Approximate First Order Internalization and Degradation

Consider the scheme shown in Figure A2 below, where A represents a species at the plasma membrane, B represents an internalized species, and C represents a degraded species. Species A internalizes with the rate constant k_{int} and becomes species B, and species B recycles back into species A with rate constant k_{rec} or degrades with rate constant k_{deg}. The differential equations for species A and B are,

$$\frac{d[A]}{dt} = -k_{\text{int}}[A] + k_{rec}[B]; \text{¿} \frac{d[B]}{dt} = k_{\text{int}}[A] - k_{rec}[B] - k_{deg}[B] \quad ,$$

where brackets denote concentration. We would like to relate the change in C directly to the change in A with an effective rate constant, such that

$$\frac{d[C]}{dt} = k_{eff}[A] \quad .$$

By mass balance, we know that

$$\frac{d[C]}{dt} = -\left(\frac{d[A]}{dt} + \frac{d[B]}{dt}\right) \quad ,$$

and from above that

$$\frac{d[A]}{dt} + \frac{d[B]}{dt} = -k_{deg}[B] \quad ,$$

therefore

$$\frac{d[C]}{dt} = k_{eff}[A] = k_{deg}[B]$$

If the internalization and recycling steps are fast relative to degradation, then we can assume that the internalization/recycling reaction reaches equilibrium such that

$$k_{\text{int}}[A] = k_{rec}[B] \Rightarrow [B] = \frac{k_{\text{int}}[A]}{k_{rec}} \quad .$$

Substitution for [B] gives

$$\frac{d[C]}{dt} = k_{eff}[A] = k_{deg}\frac{k_{\text{int}}[A]}{k_{rec}} \quad ,$$

therefore,

$$k_{eff} = \frac{k_{deg} k_{\text{int}}}{k_{rec}} \quad .$$

Figure S2. General internalization and degradation scheme.