CHAOTIC SYNCHRONIZATION AND SECURE COMMUNICATION VIA SLIDING-MODE OBSERVER

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Information signal embedded in a chaotic transmitter can be recovered by a receiver if it is a replica of the transmitter. In this paper, a new aspect of chaotic communication is introduced. A sliding-mode observer replaces the conventional chaotic system at the receiver side, which does not need information from the transmitter. So the uncertainties in the transmitter and the transmission line do not affect the synchronization, the proposed communication scheme is robust with respect to some disturbances and uncertainties. Three chaotic systems, Duffing equation, Van der Pol oscillator and Chua’s circuit, are provided to illustrate the effectiveness of the chaotic communication.

Keywords: Synchronization; chaotic communication; sliding mode observer.

1. Introduction

In the past years, synchronization of chaotic systems problem has received a great deal of attention among scientists in many fields [Pecora & Carroll, 1990; Morgul & Solak, 1996; Parlitz et al., 1992; Liao & Huang, 1999]. As it is well known, the study of the synchronization problem for nonlinear systems has been very important from the nonlinear science point of view, in particular, the applications to biology, medicine, cryptography, secure data transmission and so on. In general, synchronization research has been focused on two areas. The first relates to the employment of state observers, where the main applications pertain to the synchronization of nonlinear oscillators. The second one is the use of control laws, which allows to achieve the synchronization between nonlinear oscillators, with different structure and order [Wu & Chua, 1994]. Of particular interest is the connection between the observers for nonlinear systems and chaos synchronization, which is also known as master-slave configuration. Thus, chaos synchronization problem can be posed as an observer design procedure, where the coupling signal is viewed as output and the slave system is regarded as observer.

The general idea for transmitting information via chaotic systems is that, an information signal is embedded in the transmitter system which produces a chaotic signal, the information signal is recovered when the transmitter and the receiver are identical. Since Pecora and Carroll’s observation on the possibility of synchronizing two chaotic systems [Pecora & Carroll, 1990] (so-called drive-response configuration), several synchronization schemes have been developed. Synchronization can be classified into mutual synchronization (or bidirectional coupling) [Ushio, 1999] and master-slave synchronization (or unidirectional coupling) [Pecora & Carroll, 1990]. The chaos-based secure communications have updated their fourth generation [Tao, 1999]. The continuous synchronization
is adopted in the first three generations while the impulsive synchronization is used in the fourth generation. Less than 94 Hz of bandwidth is needed to transmit the synchronization signal for a third-order chaotic transmitter in the fourth generation while 30-kHz bandwidth is needed to transmit the synchronization signals in the other three generations [Yang & Chua, 1997].

There are many applications to chaotic communication [Hasler, 1998] and chaotic network synchronization [Chow et al., 2001]. The techniques of chaotic communication can be divided into three categories (a) chaos masking [Kocarev et al., 1992], the information signal is added directly to the transmitter; (b) chaos modulation [Boutayeb et al., 2002; Hasler, 1998; Liao & Huang, 1999; Wu & Chua, 1994], it is based on the master-slave synchronization, where the information signal is injected into the transmitter as a nonlinear filter; (c) chaos shift keying [Parlitz et al., 1992], the information signal is supposed to be binary, and it is mapped into the transmitter and the receiver. In these three cases, the information signal can be recovered by a receiver if the transmitter and the receiver are synchronized. In order to reach synchronization, the receiver should be a replica of the transmitter [Hasler, 1998].

Linear and nonlinear observers in control theory literatures can be applied to design receivers. The receiver is regarded as a chaotic observer, which has two parts — a duplicated chaotic system of the transmitter and an adjustable observer gain [Liao & Huang, 1999]. Some modifications were made when it is difficult to obtain a replica of the synchronization. For example, the transmitter and the receiver are set into the same chaotic structures, parameter identification methods can be used to construct the chaotic receiver [Huijberts et al., 2000]; when there are uncertainties in synchronization (the transmitter is not known exactly, there is noise in the transmission line, etc.), the transmitter and the receiver could be established in the same fuzzy models, fuzzy model-based design method was applied to reach synchronization [Lian et al., 2001]; stability analysis of observer-based chaotic communication with respect to uncertainties can be found in [Alvarez et al., 2002; Boutayeb et al., 2002; Martinez-Guerra et al., 2006].

