Diversifying Risk Parity

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Risk-Based Portfolio Construction

- Given perfect foresight the Markowitz (1952) approach is the rationale of choice to generate efficient portfolios with an optimal risk-return trade-off
- Mean-variance optimization is confounded by estimation risk, especially the one in estimates of expected returns
- One solution: Refrain from estimating returns and resort to risk-based allocation techniques
- Minimum-variance portfolios are characterized by low volatility and still provide quite favorable return figures raising investors' interest in risk-based concepts

Diversification Pays...but How to Diversify?

- Minimum-variance portfolios typically load on low-volatility assets rendering them rather concentrated in few assets
- What about diversification? How to define diversification?

Literature review

- *Number of assets* (Evans and Archer, 1968)
- *Herfindahl Index* (Persson, 1993)
- *Entropy of portfolio weights* (Bera and Park, 2008)

Diversifying weights versus diversifying risk

Using a PCA of the portfolio assets Meucci (2009) extracts the uncorrelated risk sources and determines the *effective number of uncorrelated bets*

Diversified Risk Parity Strategies

Maximum diversification obtains for a risk parity strategy along the uncorrelated risk sources which we dub *diversified risk parity (DRP)*

Lohre, Opfer, and Orszag (2012)

- Asset allocation study
- Demystifying uncorrelated risk sources
- Horse race: *DRP* vs. $1/N$, *minimum-variance*, *risk parity*, or the *most-diversified portfolio*

Lohre, Neugebauer, and Zimmer (2012)

- Equity portfolio selection within S&P 500
- Demystifying uncorrelated risk sources, horse race
- Link to equity factor portfolios

Managing Diversification, Meucci (2009)

- Consider a portfolio of N assets with returns $\mathbf{R}$. Given weights $\mathbf{w}$ the resulting portfolio return is $R_w = \mathbf{w}'\mathbf{R}$
- Diversification pays when combining low-correlated assets: Construct uncorrelated risk sources by applying a principal components analysis (PCA) to the VCV $\mathbf{\Sigma}$
- From the spectral decomposition theorem it follows that

$$\mathbf{\Sigma} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}' \tag{1}$$

  where $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, ..., \lambda_N)$ consists of $\mathbf{\Sigma}$'s eigenvalues
- The columns of matrix $\mathbf{E}$ represent the eigenvectors of $\mathbf{\Sigma}$ which define a set of N uncorrelated *principal portfolios* with variance λ_i for $i = 1, ..., N$ and returns $\tilde{R} = \mathbf{E}'\mathbf{R}$

Multi-Asset Data

Standard multi-asset data ranging from Dec 1987 to Sep 2011:

- Government Bonds: Most favorable risk-adjusted return
- Remaining asset classes with similar return but higher volatility
- Correlations generally low but not zero

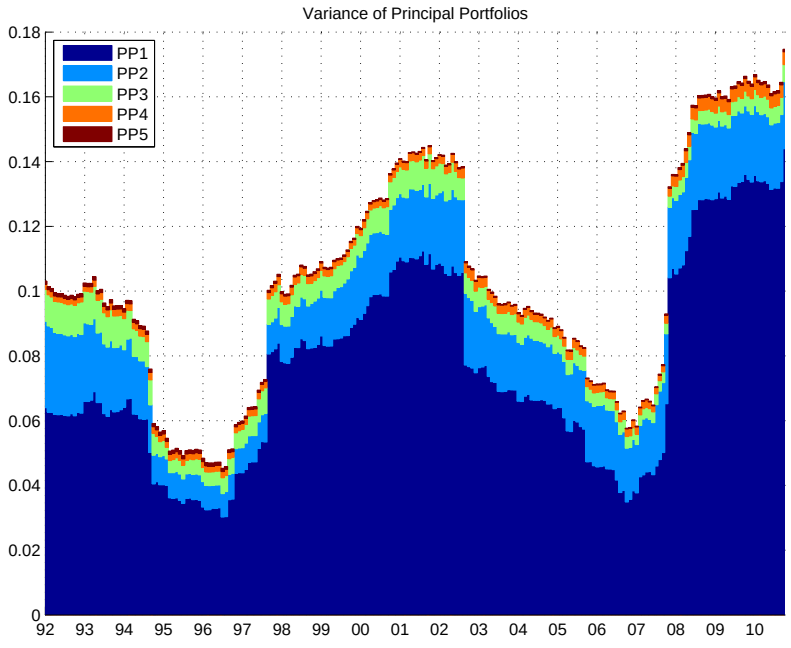
	Return p.a.	**Vola** p.a.	**SR**	**Correlation Matrix**				
				Bonds	Equities		Comm.	Credit
					Dev.	Emg.		
JPM Global Bond	6.9%	3.8%	0.96	**1.00**				
MSCI World	5.9%	14.5%	0.18	0.01	**1.00**			
MSCI Emerging Markets	8.3%	24.2%	0.21	-0.05	0.74	**1.00**		
DJ UBS Commodities	5.7%	15.5%	0.16	-0.18	0.18	0.32	**1.00**	
Barclays U.S. Aggr. Credit	6.9%	5.3%	0.69	0.53	0.25	0.20	0.09	**1.00**

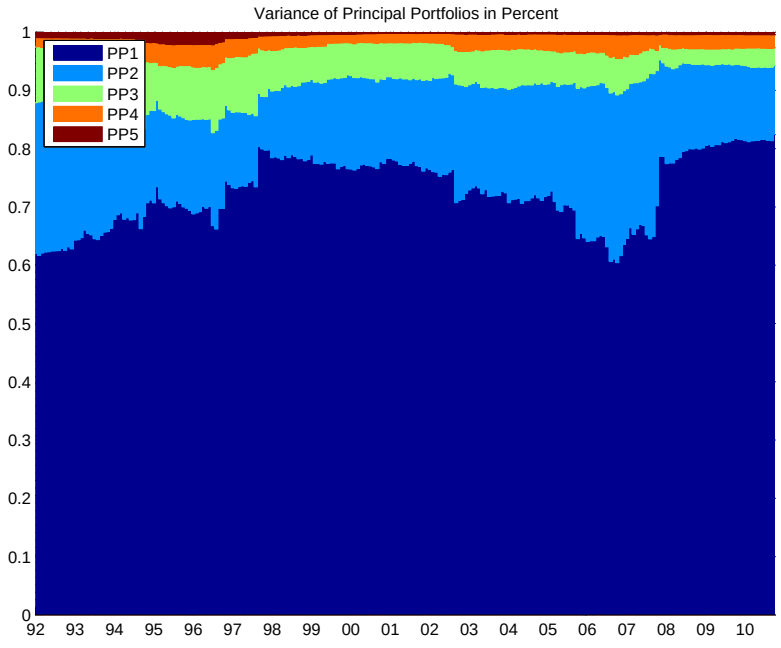
- Apply PCA to generate uncorrelated principal portfolios

