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# EFFICIENTLY MODELING AND CONTROLLING COMPLEX DYNAMIC SYSTEMS USING EVOLUTIONARY FUZZY COGNITIVE MAPS (INVITED PAPER)

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Abstract. Fuzzy Cognitive Maps (FCMs) have gradually emerged as a powerful modeling and simulation technique applicable to numerous research and application fields. This primarily springs from the inherent capabilities of the conventional Cognitive Maps (CMs), most important of which are the flexibility, the abstractive reasoning and the white-box inference engine; however, extensions to the underlying theory are more than anything needed because of the feeble mathematical structure of FCMs and mostly the desire to assign advanced characteristics not met in other computational methodologies. Under this standpoint, four core issues are discussed and respective solutions are proposed; the first one concerns the case of multi-stimulus situations (parallel stimulation of many FCM concepts), the second one focuses on the design of a learning algorithm (using evolution strategies), and finally the generic real-world phenomena of conditional effects and synergies are properly modeled to support the inference mechanism of FCMs. 

## 1. Introduction

Fuzzy Cognitive Maps (FCMs) constitute a novel, yet attractive approach that encompasses advantageous modeling features. The most pronounced of such features are the flexibility concerning system design and control, the comprehensible (white-box) structure and operation, the adaptability to problem-specific prerequisites and the capability for abstractive representation and fuzzy reasoning. Many theoretical studies [1, 18, 19, 20, 23, 24, 28,

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30, 33, 39, 42, 46, 48, 49] and diverse applications in a variety of engineering domains (analysis of electrical circuits, fault diagnosis, manufacturing organization, systems control, context dependent processing systems) and decision making (stock investment, strategic planning, modeling of organizational behavior, educational, social and psychological systems, games, virtual worlds) [6, 7, 9, 10, 11, 15, 17, 21, 22, 26, 31, 32, 34, 37, 38, 40, 41, 43, 44, 47, 50] support the aforementioned statement. However, the FCM background remains an open field of analytical research mostly because of the existence of weaknesses, such as the abstract estimation of initial concept values, the lack of an efficient mechanism for the development and fine-tuning of the maps, and the questionable reasoning in case of parallel stimulations (multi-stimulus situations). The current problematic operation mode of FCMs is plainly described by Miao and Liu in [30]: "*... the current techniques for constructing and analyzing FCMs are inadequate and infeasible in practice. Furthermore, as the FCM is a typical, nonlinear system, the combination of several inputs or initial states may result in new patterns with unexpected behaviors. Systematic and theoretical approaches are required for the analysis and design of FCMs...*". To summarize, taking into account the whole research in the domain of fuzzy cognitive maps, it becomes obvious that there still exists ample room for improvements and new approaches that will lead to the formulation of a concrete functional framework.

Two of the aforementioned weaknesses of FCMs, that is, the operation under multi-stimulus environments and the deficiency of an optimization and/or automated synthesis process, are discussed in the current context. As regards the management of multi-stimulus situations, an innovative algorithm that transforms the conventional inference engine of FCMs is described herein, while for the development of a learning process, a generic "structure evolution" scheme, based on the principle of Evolutionary Computation and specifically on Evolution Strategies (ES), is explicated. In addition, the FCM inference procedure is enhanced through the modeling of two quite interesting real-world phenomena: the synergies and the conditional effects. Appropriate methodological modifications and new advanced causal relational attributes are presented and discussed in detail.

The remainder of this article is organized as follows: In Section 2, the theoretical background of the simple and fuzzy cognitive maps is described and, in Section 3, the algorithm resolving the cases of parallel stimulations is presented. In Section 4, the idea of a learning mechanism applied for the map design and fine-tuning is analyzed, while in Sections 5 and 6, the algorithmic extension of synergies and conditional effects are developed and explicated. Finally, in Section 7, the core conclusions drawn from the theoretical analysis are highlighted and future research directions are outlined.

## 2. Theoretical Background of CMs & FCMs

Research on Cognitive Maps (CMs) began during the $19^{th}$century and the first approaches sprung from findings in physic-psychological experiments tried to trace and interpret the functionalities of various mental and cognitive tasks, abilities and phenomena in animals and humans. Tolman in 1948 [45] and later Axelrod in 1976 [1] described CMs in a more formal and systemic manner until Kosko in 1986 [24] set the basis of their contemporary fuzzy form. Their first use and scope - which was cognitive representation- expanded to the analysis of systems described by interrelated concepts, through a diversification of applications.

A CM represents a whole set of cause-effect relationships (measured by causality) that are perceived to exist among specific factors (concepts) and constitutes a systematic process for the assessment of the impact produced by changes in the state of some (or all) elements on the state of the entire system. At first, a question on the meaning arises: CMs estimate the effect of "the change of the state" or just "the state" of the factors over the whole system? Although the option "the change of the state" is the most frequently used, its meaning is somehow restricting, as there are systems, like economy, where the changes are not abrupt, are gradually increasing or decreasing, last for a long period, and their effect is lagged. The distinction of the two approaches is demonstrated in the following example. Suppose that a normal (or acceptable) level of inflation is 1.5% and that its current level is 5%. Moreover, assume that the condition of economy is analyzed in an annual basis. Considering that next year inflation will fall to 4% and after 5 years it will be gradually diminished to its normal level, the difference of the two approaches is clear:

- If one perceives the change of inflation as an abrupt phenomenon, then he can't follow its progress. Next year, one must assert that inflation fell 1% comparatively to this year. Therefore, if one accepts the value -1%, and therefore neglecting that inflation will be still at the abnormal level 4%, then it's wise to interpret the ensuing results according to current year's condition and not relatively to the normal level 1.5%.
- Contrary to that, if one perceives inflation and the other concepts related to it as parameters that are deviate from their usual (or favorable) levels then this approach will be more close to the common sense (inflation is approximately "high", "low" etc.). In this case,

the interpretation of the results must be conducted considering the normal levels of the concepts included in the given system.

To conclude, none of the two approaches is considered wrong or inappropriate but the applied settings and the produced results must be interpreted with the analogous manner. The distinction of the two perceptions is underlined as a confusion that can easily emerge.

