Orthogonal Sequences Based Multi-CFO Estimation and Semi-Blind ICA Based Equalization for Multiuser CoMP Systems

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Abstract. We propose a low-complexity carrier frequency offset (CFO) estimation approach and an independent component analysis (ICA) based semi-blind equalization structure for the orthogonal frequency division multiplexing (OFDM) based multiuser coordinated multi-point (CoMP) systems. A group of orthogonal sequences are employed to enable separation and simultaneous estimation of multiple CFOs at base stations, dividing a complex multi-dimensional search into series of low-complexity mono-dimensional searches. Then, a small number of pilot symbols attached to the source data, are used to resolve the ambiguity problem in the ICA equalized signals. It has a low complexity, as the permutation and quadrature ambiguity can be eliminated simultaneously, rather than sequentially by the previous precoding aided method. Simulation results show that the proposed semi-blind equalization structure has a bit error rate (BER) performance close to the ideal cases of the zero forcing (ZF) and minimum mean square error (MMSE) based equalizations with perfect channel state information (CSI) and no CFO, and also significantly outperforms the constant amplitude zero autocorrelation (CAZAC) sequences based CFO estimation method.

Keywords: carrier frequency offset (CFO), coordinated multi-point (CoMP), orthogonal frequency division multiplexing (OFDM), independent component analysis (ICA).

1. Introduction

Inter-cell interference is a major bottleneck for achieving a very high data rate in wireless communications [1]-[2]. Many methods have been considered to manage interference in wireless networks. Previously, adjacent cells are operating on different frequencies to effectively reduce the inter-cell interference. However, as the demand for the high mobile data rates is exponentially growing, higher spectral efficiency is driven [3]-[4]. One of the best ways to manage interference and to improve the spectral efficiency in wireless networks, is to
allow base stations to connect to each other through a backhaul link, where they could exchange the messages and jointly process the received multiple users’ signals on the same frequency band. This technology is called the uplink co-ordinated multi-point reception (CoMP) [2], which has received much attention recently, and has been adopted by the 3GPP LTE-Advanced standard [2] as a key technology to reduce inter-cell interference and to improve the service coverage. CoMP can explore the interferences between the cells, which is different from other existing methods by treating them as noise [3]-[6]. Therefore, CoMP enables vast spectral efficiency in current wireless communication systems structure.

1.1. Related Work

Channel estimation and multiuser detection play an important role in multiuser CoMP systems. Two types of approaches, namely training based methods [7]-[9] and blind methods [10]-[15], are commonly used in wireless communications to estimate the channel state information (CSI) and to recover the transmit signals. On the one hand, transmitting training signals or pilots reduces the spectral efficiency, especially in wireless communication systems with very scarce bandwidth resource. Most of previous works related to the training sequence bring considerably extra bandwidth. [7] and [8] propose an adaptive MIMO single carrier frequency-domain equalization. However, their training overhead are up to 13% and 10%, respectively. In [9], the training sequences are used to initialize the equalizer coefficients to combat the Doppler shift. Although the training overhead is as low as 0.05%, the actual training sequences are up to 160 for one transmit antenna at each time of initialization. On the other hand, the pure blind methods increase the computational complexity, particularly at the receiver. If little prior knowledge or information is available at the receiver, the semi-blind approaches [16]-[18] can be employed as a good trade-off relationship between the number of training symbols and the computational complexity.

Blind source separation method can obtain the channel state information, without introducing extra training sequences. The equalizer coefficient can be directly obtained from the statistics of the received data, with the no need to use extra bandwidth and spectral. In fact, blind source separation method is divided into second order statistics (SOS) [12]-[15] and higher order statistics (HOS) [9], [16]-[18]. The Higher Order Statistic is more against the Gaussian noise, as the fourth or higher cumulants of Gaussian noise are equal to zero. In [12]-[13], a non-redundant linear precoding is applied to the source data, and the blind channel estimation is obtained, by exploring the signal covariance matrix via the precoding, based on the second order statistics of the received signals at the receiver. This work is extended to the MIMO case in [14] and [15]. However, all the papers above are based on the SOS only, which is sensitive to the Gaussian noise.

Independent component analysis [10]-[11], is one of the higher order statistics (HOS) based blind source separation approaches, which can estimate the
data directly from the statistics of the received signals, by maximizing the non-Gaussianity of the received signals, without knowing the CSI, as long as the received signal corresponds to a linear mixture. However, as a common drawback of blind approaches, phase, quadrature and permutation ambiguity still exist in the ICA based approaches. In [9], ICA was applied on all subcarriers, and the statistical correlation between subcarriers is used to resolve the frequency dependent permutation. However, this approaches propagate bit errors and ambiguity errors across subcarriers. In [16] and [17], a large number of training sequences are used to eliminate the ambiguity, which, however, reduces the spectral efficiency of the system. In [18], the precoding is employed at the transmitter, by superimposing the reference data to the source data. At the receiver, the permutation and phase ambiguity is eliminated, by using the correlation property between the reference symbols, without consuming extra spectral. However, it is very sensitive to the precoding constant and the frame length, and has a good bit error rate (BER) performance only when data frame size is large enough.