Demystifying Principal Portfolios

Asset Class	PP1	PP2	PP3	PP4	PP5
Panel A: December 1987 to September 2011					
JPM Global Bond	-0.01	0.05	-0.04	**0.51**	**0.86**
MSCI World	**0.43**	*0.23*	**-0.86**	-0.13	0.03
MSCI Emerging Markets	**0.87**	0.16	**0.47**	0.02	0.01
DJ UBS Commodities	*0.24*	**-0.96**	-0.14	0.01	0.05
Barclays U.S. Aggr. Credit	0.04	0.01	-0.12	**0.85**	**-0.51**
Variance	7.7%	2.2%	0.8%	0.3%	0.1%
Percent Explained	69.8%	19.9%	6.9%	2.8%	0.7%
Cumulative	69.8%	89.7%	96.5%	99.3%	100.0%

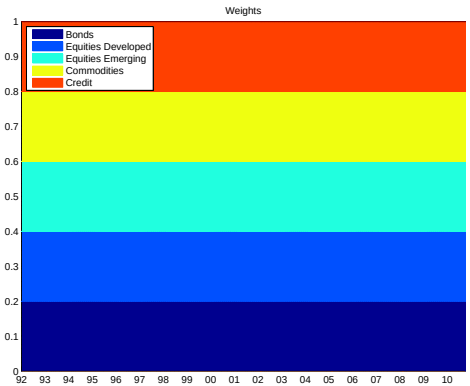
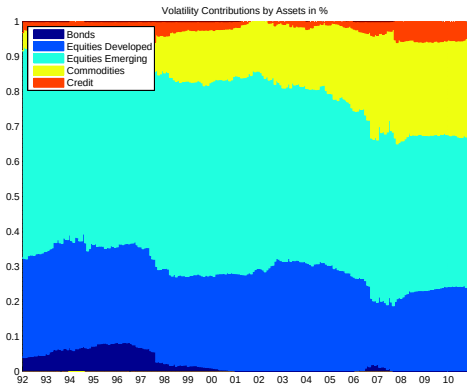
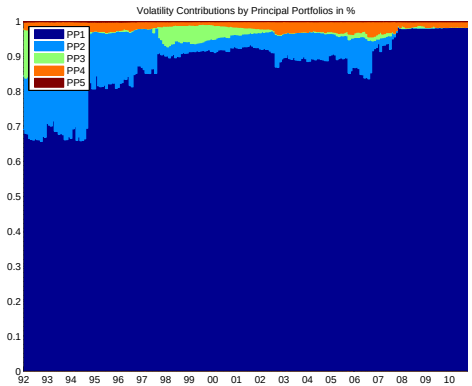
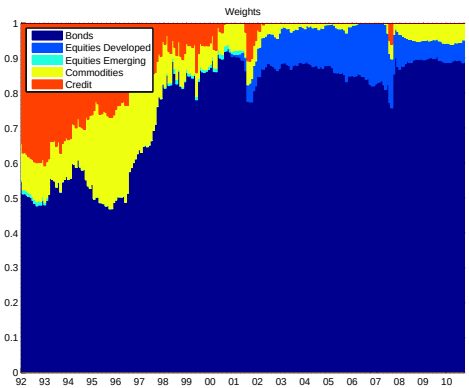
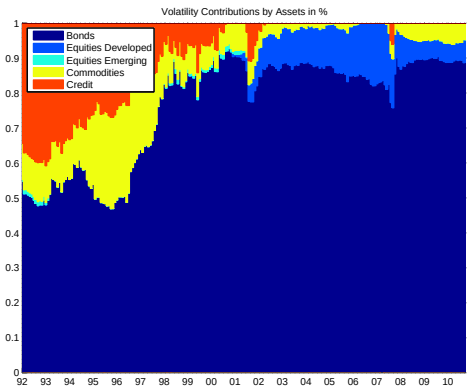
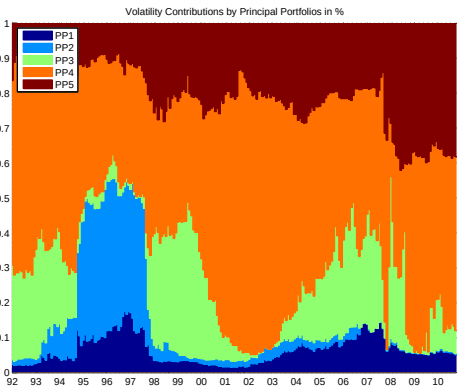
Variances of Principal Portfolios over Time





- PP1 fairly dominant, accounts for at least 60% of variation
- PP2 and PP3 represent some 20% and 10% of the variation, PP4 and PP5 account for a minor fraction
- At the end PP1 accounts for 80% of overall variability indicating contagion effects emanating from the 2008 crisis

Decomposing Risk by Principal Portfolios

	Weights	Vola by Assets	Vola by PPs
1/N			
MV			

- $1/N$ is concentrated in the first principal portfolio
- *MV* is highly concentrated in low-volatility assets

Effective Number of Uncorrelated Bets

Meucci (2009)

- A portfolio is well-diversified when the p_i are *"approximately equal and the diversification distribution is close to uniform"*
- Apply a dispersion metric to the diversification distribution:

$$\mathcal{N}_{Ent} = \exp\left(-\sum_{i=1}^{N} p_i \ln p_i\right) \tag{2}$$

- $\mathcal{N}_{Ent}$ intuitively is the *effective number of uncorrelated bets*:
  - $\mathcal{N}_{Ent} = 1$ holds for a completely concentrated portfolio
  - $\mathcal{N}_{Ent} = N$ holds for a portfolio that is completely homogenous in terms of uncorrelated risk sources

Diversifying Risk Parity

- The maximum diversification portfolio is a risk parity strategy that is budgeting risk along the uncorrelated risk sources rather than the underlying portfolio assets
- We dub this strategy *diversified risk parity (DRP)* and its weights $\mathbf{w}_{DRP}$ obtain by solving

