_1), (t^Y_1, p^X_1), (t^O_2, p^Y_1), (p^X_0, t^O_2), (p^O_0, t^O_2), (t^O_2, p^O_0)\}
\end{align*}
\]

Next let us consider the case of \( M_X = [p^X_1] - [p^X_2] - P_Y \). If \( t^1_1 \) fires then \( M_Z[\tau_1(N_Z), \ell_1(t)(=\ell_1(t))] \) holds. Since \( i \not= i \) and \( i \not= i \), \( M'_X = M_Z \), i.e. \( M_X \cup M'_Z \) holds.

Condition (iii): If \( X \not= X \) then there is a marking \( M_X \) such that \( [p^X_2]^{-\infty} M_X \). Since \( N_X \) is sound, \( M_Z[\tau_1(N_Z), \tau^\dagger(p^X_2)] \) holds. Otherwise \( [p^X_2]^{\infty} M_Z \) immediately holds. Thus \( (N_X, p^X_2) \not\rightarrow (\tau_1(N_Z), [p^X_2]) \) holds. This implies \( N_Z \) is a subclass under projection inheritance of \( N_X \). Q.E.D.

This theorem means that every WF-net of the nested model, which is obtained by connecting a sound EFC WF-net \( N_T \) with another sound EFC WF-net \( N_X \) at an action of \( N_X \), is a subclass under projection inheritance of \( N_X \). Applying it to Example 2, we can know that \( N_4 \) is a subclass under projection inheritance of \( N_1 \).

Let us consider the computation complexity of the following problem: Given a WF-net \( N_S \) of the nested model, which is obtained by connecting a sound EFC WF-net \( N_V \) with another sound EFC WF-net \( N_X \) at an action of \( N_X \), to decide whether \( N_S \) is a subclass under projection inheritance of \( N_X \). We know from Theorem 2 that the answer of the problem is always yes. Thus the problem can be solved in \( O(1) \) time.

Example 3: Connecting WF-net \( N_S \) shown in Fig. 1(b) with WF-net \( N_1 \) shown in Fig. 1(a) via the following synchronization net \( N_S = (P_5, T_5, A_5, \ell_5) \), we can obtain a WF-net \( N_6 \) of the parallel synchronized model, which is shown in Fig. 4.

\[
\begin{align*}
P_5 &= \{p^X_1\} \\
T_5 &= \{t^1_1, t^1_1, t^2_2\} \\
A_5 &= \{(t^1_2, p^X_1), (t^1_2, p^X_1), (p^X_1, t^2_2)\} \\
\ell_5 : i(\in T_5) \rightarrow \{ \begin{array}{ll}
\ell_1(t) & \text{if } t \in T_1 \\
\ell_2(t) & \text{if } t \in T_2
\end{array} \}
\end{align*}
\]

\( N_6 \) is a subclass under projection inheritance of both \( N_1 \) and \( N_5 \). The proof is given in the next section. On the other hand, if arc \((t^1_2, p^X_1)\) is removed, the resultant WF-net, denoted by \( N'_6 \), is not a subclass under projection inheritance of both \( N_1 \) and \( N_5 \). This is because in \( N'_6 \), a firing of \( t^2_2 \) results in a dead marking \( [p^X_1] \), which cannot be transformed to \( [p^O_0] \), i.e. Condition (iii) of branching bisimilarity cannot be satisfied. This means that some WF-nets of the parallel synchronized model are not a subclass under projection inheritance.

4. Verification Condition for Acyclic WF-Nets of the Parallel Synchronized Model

We give a necessary condition of projection inheritance for acyclic EFC WF-nets of the parallel synchronized model, and then prove that the necessary condition can be verified in polynomial time. We also show that the necessary condition is a sufficient condition if the WF-net is obtained by connecting SM WF-nets.
4.1 Necessary Condition