Orthogonal frequency division multiplexing (OFDM) [19]-[28] is an effective technology to combat frequency selective fading, and has been adopted in the fourth generation (4G) wireless communication systems. One of the main drawbacks of OFDM systems is their sensitivity to the carrier frequency offset (CFO), due to the imperfect local oscillators between transmitter and receiver, destroying the orthogonality among the subcarriers, and inducing the additional inter-carrier interference (ICI). In the OFDM based multiuser-CoMP system, multiple CFOs exist between mobile terminals and base stations, introducing multiple access interference (MAI). The optimum maximum likelihood (ML) based CFO estimation method is proposed in [19]. However, it can only be used in single CFO systems, and its complexity is prohibitive in real systems with multiple CFOs, due to the exhaustive multi-dimensional search. In [20], a sub-optimal CFO estimation, using constant amplitude zero autocorrelation (CAZAC) sequences, is proposed. However, there is a mutual interference due to multiple CFOs at each base station, resulting in a biased CFO estimation. Space time block codes transmission with multiple CFOs are proposed in cooperative wireless communication in [21]. However, it does not consider the multi-CFO estimation and the channel estimation. An optimum ML based estimator is proposed for downlink multiuser CoMP systems in [22]. However, its complexity is very high. An approach of approximation for ICI matrix based on the Hadamard sequence is proposed in [23], However it is a biased CFO estimation method. Moreover, in [24]-[28], they require transmitting a large number of training sequences for accurate CFO estimation.

1.2. Main Contributions of this Work

In this paper, we propose a low-complexity multi-CFO estimation method and an ICA based semi-blind equalization structure for multiuser-CoMP OFDM systems. Our work is different in the following aspects:
First, a group of orthogonal sequences are employed for separation of multiple CFOs during the preamble, based on their orthogonal cross-correlation. The simultaneous multi-CFO estimation is performed at each base station, by using the property of the CFO-induced inter-carrier interference (ICI). This is different from other existing works, as a multi-dimensional search is divided into a series of low-complexity mono-dimensional searches.

Second, an ICA based semi-blind equalization structure in CoMP OFDM systems is proposed, where a small number of pilot symbols are used to eliminate the permutation and quadrature ambiguity induced by ICA, based on exploring the cross correlation between the original and equalized pilot symbols. It also requires a lower complexity than the method in [18], as the permutation ambiguity and the quadrature ambiguity can be solved simultaneously, rather than sequentially as in the precoding aided approach.

Simulation results show that the proposed orthogonal sequences based multi-CFO estimation and ICA based semi-blind equalization scheme provides a BER performance close to the ideal case with perfect CSI and no CFO, and has a much better mean square error (MSE) performance than the CAZAC sequence based CFO estimation method [20].

The system model is presented in Section 2. The orthogonal sequences based multiple CFOs estimation is described in Section 3. The ICA based semi-blind structure is proposed in Section 4. Section 5 is the complexity analysis. Simulation results are shown in Section 6. Section 7 draws the conclusion.

Throughout the paper, we use bold symbols to represent vectors/matrices, and use superscripts $\ast$, $T$ and $H$ to denote the complex conjugate, transpose, and complex conjugate transpose of a matrix, respectively. $I_N$ and $0_N$ are an $(N \times N)$ identity matrix and an $(N \times N)$ all-zero matrix, respectively. $\text{diag}\{x\}$ denotes a diagonal matrix whose diagonal elements are entries of vector $x$. $[X]_{u,v}$ is the entry of matrix $X$ in the $u$-th row and $v$-th column. The real part of a complex is denoted by $\Re\{\cdot\}$. $|\cdot|$ denotes the absolute value of a complex number. The angle of a complex is expressed as $\angle\{\cdot\}$. $||\cdot||$ is the Frobenius norm.

2. SYSTEM MODEL

We consider an uplink multiuser-CoMP OFDM system. A total of $K$ users transmit different signals simultaneously, using the same set of $N$ subcarriers to the $M$ separated base stations. A single antenna is assumed for each base station and each user. Each receiver experiences multiple CFOs from $K$ users. The source signal of each user is transmitted in frames, containing $N_s$ OFDM blocks, while each OFDM block is prepended with a cyclic prefix (CP) of length $L_{cp}$ before the transmission, and removed at the receiver to avoid inter-block interference.

