The Weinstein conjecture in the uniruled manifolds

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Abstract

In this note we prove the Weinstein conjecture for a class of symplectic manifolds including the uniruled manifolds based on Liu-Tian's result.

Key words : Weinstein conjecture, Gromov-Witten invariants, uniruled manifold.

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Since 1978 A. Weinstein proposed his famous conjecture that *every hypersurface of contact type in the symplectic manifolds carries a closed characteristic*([We]), many results were obtained (cf.[C][FHV][H][HV1][HV2][LiuT][Lu1][Lu2][V1][V2][V3]) after C.Viterbo first proved it in $(\mathbb{R}^{2n}, \omega_0)$ in 1986([V1]). Not long ago Gang Liu Gang and Gang Tian established a deep relation between this conjecture and the Gromov-Witten invariants and got several general results as a corollary([LiuT]).

Assume S to be a hypersurface of contact type in a closed connected symplectic manifold (V, ω) separating V in the sense of [LiuT], i.e. there exist submanifolds V_+ and V_- with common boundary S such that $V = V_+ \cup V_-$ and $S = V_+ \cap V_-$, then the following result holds.

Theorem 1([LiuT]) *If there exist $A \in H_2(V;\mathbb{Z})$ and $\alpha_+,\ \alpha_- \in H_*(V;\mathbb{Q})$, such that (i) supp$(\alpha_+) \hookrightarrow$ int(V_+) and supp$(\alpha_-) \hookrightarrow$ int(V_-), (ii) the Gromov-Witten invariant $\Psi_{A,g,m+2}(C;\alpha_-,\alpha_+,\beta_1,\cdots,\beta_m)$ $\neq 0$ for some $\beta_1,\cdots,\beta_m \in H_*(V;\mathbb{Q})$, then S carries at least one closed characteristic.*

Recall that for a given $A \in H_2(V;\mathbb{Z})$ the Gromov-Witten invariant of genus g and with m+2 marked points is a homomorphism

$$\Psi_{A,g,m+2} : H_*(\overline{\mathcal{M}}_{g,m+2};\mathbb{Q}) \times H_*(V;\mathbb{Q})^{m+2} \to \mathbb{Q},$$

(see [FO][LiT][R][Si]). Though one so far does not yet know whether the GW invariants defined in the four papers agree or not we believe that they have the same vanishing or nonvanishing properties, i.e., for any given classes $C \in H_*(\overline{\mathcal{M}}_{g,m+2};\mathbb{Q})$ and $\beta_1,\cdots,\beta_{m+2} \in H_*(V;\mathbb{Q})$ one of this

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four versions vanishes on $(C;\beta_1,\cdots,\beta_{m+2})$ if and only if any other three ones vanishes on it. In addition, the version of [R] is actually a homomorphism from $H_*(\overline{\mathcal{M}}_{g,m+2};\mathbb{R})\times H_*(V;\mathbb{R})^{m+2}$ to $\mathbb{R}$. However, using the facts that $H_*(M;\mathbb{Q})$ is dense $H_*(M;\mathbb{R})$ for $M=V,\overline{\mathcal{M}}_{g,k}$ and that $\Psi_{A,g,m+2}$ is always a homomorphism one can naturally extend the other three versions to the homomorphisms from $H_*(\overline{\mathcal{M}}_{g,m+2};\mathbb{R})\times H_*(V;\mathbb{R})^{m+2}$ to $\mathbb{R}$. Below we always mean the extended versions when they can not clearly explained in the original versions. Our main result is

Theorem 2 *For a connected closed symplectic manifold (V,ω), if there exist $A\in H_2(V;\mathbb{Z})$ and $\beta_1,\cdots,\beta_{m+1}\in H_*(V;\mathbb{Q})$ such that $\Psi_{A,g,m+2}(C;[pt],\beta_1,\cdots,\beta_{m+1})\neq 0$ for $(g,m)\neq(0,0)$ and the single point class $[pt]$, then every hypersurface of contact type S in the symplectic manifold V separating V carries a closed characteristic. Specially, if $g=0$ we can also guarantee that S carries one contractible (in V) closed characteristic.*

In case $g=0$ it is not difficult to prove that Proposition 2.5(5) and Proposition 2.6 in [RT] still hold for any closed symplectic manifold (V,ω) with the method of [R]. That is,