Considering the CM structure, it is a network where the nodes represent the concepts and the links represent the cause-effect relationships between the concepts. An example of a CM representing an industrial control system in a plain manner is depicted in Fig. 1. In CMs, links between nodes may obtain only two values, $+1$ (or simply "+") and $-1$ (or simply "$-$"). The nodes may take the values $-1$, 0 and $+1$. A concept is a fuzzy union of some fuzzy quantity set Qi and the associated dis-quantity set ∼Qi [24]. In this way the positive node values means membership to the quantity set Qi while the absolute negative values means membership to the dis-quantity set ∼Qi. Usually in real-world systems, positive node values means an increase or positiveness of the concept state that the node represents while the value $-1$ means a decrease or negativeness of the corresponding concept state. Based on Fig. 1, the value $+1$ in the link between the nodes "Overflow" and "Controller C" means that an increase (decrease) of the estimation about the "Overflow" of the liquid in the tank prompts the open/increase (close/decrease) of the "Controller C". On the other hand, the value $-1$ in the link between the "Shortage" and the "Controller C" means that an increase (decrease) of the estimation about the "Shortage" prompts the close/decrease (open/increase) of the "Controller C". It is important to point out that cause-effect relationships are only these represented with the links on the CM, with a specific at all times direction. A problematic subject in CM analysis and design concerns the use of self-feedback links on the nodes. Although there is not a clear-cut practice and theoretical proof about this subject, most of researchers support neglecting self-feedback in order to achieve a stable reasoning mechanism. In other words, most of CM and FCM-based systems are built assuming that only the direct or indirect effects of the other nodes alter the initial state of the nodes.

Fuzzy Cognitive Maps (FCMs) constitute an extension of CMs, mainly, to avoid the binary logic that CMs enclose. FCMs emanate from the combination of CM and fuzzy logic principles and provide a more realistic and accurate representation of the real-world systems than CMs do. Nodes and links in a FCM have values in the interval $[-1, 1]$, while their positive and negative values have the same meaning as previously. For example, the CM depicted in Fig. 1 turns to the FCM depicted in Fig. 2. Concerning the

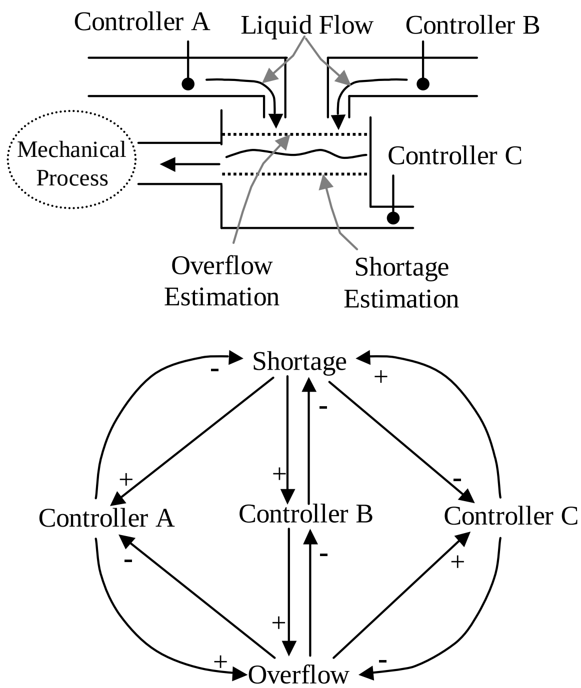


FIGURE 1. An Industrial Control Case and the Corresponding CM.

nodes, if a node value is positive and very high (e.g.+0.9) then the ensuing conclusion implies that the state of the corresponding concept increased very much or it's at a highly abnormal positive level, while if this positive magnitude is relatively low (e.g.+0.2) then the increase of the concept state is relatively small. Similarly, a high (absolute) negative value (e.g.$-0.9$) means a significant decrease of the concept state or a decline in extremely negative abnormal levels, while a relatively low (absolute) negative node value (e.g.$-0.2$) means that the decrease of the corresponding concept state is relatively small. It must be emphasized that the node values may be restricted to fall in an interval smaller than $[-1, +1]$, that is, developers may apply upper and lower thresholds in node values (e.g. $[-0.1, +0.8]$) so as to adapt FCM to problem-specific conditions.

A closer look at the node values estimation reveals the conception that they do not express absolute or relative changes of the actual magnitude of the corresponding concept but declare degrees of "significance of change" in the context of the whole system, with its own prerequisites and limitations.

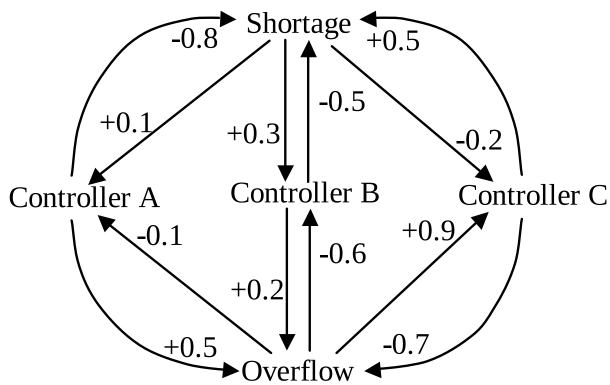


FIGURE 2. FCM for Industrial Control.

This fact becomes even more obvious when referring to concepts expressed in qualitative terms. Each concept in FCMs can be considered a linguistic variable with name "change of state" or "deviation from normal level" while the values that are eventually assigned to each node are about a variable with name "significance of change". Both of the two variables may be expressed by a list of fuzzy sets (membership functions-MFs) that are usually defined by experts. These two variables have to be connected through a "positive" manner. This assumption means that there is an underlying function connecting the two variables that is continuous with non-negative first derivatives (an increase of real world concept values must correspond to an increase or at least a stability of the significance of change). This prerequisite can be fulfilled through the appropriate construction of the rule base connecting the two variables and the special parameters of the selected MFs. The use of rule bases and MFs instead of simple (abstract) node values given by experts undoubtedly establishes a more credible, realistic and user-friendly way to assess node values. In case of having a notion about some pairs of node value changes and their respective significance in a given system, then a whole optimization problem may be defined in order to extract the exact or even an approximation of the rule base and the MFs that cover the developers' needs. Such a complex problem may be confronted with the use of rule-based neural networks or/and evolutionary algorithms.