We define the $k$-th user’s signal vector as $s_k(i) = [s_k(0, i), s_k(1, i), \ldots, s_k(N-1, i)]^T$, where $s_k(n, i)$ denotes the symbol on the $n$-th subcarrier ($n = 0, 1, \ldots, N-1$)
In this section, we describe the orthogonal sequences based multi-CFO estimation method. Let $S_{thk}$ denote the $k$-th user's orthogonal training sequence.
matrix of size \((N_p \times P)\), where \(N_p\) is the number of subcarriers and \(P\) is the training sequence length during the preamble. The \(k\)-th user’s ICI matrix along with the channel frequency response matrix, i.e., \(\mathbf{C}^{m,k} \mathbf{H}^{m,k}\), at the \(m\)-th base station can be separated from other users’ signals, accomplished by the application of a group of orthogonal sequences on \(N_p\) subcarriers and \(K\) users during the preamble. The properties listed in Section 2 are used to estimate the channel frequency response matrix \(\mathbf{H}^{m,k}\), and then the \(k\)-th user’s ICI matrix \(\mathbf{C}^{m,k}\) at the \(m\)-th base station is decoupled. Finally, multiple CFOs are searched for from a range of trail CFO values in the frequency domain at \(M\) base stations simultaneously.

With the known \(k\)-th user’s orthogonal training sequence matrix \(\mathbf{S}^k_{\text{oth}}\), the combination of the ICI matrix and the channel frequency response matrix \(\mathbf{U}^{m,k}\) between the \(k\)-th user and the \(m\)-th receiver is separated from other users’ signals at the \(m\)-th receiver as

\[
\mathbf{U}^{m,k} = \frac{1}{\mathbf{Y}^m_{\text{orth}} \mathbf{S}^k_{\text{oth}}} \mathbf{U}^{m,k} \mathbf{H}^{m,k}
\]  

where \(\mathbf{Y}^m_{\text{orth}}\) is the received training sequences matrix of size \(N_p \times P\) during the preamble at the \(m\)-th receiver. In order to allow multi-CFO separation at each base station, the orthogonal training matrices \(\mathbf{S}^k_{\text{oth}}\) and \(\mathbf{S}^{k'}_{\text{oth}}\) of the \(k\)-th user and the \(k'\)-th user must satisfy:

\[
\mathbf{S}^k_{\text{oth}} \mathbf{S}^{k'}_{\text{oth}} \mathbf{H}^{m,k} = \begin{cases} 
\mathbf{I}_N & k = k' \\
\mathbf{0}_N & k \neq k'
\end{cases}
\]

We found out that these sequences can be obtained from a group of well studied Hadamard matrices [18], [30]-[31], where any two different rows are orthogonal to each other. We select a unique row in a Hadamard matrix, and apply it onto each subcarrier of different users during the preamble. Note that, the training sequences are obtained from a Hadamard matrix which can be created only for the case where \(P\) is a multiple of 4 [30]-[31].

Shown from (4), although any two different users’ training sequences matrices are orthogonal to each other on all subcarriers, and thus each user’s CFO-induced ICI matrix can be separated from other users’ signals, the separated ICI matrix of each user is still coupled with the channel frequency response matrix. However, the ICI matrix is a circulant matrix, and the channel frequency response matrix is a diagonal matrix. Based on the properties of the ICI matrix and the channel frequency response matrix, \(\mathbf{H}^{m,k}\), the estimate of the channel frequency response matrix between the \(m\)-th receiver and the \(k\)-th user, is written as

\[
\mathbf{H}^{m,k} = \text{diag}\left\{ \sum_{u=0}^{N-1} [\mathbf{U}^{m,k}]_u \right\}
\]

where \([\mathbf{U}^{m,k}]_u\) is the \(u\)-th row of matrix \(\mathbf{U}^{m,k}\), and \(\sum_{u=0}^{N-1} [\mathbf{U}^{m,k}]_u\) is an \((N \times 1)\) vector.
Then, the CFO-induced ICI matrix \( \hat{C}^{m,k} \) between the \( k \)-th user and the \( m \)-th base station is obtained with very low complexity as:

\[
\hat{C}^{m,k} = U^{m,k} H^{m,k}^{-1}
\]  

(6)

