Motion and Force Control of Cooperative Robotic Manipulators with Passive Joints

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Abstract

In this paper, robotic systems when two or more underactuated manipulators are working in cooperative way are studied. The underactuation effects on object to be controlled and on load capacity of the cooperative arms are analyzed. A hybrid control of motion and squeeze force is proposed. For the motion control, a Jacobian matrix that relates the torques in the actuated joints to the resulting force in the load is obtained. In addition, a method to compute the dynamic load-carrying capacity of cooperative manipulators with passive joints is presented. Results of the control system are verified in simulations and in an actual system formed by two cooperative arms.

I. INTRODUCTION

Cooperative manipulators have been receiving an increasing attention from the robotics community in the last decades [1], [2], [3], [4], [5], [6], [7], [8], [9]. As in the human case, where the use of two arms represents an advantage over the use of only one arm in several cases, two or more robots can execute tasks that are difficult or even impossible for only one robot [7]. Among these tasks, it can be cited the assembly of structures and the manipulation of heavy, large, or flexible loads.

The control of cooperative manipulators is a complex task due to the interaction between the arms caused by the kinematic and internal forces constraints. The control should be coordinated and the squeeze (internal) forces in the object should be minimized to avoid damage to the load. Several solutions have been proposed to deal with the control problem in fault-free cooperative manipulators rigidly connected to an undeformable load. Such solutions include the master/slave strategy [10], the optimal division of the load control [2], [5], the definition of new task objectives or variables [11], [12], and the hybrid control of motion and squeeze in the object [6], [13], [14]. Despite of presenting good performance for the fault-free case, these controllers, in general, cannot be utilized when one or more joints of the cooperative manipulators are passive.

The authors would like to thank Marcel Bergerman from the Genius Institute of Technology for his contributions, and FAPESP (Proc. 99/10031-1, 04/04289-6) for the financial support.

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Passive joints, which are the joints without actuation, can appear in robotic manipulators as an inherent characteristic of design or as a result of free-swinging joint failures [15]. The design of robots with passive joints is motivated by the necessity of minimization of weight, size, and energy (e.g. snake-like robots). Free-swinging joint failures occur, for example, due to failures in joint actuators, controllers, and power supply (e.g. a lost of electric power in electric motors or a rupture seal on hydraulic actuators) [16]. In some cases, it is necessary to provide fault tolerance to the robot, which can be reached by the control reconfiguration after fault isolation.

Cooperative underactuated manipulators have some important properties that distinguish them from single underactuated manipulators. Though a single manipulator with passive joints is, in general, a nonholonomic system, underactuated cooperative manipulators rigidly connected to an undeformable load have the holonomic property when the number of actuated joints, \( n_a \), is equal or greater than the number of coordinates of motion in the load, \( k \). The analysis of the manipulability of cooperative systems with passive joints, which is studied in [1], can be utilized to choose which joints should be passive, when this is a desired characteristic [3]. In [4], a smooth control law based on the classical PD plus gravity compensation scheme is developed for the motion control of the load when two cooperative arms with \( n_a \geq k \) are present. A Jacobian matrix \( Q(q) \) which relates the velocities of the actuated joints and the load velocity is used to relate the torques of the actuated joints and the resulting force in the load. As only \( k \) actuated joints are necessary for the control of the load position, the authors suggest that the remaining actuated joints can be used to minimize the difference between the squeeze forces and their desired values if \( n_a > k \). They indicate some directives to solve this problem but do not present an explicit solution.

By the other hand, as one of the main justifications for the use of more than one arm is the manipulation of heavy loads, it is important to recalculate the Dynamic Load-Carrying Capacity (DLCC) in order to verify if the system with passive joints can execute the task. When the robots lose one or more actuators, the DLCC generally decreases. The DLCC is defined as the maximum load that can be carried by the system in a specified trajectory. To the best of the authors’ knowledge, the DLCC was analyzed only for full-actuated cooperative systems [9].