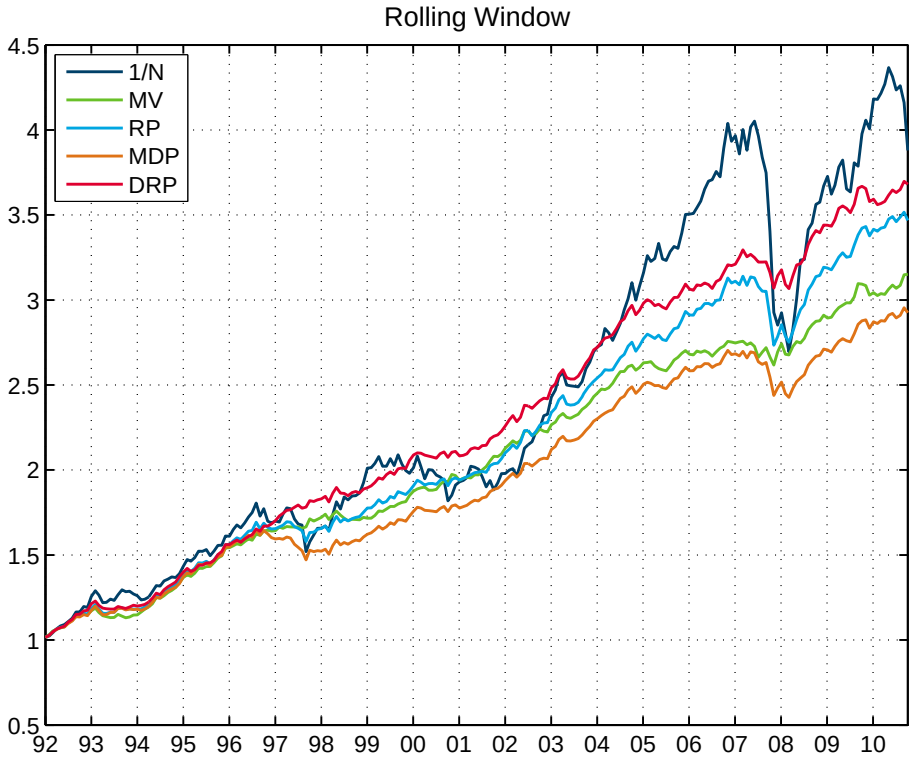
$$\mathbf{w}_{DRP} = \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \mathcal{N}_{Ent}(\mathbf{w}) \tag{3}$$

  where the weights $\mathbf{w}$ can be subject to a set of constraints $\mathcal{C}$
- Moreover, the framework allows for a litmus test of competing techniques like $1/N$, *minimum-variance, risk parity*, or the *most-diversified portfolio* of Choueifaty and Coignard (2008)

Risk-based Asset Allocation

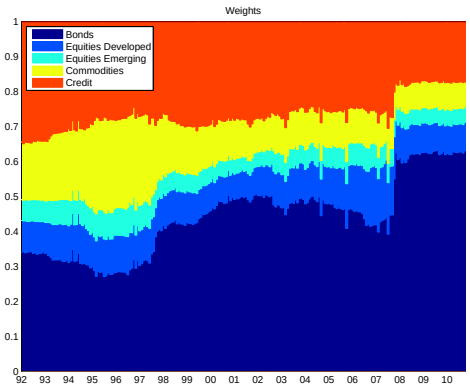
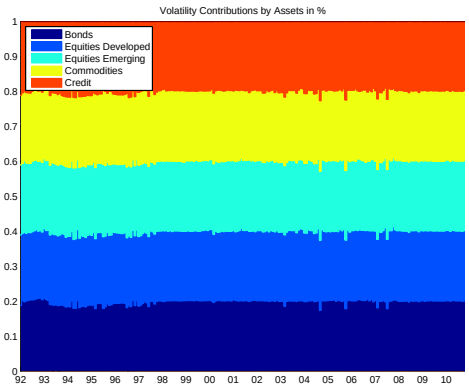
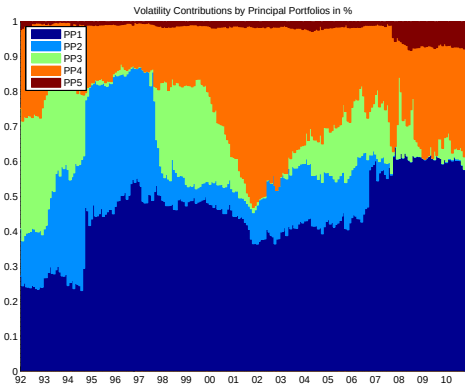
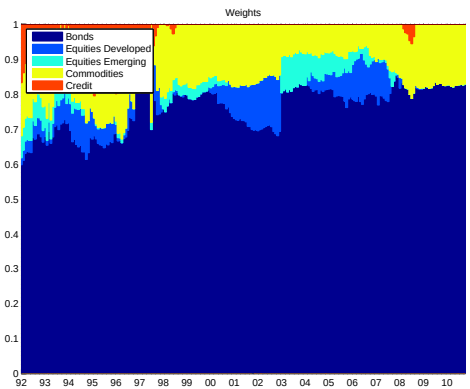
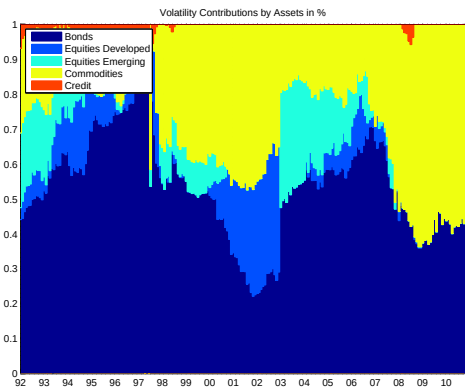
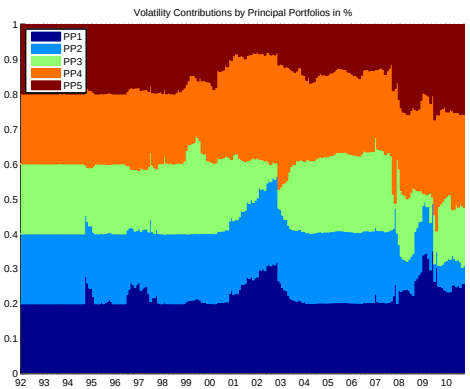
- Constructing the diversified risk parity strategy we determine the principal portfolios via rolling window estimation
- The first PCA consumes 60 months of data, thus, the strategy performance can be assessed from Jan 1993 to Sep 2011
- For benchmarking the diversified risk parity strategy we consider four alternatives
  1. $1/N$
  2. Minimum-variance (MV)
  3. Risk parity (RP)
  4. Most-diversified portfolio (Choueifaty/Coignard, 2008) (MDP)
- We enforce full investment and positivity constraints and rebalance all strategies at a monthly frequency