We utilize soundness for verification of projection inheritance for acyclic EFC WF-nets of the parallel synchronized model. Let $N_Z$ be an acyclic EFC WF-net of the parallel synchronized model, which is obtained by connecting an acyclic sound EFC WF-net $N_X$ with another acyclic sound EFC WF-net $N_Y$ via a synchronization net $N_s$. We predict that if $N_Z$ is not sound then $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$. This prediction is based on the following: Let us assume that $N_Z$ is not sound. $N_Z$ does not satisfy Conditions (i), (ii), and (iii) of soundness. If $N_Z$ does not satisfy Conditions (i) and/or (ii) of soundness then there exists a marking in $R(N_Z, [p^Z_I])$ which cannot be transformed to $[p^Z_O]$. This implies that $N_Z$ cannot satisfy Condition (iii) of branching bisimilarity. If $N_Z$ does not satisfy Condition (iii) of soundness then there exists a dead transition corresponding to an external action in $(N_Z, [p^Z_I])$. This implies that $N_Z$ cannot satisfy Condition (i) or (ii) of branching bisimilarity. Thus $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$. We can obtain the following theorem.

**Theorem 3:** Let $N_Z$ be an acyclic EFC WF-net of the parallel synchronized model, which is obtained by connecting an acyclic sound EFC WF-net $N_X$ with another acyclic sound EFC WF-net $N_Y$ via a synchronization net $N_s$. $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$ if $N_Z$ is not sound.

As a preparation for proving Theorem 3, we give the following lemma.

**Lemma 1:** Let $N_Z$ be an acyclic WF-net of the parallel synchronized model, which is obtained by connecting an acyclic sound EFC WF-net $N_X$ with another acyclic sound EFC WF-net $N_Y$ via a synchronization net $N_s$. $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$ if there is a dead marking besides $[p^Z_O]$ in $(N_Z, [p^Z_I])$.

Proof of Lemma 1: Let us assume that $N_Z$ is a subclass under projection inheritance of both $N_X$ and $N_Y$. There is a branching bisimulation relation $R_X$ between $R(\tau_{a_sT_X}(N_Z), [p^X_I])$ and $R(N_X, [p^Z_I])$. There is a branching bisimulation relation $R_Y$ between $R(\tau_{a_sT_Y}(N_Z), [p^Z_I])$ and $R(N_Y, [p^Y_I])$. Since $N_X$ and $N_Y$ are sound, $[p^X_O]$ and $[p^Y_O]$ are respectively only one dead marking of $(N_X, [p^X_I])$ and $(N_Y, [p^Y_I])$. Let $M^Z_{dead}$ be a dead markings besides $[p^Z_O]$ in $(N_Z, [p^Z_I])$. Let us consider the following four cases: (a) $M^Z_{dead} R_X [p^X_O]$; (b) $M^Z_{dead} R_X [p^Y_O]$; (c) $M^Z_{dead} R_Y [p^Y_O]$; and (d) $M^Z_{dead} R_Y [p^Z_I]$. Case (a): From Condition (ii) of branching bisimilarity, we must have $M^Z_{dead} \tau_{a_sT_X}(N_Z), \tau^*_a[p^X_O]$. Since $M^Z_{dead}$ is dead, $M^Z_{dead}$ cannot be transformed to $[p^X_O]$. Therefore $R_X$ is not branching bisimulation. This contradicts the assumption.

Case (b): Let $M_X$ be a marking such that $M^Z_{dead} R_X M_X$. Since $M_X$ is not dead, there is an action fireable in $M_X$. The action is denoted by $\alpha$. From Condition (i) of branching bisimilarity, we must have $M^Z_{dead} \tau_{a_sT_X}(N_Z), \alpha$. However $\alpha$ cannot be fireable in $M^Z_{dead}$ because $M^Z_{dead}$ is dead. Therefore $R_X$ is not branching bisimulation. This contradicts the assumption.

Case (c): The contradiction to the assumption is proved in a similar way as Case (a).

Case (d): The contradiction to the assumption is proved in a similar way as Case (b).

For all the four cases, the assumption contradicts. Thus $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$.

**Q.E.D.**

Proof of Theorem 3: Let us consider the following three cases: (a) $N_Z$ does not satisfy Condition (i) of soundness; (b) $N_Z$ does not satisfy Condition (ii) of soundness; and (c) $N_Z$ does not satisfy Condition (iii) of soundness.