Robust control techniques and many traditional schemes have been applied in robust synthesis for chaotic synchronization, e.g. robust observer and $H_{\infty}$ technique are used in [Suykens et al., 1999a] and [Suykens et al., 1999b]. Since sliding-mode observer contains a sliding-mode term, it provides the robustness against an inaccurate modeling of measurements and output noises. The early works dealing with sliding mode observers which consider measurement noise were proposed by Drakunov and Utkin [1995]. They discussed the state estimation using sliding mode technique. De Carlo et al. [1996] discussed the variable structure control as a high-speed switched feedback control resulting in a sliding mode. Anulova [1986] treated an analysis of systems with sliding mode in the presence of noises. Slotine et al. [1987], successfully designed, so named, sliding-mode approach to construct observers which are highly robust with respect to noises in the input of the system. But, it turns out that the corresponding stability analysis cannot be directly applied in the situations with the output noise (or, mixed uncertainty) presence. So, it is still a challenge to suggest a workable technique to analyze the stability of identification error generated by sliding-mode (discontinuous nonlinearity) type observers [Martinez-Guerra et al., 2004; Yu et al., 2000].

In this paper, a novel design approach for chaotic communication is proposed, where the receiver is a pure sliding-mode observer. The main difference with the above methods is that the receiver is no longer a chaotic system. The uncertainty of the transmitter will not affect the synchronization. The proposed communication scheme can be more robust than both transmitter and receiver employed in chaotic systems. But the information may be recovered by the observer who does not have knowledge about the transmitter, this is a big challenge to secure communication by means of chaos. Numerical demonstrations using prototype of chaotic oscillators are also provided.

2. Chaotic Communication Based on Sliding-Mode Observer

In normal chaotic communication, the transmitter and the receiver are chaotic systems. They can be described in the form of the following nonlinear system

$$\dot{\xi} = f(\xi) + g(\xi)u$$
$$y = h(\xi)$$

where $\xi \in \mathbb{R}^n$ is the state of the plant, $u \in \mathbb{R}$ is a control input, $y \in \mathbb{R}$ is a measurable output, $f$, $g$ and $h$ are smooth nonlinear functions. Most chaotic
systems have uniform relative degree \( n \), i.e.
\[
L_g h(\xi) = \cdots = L_g L_{j}^{n-2} h(\xi) = 0, \quad L_g L_{j}^{n-1} h(\xi) \neq 0
\]
So there exists a mapping
\[
\eta = T(\xi)
\]
which can transform the system (1) into the following normal form [Isidori, 1995]
\[
\dot{\eta}_i = \eta_{i+1}, \quad i = 1, \ldots, n-1
\]
\[
\dot{\eta}_n = \Phi(\eta, u)
\]
\[
y = \eta_1
\]
where \( \Phi(\cdot) \) is a continuous nonlinear function.

First, we discuss a simple case, the transmitter and the receiver are second order chaotic oscillators, for example, Duffing Equation and Van der Pol Oscillator. When \( n = 2 \), (3) becomes
\[
\dot{\eta}_1 = \eta_2
\]
\[
\dot{\eta}_2 = \Phi(\eta_1, \eta_2, u)
\]
\[
y = \eta_1 + s
\]
Duffing equation describes a specific chaotic circuit [Chen & Dong, 1993]. It can be written as
\[
\dot{\eta}_1 = \eta_2
\]
\[
\dot{\eta}_2 = p_1 \eta_1 - p_2 \eta_1^3 - p \eta_2 + q \cos(\omega t) + u_t
\]
where \( p, p_1, p_2, q \) and \( \omega \) are constants, \( u_t \) is a control input. It is known that the solution of (5) exhibits almost periodic and chaotic behavior. In the uncontrolled case, if we select \( p_1 = 1.1, p_2 = 1, p = 0.4, q = 2.1, \omega = 1.8 \), the Duffing oscillator (5) has a chaotic response as in Fig. 1.