(i) $\Psi_{0,0,k}([pt];\alpha_1,\cdots,\alpha_k)=\alpha_1\cap\cdots\cap\alpha_k($ the intersection number);

(ii) for the product manifold $(V,\omega)=(V_1\times V_2,\omega_1\oplus\omega_2)$ of any two closed symplectic manifolds (V_1,ω_1) and (V_2,ω_2) it holds that

$$\Psi^V_{A_1\otimes A_2,0,k}([pt];\alpha_1\otimes\beta_1,\cdots,\alpha_k\otimes\beta_k)=\Psi^{V_1}_{A_1,0,k}([pt];\alpha_1,\cdots,\alpha_k)\Psi^{V_2}_{A_2,0,k}([pt];\beta_1,\cdots,\beta_k).$$

Thus if $\Psi^{V_2}_{A_2,0,m+1}([pt];[pt],\beta_1,\cdots,\beta_m)\neq 0$ we get

$$\Psi^V_{A_1\otimes A_2,0,m+1}([pt];[pt],\alpha_1\otimes\beta_1,\cdots,\alpha_m\otimes\beta_m)\neq 0$$

for $A_1=0$ and $\alpha_1=\cdots=\alpha_m=[V_1]$. This leads to

Corollary 3 *Weinstein conjecture holds in the product symplectic manifolds of any closed symplectic manifold and a symplectic manifold satisfying the condition of Theorem 2 for $g=0$.*

Recall that a smooth Kahler manifold (M,ω) is called *uniruled* if it can be covered by rational curves. Y. Miyaoka and S. Mori showed that a smooth complex projective manifold X is uniruled if and only if there exists a non-empty open subset $U\subset X$ such that for every $x\in U$ there is an irreducible curve C with $(K_X,C)<0$ through x([MiMo]). Specially, any *Fano* manifold is uniruled([Ko]). The complex projective spaces, the complete intersections in it, the Grassmann manifolds and more general flag manifold are the important examples of the Fano manifolds. In [R, Prop. 4.9] it was proved that if a smooth Kahler manifold M is symplectic deformation equivalient to uniruled manifold, M is uniruled. Actually, as mentioned there, Kollar showed that on the uniruled manifold (M,ω) there exists a class $A \in H_2(V;\mathbb{Z})$ such that

$$\Phi_{A,0,3}([pt];[pt],\beta_1,\beta_2) \neq 0 \tag{1}$$

for some classes β_1 and β_2(see [R] for more general case). In [Le], Hong Van Le checked this for $\mathbb{C}P^k$-bundle over Grassmannian manifold $G(n,k)$. Combing these with Corollary 3 we get

Corollary 4 *Every hypersurface S of contact type in the uniruled manifold V or the product of any closed symplectic manifold and an uniruled manifold carries one contractible (in V) closed characteristic.*

The ideas of proof are combing Liu-Tian's Theorem 1 above, the properties of the Gromov-Witten invariants and Viterbro's trick of [V4].

Proof of Theorem 2 Under the assumptions of Theorem 2, the reduction formula of the Gromov-Witten invariants([Prop. C, R]) implies that

$$\Psi_{A,g,m+3}(\pi^*(C);[pt],PD([\omega]),\beta_1,\cdots,\beta_{m+1}) = \omega(A)\cdot\Psi_{A,g,m+2}(C;[pt],\beta_1,\cdots,\beta_{m+1}) \neq 0 \tag{2}$$

since A contains the nontrivial pseudoholomorphic curves. To use Theorem 1 we need to show that there exists a homology class $\gamma \in H_2(V;\mathbb{R})$ with support $supp(\gamma) \hookrightarrow int(V_+)$(or $int(V_-)$) such that

$$\Psi_{A,g,m+3}(\pi^*(C);[pt],\gamma,\beta_1,\cdots,\beta_{m+1}) \neq 0. \tag{3}$$