Performance of Risk-Based Strategies



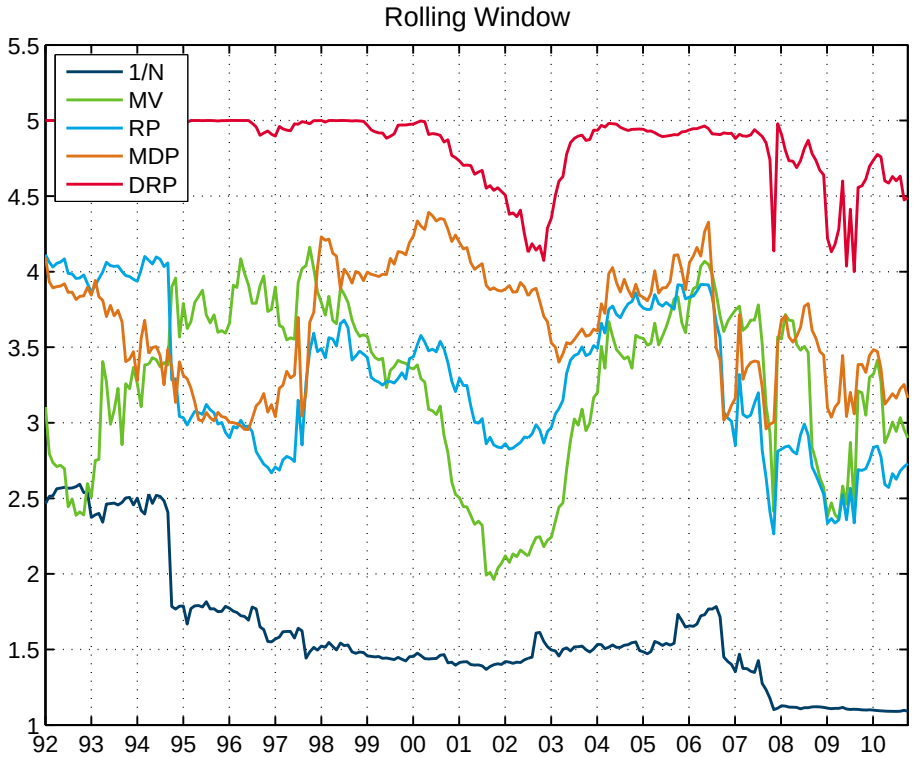
- $1/N$ with highest return and volatility gives lowest SR, MV with reasonable return at lowest volatility gives high SR
- RP: Middle-ground between $1/N$ and MV, MDP still ok
- DRP: Highest SR and convincing MDDs!

Diversified Risk Parity versus Risk Parity

	Weights	Vola by Assets	Vola by PPs
RP			
DRP			

- DRP reacts more timely to changes in risk structure
- Despite constraints DRP is well meeting its objective
- RP is rendered highly concentrated in terms of PPs at the end of the sample period

Diversification throughout Time



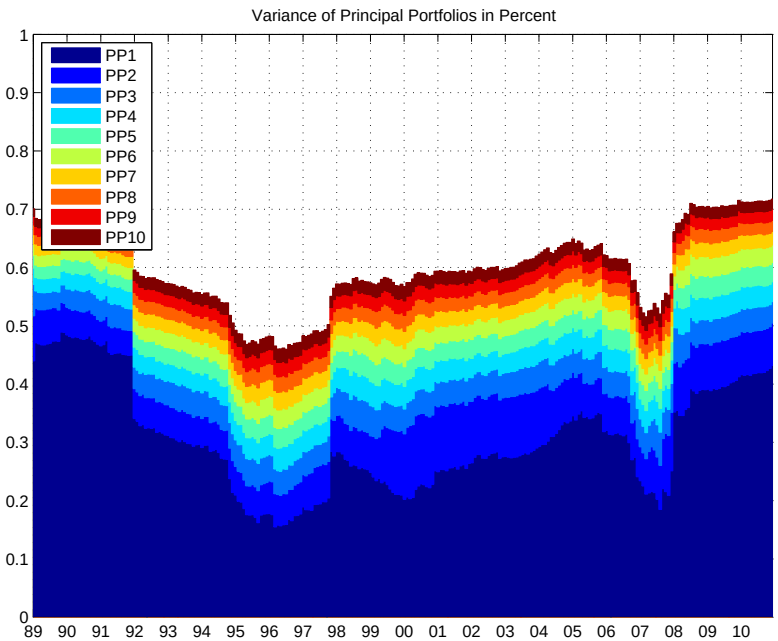
- $1/N$ dominated by the other strategies
- DRP maintains the highest number of bets throughout time
- MV and RP represent 3 bets, MDP close to 4, however, these strategies are losing ground at the end of the sample period

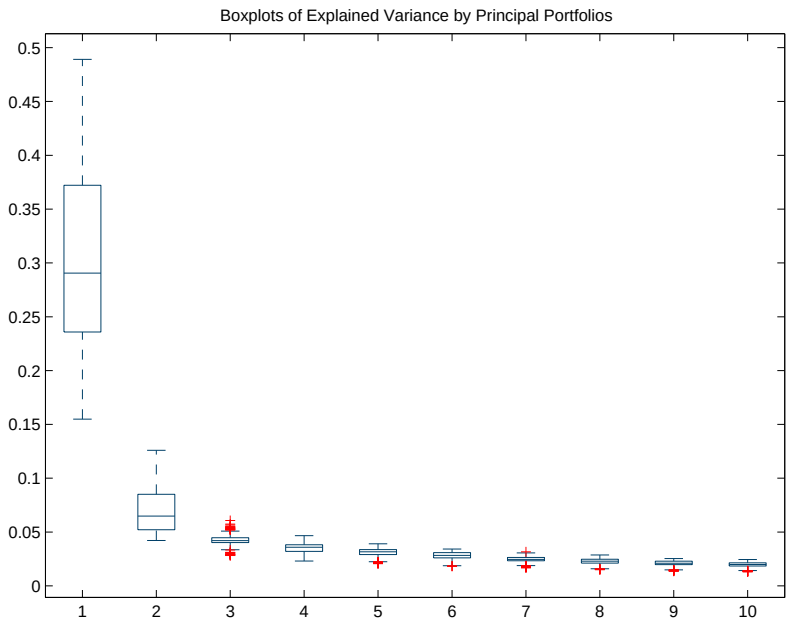
Diversified Risk Parity for Equity Portfolio Selection

- Diversifying *across* asset classes seems reasonable
- What about diversification *within* an asset class like equities?
- Significant exposure to a single (market) risk factor is a well-known issue which is usually addressed by means of diversification across sectors or styles
- The presented framework for achieving maximum diversification is highly appropriate
- We especially
  - determine the number of relevant risk sources
  - associate these principal portfolios to sectors and equity factors
  - document a dynamic DRP strategy to provide convincing performance and diversification characteristics