The Van der Pol oscillator can be described as [Venkatasubramanian, 1994]
\[
\dot{\eta}_1 = \eta_2
\]
\[
\dot{\eta}_2 = a_1[(1 - a_2 \eta_1^2) \eta_2 - a_3 \eta_1] + u_t
\]
In the uncontrolled case, if we select \( a_1 = 1.5, a_2 = 1, a_3 = 1 \), the Van der Pol oscillator (6) has a chaotic response as in Fig. 2.

In this paper, chaos modulation [Alvarez et al., 2002; Boutayeb et al., 2002; Liao & Huang, 1999; Morgul & Solak, 1996] is used for communication, where the information signal \( s \) is embedded into the output of the chaotic transmitter. The transmitter is a slight modification of the normal chaotic systems (4) as follows:
\[
\dot{\hat{\eta}}_1 = \hat{\eta}_2
\]
\[
\dot{\hat{\eta}}_2 = \hat{\Phi}(\eta_1, \eta_2)
\]
\[
y = \eta_1 + s
\]
where the output \( y = \eta_1 + s \) is chaotic masking.

In this paper, we discuss a new observer-based receiver, we propose the following sliding-mode observer for the receiver
\[
\dot{\hat{\eta}}_1 = \hat{\eta}_2 + m \tau^{-1}\text{sign}(y - \hat{y})
\]
\[
\dot{\hat{\eta}}_2 = m^2 \tau^{-2}\text{sign}(y - \hat{y})
\]
where \( \hat{\eta}_1, \hat{\eta}_2 \) are the states on the receiver side and \( \hat{y} \) the estimate of the output \( y \). \( m \) and \( \tau \) are a small positive parameters \( m > 0, 0 < \tau < 1 \), the sign

![Fig. 1. Chaotic behavior of Duffing equation with \( x(0) = [0, 0]^T \).](image1)

![Fig. 2. Chaotic behavior of Van der Pol oscillator with \( x(0) = [1, 1]^T \).](image2)
function is defined as
\[
\text{sign}(y - \hat{y}) = \begin{cases} 
1 & (y - \hat{y}) > 0 \\
-1 & (y - \hat{y}) < 0 \\
0 & (y - \hat{y}) = 0 
\end{cases}
\]

The schematic diagram of the chaotic communication based on sliding-mode observer is shown in Fig. 3.

The receiver (8) proposed in this paper is very easy to be applied and robust with respect to the uncertainty in the transmitter side. For example, the parameters \(a_1, a_2, a_3\) that are not known exactly, will affect the recovery accuracy of the information signal.

Let us define the synchronization error as
\[
e_1 = \eta_1 - \hat{\eta}_1
\]
\[
e_2 = \frac{1}{m}(\eta_2 - \hat{\eta}_2)
\]

The recovered signal at receiver is
\[
\hat{s} = y - \hat{y} = e_1 + s
\]

By (7) and (8) the synchronization error can be formed as
\[
\dot{e} = A_\mu e - K \text{sign}(Ce + s) + \Delta f
\]

with
\[
A_\mu = \begin{pmatrix} -\mu & m \\ 0 & -\mu \end{pmatrix}, \quad \mu > 0,
\]
\[
K = m\tau^{-1} \begin{pmatrix} 1 \\ m\tau^{-1} \end{pmatrix}, \quad \Delta f = \begin{pmatrix} \mu e_1 \\ \frac{\Phi}{m} + \mu e_2 \end{pmatrix},
\]
\[
C = \begin{pmatrix} 1 & 0 \end{pmatrix}
\]

\(\mu\) is a regularizing parameter, \(\Delta f\) is an uncertainty term (or unmodeled dynamic term).