To this goal we note S to be a hypersurface of contact type, and thus there exists a Liouville vector field X defined in a neighborhood U of S, which is transverse to S. The flow of X define a diffeomorphism Φ from $S\times(-3\epsilon,3\epsilon)$ onto an open neighborhood of S in U for some $\epsilon>0$. Here we may assume $\Phi(S\times(-3\epsilon,0]) \subset V_+$ and $\Phi(S\times[0,3\epsilon)) \subset V_-$. For any $0<\delta<3\epsilon$ let us denote by $U_\delta := \Phi(S\times[-\delta,\delta])$. We also denote by $\alpha = i_X\omega$, then $d\alpha=\omega$ on U. Choose a smooth function $f: V\to\mathbb{R}$ such that $f|_{U_\epsilon}\equiv 1$ and vanishes outside $U_{2\epsilon}$. Define $\beta := f\alpha$. This is a smooth 1-form on V, and $d\beta=\omega$ on U_ϵ. Denote by $\widehat{\omega}=\omega-d\beta$. Then $\widehat{\omega}|_{U_\epsilon}\equiv 0$ and thus cohomology classes $[\omega]=[\widehat{\omega}]$ is in $H^2(V,U_\epsilon)$. Now from the naturality of Poincare-Lefschetz duality ([p.296, Sp]): $H_{2n-2}(V-U_\epsilon)\cong H^2(V,U_\epsilon)$ it follows that we can choose a cycle representive $\widehat{\gamma}$ of $\gamma := PD([\omega])$ with support $supp(\widehat{\gamma}) \hookrightarrow int(V-U_\epsilon)$. Notice that $V-U_\epsilon \subset V-S = int(V_+)\cup int(V_-)$ and $int(V_+)\cap int(V_-)=\emptyset$. We can denote by $\widehat{\gamma}_+$ and $\widehat{\gamma}_-$ the union of connected components of $\widehat{\gamma}$ lying $int(V_+)$ and $int(V_-)$ respectively. Then the homology classes determined by them in $H_*(V,\mathbb{R})$ satisfy: $[\widehat{\gamma}_+]+[\widehat{\gamma}_-]=\gamma$. Thus $[\widehat{\gamma}_+]$ and $[\widehat{\gamma}_-]$ have at least one nonzero class. By the property of the Gromov-Witten invariants we get

$$\Psi_{A,g,m+3}(\pi^*(C);[pt],\gamma,\beta_1,\cdots,\beta_{m+1}) = \Psi_{A,g,m+3}(\pi^*(C);[pt],[\widehat{\gamma}_+],\beta_1,\cdots,\beta_{m+1}) \tag{4}$$

$$+ \quad \Psi_{A,g,m+3}(\pi^*(C); [pt], [\widehat{\gamma}_-], \beta_1, \cdots, \beta_{m+1}) \neq 0.$$

Hence the right side of (4) has at least one nonzero term. Without loss of generality we assume that

$$\Psi_{A,g,m+3}(\pi^*(C); [pt], [\widehat{\gamma}_+], \beta_1, \cdots, \beta_{m+1}) \neq 0.$$

Then Theorem 1 and the remarks in Appendix lead to the conclusion. □

Remark 5 Actually we believe that Theorem 1 still holds provided the hypersurface S of contact type therein is replaced by the stable hypersurface in the sense of [HV2]. Hence the hypersurface S of contact type in our results above may be replaced by the stable hypersurface for which the symplectic form is exact in some open neighborhood of it.

Appendix: Remarks on Liu-Tian's paper

In this appendix we will show that Liu-Tian's arguments may actually get a nontrivial contractible(in V) closed characterstic if $g = 0$ in Theorem 1.1 of [LiuT]. Specially, the closed characterstics obtained in their Theorem 1.3-1.4 are contractible in V. That is, under such assumptions the following stronger version of the Weinstein conjecture holds.

SWC: *Every hypersurface S of contact type in a symplectic manifold (V, ω) carries a* **contractible***(in V) closed characteristic.*

Without special statements we follow the notations in [LiuT] below. Though the arguments are almost the repeat of §2 in [LiuT] we still give it for the sake of clarification. Let $\tilde{H}$ be the particular Hamiltonian function associated with S defined in [LiuT]. But we require a smooth function $\phi : [-\epsilon, \epsilon] \to [-\epsilon, \epsilon]$ to be strictly increasing on $[-\epsilon + \delta, \epsilon - \delta]$. For the Hamiltonian vector field $X_{\tilde{H}}$ determined by $\omega(X_{\tilde{H}}, \cdot) = d\tilde{H}$, let us consider the Hamiltonian equation

$$(A.1) \qquad \frac{dx}{dt} = X_{\tilde{H}}(x).$$

Then the following claim holds.