In reality, the value of CFO \( \Delta f^{m,k} \) is less than half of the subcarrier spacing [9],[11], so the normalized CFO is in the range of \(-0.5\) to \(0.5\). Since the ICI matrix is estimated, the CFO \( \tilde{\phi}^{m,k} \) between the \( k \)-th user and the \( m \)-th base station is estimated by searching for the minimum distance between \( \hat{C}^{m,k} \) and \( C = \text{FE}(\phi)F^H \) with the trial CFO value \( \tilde{\phi} \)

\[
\tilde{\phi}^{m,k} = \arg \min_{\phi \in (-0.5, 0.5)} \| \hat{C}^{m,k} - C \|_2
\]

\[
= \arg \min_{\phi \in (-0.5, 0.5)} \| \hat{C}^{m,k} - \text{FE}(\phi)F^H \|_2
\]  

(7)

The multiple CFOs can be estimated simultaneously, dividing the complex multi-dimensional search into a series of low-complexity mono-dimension searches. However, it is hard to compensate for multiple CFOs if not impossible [21], because each base station has received multiple CFOs from users, resulting in that the number of different CFOs may be more than the number of users’ signals. The traditional CFO compensation by pre-multiplying the CFO value is impossible at the receiver. To solve this problem, a CFO correction technique is used under the feedback model in [31]-[32], by feeding the CFOs between different users from base stations back to users, aiming at reducing the differences of transmission frequencies among users. Then, the CFO compensation is performed by pre-multiplying the inverse of the ICI matrix \( \hat{C}_m^H \) with adjusted CFO value to \( y(i) = [y_0(i), y_1(i), \ldots, y_{M-1}(i)]^T \) at each base station as \( y(i) = C_m^H y(i) \) in the \( i \)-th block. The compensation error matrix between base station \( m \) and user \( k \) is denoted as \( Q_{m,k} = C_m^H \hat{C}_{m,k} \). The compensation error matrix may become a non-identity matrix, meaning that a subcarrier frequency component is affected by other subcarrier frequency components.

4. ICA Based Semi-Blind Equalization

With accurate CFO estimation and compensation, the received signals become a linear mixture of the transmitted signals. The separated base stations can jointly detect and equalize the multiple user’s signals by employing ICA [10]-[11] on each subcarrier. However, as the instinct drawback of ICA models, the permutation ambiguity and phase ambiguity exist in the ICA separated signals. In order to eliminate the ambiguity, the de-rotation process is first used to correct some possible phase shifts in the ICA separated signals, enabling the signal to have a same phase rotation in a data frame on a separated substream. Second, the pilots of length \( N_{pil} \), known at the receiver, are extracted to resolve the
Yufei Jiang, Xu Zhu, Enggee Lim and Yi Huang

permutation and quadrature ambiguity, by using the cross-correlation between the equalized and original pilot symbols. The received signals vector $\mathbf{Y}(n, i) = [Y_0(n, i), Y_1(n, i), \ldots, Y_{M-1}(n, i)]^T$ on the $n$-th subcarrier is re-expressed as

$$
\mathbf{Y}(n, i) = \mathbf{Q}(n)\mathbf{H}(n)\mathbf{s}(n, i) + \mathbf{Z}(n, i)
$$

where $\mathbf{s}(n, i) = [s_0(n, i), s_1(n, i), \ldots, s_{K-1}(n, i)]^T$ is the $K$ users’ signals vector on the $n$-th subcarrier, $\mathbf{Q}(n)$ is the CFO estimation error matrix on the $n$-th subcarrier, $\mathbf{H}(n)$ and $\mathbf{Z}(n, i)$ are the $M \times K$ channel frequency response matrix and $M \times 1$ noise vector on the $n$-th subcarrier, respectively.

4.1. ICA Based Frequency-Domain Equalization

ICA aims to extract the original source data from the linear mixture based on the statistics of received signals. To obtain the spatially uncorrelated signals, principal component analysis (PCA) [10]-[11] is first used to whiten the received signals, as a standard ICA preprocessing step. The whitening matrix $\mathbf{W}(n)$ is obtained from the eigenvalue decomposition of the autocorrelation matrix of received signals as

$$
\mathbf{R}_{yy}(n) = \mathbb{E}\{\mathbf{Y}(n, i)\mathbf{Y}^H(n, i)\} = \mathbf{U}(n)\Lambda(n)\mathbf{U}^H(n)
$$

The whitening matrix is given as

$$
\mathbf{W}(n) = \Lambda^{-1/2}(n)\mathbf{U}^H(n)
$$

such that

$$
\mathbf{W}(n)\mathbb{E}\{\mathbf{Y}(n, i)\mathbf{Y}^H(n, i)\}\mathbf{W}^H(n) = \mathbf{I}_K
$$