The following assumptions are used for our theoretical result:

**A1.** There exist non-negative constants \(L_{0f}, L_{1f}\) such that for any \(e\) the following generalized quasi-Lipschitz conditions holds
\[
\|\Delta f\| \leq L_{0f} + (L_{1f} + \|A_\mu\|)\|e\|
\] (12)

**A2.** Information signal is assumed to be bounded as \(\|s\|_\Lambda^2 = s^T\Lambda s \leq (\bar{s})^2 < \infty\) where \(\Lambda\) is a symmetric definite positive matrix.

**A3.** There exists a positive definite matrix \(Q_0 = Q_0^T > 0\) such that the following matrix Riccati equations
\[
PA_\mu + A_\mu^T P + PRP + Q = 0
\] (13)

where
\[
R = \Lambda_f^{-1} + 2\|\Lambda_f\|L_{1f}I, \quad 0 < \Lambda_f = \Lambda_f^T,
\]
\[
Q = Q_0 + 2(L_{1f} + \|A_\mu\|)^2 I
\] (14)

has a positive definite solution \(P = P^T > 0\). Since \(P^T > 0\), there exists \(k > 0\) such that \(K = kP^{-1}C^T\).

**Remark 1.** Note that the dynamic \(\Phi(\eta_1, \eta_2)\) in (7) is Lipschitz with respect to \(\eta_1\) and \(\eta_2\), so for chaotic systems the assumption A1 is satisfied. The measurement is corrupted by a reasonable signal \(s\) which is bounded, it is assumption A2. To calculate the solution to the Riccati equation (13), the following parameters have been selected
\[
\Lambda_f = \lambda_f I, \quad L_{1f} = \mu, \quad R = (\lambda_f^{-1} + 2\lambda_f\mu)I
\]
\[
Q_0 = q_0 I, \quad Q = (q_0 + 8\mu^2)I
\] (15)
with $\lambda_f = 20$, $\mu = 0.0001$, $q_0 = \mu^2$, we obtain

$$P = 10^{-3} \begin{bmatrix} 3.16099 & -0.22096 \\ -0.22096 & 3.16099 \end{bmatrix} > 0$$

which satisfies assumption A3.

**Theorem 2.** The sliding-mode observer-based receiver (8) can recover the information signal $s$ which is embedded in the chaotic transmitter (7), the signal recovery error $\hat{s} = s - \hat{s}$ converges to the following residual set

$$D_s = \{ \hat{s} | \| \hat{s} \|_P \leq \bar{p}(k) \}$$

where $P$ is a solution of the Riccati equations (13)

$$\bar{p}(k) = \left( \frac{\rho(k)}{\sqrt{(k\alpha_p)^2 + \rho(k)\alpha_Q + k\alpha_p}} \right)^2$$

where

$$\rho(k) = 2\|\Lambda_f\|L_{0f}^2 + 4k(\sqrt{n\Lambda^{-1}})f$$

$$k\alpha_p = k(\lambda_{\text{min}}(P^{-1/2}C^TP^{-1/2}))$$

$$\alpha_Q = \lambda_{\text{min}}(P^{-1/2}Q^TP^{-1/2}) > 0$$

where $n$ is the dimension of the chaotic system.

**Proof.** The Lyapunov function candidate $V(e)$ as

$$V(e) = \|e\|_P^2 = e^TPe, \quad 0 < P = P^T$$

and using the matrix inequality

$$X^TY + Y^TX \leq X^T\Lambda_fX + Y^T\Lambda_f^{-1}Y$$

valid for any $X, Y \in \mathbb{R}^{nxm}$, $0 < \Lambda_f = \Lambda_f^T$, it follows

$$\dot{V}(e) = 2e^TP\dot{e}$$

$$= 2e^TP(A_\mu e - K\text{sign}(Ce + s) + \Delta f)$$

$$\leq 2e^TPA_\mu e - 2ke^TC^T\text{sign}(Ce + s) + 2e^TP\Delta f$$

$$\leq e^T(PA_\mu + A_\mu^TP)e - 2e^TC^T\text{sign}(Ce + s)$$

$$+ e^TP\Lambda^{-1}Pe + X^T\Lambda_f^{-1}X = e^T(PA_\mu + A_\mu^TP)e - 2e^TC^T\text{sign}(Ce + s)$$