As far as the FCM cause-effect relationships or the network links are concerned, high (absolute) values of causality signify strong cause-effect relationships between the concepts. For instance, in Fig. 1, relationship from "Controller A" to "Overflow" may be less strong than that from "Controller C" to "Overflow". In case of a mediocre positive change of "Controller A", the estimation "Overflow" will change positively only a little, while in case of a mediocre positive change of "Controller C", the estimation "Overflow" will change negatively for a high degree. Therefore, it is obvious the necessity for the use of different (absolute) values for the links among the concepts. This modification is a prerequisite for an accurate approach of any given system.

The term "cause-effect relationship" includes an extremely important feature that does not become easily realized; it encloses a list of fuzzy rules and does not express just a premise. In case of a relationship $A(+1) \rightarrow B$, the following rules exist:

- If $A$ changes a little positively/negatively then the impact on $B$ is a little significant positively/negatively.
- If $A$ changes moderately then the impact on $B$ is moderately significant.

So, it becomes obvious that FCM mechanism is able to manage all these rules in parallel, contrary to the typical practice of a fuzzy rule-based system that would impose the construction of a huge base with an enormous number of rules, inputs and outputs.

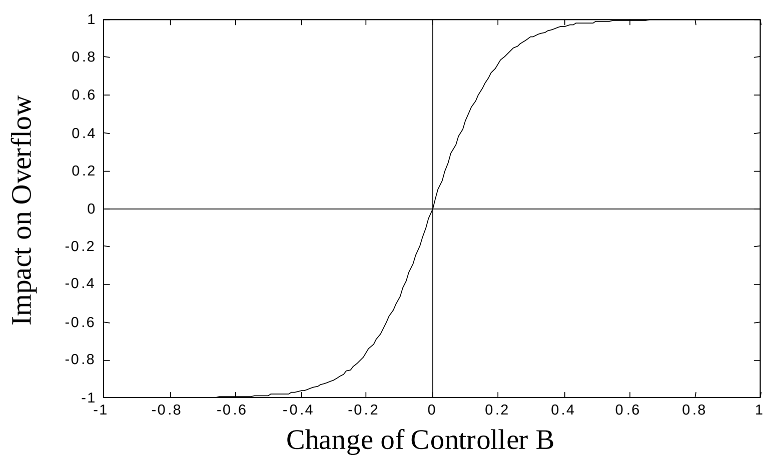


FIGURE 3. Controller $B$ →Overflow.

Assigning just one value for causality implies that cause-effect relationships demonstrate a linear behavior. However, this may not be the case. For example, "Controller B" (Fig. 2) may have an impact on "Overflow" (just an hypothesis) through a manner expressed by a hyperbolic tangent sigmoid function (Fig. 3) because it is considered an extremely sensitive factor whose even insignificant changes cause a harsh impact on the liquid level. Potential and powerful functional forms are the linear, the stepwise, the sigmoid, the gaussian and even the exponential family of curves (Fig. 4). Such functions are easily incorporated to the FCM inference mechanism, so their use is considered favorable than tiresome.

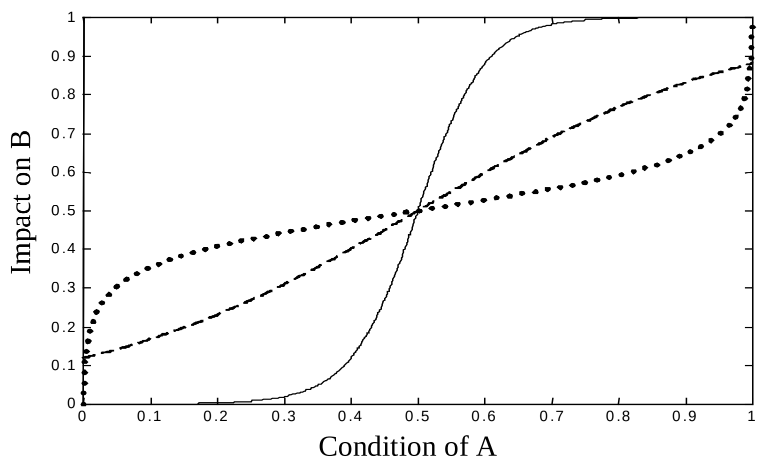


FIGURE 4. Types of Positive Relationships ($A \rightarrow$B).

In general, instead of assigning a crisp value for causality at each cause-effect relationship, the adoption of fuzzy ones is proposed herein. According to the requirements and characteristics of each application, a deep investigation is needed in order to accurately evaluate the nature of the cause-effect relationships, which may be expressed through hybrid functional forms, fuzzy rule bases or more generally case-sensitive fuzzy relations.

A question that arises is about how to determine a function or a crisp value for the causality of the cause-effect relationships keeping a high degree of credibility. Current practice implies experts' intervention, which unavoidably encompasses subjective reasoning. A solution is described in the Section 4: the concept of FCM learning/training and, most important, the scheme of structure evolution as an integrated process for constructing FCMs from scratch. The philosophy of the described methodology rests on the training procedure applied to typical learning systems (use of examples – facts - input/output pairs), combined with the powerful optimization capabilities of evolution strategies (ES). Flexibility, adaptability and robustness of ES, accompanied of accurate input/output examples (which demands good knowledge of the given system), guarantee the successful implementation of the proposed procedure. An additional learning algorithm has been proposed which in fact it is a modified version of Hebbian learning algorithm, which is widely used in Neural Nets [10].

Inference mechanism (in its simplest form) involves mathematical matrix manipulations that are based on the *concept vector* and the *adjacency matrix*. The concept vector includes as many elements (dimension) as the number of concepts in the given system and its values represent the concepts states. The adjacency matrix is used for the mathematical representation of the cause-effect relationships among concepts. Fig. 5 depicts the concept vector and the adjacency matrix of a hypothetical FCM.

| A (1) | B (2) | C (3) | D (4) | E (5) | F (6) |
|---|---|---|---|---|---|
| +1 | +0.6 | 0 | 0 | -0.7 | 0 |

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| (1) | 0 | 0.5 | 0 | 0 | 0 | 0.5 |
| (2) | 0 | 0 | 0.4 | − 0.5 | 0 | − 0.7 |
| (3) | − 0.6 | 0 | 0 | − 0.2 | 0 | 0 |
| (4) | 0 | 0 | 0 | 0 | 0.1 | 0.3 |
| (5) | 0 | 0 | 0 | 0 | 0 | 0.4 |
| (6) | 0 | 0 | 0 | 0 | 0 | 0 |

FIGURE 5. Concept vector and adjacency matrix.