Claim *Any non-trivial closed orbit x of (A.1) will lie on some level hypersurface $S_t = \tilde{H}^{-1}(t)$, $-\epsilon + \delta < t < \epsilon - \delta$. Conversely, any closed characteristic lying some hypersurface S_t, $t \in (-\epsilon + \delta, \epsilon + \delta)$, can become a non-trivial closed orbit x of (A.1) after suitably parametrized.*

Denote $\mathcal{P}(S)$ by the set of closed characteristics on S. Then the flow ψ^t of X gives a 1-1 corresponding between $\mathcal{P}(S)$ and $\mathcal{P}(S_t)$ by $P \mapsto \psi^t(P)$, and $P \in \mathcal{P}(S)$ is contractible in V if and only if $\psi^t(P)$ is contractible in V. Therefore, similar to Main Assumption (I) in [LiuT] we make the following assumptions.

Assumptin : (A.1)*has no nontrivial closed orbit which is contractible in* V *(* $\Longleftrightarrow$ *There is no nontrivial closed characteristic on* S *which is contractible in* V *(* $\Longleftrightarrow$ *There is no nontrivial closed characteristic on* S_t *which is contractible in* V *for any* $t \in (-\epsilon+\delta, \epsilon+\delta)$*))*.

We have the refined version of Lemma 2.1 in [LiuT].

Lemma *Let* $\tilde{H}$ *be as above. Then there exists a Morse function* H *on* V *such that*

(i) *H has the same level sets as $\tilde{H}$ on $\Psi(S \times (-\epsilon+2\delta, \epsilon-2\delta))$. Prescisely speaking, for any $s \in (-\epsilon+2\delta, \epsilon-2\delta)$ there exista a unique $\tilde{s} \in (-\epsilon+2\delta, \epsilon-2\delta)$ such that $\tilde{H}^{-1}(\tilde{s}) = H^{-1}(s)$. Conversely, for any $\tilde{s} \in (-\epsilon+2\delta, \epsilon-2\delta)$ there exists a unique $s \in (-\epsilon+2\delta, \epsilon-2\delta)$ such that $\tilde{H}^{-1}(\tilde{s}) = H^{-1}(s)$.*

(ii) *H is C^0-close to $\tilde{H}$ so that for any critical points c_- in V_- and c_+ in V_+,*

$$0 < \frac{\omega(A)}{H(c_+) - H(c_-)} < \frac{\omega(A)}{\tilde{H}(c_+) - \tilde{H}(c_-)} + \frac{1}{2} = \lambda_0$$

where $\lambda_0 := \frac{\omega(A)}{2\epsilon} + \frac{1}{2}$ and A is an effective second homology class in the sense that it can be represented by some J-holomorphic sphere;

(iii) *When $\tilde{H}$ satisfies the* **Assumptions** *above, for any $0 < \lambda \leq \lambda_0 + \frac{1}{2}$, $H_\lambda = \lambda \cdot H$ has no non-trivial contractible (in V)closed orbits of period one.*

Proof For ϵ and $\delta \ll \epsilon$ as above we take $r > 0$ such that

$$0 < \frac{\omega(A)}{2\epsilon - 2r - 4\delta} < \lambda_0.$$

Denote by $\tilde{H}_+ = \tilde{H}|V^+_{\epsilon-2\delta}$ and $\tilde{H}_- = \tilde{H}|V^-_{-\epsilon+2\delta}$. Here

$$V^+_t = V_+ \setminus \Phi(S \times [0,t]), \quad \partial V^+_t = S_t,\ t \geq 0 \quad \text{and} \quad V^-_t = V_- \setminus \Phi(S \times [t,0]), \quad \partial V^-_t = S_t,\ t \leq 0.$$

Notice that $\nabla\tilde{H}_+(x) \neq 0$ for any $x \in \partial V^+_{\epsilon-2\delta}$ and $\nabla\tilde{H}_-(x)$ for any $x \in \partial V^-_{-\epsilon+2\delta}$. We can choose C^2-small smooth nonnegative functions $\tilde{G}_+ : V^+_{\epsilon-2\delta} \to \mathbb{R}$ vanishing near $S_{\epsilon-2\delta}$, and $\tilde{G}_- : V^-_{-\epsilon+2\delta} \to \mathbb{R}$ vanishing near $S_{-\epsilon+2\delta}$ such that $\tilde{F}_+ = \tilde{H}_+ + \tilde{G} : V^+_{\epsilon-2\delta} \to \mathbb{R}$ and $\tilde{F}_- = \tilde{H}_- + \tilde{G}_- : V^-_{-\epsilon+2\delta} \to \mathbb{R}$ are Morse functions. Notice that $\tilde{F}_+ > \epsilon - 2\delta$ on $V^+_{\epsilon-2\delta}$ and $\tilde{F}_- < -\epsilon + 2\delta$ on $V^-_{-\epsilon+2\delta}$. We have the decompositions