In this paper, the problems of controlling two or more cooperative manipulators with passive joints are addressed. The main contributions of this paper are:

1) The proposition of a stable motion control method for the general case where two or more manipulators with passive joints are present in the cooperative system. The control of several cooperative manipulators with passive joints, instead of only two as in [4], is made possible by the introduction of a new Jacobian matrix \( Q(q) \). Besides the proposed controller assuring the stability of the controlled system, it has the advantage that the control law does not use inertia matrices of the robots.

2) The extension of the squeeze force control, independent of the motion control, to the case where there exist passive joints in the cooperative system. The proposed control strategy for the cooperative systems with passive joints presents a hybrid nature where the object motion and \( n_a - k \) components of the squeeze forces are independently treated.

3) A method to measure the load capacity of cooperative systems with passive joints.
This paper is organized as follows: Section II describes the kinematics and the dynamics of cooperative manipulators; Section III develops a controller for the system with passive joints; Section IV presents a procedure to compute the DLCC for underactuated cooperative manipulators; Section V shows the results of the control system in simulations and in an actual system; and, finally, Section VI presents the conclusions.

II. COOPERATIVE MANIPULATORS

Consider a multi-robot system with \( m \) robots rigidly connected to an undeformable load (see Figure 1, where the case \( m = 2 \) is chosen for easy representation). The equation of motion for the \( i \)-th arm of a fault-free multi-robot system with \( m \) robots rigidly connected to an undeformable load is given by

\[
M_i(q_i)\ddot{q}_i + g_i(q_i) + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i - J_i(q_i)^T h_i
\]

where \( \dot{q}_i \) is the vector of joint angles (the cooperative system is formed by \( m \) arms, \( i = 1, \ldots, m \)), \( \tau_i \) is the vector of applied torques, \( M_i(q_i) \) is the inertia matrix, \( C_i(q_i, \dot{q}_i) \) is the matrix of centrifugal and Coriolis terms, \( g_i(q_i) \) is the vector of gravitational terms, \( J_i(q_i) \) is the geometric Jacobian (from joint velocity to end-effector velocity), \( h_i = [f_i^T, \eta_i^T]^T \) is the force vector at the end-effector, \( f_i \) is the vector of spatial forces at the end-effector, and \( \eta_i \) is the vector of torques at the end-effector; the friction terms were not shown for simplicity. The combined dynamics of all arms can be written in only one equation as

\[
M(q)\ddot{q} + g(q) + C(q, \dot{q})\dot{q} = \tau - J(q)^T h
\]

where \( q = [q_1^T, q_2^T, \ldots, q_m^T]^T \), \( \tau = [\tau_1^T, \tau_2^T, \ldots, \tau_m^T]^T \), \( h = [h_1^T, h_2^T, \ldots, h_m^T]^T \), \( g = [g_1^T, g_2^T, \ldots, g_m^T]^T \).

The equation of motion for the manipulated object is given by

\[
M_o \ddot{x}_o + b_o(x_o, \dot{x}_o) = J_o(x_o)^T h
\]

where \( x_o = [p_o^T, \phi_o^T]^T \) is the \( k \)-dimensional vector of position and orientation at the origin of the frame CM attached to the center of mass of the load (Figure 1), \( p_o \) is the vector of load position, \( \phi_o \) is the minimal representation of orientation of the load, \( v_o = [p_o^T, \omega_o^T]^T \) is the \( k \)-dimensional vector of linear and angular velocities of the
Fig. 1. Two 3-joint cooperative manipulators. The second joint of the arm on the left is passive.

load, \( \mathbf{b}_o(\mathbf{x}_o, \mathbf{v}_o) \) is the vector of centrifugal, Coriolis, and gravitational terms, \( \mathbf{M}_o \) is the load inertia matrix, and \( \mathbf{J}_o(\mathbf{x}_o) = \begin{bmatrix} \mathbf{J}_o1(\mathbf{x}_o)^T & \ldots & \mathbf{J}_om(\mathbf{x}_o)^T \end{bmatrix}^T \), where \( \mathbf{J}_o(\mathbf{x}_o) \) converts velocities of the load into velocities of the end-effector of arm \( i \) and is given by

\[
\mathbf{J}_o(\mathbf{x}_o) = \begin{bmatrix} \mathbf{I} & \mathbf{a}_{o_i} \times \\
0 & \mathbf{I} \end{bmatrix}
\]