Assume a change (stimulation) of concept "C" with a value +0.8. At first, the initial values of concept vector elements are determined. So, the concept vector becomes: [00+0.8000]. Next, the concept vector is multiplied with the adjacency matrix (iteration 1):

$$(1)\begin{bmatrix} 0 \\ 0 \\ 0.8 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\top} \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.4 & -0.5 & 0 & -0.7 \\ -0.6 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -0.48 \\ 0 \\ 0 \\ -0.16 \\ 0 \\ 0 \end{bmatrix}^{\top}$$

On the ensuing concept vector, two procedures are applied: (a) the element that corresponds to the concept that was initially stimulated takes its initial value, so the concept vector becomes: $[-0.480+0.8-0.1600]$, and (b) if the value of any element lies out of the interval $[-1, 1]$ then it is truncated. The adjusted concept vector is multiplied again with the adjacency matrix (iteration 2):

$$\begin{bmatrix} -0.48 \\ 0 \\ 0.8 \\ -0.16 \\ 0 \\ 0 \end{bmatrix}^{\top} \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.4 & -0.5 & 0 & -0.7 \\ -0.6 & 0 & 0 & -0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -0.48 \\ -0.24 \\ 0 \\ -0.16 \\ -0.016 \\ -0.288 \end{bmatrix}^{\top}$$

(2)

At the produced concept vector, the aforementioned two procedures are applied again. This algorithm follows the previous steps until the values of the concept vector remain constant after each iteration. The final concept vector expresses the stable condition of the whole given system.

Admittedly, t-norm and t-conorm operators can be easily incorporated in the inference mechanism. However, this alternative may be incorporated provided that the cumulative behavior of cause-effects is sustained. For instance, the most well known t-norm and t-conorm operators, the minimum and the maximum respectively, have to be avoided as they cause distortions when handling and finally interpreting multiple effects. The rationale underlying cognitive maps supports the conception of cumulative cause-effects and that's why the use of the naive summation and multiplication form, which fulfill the aforementioned requirement, has prevailed through the diverse applications of FCMs. Undoubtedly, even these forms entail significant pitfalls, especially in cases where the discrete sums take large values like 5, 10 or 20 and therefore reasoning becomes doubtful. Therefore, the selection of operators for the inference mechanism is an open issue that has to be addressed based on the logic of the additive/cumulative behavior of cause-effects.

## 3. Handling Multi-Stimulus Environments

The majority of the theoretical and case studies regarding FCMs treat them as "what-if" analysis tools but with a common feature; only one node is considered to be initially stimulated at each testing phase. Admittedly, this practice is somehow inappropriate for a complete description and documentation of the possible conditions that may be produced by the simulated environments. At the same time, the accuracy related to the approximation of the systems behavior seems to be prohibited, as many real world situations may be put aside. The conventional inference mechanism of FCMs cannot handle effectively multi-stimulus environments. This conclusion is drawn considering the fact that at each step of the inference algorithm the initially set node values are considered constant, that is, they are not updated through the activation of adjacent cause-effect relationships. The separation of the initial stimulations and thus, the production and handling of the distinct results is a promising approach. In order to examine the proposed scheme in more detail, consider a system with n concepts where k of them are initially activated in parallel/synchronously. The conventional inference engine rests on the following generic equation:

$$\underset{(k\times n)}{I} \otimes \underset{(n\times n)}{AM} = \underset{(k\times n)}{O}, k \leq n. \tag{3}$$

where $I^{\top} = \begin{bmatrix} S_1^{\top} & S_2^{\top} & . & . & S_k^{\top} \end{bmatrix}$, $O^T = \begin{bmatrix} C_1^{\top} & C_2^{\top} & . & . & C_k^{\top} \end{bmatrix}$, $AM$ is the Adjacency Matrix, $S_j = [s_1^j \ ... \ s_n^j]$ is the initial concept vector when only node $j(1 \leq j \leq k)$ is stimulated and $C_j = [c_1^j \ ... \ c_n^j]$ is the corresponding final concept vector. The rows of matrix I have only one non-zero value (only one node is stimulated at each time), while the columns have at most one non-zero value (some nodes may not be initially stimulated). Using the values of matrix O, the following matrices can be defined (the ":" notion is used as conditional operation):

$$\underset{(1\times n)}{S\overset{+}{U}M} \text{ where } SUM^{+}_{(i)} = \sum_{j=1}^{k} \{O_{(j,i)} : O_{(j,i)} > 0\}, 1 \leq i \leq n. \tag{4}$$

$$\underset{(1\times n)}{S\overset{-}{U}M} \text{ where } SUM^{-}_{(i)} = \sum_{j=1}^{k} \{|O_{(j,i)}| : O_{(j,i)} < 0\}, 1 \leq i \leq n. \tag{5}$$

$$\underset{(1\times n)}{M\overset{+}{A}X} \text{ where } MAX^{+}_{(i)} = \max_{1\leq j\leq k} \{O_{(j,i)} : O_{(j,i)} > 0\}, 1 \leq i \leq n. \tag{6}$$

$$\underset{(1\times n)}{M\overset{-}{A}X} \text{ where } MAX^{-}_{(i)} = \max_{1\leq j\leq k} \{|O_{(j,i)}| : O_{(j,i)} < 0\}, 1 \leq i \leq n. \tag{7}$$