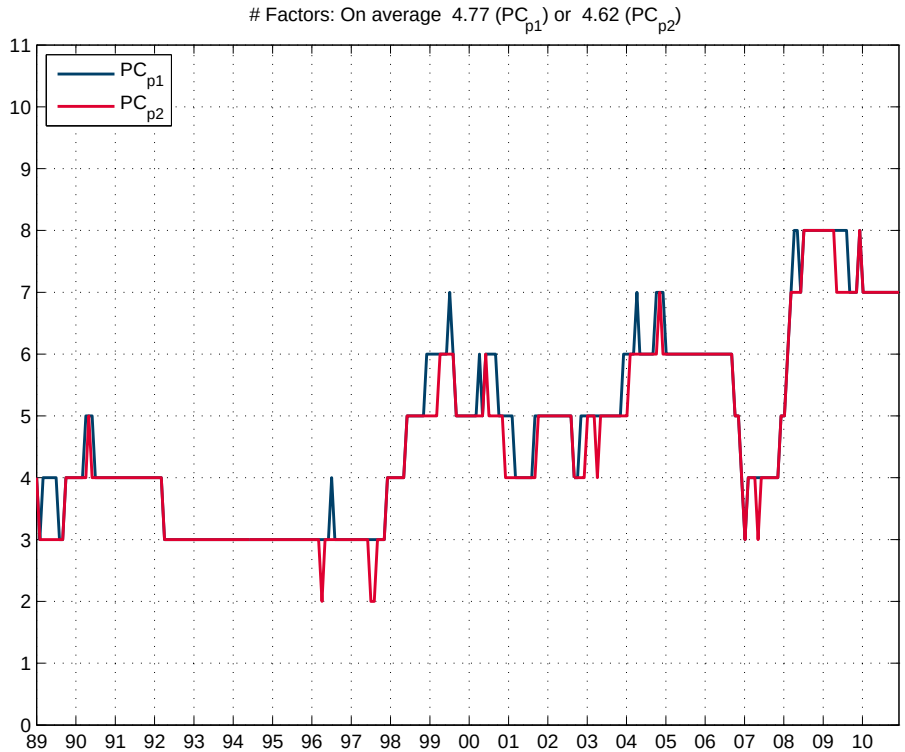
Extracting Principal Portfolios from the S&P 500

- Determine principal portfolios from Oct 1989 to Sep 2011
- In a given month, the PCA is restricted to the then active 500 constituents of the S&P 500
- In total, we deal with 1037 companies over the sample period



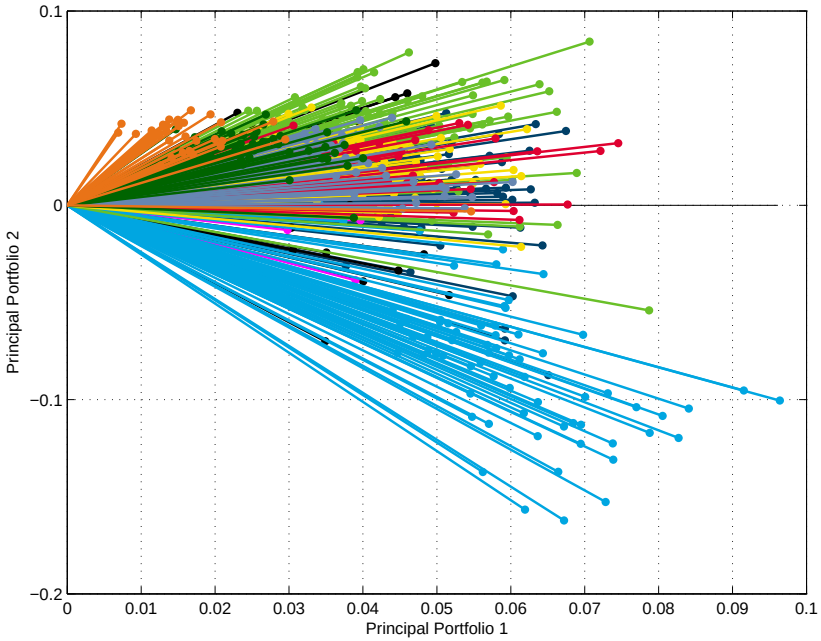


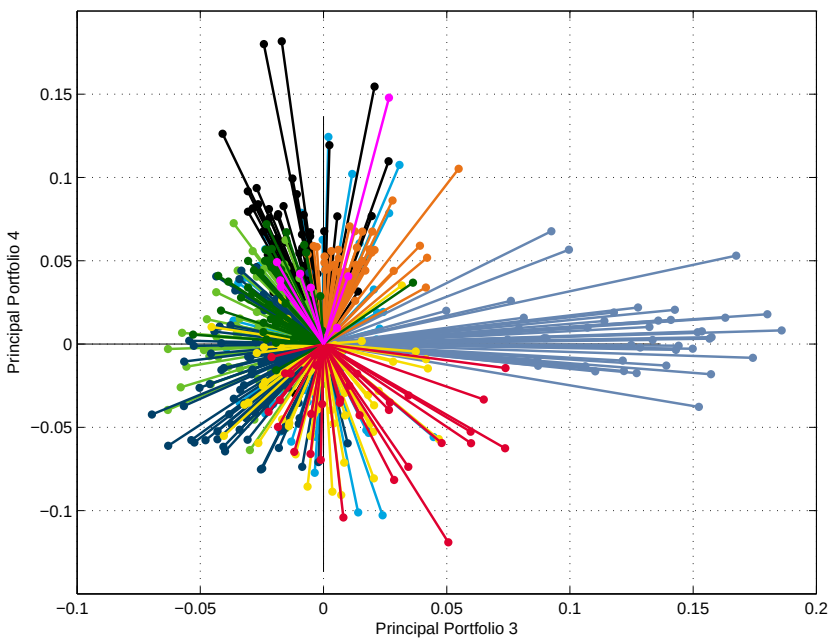
How many risk sources are embedded in the S&P 500?



- Determine a reasonable number of principal portfolios using the PC_{p1} and PC_{p2} criteria of Bai and Ng (2002)
- It is hardly reasonable to allocate any risk budget to higher principal portfolios

Demystifying Principal Portfolios: Sectors?





- PPs arising from a PCA over the most recent 60 months
- PP1 qualifies for a common market factor, PP2 is short IT and long most of the remaining sectors
- PP3 is long Energy and short in Financials, Consumer Discretionary/Staples; PP4 is long Utilities, Health Care, and Telecoms and short Materials and Industrials

Demystifying Principal Portfolios: Factors?

Characterize PPs in an extended Fama-French setting:

$$R_{PPi,t} = \alpha + \beta_1 R_{M,t} + \beta_2 R_{Size,t} + \beta_3 R_{Value,t} + \beta_4 R_{Mom,t} + \beta_5 R_{Vola,t} + \beta_6 R_{Liqui,t} + \varepsilon_t$$

	PP1	*PP2*	*PP3*	*PP4*	*PP5*	*PP6*	*PP7*	*PP8*
Coefficients								
Alpha	**-0.07**	**-0.19**	*-0.08*	*0.06*	-0.03	0.04	0.03	*0.05*
Market	**25.95**	**9.59**	-2.21	**2.15**	**2.23**	0.33	0.42	-1.03
Size	**-20.75**	*-10.41*	**-14.02**	-2.31	-2.58	6.50	**4.86**	**7.00**
Value	**16.61**	**35.79**	**20.62**	**-16.23**	5.36	3.65	**-8.87**	**-10.10**
Mom	**-4.05**	**-6.60**	**9.15**	**7.80**	-0.35	-2.70	-1.49	-0.07
Vola	**5.16**	**-30.63**	**8.36**	-3.31	**5.66**	0.91	-0.95	2.48
Liqui	4.21	**-23.96**	2.07	4.89	**-17.35**	**-19.04**	4.43	7.61
Adj. R^2	94.0%	57.6%	13.0%	22.2%	34.6%	9.6%	8.3%	8.0%