where \( \mathbf{I} \) is the identity matrix, \( \mathbf{a}_{o_i} \) is the vector from the origin of frame CM to the contact point between the load and the end-effector of arm \( i \) (Figure 1), and \( \times \) denotes cross product in a coordinate representation. In the three-dimensional space, \( \mathbf{p}_o = [x_o \ y_o \ z_o]^T \), and either Euler angles or RPY (roll-pitch-yaw) angles can be chosen as the minimal representation of the load orientation, i.e., \( \phi_o = [\varphi_o \ \nu_o \ \psi_o]^T \). The velocities \( \mathbf{v}_o \) can be calculated by \( \mathbf{v}_o = \mathbf{T}(\mathbf{x}_o)\dot{\mathbf{x}}_o \) [17], where \( \mathbf{T}(\mathbf{x}_o) \) is a transformation matrix used to relate the angular velocities to the derivative of the minimal representation of the orientation (Euler angles or RPY angles) in the 3-dimensional space (\( \mathbf{T}(\mathbf{x}_o) = \mathbf{I} \) for planar manipulators).

As it is possible to compute the positions and orientations of the load using the positions of the joints of any arm, the cooperative system presents the following kinematic constraint

\[
\mathbf{x}_o = \varphi_1(\mathbf{q}_1) = \varphi_2(\mathbf{q}_2) = \ldots = \varphi_m(\mathbf{q}_m) \tag{4}
\]

where \( \varphi_i(\mathbf{q}_i) \) is the vector of the position and orientation of the load computed via the joint positions of arm \( i \), i.e., the direct kinematics of arm \( i \). The velocities of the load are constrained by the equalities

\[
\mathbf{v}_o = \mathbf{D}_1(\mathbf{q}_1)\dot{\mathbf{q}}_1 = \mathbf{D}_2(\mathbf{q}_2)\dot{\mathbf{q}}_2 = \ldots = \mathbf{D}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m \tag{5}
\]

where \( \mathbf{D}_i(\mathbf{q}_i) = \mathbf{J}_o(\mathbf{x}_o)^{-1}\mathbf{J}_i(\mathbf{q}_i) \) is the Jacobian relating joint velocities of arm \( i \) and load velocities.

With the rigidity assumption, the forces of the end-effectors projected to frame CM (\( \mathbf{h}_o = \mathbf{J}_{oq}^T\mathbf{h} \), where the \( \mathbf{J}_{oq}^T \) is the projection matrix) can be decomposed as

\[
\mathbf{h}_o = \mathbf{h}_{os} + \mathbf{h}_{om} \tag{6}
\]
where \( h_{os} \in X_s \) is the vector of the squeeze forces [18] and \( h_{om} \in X_m \) is the vector of the motion forces, i.e., the forces and torques in the object that contribute only to the motion. The subspace \( X_m \) is called move subspace and its orthogonal complement \( X_s \) is called squeeze subspace. It is important to observe that \( \dim(X_m) = k \) and \( \dim(X_s) = k(m - 1) \). The squeeze subspace is given by the kernel of \( A^T = [I \ldots I] \), \( X_s = \ker(A^T) \). \( A^T \) transforms the \( mk \)-dimensional vector \( h_o \) into the \( k \)-dimensional vector of the resulting force at the object frame CM.

Besides, joint torques in the form \( D(q)^T h_{sc} \), where \( D(q) = [D_1(q_1) \ D_2(q_2) \ldots D_m(q_m)] \) and \( h_{sc} \in X_s \), do not affect the motion if the arms configurations are not singular. However, the motion of the arms affects the squeeze forces due to the squeeze components of the d’Alembert (inertial) forces. In this way, the vector or the squeeze forces can be decomposed as

\[
h_{os} = h_{sm} + h_{sc} \quad (7)
\]

where \( h_{sm} \) is the term of the squeeze forces induced by the motion and \( h_{sc} \) is the squeeze component that is not affected by the motion.

Thus, in [13], a stable motion control with compensation of the gravitational torques is designed for the full-actuated cooperative system ignoring the squeeze forces and a squeeze control is designed considering the component of the squeeze forces, caused by the motion (\( h_{sm} \)), as disturbance. In the next section, it is developed a control strategy for multiple manipulators with passive joints.