JADE [10]-[11], one of the well-established ICA algorithms, based on the joint diagonalization of cumulant matrices of the received signal, is applied on each frame of length $N_s$ to get the unitary matrix $\mathbf{V}(n)$ on the $n$-th subcarrier. The ICA equalized signals vector $\mathbf{\tilde{s}}(n, i) = [\tilde{s}_0(n, i), \tilde{s}_1(n, i), \ldots, \tilde{s}_{K-1}(n, i)]^T$ on the $n$-th subcarrier and in the $i$-th OFDM block is given by:

$$
\mathbf{\tilde{s}}(n, i) = \mathbf{V}(n)\mathbf{W}(n)\mathbf{\tilde{Y}}(n, i) + \mathbf{\tilde{Z}}(n, i)
$$

where $\mathbf{\tilde{Z}}(n, i) = \mathbf{V}(n)\mathbf{W}(n)\mathbf{Z}(n, i)$ is the output noise matrix from the ICA based equalization. However, there exists an ambiguity matrix $\mathbf{G}(n)$ in $\mathbf{\tilde{s}}(n, i)$, i.e.

$$
\mathbf{\tilde{s}}(n, i) = \mathbf{G}(n)\mathbf{s}(n, i)
$$

It is assumed that the $N_s$ blocks in a data frame on the $n$-th subcarrier has the same ambiguity matrix $\mathbf{G}(n)$ composed of two indeterminacies as

$$
\mathbf{G}(n) = \mathbf{P}(n)\mathbf{D}(n)
$$

where $\mathbf{P}(n)$ denotes the permutation ambiguity matrix and $\mathbf{D}(n)$ denotes the quadrant ambiguity matrix. The ambiguity which remains in the ICA equalized signals can be resolved by the proposed pilot aided method described below.
4.2. Phase Deviation

A de-rotation process can correct possible phase deviations in the ICA equalized signals, to ensure that all of the symbols, including the separated pilots and source symbols on the \( n \)-th subcarrier in a frame, have a same phase rotation on each separated substream. The equalized signals vector \( \mathbf{s}(n, i) = [\hat{s}^0(n, i), \hat{s}^1(n, i), \cdots, \hat{s}^{K-1}(n, i)]^T \) after phase correction is denoted as

\[
\mathbf{s}(n, i) = \mathbf{\hat{s}}(n, i)[\alpha_k(n)/|\alpha_k(n)|]
\]

where

\[
\alpha_k(n) = \left( \frac{1}{N_s} - \frac{1}{1} \sum_{i=0}^{N_s} [\hat{s}^k(n, i)]^4 \right)^{-\frac{1}{4}} e^{j\frac{\pi}{4}}
\]

is the de-rotation factor obtained from the statistical characteristics of \( \mathbf{\hat{s}}(n, i) \) with QPSK modulation. The de-rotation process introduces a phase rotation of \( \theta \) \((\theta \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\})\) in \( \mathbf{s}(n, i) \), which might vary from frame to frame, and from user to user.

4.3. Permutation and Quadrature Ambiguity Resolution

After the de-rotation process, the data in a frame for each separated user on a subcarrier has the same quadrature and permutation ambiguity, which can be resolved by utilizing the cross correlation property between the original and equalized pilot symbols. The cross correlation \( \rho_{\pi_k, k}(n) \) between the ICA equalized pilot symbols \( \hat{s}_{\text{pil,} \pi_k}(n, i) \) and the original pilot symbols \( s_{\text{pil,} k}(n, i) \) on the \( n \)-th subcarrier for the \( k \)-th user is defined as

\[
\rho_{\pi_k, k}(n) = \frac{1}{N_{\text{pil}}} \sum_{i=0}^{N_{\text{pil}}-1} \left\{ [\hat{s}_{\text{pil,} \pi_k}(n, i)]^* s_{\text{pil,} k}(n, i) \right\}
\]

where \( \theta_{\pi_k} = \frac{\pi}{2} l \quad l \in \{0, 1, 2, 3\} \) is the possible phase rotation for the \( \pi_k \)-th separated user on the \( n \)-th subcarrier. In the noiseless case, i.e. \( \sigma^2 = 0 \), we have

\[
\rho_{\pi_k(n), k}(n) = \begin{cases} 
1 & \pi_k(n) = k \\
0 & \pi_k(n) \neq k
\end{cases}
\]

The phase rotation of \( \theta_{\pi_k(n)} \) on the \( \pi_k(n) \)-th user can be found by

\[
e^{j\theta_{\pi_k(n)}} = \frac{s_{\text{pil,} k}(n, i)}{\hat{s}_{\text{pil,} \pi_k}(n, i)}
\]