Risk-based Equity Strategies

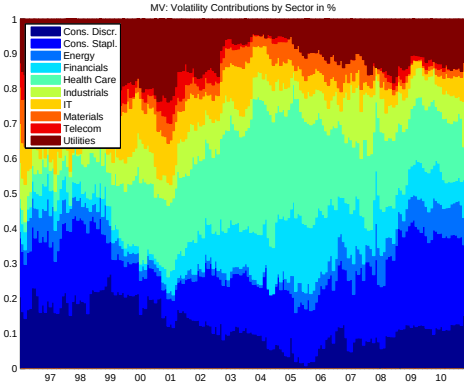
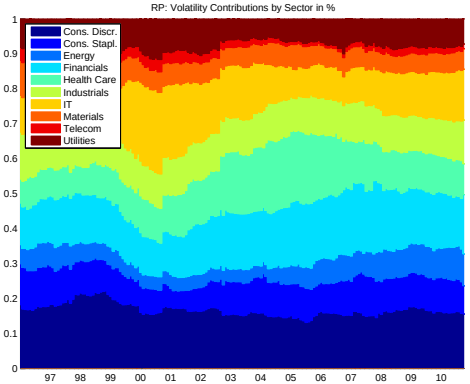
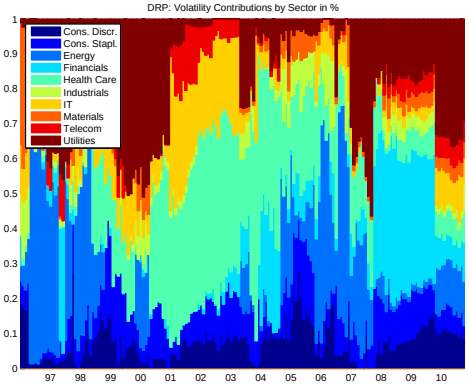
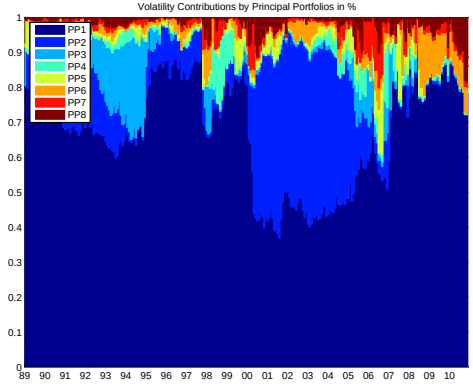
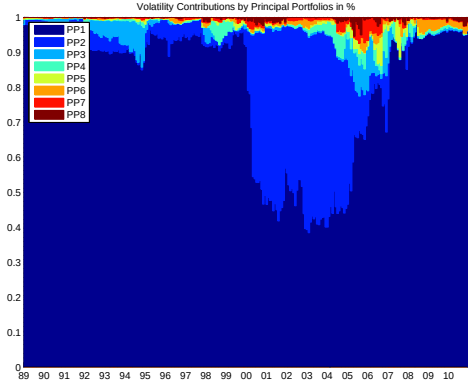
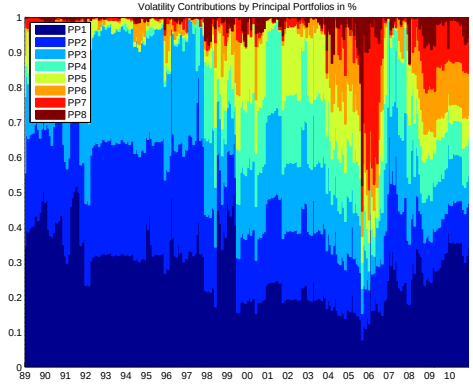
- Constructing the diversified risk parity strategy we obtain PPs by PCA estimation over a rolling 60 months window
- Strategy performance from Oct 1989 to Sep 2011
- For benchmarking the diversified risk parity strategy we consider four risk-based alternatives next to the S&P 500:
  1. $1/N$
  2. Minimum-variance (MV)
  3. Risk parity (RP)
  4. Most-diversified portfolio (Choueifaty/Coignard, 2008) (MDP)
- We enforce full investment and positivity constraints together with maximum stock weights of 5% and rebalance all strategies at a monthly frequency

Performance of Risk-Based Strategies

Statistic	**Index**	**Risk-Based Allocations**				
	S&P 500	*1/N*	*MV*	*RP*	*MDP*	*DRP*
Risk and Return Figures						
Return p.a.	7.5%	9.9%	8.1%	9.2%	7.9%	11.0%
Vola p.a.	13.8%	17.2%	11.8%	14.0%	13.1%	15.1%
Sharpe Ratio	0.28	0.36	0.38	0.39	0.33	0.49
Max DD	-47.5%	-55.9%	-38.2%	-47.6%	-39.6%	-35.8%
# Assets	500.0	500.0	36.2	500.0	38.1	43.4
Turnover	0.4%	2.2%	14.7%	3.7%	16.2%	25.3%

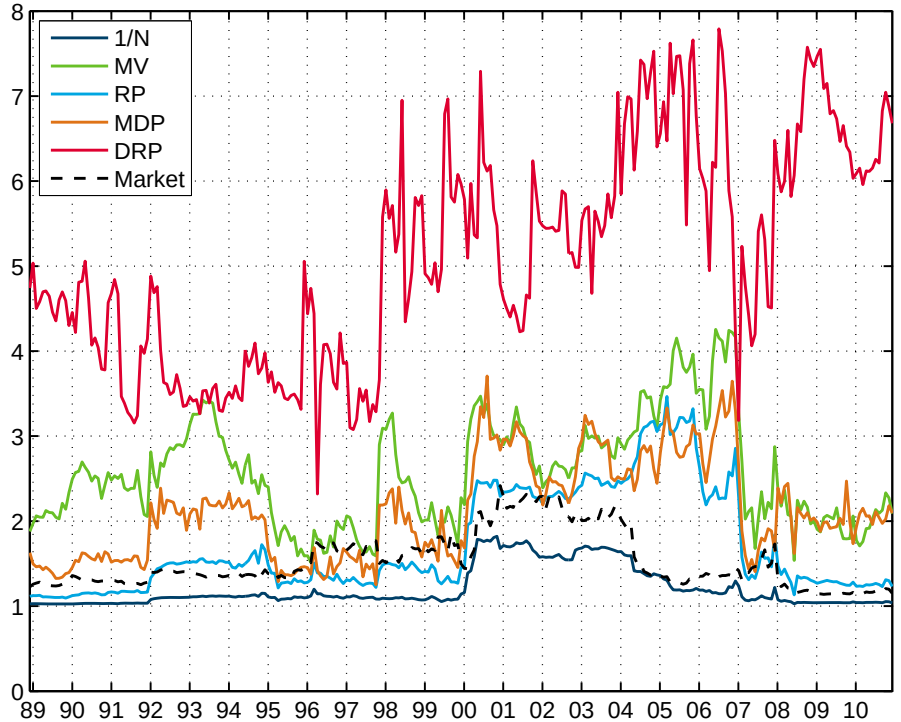
- $1/N$: High return at the highest volatility gives medium SR
- MV: Reasonable return at the lowest volatility gives higher SR
- RP: In between $1/N$ and MV with large MDD
- MDP: MDP lags in terms of returns and SR
- DRP: Highest SR and smallest MDD!