Case (a): Since $N_Z$ is acyclic, $[p^Z_I]$ is eventually transformed to a dead marking. If Condition (i) is not satisfied then there exists a dead marking besides $[p^Z_O]$. This means from Lemma 1 that $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$.

Case (b): If Condition (ii) is not satisfied then there exists a marking $M_Z$ in $r(N_Z, [p^Z_I])$ such that $M_Z \neq [p^Z_O]$. Since there is no sink transition in $N_Z$, $M_Z$ is eventually transformed not to $[p^Z_O]$ but to another dead marking. This means from Lemma 1 that $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$.

Case (c): Let $I_{dead}$ be a dead transition in $(N_Z, [p^Z_I])$. If $I_{dead} \neq I^1$ or $I^2$ then $[p^Z_O]$ is not reachable from $[p^Z_I]$. This implies that Condition (iii) of branching bisimilarity cannot be satisfied. If $I_{dead} \in T_X$ (or $T_Y$) then the external action corresponding to $I_{dead}$ cannot be executed in $(N_Z, [p^Z_I])$. This implies that Condition (i) or (ii) of branching bisimilarity cannot be satisfied. Therefore $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$.

Thus if $N_Z$ is not sound then $N_Z$ is not a subclass under projection inheritance of both $N_X$ and $N_Y$.

**Q.E.D.**

This theorem shows that soundness is a necessary condition of projection inheritance for acyclic EFC WF-nets of the parallel synchronized model. It is known from Property 1 that soundness of EFC WF-nets can be decided in polynomial time. Consequently, this theorem enables us to prove in polynomial time that if a given WF-net of the parallel synchronized model is acyclic EFC but is not sound, the WF-net does not satisfy projection inheritance.

4.2 Necessary and Sufficient Condition

We show that the proposed necessary condition becomes a sufficient condition by imposing a structural restriction. The structural restriction is that an acyclic WF-net of the parallel synchronized model is constructed by connecting acyclic SM WF-nets.
Theorem 4: Let $N_Z$ be an acyclic EFC WF-net of the parallel synchronized model, which is obtained by connecting an acyclic SM WF-net $N_X$ with another acyclic SM WF-net $N_Y$ via a synchronization net $N_S$. $N_Z$ is a subclass under projection inheritance of both $N_X$ and $N_Y$ if $N_Z$ is sound. □

As a preparation for proving Theorem 4, we give the following lemma.

Lemma 2: Let $N_X = (P_X, T_X, A_X, \ell_X)$ be a single path WF-net, $N_Y = (P_Y, T_Y, A_Y, \ell_Y)$ an acyclic MG WF-net of which $N_X$ is a subnet, i.e., $P_X \subseteq P_Y$, $T_X \subseteq T_Y$, $A_X \subseteq A_Y$, $\ell_X \subseteq \ell_Y$. $N_Y$ is a subclass under projection inheritance of $N_X$. □

Proof of Lemma 2: Since $N_Y$ is single path, there exists only one firing sequence transforming $[p_X^0]$ to $[p_Y^0]$ in $(N_X, [p_X^0])$. Let $P_X = \{p_X^1, p_X^2, \ldots, p_X^{i-1}\}$, $P_Y = \{p_Y^1, p_Y^2, \ldots, p_Y^m\}$, and $A_X = \{a_X^1, a_X^2, \ldots, a_X^k\}$, $A_Y = \{a_Y^1, a_Y^2, \ldots, a_Y^l\}$.

The firing sequence is denoted by $\sigma_X = t_X^1, t_X^2, \ldots, t_X^i$. Since $N_X$ is acyclic MG, every transition fires only once in any firing sequence $\sigma_Y$ transforming $[p_X^0]$ to $[p_Y^0]$. Since $N_Y$ includes $N_X$ as a subnet, strictly speaking as a path, $\sigma_Y$ is denoted by

$$\sigma_Y = \sigma_X t_Y^1, \sigma_X t_Y^2, \ldots, \sigma_X t_Y^m$$

where $\cup_{i=0}^m |\sigma_i|| = T_Y - T_X$.