In the presence of noise, we can search for the largest real part of cross correlation to find the correct order \( \pi_k^{\text{corr}}(n) \) and the phase rotation \( \theta_{\pi_k^{\text{corr}}(n)} \) si-
Yufei Jiang, Xu Zhu, Enggee Lim and Yi Huang

multaneously for the separated $\pi_k$-th user:

$$
[\theta_{\pi_k}^{\text{cor}}(n), \pi_k^{\text{cor}}(n)] = \arg \max_{\theta_{\pi_k}(n), \pi_k(n)} \Re\{\rho_{\pi_k}(n,k)\}
$$

$$
= \arg \max_{\theta_{\pi_k}(n), \pi_k(n)} \Re\left\{ \frac{1}{N_{\text{pil}}} \sum_{i=1}^{N_{\text{pil}}} \left\{ [s_{\text{pil},\pi_k}(n,i)] e^{j\theta_{\pi_k}(n)} \cdot s^*_{\text{pil},k}(n,i) \right\} \right\}
$$

(20)

$\pi_k^{\text{cor}}(n)$ is considered as the correct order which can be arranged into the matrix of

$$
P^{-1}(n) = \text{diag}\{[\pi_0^{\text{cor}}(n), \pi_1^{\text{cor}}(n), \cdots, \pi_{K-1}^{\text{cor}}(n)]\}
$$

and $\theta_{\pi_k}^{\text{cor}}(n)$ is the correct phase rotation matrix as

$$
D^{-1}(n) = \text{diag}\{e^{j\theta_{\pi_0}^{\text{cor}}(n)}, e^{j\theta_{\pi_1}^{\text{cor}}(n)}, \cdots, e^{j\theta_{\pi_{K-1}}^{\text{cor}}(n)}\}
$$

(22)

The $\pi_k^{\text{cor}}(n)$ and $\theta_{\pi_k}^{\text{cor}}(n)$ are considered as the highest possibility of being the correct order and the correct phase rotation, when the real part is the largest one.

5. Complexity Analysis

In this section, the complexities of the proposed CFO estimation approach and the pilot aided ICA based equalization are presented in terms of the number of complex multiplications. The complexity of the ML based CFO estimation method [19] and the precoding aided and ICA based equalization structure [8] are also analyzed for comparison.

$q = 1/\Delta$ is the homogenization of step size $\Delta$ between $-0.5$ and $0.5$ for the CFO search.

As shown in Table 1, the proposed CFO estimation method has a much lower complexity than the ML based CFO estimation approach which requires an exhaustive search with prohibitive complexity for real systems and is impractical for real systems. The multiple CFOs are separated with the complexity order of $N^2MKP$, while the matrix inversion dominates the computational complexity in the ML based CFO estimator, and has $N^2K^3P^3M$ times higher complexity than the proposed CFO estimation method. For the CFO search, the complexity of the ML based CFO estimator increases exponentially with the number of base stations and users, i.e., $q^{MK}$ multiplications are required, while the hadamard sequences based multi-CFO estimation has the computational complexity of $qK$, which increases linearly with the increase of the number of users. The proposed CFO estimation method benefits from dividing a complex multi-dimensional into a number of low-complexity one-dimensional searches. Also, the complexity of the simultaneous multi-CFO estimation is independent of the number of base stations.

Both of the proposed pilot aided method and the precoding aided method [18] use JADE [10]-[11] for equalization. However, they use different indeterminacy resolutions for permutation and quadrature ambiguity.
The proposed pilot aided semi-blind equalization requires a much lower complexity than the precoding aided method. The proposed semi-blind equalization uses a small number of pilot symbols to eliminate the ambiguity, with only a complexity of $4N_{\text{pil}}K$. While the precoding aided semi-blind structure requires a pair of precoding-decoding process at both transmitter and receiver side, and has a good BER performance only when the frame size is big, because a large number of received blocks are required to eliminate the ambiguity. Also, the permutation and quadrature ambiguity can be resolved simultaneously by the proposed method, rather than sequentially as in [18]. Therefore, the proposed pilot aided and ICA based equalization has lower computational complexity.