These matrices support the proposed algorithm, which is the following:
∀j (1≤j≤k),
  Stimulate $S_j$, Store $C_j$
∀i (1≤i≤n),
  Assess $SUM^{+}_{(i)}$, $SUM^{-}_{(i)}$,$MAX^{+}_{(i)}$ and $MAX^{-}_{(i)}$

$$TOTAL^{+}_{(i)} = (1-\beta) \quad MAX^{+}_{(i)} + \beta SUM^{+}_{(i)}. \tag{8}$$

$$TOTAL^{-}_{(i)} = (1-\beta) \quad MAX^{-}_{(i)} + \beta SUM^{-}_{(i)}. \tag{9}$$

$$FINAL_{(i)} = TOTAL^{+}_{(i)} + TOTAL^{-}_{(i)}. \tag{10}$$

The concept vector FINAL is formed taking into consideration all discrete positive and negative effects and gives the new equilibrium of the whole system. The parameter $\beta$ ($0 \leq \beta \leq 1$), *the cumulative behavior parameter*, is user-defined and adjusts the degree of influence of both the positive and negative maximum effects in relation to the cumulative effects. Values of parameter $\beta$ close to 1 reduce the level of significance of the maximum effects (the cumulative behavior becomes stronger) while small values reinforce the influence of the strongest effects ("winner takes all" behavior). In fact, the proposed algorithm constitutes a convex combination of the vectors MAX and SUM, sustaining simultaneously its useful properties [25, 27]. It is interesting to notice that the proposed scheme presents the following advantageous features:

**-:** It exploits the maximum positive and negative effects;
**-:** It sustains the additive/cumulative behavior of cause-effects;
**-:** It provides the developers with a parameter $\beta$ to adjust the influence of the maximum effects;
**-:** It discriminates positive and negative effects without causing confusion and producing misleading results.

## 4. FCM Learning based on Evolution Strategies

To create a fuzzy cognitive map, without the intervention of experts [37,42], the possibility to apply the principles of evolutionary computation arises [18]. The main idea behind the proposed approach concerns the design and development of algorithms that support *structure evolution*; this term involves the systematic process of cognitive map design (determination of what cause-effect relationships take place) and fine-tuning of the causality attached to the perceived relationships of given systems.

Evolutionary computation [2, 3, 4, 5, 8, 12, 13, 14] is the research area investigating the development of optimization algorithms that imitate the principles of natural evolution. The idea to use the principles of organic evolution was developed independently by Holland [16], who introduced the theory of Genetic Algorithms (GAs), and by Rechenberg and Schwefel who described the Evolution Strategies [29, 35, 36]. Considering an optimization problem, evolutionary algorithms assume the existence of a *population of individuals* - $P(t)$ at generation t - each of which represents a search point in the space of potential solutions. Individuals are vectors of length proportionate to the number of *object variables* constituting the optimization problem. The representation of the individuals is strictly related to the *phenotype* and the *genotype* spaces. Individuals take their initial values in a mostly random manner and afterwards evolve successively to better regions of the search space (according to their *fitness* value, which is estimated applying a problem-dependent function). The most widespread evolution processes are *recombination*, *mutation* and *selection*. During recombination, pairs of (or more than two) individuals combine their genetic information while, during mutation, a portion of the individuals undergoes random changes in some of their characteristics. Mutation is extremely important for an effective evolution operation as it is a means for obtaining new information not included in the ancestors and therefore new subspaces of the search space can be explored. Finally, selection aims to improve the average quality of the population in order to reinforce the search in promising areas in the space of potential solutions. In other words, at each generation some of the existing individuals are selected in order to form the population of the next generation. The individuals with the highest fitness values are more preferable and consequently obtain higher probabilities to survive in the next generation. A typical EA structure is as follows:

Evolutionary Algorithm

$t$=0
Initialize: P($t$)
Evaluate: P($t$)
*While* (STOP_CRITERION not satisfied)
  Recombine: $P'(t)$ = r $(P(t))$
  Mutate: $P''(t)$ = m $(P'(t))$
  Evaluate: $P''(t)$
  Select: $P(t+1)$ = s $(P''(t))$
  $t = t+1$
*End While*

An optimization (minimization) problem requires finding the parameters $\mathbf{x} \in M \subseteq \mathbb{R}^n$ that minimize the value of the objective function $f(\mathbf{x})$, where $f$: $M \to \mathbb{R}$. If the global minimum of $f$ is achieved with the vector $\mathbf{x}^*$ then $\forall \mathbf{x} \in M \Rightarrow f(\mathbf{x}^*) \leq f(\mathbf{x})$. An evolution strategy requires the existence of a population with $\mu$ individuals that are real-valued (float numbers) vectors with n elements. The vector elements ($x_i \in \mathbb{R}$, $1 \leq i \leq n$) are the object variables. The representation of the variables as float numbers (which are usually their real-world values) and not as complicated coded strings constitutes a great advantage of evolution strategies. According to primary approaches, mutation in evolution strategies is performed independently on each vector element by adding a normally distributed random value with mean value 0 and standard deviation $\sigma : x_i' = x_i + \sigma N_i(0,1)$, where $N_i(0,1)$ is a normally distributed random number with mean value equal to 0 and standard deviation equal to 1. Although this general form of mutation is sufficient to ensure the existence of variation in the population, further modifications have been proposed. The most extended form [2, 5, 36] requires that each vector element corresponds to a strategy parameter $\sigma_i(\sigma_i \in \mathbb{R}_+^n$, $1\leq i \leq n)$ that also undergoes variations and determines the vector element as follows:

$$\sigma_i^{'} = \sigma_i \cdot e^{\left(\tau^{'} N(0,1) + \tau N_i(0,1)\right)}, x_i^{'} = x_i + \sigma_i^{'} \cdot N_i(0,1). \tag{11}$$

Parameter $\tau$ is used in order to impose different variation in each vector element separately, while parameter $\tau\prime$ is used in order to differentiate the whole population from generation to generation.

For recombination, multiple techniques exist with most prominent types the discrete (random choice for each offspring element between those of the two parents) and the intermediate (each offspring element is the arithmetic average of the corresponding parental elements). Last, for the antagonistic selection, two basic strategies that have been reported; $(\mu,\lambda)$ and $(\mu+\lambda)$. Both strategies indicate that $\mu$ parents create $\lambda\geq\mu$ offspring but the difference between them appears when the individuals for the next generation must be determined. The $(\mu,\lambda)$ strategy imposes the elimination of the parental population and its complete replacement with the best $\mu$ offspring. On the other hand, with the $(\mu+\lambda)$ strategy, the best $\mu$ individuals of the whole pool of parents and offspring are selected.