Risk Decomposition by Sectors or Principal Portfolios

	MV	RP	DRP
Vola by Sectors			
Vola by PPs			

- MV: Defensive sectors, concentrated in PP1 and PP2
- RP: Close to $1/N$, highly concentrated
- DRP: Active sector allocation, tracks the number of relevant bets, balanced risk decomposition across PPs

Diversification throughout Time



- $1/N$ dominated by the other strategies
- DRP maintains the highest number of bets throughout time
- MDP and RP represent around 2 bets, MV slightly better, however, these strategies are losing ground at the end of the sample period

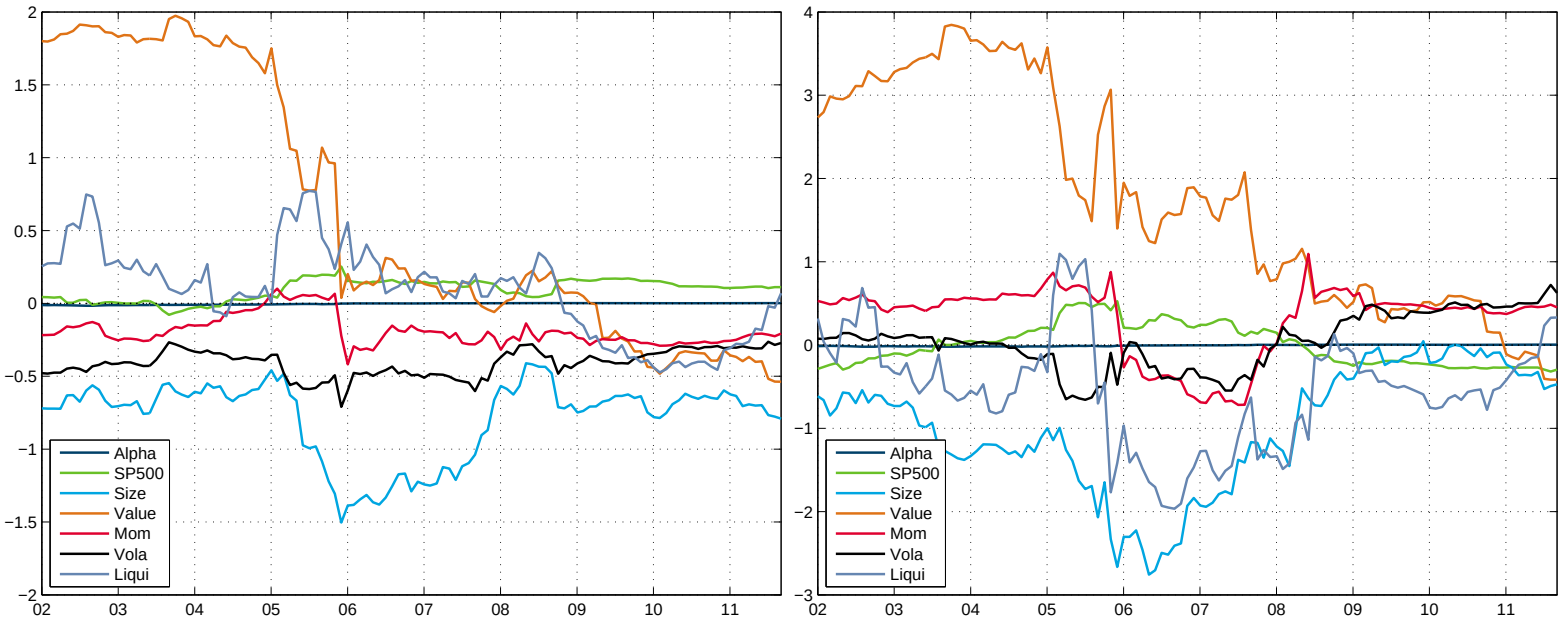
Explaining the Performance of Risk-Based Strategies

$$R_{RBS,t} = \alpha + \beta_1 R_{M,t} + \beta_2 R_{Size,t} + \beta_3 R_{Value,t} + \beta_4 R_{Mom,t} + \beta_5 R_{Vola,t} + \beta_6 R_{Liqui,t} + \varepsilon_t$$

	Index	**Risk-Based Allocations**				
	S&P 500	*1/N*	*MV*	*RP*	*MDP*	*DRP*
Coefficients						
Alpha	0.09%	-0.21%	-0.18%	**-0.26%**	-0.27%	-0.22%
Market	-	**0.15**	-0.05	**0.08**	-0.08	**-0.19**
Size	**1.55**	**-0.65**	**-0.53**	**-0.59**	**-0.67**	-0.50
Value	**1.08**	**1.25**	0.28	**0.93**	0.45	**1.80**
Momentum	*0.25*	**-0.43**	-0.13	**-0.31**	-0.10	0.19
Volatility	**1.72**	-0.03	**-0.72**	**-0.51**	**-0.39**	0.08
Liquidity	**1.02**	-0.09	0.21	-0.16	0.54	-0.56
Adjusted R^2	71.7%	47.2%	43.7%	42.1%	25.8%	17.0%

- MV, RP, and MDP load on the low-volatility anomaly
- DRP small adj. R^2, large value tilt

Equity Factor Exposure over Time



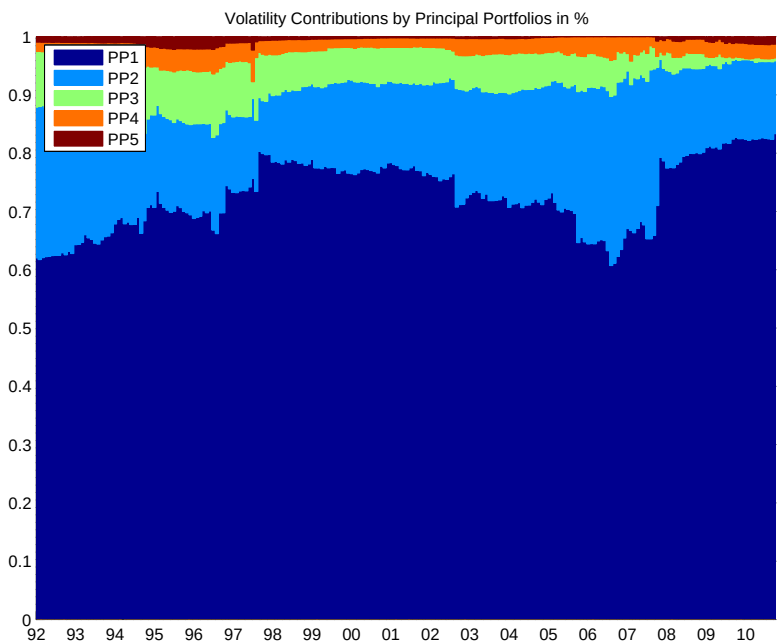
- RP with value and small cap tilt. Constantly loading on volatility factor
- DRP with sizable value tilt that is diminishing over time, no volatility factor exposure, time-varying momentum exposure