6. Simulation Results

We use simulation results to demonstrate the performance of the proposed orthogonal sequence based CFO estimation and pilot aided ICA based signal equalization for multiuser-CoMP OFDM systems with $K=2$ users and $M=2$ base stations. A CP of length $L_{\text{cp}}=16$ is used and the number of subcarriers is $N=64$ for the OFDM systems. Each data frame with QPSK modulation consists of
Fig. 2. MSE performance of the proposed Hadamard pilot based multiple CFOs estimation for multiuser OFDM CoMP systems
Table 1. Computational Complexity

<table>
<thead>
<tr>
<th>CFO Estimation</th>
<th>Orthogonal sequences</th>
<th>Maximum likelihood [19]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td>Complexity order</td>
<td>Item</td>
</tr>
<tr>
<td>CFO separation</td>
<td>( N^2MKP )</td>
<td>Problem formulation for CFO ((NKP)^2M^2)</td>
</tr>
<tr>
<td>CFO search</td>
<td>( qK )</td>
<td>CFO search</td>
</tr>
<tr>
<td>Ambiguity Elimination</td>
<td>( 4NN_{pil}K )</td>
<td>Reordering ( NN_s(K!) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quadrature elimination</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( NK )</td>
</tr>
</tbody>
</table>

Fig. 3. Impact of different CFOs on the MSE of CFO estimation

\( N_s = 256 \) symbol blocks, where the first 6 blocks \( (N_{pil}=6) \) of each frame are used as pilots, resulting in a low training overhead of 2.3%. The Clarke’s block fading channel model [1] with an exponential power delay profile is employed, where the CSI remains constant during a frame. The root mean square (RMS) delay spread of the channel is 86 ns. The resulting channel impulse response length is \( L=7 \). The orthogonal sequences of length \( P=32 \) are assigned across \( N_p = 16 \) subcarrires to separate and estimate the multiple CFOs at each base.
Fig. 4. Impact of the number of users on the normalized MSE performance of the CFO estimation.

station during the preamble. A step size $\Delta = 0.001$ results in a search of $q = 1000$ possible CFO values within the range of -0.5 to 0.5 for the CFO search.

Fig. 1 demonstrates the BER performance of the proposed orthogonal sequences based multi-CFO estimation and ICA based equalization scheme, compared with the zero forcing (ZF) based and the minimum mean square error (MMSE) based equalization methods. The precoding aided semi-blind equalization method [18] is also used for comparison. The CFOs are set to [-0.3 0.3; 0.3 -0.3] and the precoding constant is set to 0.25 [18]. The proposed multi-CFO estimation and ICA based structure provides a BER performance close to the ZF based and the MMSE based equalization methods with perfect CSI and no CFO. The pilot aided and ICA based structure outperforms the precoding aided method by around 1 – 2 dB, showing the effectiveness of our proposed method in ambiguity elimination.

Fig. 2 demonstrates the MSE performance of the proposed orthogonal training sequence based multi-CFO estimation for multiuser OFDM CoMP systems, with training sequences length $P = 16$ and $P = 32$. The CFO estimation MSE is defined as

$$MSE = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} (\phi_{m,k} - \hat{\phi}_{m,k})^2$$

(23)
Fig. 5. Impact of pilot length on the MSE performances of the proposed multi-CFO estimation
Yufei Jiang, Xu Zhu, Enggee Lim and Yi Huang

The CFOs are set to [-0.3 0.3; 0.3 -0.3]. The proposed CFO estimation method significantly outperforms the CAZAC sequences based method in [20] in terms of the MSE performance, especially at high signal-to-noise ratio (SNR). This is because our proposed method can successfully separate multiple user’s ICI matrices, and achieve multi-CFO estimation at each base station, while the CAZAC sequence based CFO estimation can not completely eliminate the MAI caused by different CFOs at base station.

Fig. 3 shows the impact on the MSE performance of multi-CFO estimation with different sets of CFOs over a range of SNRs. The MSE performances improve as the SNR increases and have no error floor at high SNR region. The CFOs as [-0.1 0.1; 0.1 -0.1] has the best MSE performance, while the CFOs as [-0.5 0.5; 0.5 -0.5] has the worse performance. Their performance gap implies that the performance becomes better when the CFO value is closer to zero.

![Graph showing the impact of frame size on the BER performances of pilot aided and precoding aided semi-blind equalization structures.](image)

**Fig. 6.** Impact of frame size on the BER performances of pilot aided and precoding aided semi-blind equalization structures (EQ: equalization)

The impact of the number of users on the proposed multi-CFO estimation is showed in Fig. 4, where the number of users varies from 2 to 4. For fair comparison, CFO values are set to 0.1 or -0.1 only, and the MSE for the proposed
CFO estimation is normalized to

\[
\text{NMSE} = \frac{\sum_{m=0}^{M-1} \sum_{k=0}^{K-1} [\phi_{m,k} - \hat{\phi}_{m,k}]^2}{\sum_{m=0}^{M-1} \sum_{k=0}^{K-1} |\phi_{m,k}|^2}
\]  

(24)

The proposed CFO estimation presents a robust performance against the increase of the number of users.

Fig. 5 shows the impact of the pilot length on performance of the proposed CFO estimation method with different SNRs. The MSE performance for CFO estimation improves with the increase of pilot length, until the pilot length reaching a around \( P = 32 \) for different SNRs. It then becomes steady.