Concentrating on the FCM learning process (training), it calls for representative data sets that approximate the structure and operation of the given systems. These data sets will guide the applied evolution strategy to determine the appropriate FCM link values. The training phase of FCMs is to some extent similar to that of neural networks, that is, it is based on input/output pairs that are called examples. The inputs are considered initial stimulators of the systems and the outputs indicate the final states of the corresponding concepts after the applied stimulations. The distinction of FCM concepts as inputs or outputs depends on the focus of the map developer/ designer or user. In fact, all the concepts of a given system may constitute the inputs and outputs at the same time. For the purpose of FCM training, each individual in the simulated ES is a vector with $n$ elements, where $n$ equals to the number of the causeeffect relationships that is to be estimated, and its fitness is computed taking into account the degree to which the outputs that are produced by the given initial stimulations are close to the outputs indicated by the predetermined input/output pairs:

$$\text{fitness} = \sum_{k=1}^{\text{examples}} \sum_{i=1}^{\text{outputs}} \left| \text{output}_i^{(\text{k})\text{estimated}} - \text{output}_i^{(\text{k})\text{real}} \right| \quad (12)$$

## 5. Managing Synergies

Synergies refer to situations where the parallel occurrence of multiple causes leads to total effects that are larger than the simple aggregation of the expected individual effects. These phenomena, which have been widely examined in management science, attract the research attention when approximating cognitive processes or modeling highly abstractive and qualitatively expressed systems. As the core functional purpose of FCMs remains the abstractive qualitative reasoning, the introduction of a formalism managing the synergistic effects is considered quite favorable.

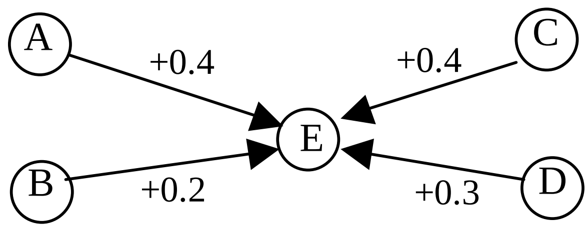


Figure 6. Synergy behavior among links A-E, B-E and D-E.

Synergistic effects imply some kind of correlation (or association) between them in such a way that their interrelated change or simply their parallel existence finally leads to a multiplicative behavior (in this situation we refer to a positive synergy) or, on the contrary, to a diminishing/competitive one (negative synergy). Looking at the FCM-based system in Fig. 6, we assume that there is a positive synergy among the cause-effect relations A-E, B-E, and D-E. For the specific problem, let's say that the *strength of the synergy* is 0.5, which means that the simultaneous stimulation of concepts A, B and D will produce an effect that will be 1.5 times $(= 1 + 0.5)$ their aggregate effect, then the expected outcome would be

$$C_E = 0.5 * 0.4 + (1 + 0.5) * (0.5 * 0.4 + 0.5 * 0.2 + 0.5 * 0.3) = 0.875.$$

On the other hand, if one assumes a negative synergy among the same relations with strength 0.4, then the expected result should be $C_E$=0.5*0.4+(1-0.4)*(0.5*0.4+0.5*0.2+0.5*0.3)=0.47.

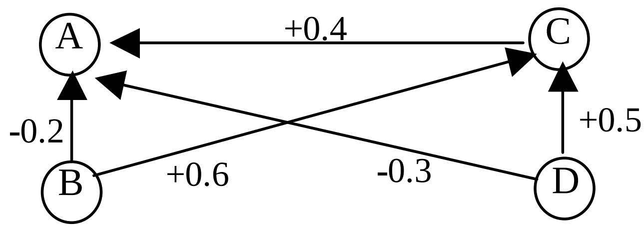


FIGURE 7. Test FCM.

Let's consider the FCM depicted in Fig. 7 and its corresponding adjacency matrix (AM) given in Fig. 8. To describe what synergies take place, the Synergy Matrix (SM), such the one depicted in Fig. 9, is created. From this it is implied that there is a synergy among the cause-effect relationships B-A, C-A and D-A, and also between the relationships B-C and D-C. Obviously, the values $+1$ in a column of the SM declare the participation of the corresponding cause-effect relationships to the existing synergy.

Afterwards, the strengths of the defined synergies have to be declared. This is provided by the *Synergy Strength Vector* (SSV, Fig. 10). It can be easily extracted from the SSV that the synergy of relationships B-A, C-A and D-A is positive at a degree 0.2 while that of relationships B-C and D-C is negative at a degree 0.8 (in absolute terms). In fact, the values of the SSV may not be restricted to fall in a specific interval and therefore can take any real values that developers consider fit to their own problem.

$$AM = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2 & 0 & +0.6 & 0 \\ +0.4 & 0 & 0 & 0 \\ -0.3 & 0 & +0.5 & 0 \end{bmatrix}$$

FIGURE 8. Adjacency Matrix.

In case of synergies that involve more than 2 cause-effect relationships, the non-occurrence of some of the constituent effects has an instant implication on the synergy strength. That is, the synergy strength must be properly adapted. Of course, when only one relation is stimulated then the synergy does not take place at all. Keeping this remark in mind, the *Adjusted Synergy Strength Vector* (ASSV) and the *Adjusted Synergy Matrix* (ASM) are defined as ($\forall 1 \leq i, j \leq n$):

$$\text{Synergy Matrix (SM)} = \begin{array}{c} \\ (A) \\ (B) \\ (C) \\ (D) \end{array} \begin{array}{c} \begin{array}{cccc} (A) & (B) & (C) & (D) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

FIGURE 9. Synergy Matrix.

Synergy Strength Vector (SSV)=[0.2 0 -0.8 0]

FIGURE 10. Synergy strength vector.

$$ASSVj = \frac{\max\left\{0, \left|\sum_{i=1}^{n} \{\text{sign}(c_i) * \text{sign}(SM_{ij}) * \text{sign}(AM_{ij})\}\right| - 1\right\}}{\max\left\{1, \sum_{i=1}^{n} \{SM_{ij}\} - 1\right\}} \times SSV_j. \tag{13}$$

$$ASM_{ij} = SM_{ij} * ASSV_j. \tag{14}$$

$$Adj.S.S.V.(ASSV) = [ASSV_1 \cdots ASSV_4]$$

FIGURE 11. Adj. synergy strength vector.

$$Adj.S.M.(ASM) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ ASSV_1 & 0 & ASSV_3 & 0 \\ ASSV_1 & 0 & 0 & 0 \\ ASSV_1 & 0 & ASSV_3 & 0 \end{bmatrix}$$

FIGURE 12. Adjusted synergy matrix.