$$\tilde{F}_+ = \epsilon - 2\delta + \bar{F}_+, \quad \tilde{F}_- = -\epsilon + 2\delta + \bar{F}_-.$$

Here $\bar{F}_+|S_{\epsilon-2\delta} \equiv 0$ and $\bar{F}_-|S_{-\epsilon+2\delta} \equiv 0$. For $\tau \geq 0$ we define $\tilde{F}^\tau_+ = \epsilon - 2\delta + \tau\bar{F}_+$ and $\tilde{F}^\tau_- = -\epsilon + 2\delta + \tau\bar{F}_-$. Then it holds that

$$\tilde{F}^\tau_+|V^+_{\epsilon-2\delta} > \epsilon - 2\delta, \quad \tilde{F}^\tau_-|V^-_{-\epsilon+2\delta} < -\epsilon + 2\delta;$$

$$\|\tilde{F}^\tau_+ - (\epsilon - 2\delta)\|_{C^2} = \tau\|\bar{F}_+\|_{C^2}, \quad \|\tilde{F}^\tau_- - (-\epsilon + 2\delta)\|_{C^2} = \tau\|\bar{F}_-\|_{C^2}.$$

Take $\tau_0 > 0$ so small that for any $0 \le \tau \le \tau_0$ the functions $\tau \bar{F}_+$ and $\tau \bar{F}_-$ have no nontrivial closed orbits of period one in $V^+_{\epsilon-2\delta}$ and $V^-_{-\epsilon+2\delta}$ respectively. Fix a $0 < \tau_1 \le \tau_0$ such that (i) $\tau_1(\lambda_0 + \frac{1}{2}) < \tau_0$, (ii) for any critical points c_+ of $\bar{F}_+$ and c_- of $\bar{F}_-$ it holds that

$$\tau_1|\bar{F}_+(c_+) - \bar{F}_-(c_-)| < 2r.$$

Denote by $H_+ = \tilde{F}^{\tau_1}_+$ and $H_- = \tilde{F}^{\tau_1}_-$ and extend $H_+ \cup H_-$ to V as follows:

$$H(x) = \begin{cases} H_+(x) & \text{as } x \in V^+_{\epsilon-2\delta}, \\ \tilde{H}(x) & \text{as } x \in \cup_{-\epsilon+3\delta<t<\epsilon-3\delta} S_t, \\ H_-(x) & \text{as } x \in V^-_{-\epsilon+2\delta}, \\ \psi(t) & \text{as } x \in S_t,\ t \in (\epsilon-3\delta, \epsilon-2\delta) \cup (-\epsilon+2\delta, -\epsilon+3\delta); \end{cases}$$

where $\psi : \mathbb{R} \to \mathbb{R}$ is a strictly increasing smooth function and

(1) on $[\epsilon - 3\delta, \epsilon - 2\delta)$ it connects smoothly the functions $\psi_1(t) = t,\ t \le \epsilon - 3\delta$ and $\psi_2(t) = (\epsilon - 2\delta) + \tau_1(t - (\epsilon - 2\delta)),\ t \ge \epsilon - 2\delta$.

(2) on $(-\epsilon+2\delta, -\epsilon+3\delta]$ it connects smoothly the functions $\psi_3(t) = (-\epsilon+2\delta)+\tau_1(t-(-\epsilon+2\delta)),\ t \ge -\epsilon + 2\delta$ and $\psi_4(t) = t,\ t \ge -\epsilon + 3\delta$.

It is easily checked that $H : V \to \mathbb{R}$ is a smooth function. Denote by $H_\lambda := \lambda \cdot H$ for $0 \le \lambda \le \lambda_0 + \frac{1}{2}$. We prove it to satisfy the condition (iii) of the Lemma above.