We must prove that the following relation $R$ between $R(N_X, [p_X^1])$ and $R(T_I(N_Y), [p_Y^1])$ with $I = T_2(T_Y - T_X)$ satisfies Conditions (i)–(iii) of branching similarity:

$$R = \{(M_X, M_Y) \in R(N_X, [p_X^1]) \times R(T_I(N_Y), [p_Y^1]) \mid M_X = M_Y \bar{P}_X\}$$

Condition (i): If $t_X^i$ fires, i.e., $M_X = [p_X^{i-1}][N_X, \ell_X(t_X^i)]$, $M'_X = [p_X^i]$, then $M_Y[T_I(N_Y), \ell_Y(t_Y(t))] M'_Y[T_I(N_Y), \ell_Y(t_Y(t))] (\sigma_X t_Y^m)$ holds. Since $R_\Sigma^{M'_X} \equiv \Sigma^{[p_X^{i-1}]} = M_X$, $M_X R \Sigma M'_Y$ holds.

Since $X, t_X^i \equiv t_X^i$ and $T_Y, t_Y^i \equiv T_Y, t_Y^i$, $M_X R \Sigma M'_Y$ holds.

Condition (ii): If $t_Y^i$ fires, i.e., $M_Y[T_I(N_Y), \ell_Y(t_Y(t))] M'_Y$, then $M_X[N_X, \ell_X(t_X(t))] M'_X$ holds. Since $X, t_X^i \equiv t_X^i$ and $T_Y, t_Y^i \equiv T_Y, t_Y^i$, $M_X R \Sigma M'_Y$ holds. If another transition $t \in (T_Y - T_X)$ fires, i.e., $M_Y[T_I(N_Y), \ell_Y(t)] M'_Y$, then $M_X M'_X$ holds because $T_2P_X = M_Y P_Y$.

Condition (iii): It obviously holds, because $M_X R \Sigma M'_Y$.

Thus $(N_X, [p_X^1]) \sim_{\sigma} (T_I(N_Y), [p_Y^1])$ holds. This implies $N_Y$ is a subclass under projection inheritance of $N_X$. Q.E.D.

Proof of Theorem 4: $N_Z$ is a sound EFC WF-net, so $(N_Z, [p_Z^0])$ is live and safe from Theorem 1 of Ref. [3]. From Theorem 14 of Ref. [4], we can view $N_Z$ as an interconnection of strongly connected MG-components\(^1\). Let $\ell$ be the number of the strongly connected MG-components. All the strongly connected MG-components share $\tau'$ as an articulation point. Therefore for each of the strongly connected MG-components, we can obtain a MG WF-net by removing $\tau'$ from the MG-component. Those MG WF-nets are denoted by $N_Z_1, N_Z_2, \ldots, N_Z_\ell$.

In a similar way as $N_Z$, since $N_X$ is a SM WF-net, we can view $N_X$ as an interconnection of strongly connected MG-components. Let $m$ be the number of the strongly connected MG-components. Therefore for each of the strongly connected MG-components, we can obtain a MG WF-net, strictly speaking a single path WF-net, by removing $\tau'$ from the MG-component. Those single path WF-nets are denoted by $N_{X_1}, N_{X_2}, \ldots, N_{X_m}$. In a similar way as $N_X$, we can also view $N_Y$ as an interconnection of single path WF-nets. Let $n$ be the number of the single path WF-nets. Those single path WF-nets are denoted by $N_{Y_1}, N_{Y_2}, \ldots, N_{Y_n}$.

$R(N_Z, [p_Z^0])$ be a single path starting from $[p_Z^0]$. A minimal T-invariant\(^2\) of $N_Z$ corresponds to one of the strongly connected MG-components from Theorem 5.17 of Ref. [9]. Therefore for each $i \in \{1, 2, \ldots, \ell\}$, $R(N_Z, [p_Z^0]) \subseteq R(N_{X_i}, [p_{X_i}^0])$ holds. Since $N_Z$ can be viewed as an interconnection of $N_{X_1}, N_{X_2}, \ldots, N_{X_m}$, we can say $R(N_Z, [p_Z^0]) = \bigcup_{i=1}^\ell R(N_{X_i}, [p_{X_i}^0])$. This implies $R(N_X, [p_X^0]) = \bigcup_{i=1}^\ell R(T_I(N_{X_i}), [p_{X_i}^0])$.