The values of the ASSV and the ASM are portions of, or at most equal to, the initially defined synergy strengths and, therefore, they denote the actual degree to which the synergy effects will take place (the higher the absolute values of the ASSV and the ASM, the stronger the synergies). The last step

$$AAM = \begin{bmatrix} 0 * (ASM_{11} + 1) & 0 * (ASM_{12} + 1) & 0 * (ASM_{13} + 1) & 0 * (ASM_{14} + 1) \\ -0.2 * (ASM_{21} + 1) & 0 * (ASM_{22} + 1) & 0.6 * (ASM_{23} + 1) & 0 * (ASM_{24} + 1) \\ -0.4 * (ASM_{31} + 1) & 0 * (ASM_{32} + 1) & 0 * (ASM_{33} + 1) & 0 * (ASM_{34} + 1) \\ -0.3 * (ASM_{41} + 1) & 0 * (ASM_{42} + 1) & 0.5 * (ASM_{43} + 1) & 0 * (ASM_{44} + 1) \end{bmatrix}$$

FIGURE 13. Adjusted adjacency matrix.

to our analysis is the modification of the FCM adjacency matrix. For this purpose, the *Adjusted Adjacency Matrix* (AAM) is defined (Fig. 13):

$$AAM_{ij} = AM_{ij} * (1 + ASM_{ij}) \tag{15}$$

or

$$AAM_{ij} = AM_{ij} * (1 + ASSV_j)^{SM_{ij}} \tag{16}$$

The assumption of synergistic effects and the incorporation in the FCM inference engine of a mechanism managing them, evidently cause an augmentation of the complexity underlying CM-based reasoning when viewing FCMs both as cognitive structures and simulation methodologies. The increased complexity lies also in the synthesis of FCM-based models as it is necessary the recognition and afterwards the estimation of the strength underlying each potential synergy. On the other hand, the proposed algorithm improves the modeling capabilities and intensifies the accuracy demanded in the design and operation mode of FCMs.

It must be clarified that the attributes of the proposed approach constitute just an extra part of the conventional FCM algorithm, which means that the non-existence of synergies doesn't excite irregular situations but rather produces identical results with the conventional modeling and inference procedures. The use of the modified inference mechanism is optional and applies only on the relationships associated with some kind of synergy. Consequently, the principle of superposition is valid.

## 6. Conditional Effects

The majority of physical and technical systems involve some kind of conditions, rules or prerequisites in their causal relational structures. It's quite common to assume that the action of some concepts over the elements of a given system is associated with the existence or not of other concepts or, on the contrary, themselves restrict the cause-effect behavior of other constituent elements. Despite the importance of the conditional effects, an analytic algorithmic-procedural frame based on formal mathematical notions is non-existent. This paper sheds some light on this aspect of FCM computational capabilities trying to enclose as much real world situations as possible keeping simultaneously complexity at low levels and provide the user/operator with new, yet really useful and valuable modeling tools.

In general, between two FCM concepts $i$ and $j$, the constraint types of Fig. 14 may be listed (C is the concept vector). Of course, such constraints can be extended to capture conditions among more than 2 concepts (If $c_i > 0$ calls for $c_j > 0$ AND $c_k < 0$ AND ...). Reconsidering the system of Fig. 6, to accept the existence of specific effects over node E, several conditions may arise. For instance, one may assume that in order the relationship B-E to be valid, the following constraints take place:

- If $c_B > 0$ demands $c_A > 0$ AND $c_C < 0$;
- If $c_B < 0$ demands $c_A > 0$ AND $c_C < 0$;
- If $c_B = 0$ demands $c_A > 0$ AND $c_C < 0$.

Additionally, in order the relationship C-E to be valid, the following conditions must be fulfilled:

- If $c_C > 0$ demands $c_D > 0$;
- If $c_C < 0$ OR $c_C = 0$ demands $c_D = 0$.

Under this viewpoint, for each cause-effect relationship in a cognitive map, a *Conditional Effect Matrix* (CEM) may be formulated. For the example of Fig. 6, the adjacency matrix (AM) and the corresponding conditional effect matrices (CEMs) for the relationships B-E and C-E are given in Fig. 15 (the symbol "x" expresses the "don't care symbol", which means that no condition/constraint is assigned on the corresponding node).

If $c_i > 0$ calls for $c_j >, <$ or $= 0$
If $c_i < 0$ calls for $c_j >, <$ or $= 0$
If $c_i = 0$ calls for $c_j >, <$ or $= 0$

FIGURE 14. A simple form of constraint types.

To explain the meaning of a CEM, a hypothetical cause-effect relationship from a node $k$ to a node $m$ could be considered. The $CEM_k^m$, that is the conditional effect matrix corresponding to the relationship $k \to m$, captures the following information ($\forall 1 \leq i \leq n$):

- For the $1^{st}$ column of$CEM_k^m$(all conditions regarding the case $c_k > 0$)
  - **-:** If $CEM_k^m(i,1) = \{+1, -1 \text{ or } 0\}$, it means that: If $c_k > 0$ then $c_i$ must be respectively $\{>0, <0 \text{ or } =0\}$
- For the $2^{nd}$ column of $CEM_k^m$ (all conditions regarding the case $c_k < 0$)

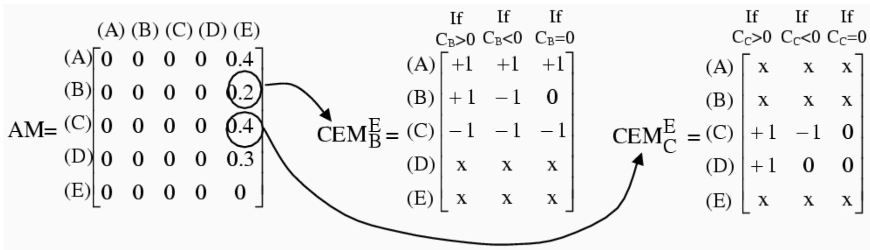


FIGURE 15. Conditional effect matrices for the relationships B-E and C-E.