Let $x = x(t)$ be a nontrivial 1-period orbit of $\dot{x} = X_{H_\lambda}(x)$ and contractible in V, then λ cannot vanish. Let $H(x(t)) \equiv c($ a constant). If $c > \epsilon - 2\delta$ it follows from the definition of H that $x(t) \in V^+_{\epsilon-2\delta}$ for all $t \in \mathbb{R}$. But on $V^+_{\epsilon-2\delta}$

$$H_\lambda(x) = \lambda H_+(x) = \lambda \tilde{F}^{\tau_1}_+(x) = \lambda(\epsilon - 2\delta) + \lambda\tau_1 \bar{F}_+(x)$$

and therefore

$$\|H_\lambda - \lambda(\epsilon - 2\delta)\|_{C^2} = \lambda\tau_1\|\bar{F}_+\|_{C^2} \le \tau_0\|\bar{F}_+\|_{C^2}.$$

By the choice of τ_0 , $X_{\lambda\tau_1\bar{F}_+}$ has no nontrivial 1-periodic orbit, and thus X_{H_λ} has no nontrivial 1-periodic orbit on $V^+_{\epsilon-2\delta}$. This leads to a contradiction. Hence $c \le \epsilon - 2\delta$. Similarly, we can prove that $c \ge -\epsilon + 2\delta$. Since we have assumed that $x = x(t)$ be a nontrivial 1-period orbit of $\dot{x} = X_{H_\lambda}(x)$ and contractible in V, then $y(t) := x(\lambda t)$ is a nontrivial $\frac{1}{\lambda}$-periodic orbit of $\dot{y} = X_H(y)$ and $H(y(t)) \equiv c$. Hence y is a nontrivial closed characteristic on S_c and also contractible in V. This contradicts to the **Assumption** above . □

Notice that H has no critical point on an open neighborhood of $\Psi(S\times[-\epsilon+2\delta,\epsilon-2\delta])$ and in order to make generic small perturbation for H so that the perturbed $\hat{H}$ satisfies Th 7.4 in [FHS] it suffice to perturb H near critical points of it. We can require that the perturbed $\hat{H}$ agree with H in a small open neighborhod of $\Psi(S\times[-\epsilon+2\delta,\epsilon-2\delta])$ and is alway more than $\epsilon-2\delta$ on $V^+_{\epsilon-2\delta}$ and less that $-\epsilon+2\delta$ in $V^-_{-\epsilon+2\delta}$.

Having these we can show that the arguments in [LiuT] atually prove the existence of contractible (in V) closed characteristic in Theorem 1 if $g=0$. In order to see this point we recall that the introduction of Definition 2.1 in [LiuT] was based on Lemma 2.2 in [LiuT]. When H satisfies the Lemma above, the Lemma 2.2 in [LiuT] showed that a sequence of $(J_{\lambda_i},H_{\lambda_i})$- maps $\{f_i\}$ of class A connecting c_- and c_+, with $\lambda_i\in[\varepsilon,\lambda_0+1/2]$ for some small ε must weakly C^∞-converges to a cuspidal $(J_{\lambda_\infty},H_{\lambda_\infty})$- map f_∞ of the same class A connecting c_- and c_+ in the sense of Definition 2.1 of [LiuT]. Here the key is that 1-periodic orbits as the joint points of different broken trajectories of f_∞ must be critical points of H because of the assumptions of H_λ. Notice that 1-periodic orbits as the joint points of different broken trajectories of f_∞ must be contractible in V. Hence our above choice of H can still gurantee that 1-periodic orbits as the joint points of different broken trajectories of f_∞ must be critical points of H. Thus in the case f_∞ is still the cuspidal (J_λ,H_λ)-map in the sense of Liu-Tian's Definition 2.1. Hence their all arguments still work.

Remark In view of the definition of ϕ on page 3 of [LiuT] "$H(c_+)-H(c_-)$" in (ii) of the Lemma above was mistakenly written as "$H(c_-)-H(c_+)$" in Lemma 2.1 of [LiuT]. Thus the equation in (i) of [LiuTian, p.11] must be changed as

$$\frac{\partial f_i^P}{\partial s}+J(f_i^P)\frac{\partial f_i^P}{\partial\theta}+\nabla H(f_i^P)=0.$$

Of course, the proofs of Lemma 7.2 and Theorem 8.1 should also be changed sutiably.

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