We must prove that there is a branching bisimulation relation $R(N_X, [p_X^1])$ and $R(T_I(N_Y), [p_Y^1])$ with $I = T_2(T_Y - T_X)$.

Each MG WF-net $N_{X_i}$ forms a structure in which two single path WF-nets $N_{Y_{i1}} \cup \{1, 2, \ldots, m\}$ and $N_{Y_{i2}} \cup \{1, 2, \ldots, \ell\}$ are coupled via some transition-to-transition paths derived from $N_X$. We can prove from Lemma 2 that $N_Z$ is a subclass under projection inheritance of $N_X$, i.e., there exists a branching bisimulation relation $R_{\Sigma}$ between $R(N_X, [p_X^1])$ and $R(T_I(N_Y), [p_Y^1])$, where $j \in \{1, 2, \ldots, m\}$ and $i \in \{1, 2, \ldots, \ell\}$. This implies that there is branching bisimulation relation $R_{\Sigma} \subseteq R_{\Sigma}$ and $R_{\Sigma} \subseteq R(T_I(N_Y), [p_Y^1])$.

This theorem shows that soundness is a necessary and sufficient condition of projection inheritance for acyclic EFC WF-nets of the parallel synchronized model that is obtained by connecting acyclic SM WF-nets. Applying it to Example 3, we can know that since $N_0$ is sound, $N_0$ is a subclass under projection inheritance of both $N_1$ and $N_2$.

Next we show that the necessary and sufficient condition can be verified in polynomial time.

Corollary 1: The following problem can be solved in polynomial time: Given an acyclic EFC WF-net of the par-
allel synchronized model, which is obtained by connecting acyclic SM WF-nets via a synchronization net, to decide whether the EFC WF-net is a subclass under projection inheritance of both the SM WF-nets.

Proof: It is immediately obtained from Property 1 and Theorem 4.

5. Conclusion

In this paper, we investigated the behavioral inheritance problem restricted to interworkflows based on the WfMC protocol. The results are shown in Table 1. We first showed that every WF-net of the chained model obtained by connecting a sound EFC WF-net $N_X$ with another EFC WF-net $N_X$ after an action of $N_X$ is a subclass under life-cycle inheritance of $N_X$; and every WF-net of the nested model obtained by connecting $N_Y$ with $N_X$ at an action of $N_X$ is a subclass under projection inheritance of $N_X$. Note that every WF-net of the nested model is sound. Next we showed that soundness is a necessary condition of projection inheritance for an acyclic EFC WF-net of the parallel synchronized model. Then we proved that the necessary condition can be checked in polynomial time. Finally, we showed that soundness is a necessary and sufficient condition of projection inheritance if the WF-net is obtained by connecting SM WF-nets. We can solve in polynomial time the behavioral inheritance problems within highlighted cells in Table 1. Furthermore we can efficiently solve those problems, because soundness can be verified by using model checking tools [10].

We argue that our results have three advantages over the previous work, e.g. [6] and [8]: (i) Our results are practical because they are based on the WfMC protocol; (ii) Our results have much expressive power. Our models include not only attaching a WF-net to another WF-net but coupling parallel WF-nets on equal terms; (iii) Our results contribute to verification of behavioral inheritance. In the previous work, there is no discussion on polynomial time verification of behavioral inheritance. We gave two verification conditions which can be checked in polynomial time. Thus we can say that our results are useful to efficiently solve many behavioral inheritance problem restricted to interworkflows based on the WfMC protocol.

As a future work, we plan to generalize the necessary and sufficient condition on projection inheritance to the other net classes, e.g. subclasses of acyclic EFC WF-nets obtained by connecting MG WF-nets.

### Table 1

<table>
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<tr>
<th>connection model</th>
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**Acknowledgement**

The authors would like to thank Mr. Tetsushi Narui for his contribution to this work.

**References**


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