$$+ (L_{0f}^2 + (L_{1f} + \|A_\mu\|)^2\|e\|^2)\|\Lambda_f\|$$

$$- 2ke^TC^T\text{sign}(Ce + s)$$

$$= e^T(PA_\mu + A_\mu^TP + PRP + Q)e - e^TQe$$

$$+ 2\|\Lambda_f\|L_{0f}^2 - 2k(Ce)^T\text{sign}(Ce + s)$$

by using

$$x^T\text{sign}[x + z] \geq \sum_{i=1}^n |x_i| - 2\sqrt{n}\|z\|$$

Then

$$\dot{V}(e) \leq -e^TQe + 2\|\Lambda_f\|L_{0f}^2$$

$$- 2k \sum_{i=1}^n |(Ce)_i| - 2\sqrt{n}\|s\|$$

$$\leq -e^TQe - 2k \sum_{i=1}^n |(Ce)_i| + \rho(k)$$

where

$$\rho(k) = 2\|\Lambda_f\|L_{0f}^2 + 4k(\sqrt{n\Lambda^{-1}})f$$

Thus

$$\dot{V}(e) \leq -\|e\|_Q - 2k\alpha_P\|e\|_P + \rho(k)$$

where

$$\left( \sum_{i=1}^n |(Ce)_i| \right)^2 \geq \sum_{i=1}^n |(Ce)_i|^2$$

$$= \|Ce\|^2$$

$$= \|CP^{-1/2}P^{-1/2}c\|^2$$

$$\geq \alpha_P e^TQe$$

(16)

with

$$\alpha_P = \lambda_{\text{min}}(P^{-1/2}C^TP^{-1/2}) \geq 0$$

(17)

So that, from (15) we obtain

$$\dot{V}(e) = -\alpha_QV(e) - \vartheta \sqrt{V(e)} + \beta$$

where

$$\alpha_Q = \lambda_{\text{min}}(P^{-1/2}Q^TP^{-1/2}) > 0,$$

$$\vartheta = 2k\alpha_p, \quad \beta = \rho(k)$$

By the theorem proposed in [Martinez-Guerra et al., 2004] and A1–A3, we can formulate the following

$$\left[ 1 - \frac{\vartheta}{\beta} \right]_+ \rightarrow 0$$

where the function $[\cdot]_+$ is defined as

$$[z]_+ = \begin{cases} z & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

The other part of the proof is similar as [Martinez-Guerra et al., 2004].

**Remark 3.** The theorem actually states that the weighted estimation error $V(e) = e^TPe$ converges to the zone $\bar{p}(k)$ asymptotically, that is, it is ultimately bounded.

$$\bar{p}(k) \geq e^TPe \geq e_1^TPe_1$$
Remark 4. Because

\[
\bar{p}(k) = \left( \frac{2\|\Lambda_f\|L_{0f}^2}{k} + 4\left(\frac{\sqrt{n\Lambda_f^{-1}}}{k}\right) + \alpha_0 + \alpha_p \right)^2
\]

Since \(P\) is bounded, we can select \(\tau\) arbitrary small (the gain of the receiver (8) becomes bigger) in order to make \(k\) very big (since \(K = kP^{-1}C^T = m\tau^{-1}(\cdot_{\tau^{-1}})\)), so the first term of (18) goes to zero. \(\Lambda_f^{-1}\) in (15) is any positive matrix, we can choose it any small such that the second term of (18) goes to zero. So \(|\dot{s} - s|\) can be arbitrary small when \(\tau \to 0\).