-: If $CEM_k^m(i,2) = \{+1, -1 \text{ or } 0\}$, it means that: If $c_k < 0$ then $c_i$ must be respectively $\{>0, <0 \text{ or } =0\}$

- For the $3^{rd}$ column of $CEM_k^m$ (all conditions regarding the case $c_k = 0$)

-: If $CEM_k^m(i,3) = \{+1, -1 \text{ or } 0\}$, it means that: If $c_k = 0$ then $c_i$ must be respectively $\{>0, <0 \text{ or } =0\}$

From this formula it is implied that $CEM_k^m(k,1) = +1$, $CEM_k^m(k,2) = -1$ and $CEM_k^m(k,3) = 0$.

Additionally to the CEM, the *Aggregate Strength Vector* (ASV), which enumerates the constraints existing in each column of a CEM, may be defined. For the two CEMs depicted in Fig. 15, the corresponding ASVs are $ASV_B^E = [\ 3 \quad 3 \quad 3\ ]$ and $ASV_C^E = [\ 2 \quad 2 \quad 2\ ]$. Having defined for each cause-effect relationship its CEM and the underlying ASV, an operator denoting the degree to which the rules/prerequisites are satisfied, according to the values of the concept vector, is needed. So, for each cause-effect relationship from node i to node j, the *Satisfaction Vector* $SFV_i^j$ and the *Conditions Strength* operator $CS_i^j$ may be defined as follows ( $\forall 1 \le i, j \le n$, $\forall 1 \le q \le 3$):

$$SFV_i^j(q) = \sum_{p=1}^{n} \text{sign}(c_p) * \text{sign}(CEM_i^j(p,q)) + \sum_{p=1}^{n} [1 - \text{sign}(\max\{|c_p|, |CEM_i^j(p,q)|\})]. \tag{17}$$

$$CS_i^j = \sum_{q=1}^{3} \left\{ 1 - \text{sign}\left(1 - \frac{SFV_i^j(q)}{ASV_i^j(q)}\right) \right\} \tag{18}$$

The values of the SFV are produced through the element-by-element comparison of the concept vector elements with the CEM values. In detail, the formula of SFV was created in order to capture the following information:

- If $c_i > 0$ AND $CEM_k^m(i,-) > 0 \Rightarrow +1$
- If $c_i < 0$ AND $CEM_k^m(i,-) < 0 \Rightarrow +1$
- If $c_i > 0$ AND $CEM_k^m(i,-) < 0 \Rightarrow -1$
- If $c_i < 0$ AND $CEM_k^m(i,-) > 0 \Rightarrow -1$
- If $c_i = 0$ AND $CEM_k^m(i,-) = 0 \Rightarrow +1$
- If $c_i = 0$ AND $CEM_k^m(i,-) > 0 \Rightarrow 0$
- If $c_i = 0$ AND $CEM_k^m(i,-) < 0 \Rightarrow 0$
- If $c_i > 0$ AND $CEM_k^m(i,-) = 0 \Rightarrow 0$
- If $c_i < 0$ AND $CEM_k^m(i,-) = 0 \Rightarrow 0$

When focusing on a cause-effect relationship from node k to node m, the vector $SFV_k^m$ has three elements with the following meaning:

- For the $1^{st}$ element of $SFV_k^m$(all conditions regarding the case $c_k > 0$)
  - **-:** If $SFV_k^m(1) = ASV_k^m(1)$, it means that: If $c_k > 0$ then all constraints are satisfied.
  - **-:** If $SFV_k^m(1) < ASV_k^m(1)$, it means that: If $c_k > 0$ then some constraints are not satisfied.
- For the $2^{nd}$ element of $SFV_k^m$(all conditions regarding the case $c_k < 0$)
  - **-:** If $SFV_k^m(2) = ASV_k^m(2)$, it means that: If $c_k < 0$ then all constraints are satisfied.
  - **-:** If $SFV_k^m(2) < ASV_k^m(2)$, it means that: If $c_k < 0$ then some constraints are not satisfied.
- For the $3^{rd}$ element of $SFV_k^m$(all conditions regarding the case $c_k = 0$)
  - **-:** If $SFV_k^m(3) = ASV_k^m(3)$, it means that: If $c_k = 0$ then all constraints are satisfied.
  - **-:** If $SFV_k^m(3) < ASV_k^m(3)$, it means that: If $c_k = 0$ then some constraints are not satisfied.

The condition strength operator just expresses the extent to which the constraints attached to each cause-effect relationship are fulfilled. According to the mathematical representation of the CEM (recall that $CEM_k^m(k,1) = +1$, $CEM_k^m(k,2) = -1$ and $CEM_k^m(k,3) = 0$), it becomes obvious that at most one column of the CEM may fulfill the prerequisites set by the developers when applying a specific concept vector. Therefore, the feasible set of values for the $CS_i^j$ is the Boolean set {0,1}; the value +1 implies that all constraints of a specific column of the CEM are satisfied while the opposite stands for the non-existence of a column representing valid constraints. After this explanation, it becomes evident the necessity for adaptation of the FCM adjacency matrix, thus the *Adjusted Adjacency Matrix* is produced ($\forall 1 \leq i, j \leq n$):

$$AAM_{ij} = AM_{ij} * CS_i^j. \tag{19}$$

## 7. Conclusions - Future Directions

Fuzzy cognitive maps are considered an emerging modeling and simulation methodology with advantageous features and significant capabilities for approximating and managing complex, not only cognitive, but also real world structures. However, research extension over the conventional mathematical formalism of FCMs is needed so as to enhance functionality and applicability. Recognizing this status, the current work sheds light on four significant and subtle aspects of FCM operation mode: the case of multi-stimulus environments, the development of a learning/optimization mechanism and the irregular situations of synergies and conditional effects. All proposed methodological approaches, which developed under the perspective of simplicity and instant applicability, improve the FCM endogenous features, eliminate a large portion of weaknesses, establish a functional reliability, and reinforce the practical value of FCMs in complicated cases. Especially, the combinatory scheme of FCMs and evolutionary computation, that allows the formulation of the structure of FCMs from zero ground and the adaptability in dynamic environments, consists a novel research field to cover needs and deficiencies in a variety of modeling and simulating problems. However, experimentalism, based on simulations of well-structured real-world systems, are necessary in order to verify the acceptable and particularly the advantageous behavior of the proposed modifications. Moreover, research must further advance and devise modeling tools able to approximate and analyze many more irregular yet useful phenomena of cognitive processes.

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