Fig. 6 demonstrates the impact of frame size on the BER performance of the pilot aided and precoding aided semi-blind equalization structures, at SNR = 10 dB and 20 dB. The performance of the pilot aided method remains constant over a wide range of frame sizes, while the precoding aided method is very sensitive to the change of frame length from small to medium. This is because the precoding aided approach requires a large frame of data to achieve a good performance. Thus, the pilot aided ICA based equalization is more robust against channel variations.

7. Conclusion

We have proposed a low-complexity orthogonal sequences based multi-CFO estimation approach, and a pilot aided and ICA based semi-blind equalization structure for multiuser-CoMP OFDM systems. Based on the properties of the orthogonal sequences extracted from the Hadamard matrix, the multiple CFOs are separated and estimated simultaneously. The multi-dimensional multi-CFO search is divided into a series of one-dimensional searches with low complexity and high accuracy. By utilizing the cross-correlation between the original and equalized pilot symbols, the permutation and phase ambiguity due to the ICA model are resolved simultaneously, without increasing the computational complexity. The proposed pilot-aided semi-blind equalization structure does not provide a BER performance closed to the ideal case with perfect CSI and no CFO, but also has a much better MSE performance than the CAZAC sequence based CFO estimation scheme [10]. The proposed CFO estimation has a robust performance against the increase of the number of users, and its MSE performance becomes better when the CFO value is closer to zero. The proposed pilot aided ICA based equalization is more robust against channel variations, than the precoding aided semi-blind structure.

Appendix: Proof of Property II of ICI Circulant Matrix

The CFO matrix is right multiplied by a unitary IDFT matrix, with the element in the \( a \)-th row and the \( v \)-th column \((a = 0, 1, \ldots, N - 1; v = 0, 1, \ldots, N - 1)\)
written as

\[
E(\phi)F^H = \frac{1}{\sqrt{N}} [Q^a W^{-av}]_{a,v}
\]  

(25)

By left multiplying the unitary DFT matrix to (25), the ICI matrix is formed, and its element \([C]_{u,v}\) in the \(u\)-th row and the \(v\)-th column \((u = 0, 1, \cdots, N - 1; v = 0, 1, \cdots, N - 1)\) is written as

\[
[C]_{u,v} = \left[ FE(\phi)F^H \right]_{u,v} \\
= \left[ \frac{1}{N} \sum_{a=0}^{N-1} W^{av} (Q^a W^{-av}) \right]_{u,v} \\
= \frac{1}{N} \sum_{a=0}^{N-1} [Q^a W^{au}]_{u,v}
\]  

(26)

Substituting \(Q = e^{j\frac{2\pi}{N}}\) and \(W = e^{-j\frac{2\pi}{N}}\) into (26) yields

\[
\sum_{u=0}^{N-1} [C]_{u,v} = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{a=0}^{N-1} e^{j2\pi a(\phi+v-u)}
\]

\[
= \frac{1}{N} \sum_{a=0}^{N-1} \left[ e^{j2\pi a(\phi+v-u)} + e^{j2\pi a(\phi+v-u-1)} + \cdots + e^{j2\pi a(\phi+v-u-(N-1))} \right]_{v}
\]

\[
= \frac{1}{N} \sum_{a=1}^{N-1} \left[ e^{j2\pi a(\phi+v-u)} + e^{j2\pi a(\phi+v-u)} + \cdots + e^{j2\pi a(\phi+v-u-(N-1))} \right]_{v}
\]

\[
= \frac{1}{N} \left[ e^{j2\pi a(\phi+v-u)} \right] \left[ 1 + e^{-j\frac{2\pi a}{N}} + e^{-j\frac{4\pi a}{N}} + \cdots + e^{-j\frac{2\pi a}{N}} \right]
\]

(27)

since \((1 + e^{-j\frac{2\pi a}{N}} + \cdots + e^{-j\frac{2\pi a(N-1)}{N}})\) in (27) is the geometric sequence with common ratio of \(e^{-j\frac{2\pi a}{N}}\) and initial term of 1, (27) is rewritten as

\[
\sum_{u=0}^{N-1} [C]_{u,v} = \frac{1}{N} \left[ N + \sum_{a=1}^{N-1} e^{j2\pi a(\phi+v-u)} \left( e^{-j\frac{2\pi a}{N}} - 1 \right) \right]_{v}
\]

(28)

Similarly, \(\sum_{v=0}^{N-1} [C]_{u,v} = 1\) and \(\sum_{v=0}^{N-1} \sum_{u=0}^{N-1} [C]_{u,v} = N\) can be proved.

References


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