Remark 5. Although we have restricted ourselves to the case of second-order chaotic system, the observer construction and convergence analysis can be extended to \(n\)-dimensional case. The chaotic transmitter is

\[
\begin{align*}
\dot{z}_j &= \eta_{j+1}, \quad j = 1, \ldots, n - 1 \\
\dot{z}_n &= H(\eta, s) \\
y &= \eta_1 + s
\end{align*}
\]

the sliding-mode observer-based receiver is constructed as

\[
\begin{align*}
\dot{\hat{z}}_j &= \hat{\eta}_{j+1} + m^j\tau^{-j}\text{sign}(y - \hat{y}) \\
\dot{\hat{z}}_n &= m^n\tau^{-n}\text{sign}(y - \hat{y})
\end{align*}
\]

(19)

where the constants \(k_j\) are chosen such that the polynomial \(\mu^n + k_{n-1}\mu^{n-1} + \cdots + k_1 = 0\) has all its roots in the open left-hand side of the complex plane. As the second-order case, it can be proved that the synchronization error \(\hat{\eta}\) converges to the normal form as (3), we cannot apply sliding-mode observer directly on the receiver, for example Chua’s circuit

\[
\begin{align*}
C_1\dot{\xi}_1 &= G(\xi_2 - \xi_1) - g(\xi_1) + u \\
C_2\dot{\xi}_2 &= G(\xi_1 - \xi_2) + \xi_3 \\
L\dot{\xi}_3 &= -\xi_2
\end{align*}
\]

where \(g(\xi_1) = m_0\eta_1 + 1/2(m_1 - m_0)[\xi_1 + B_p + |\xi_1 - B_p|]\), \(\xi_1, \xi_2, \xi_3\) denote the voltages across \(C_1, C_2\) and \(L\). It is known that with \(C_1 = 1/9, C_2 = 1, L = 1/7, G = 0.7, m_0 = -0.5, m_1 = -1.5, B_p = 1\) the circuit displays double scroll as in Fig. 4. If we make transformation \(\eta = T(\xi)\) as

\[
\begin{align*}
\eta_1 &= \xi_3 \\
\eta_2 &= -L\xi_2 \\
\eta_3 &= \frac{LG}{C_2}(\xi_2 - \xi_1) - \frac{L}{C_2}\xi_3
\end{align*}
\]

(21)

the Chua’s circuit becomes the normal form

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= \eta_3 \\
\dot{\eta}_3 &= f(\eta_1, \eta_2, \eta_3) + gu \\
y &= \eta_1
\end{align*}
\]

where \(f(\eta_1, \eta_2, \eta_3) = G/C_2\{ -\eta_3 - (G/C_1)[-2\eta_2 - (1/GL)\eta_1 - (C_2/G)\eta_3 - (1/C_1)g(-\eta_1 - (1/GL)\eta_1 - (C_2/G)\eta_3)] - (1/C_2L)\eta_1, \quad g = G/C_1C_2\). Now sliding-mode observer-based receiver (19) can be applied.
3. Numerical Simulation

We use three types of chaotic systems as transmitters, the information signal $s$ is embedded in the output of the transmitter.

(a) Duffing equation is

$$\dot{\eta}_1 = \eta_2 + \frac{1}{\tau}s$$
$$\dot{\eta}_2 = -1.1y - y^3 - 0.4\eta_2 + 2.1 \cos(1.8t) + \frac{1}{\tau^2}s$$
$$y = \eta_1 + s, \quad \eta(0) = [0, 0]^T$$

(b) Van der Pol oscillator is

$$\dot{\eta}_1 = \eta_2 + \frac{1}{\tau}s$$
$$\dot{\eta}_2 = 1.5[(1 - \eta_1^2)\eta_2 - \eta_1] + \frac{1}{\tau^2}s$$
$$y = \eta_1 + s, \quad \eta(0) = [2, -1]^T$$