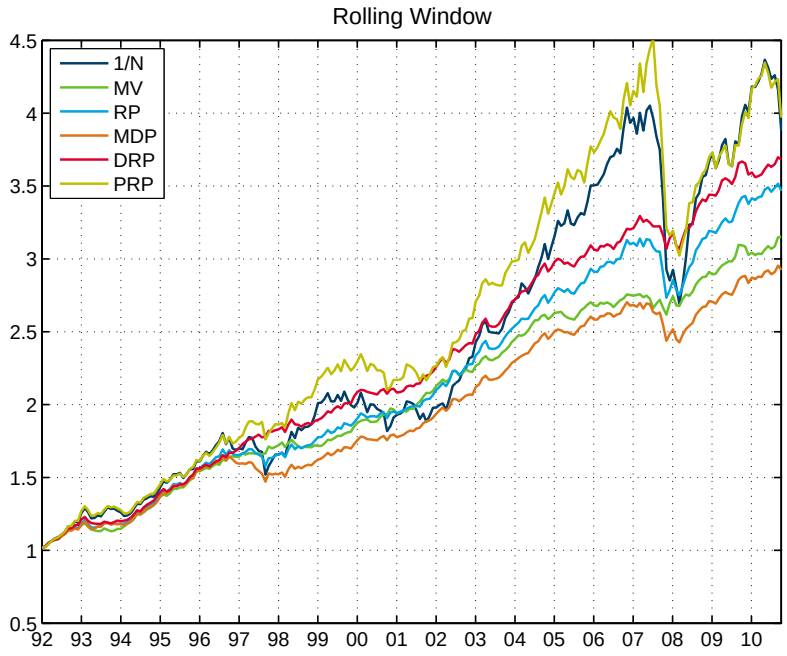
Conclusion

- We have introduced the diversified risk parity strategy that achieves maximum diversification by equally budgeting risk to each of the uncorrelated risk sources
- Besides providing convincing risk-adjusted performance DRP is meeting its diversification objective well:
  - across asset classes
  - within equities
- The competing alternatives tend to be rather concentrated in a few bets
- DRP has a built-in mechanism for tracking the prevailing risk structure thus providing a more robust way for achieving maximum diversification throughout time

Principal Risk Parity for Asset Allocation

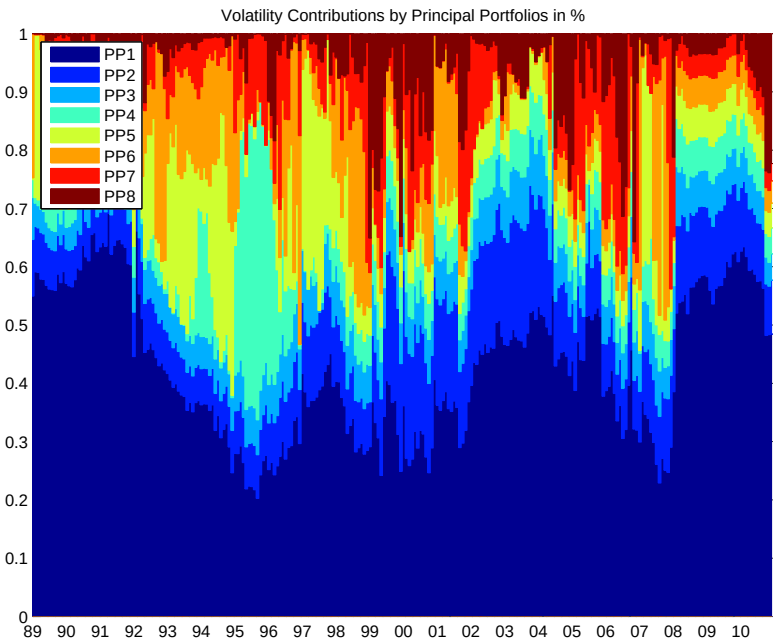
- Principal Risk Parity (PRP): Budget risk across principal portfolios proportional to their contribution to total variance
- PRP strategy is tracking closely the principal portfolio's variance decomposition over time
- Higher return at higher risk!

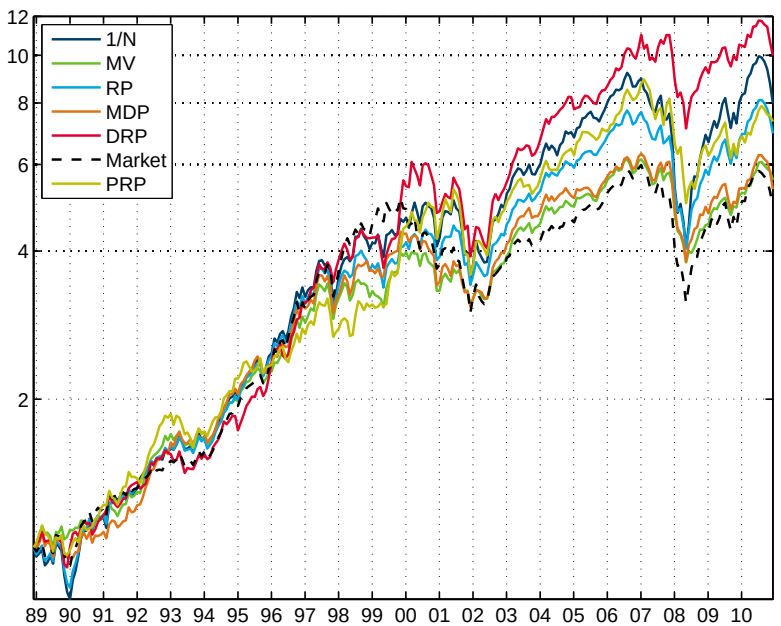




Principal Risk Parity for Equities

- Principal Risk Parity (PRP): Budget risk across principal portfolios proportional to their contribution to total variance
- PRP strategy is tracking closely the principal portfolio's variance decomposition over time
- High return at higher risk!





Risk-Based Commodity Investing

- Bernardi, Leippold, and Lohre (2012) support alternative risk parity strategies for commodities as well
- Commodities are characterized by high heterogeneity—translating into 8 relevant PPs
- DRP and PRP both provide superior performance