(c) Chua’s circuit, we use the following parameters $C_1 = 1/9, C_2 = 1, L = 1/7, G = 0.7, m_0 = -0.5, m_1 = -1.5, B_p = 1$. By the transformation (21), the Chua’s circuit (20) can be written as

$$\dot{\eta}_1 = \eta_2$$
$$\dot{\eta}_2 = \eta_3$$
$$\dot{\eta}_3 = \frac{31}{4.9}\eta_3 - \frac{310}{7}\eta_1 - \frac{22}{4.9}\eta_3 - \frac{22}{7}\eta_2 - \frac{220}{7}\eta_1$$
$$- 0.7\eta_2 - 7\eta_1 + 22g(\eta_1, \eta_2, \eta_3)$$
$$y = \eta_2, \quad \eta(0) = [1, 0, -7]^T$$

where $g(\eta_1, \eta_2, \eta_3) = |-(1/4.9)\eta_3 -(1/7)\eta_2 - (10/7)\eta_1 + 1| - |-(1/4.9)\eta_3 -(1/7)\eta_2 - (10/7)\eta_1 - 1|$. We use $\eta_2$ and $\eta_3$ as the transmitter

$$\dot{\eta}_2 = \eta_3 + \frac{1}{\tau}s$$
$$\dot{\eta}_3 = \frac{31}{4.9}\eta_3 - \frac{310}{7}\eta_1 - \frac{22}{4.9}\eta_3 - \frac{22}{7}y - \frac{220}{7}\eta_1$$
$$- 0.7y - 7\eta_1 + 22g(\eta_1, y, \eta_3) + \frac{1}{\tau^2}s$$
$$y = \eta_2 + s$$

where $\eta_1$ satisfies $\dot{\eta}_1 = \eta_2$.

Now we design the sliding-mode receiver as (8). We choose $m = 0.1, \tau = 0.01$. The sliding-mode observer-based receiver is

$$\dot{\hat{\eta}}_1 = \hat{\eta}_2 + 10\text{sign}(y - \hat{y})$$
$$\dot{\hat{\eta}}_2 = 10^2\text{sign}(y - \hat{y})$$
$$\dot{\hat{y}} = \hat{\eta}_1, \quad \eta(0) = [1, 1]^T$$

The information signal $s$ is chosen as sinusoidal signal with frequency of 100 Hz as in [Boutayeb et al., 2002] and [Liao & Huang, 1999], i.e.

$$s = 0.05 \sin(200\pi t)$$

Figures 5–7 show the communication process with three different chaotic transmitters and one receiver, here the waveform of the transmitted signal $y$ is shown in subplot (a), the convergence behavior of $s - \hat{s}$ is shown in subplots (b).

After transient process ($t > 0.1$), the maximum relative error is defined as

$$e_{\text{max}} = \frac{\max(|s - \hat{s}|)}{\max(|s|)}$$
For Duffing oscillator, $e_{\text{max}} \cong 1.5\%$. For Van der Pol oscillator, $e_{\text{max}} \cong 2\%$. For Chua’s circuit, $e_{\text{max}} \cong 1.915\%$. Although the relative errors are different, they are acceptable for signal communication. It is interesting to see that one receiver (8) can recover the information signal from three different chaotic transmitters.

Model-based observer requires complete information of the transmitter. We use linear observer [Liao & Huang, 1999] to compare with our results, see Fig. 8. We find that model-based observer gives the best performance, but if the transmitter is unknown or partially known, this kind of receiver does not work. Another advantage of model-based receiver is that it can be applied to any chaotic transmitter as in (1), but sliding mode observer is only suitable for the chaotic system which has the normal form as in (3).

4. Conclusion

In this paper, we propose a novel chaotic communication approach, where the receiver is a sliding-mode observer. The main difference with normal chaos modulation in communication is that the receiver is no longer a chaotic system. The proposed scheme is robust with respect to uncertainties. Although the communication single error can be arbitrarily small by selecting a proper observer gain in the receiver, a large observer gain will also
enlarge the transmission noise. Sliding-mode based receiver cannot work as well as the normal receivers, but it may impose security risk on the current secure communication system using chaotic communication technique when the transmitter is in the form of (3) or it can be transformed into this form.

To the best of our knowledge, this kind of receiver has not yet been applied in real application. But we hope this paper will encourage the research effort in the real chaotic communication field.

References


Tao, Y. [1999] “Chaotic secure communication systems history and new results,” Telecommun. Rev. 9, 597–634.