III. CONTROL OF COOPERATIVE ARMS WITH PASSIVE JOINTS

The cooperative manipulators control with passive joints is decomposed in motion control and in squeeze force control. A stable motion control with compensation of the gravitational torques is firstly designed ignoring the squeeze forces when \( n_a \geq k \). For this purpose, the Jacobian matrix \( Q(q) \) that relates the velocities in the active joints to the load velocities (or the torques in the actuated joints to the resulting forces in the load, by using the virtual work principle) should be calculated. Then, for the case \( n_a > k \), the squeeze control is designed considering the component of the squeeze forces, caused by the motion, as disturbance. In summary, the control law applied in the actuated joints for the system with passive joints is given by

\[
\tau_a = \tau_{mg} + \tau_s \quad (8)
\]

where \( \tau_{mg} \) is the motion control law with compensation for the gravitational torques and \( \tau_s \) is the squeeze control law.

In the following, a method to compute the Jacobian matrix \( Q(q) \) for the cooperative system with \( m > 1 \) is proposed. From (5)

\[
m \dot{q}_o = D_1(q_1) \dot{q}_1 + D_2(q_2) \dot{q}_2 + \ldots + D_m(q_m) \dot{q}_m. \quad (9)
\]

Assume that \( n_a \) joints are actuated and \( n_p \) joints are passive from the \( n \) joints of all arms. The positions of the passive joints are grouped in the vector \( q_p \) and the positions of the actuated joints are grouped in the vector \( q_a \)
(Figure 1). Partitioning (9) in quantities related to the passive and actuated joints

\[ m\dot{v}_o = \sum_{i=1}^{m} D_a(q_i)\dot{q}_a + \sum_{i=1}^{m} D_p(q_i)\dot{q}_p = D_a(q)\dot{q}_a + D_p(q)\dot{q}_p \]  

(10)

where \( a \) refers to the actuated joints and \( p \) to the passive joints. Two cases can be considered from (5). When \( m \) is even

\[ \sum_{i=1}^{m} (-1)^{i+1} D_i(q_i)\dot{q}_i = 0 \]  

(11)

that can be partitioned in order to relate the velocities of the actuated and passive joints as

\[ \sum_{i=1}^{m} (-1)^{i+1} D_a(q_i)\dot{q}_a + \sum_{i=1}^{m} (-1)^{i+1} D_p(q_i)\dot{q}_p = R_a(q)\dot{q}_a + R_p(q)\dot{q}_p = 0 \]  

(12)

(it is interesting to observe that such relation cannot be found in an individual manipulator with passive joints [4]) and when \( m \) is odd

\[ \sum_{i=1}^{m} (-1)^{i+1} D_i(q_i)\dot{q}_i = v_o \]  

(13)

that can also be partitioned in terms of the actuated and passive joints

\[ R_a(q)\dot{q}_a + R_p(q)\dot{q}_p = v_o. \]  

(14)

Using (10), (12), and (14), the velocities of the load are related to the velocities of the actuated joints by

\[ v_o = Q(q)\dot{q}_a \]  

(15)

where, if \( m \) is even

\[ Q(q) = \frac{1}{m} \left( D_a(q) - D_p(q)R_p(q)\#R_a(q) \right) \]  

(16)

\( (R_p(q)\# \) is the pseudo-inverse of matrix \( R_p(q) \)) and if \( m \) is odd

\[ Q(q) = \left( mI - D_p(q)R_p(q)\# \right)^{-1} 
\[ \left( D_a(q) - D_p(q)R_p(q)\#R_a(q) \right). \]  

(17)

One can observe that \( R_p(q) \) needs to be full rank, with dimension \( kn_p \) where \( k \geq n_p \). Thus, the following assumptions are considered in this paper

**Assumption 1**: The number of actuated joints \( n_a \) is not smaller than the dimension of the object coordinates \( k \), which is not smaller than the number of passive joints \( n_p \), i.e., \( n_a \geq k \geq n_p \).

**Assumption 2**: The matrix \( R_p(q) \) is full rank.
Assumption 2 means that singularities in $R_p(q)$, which are determined by the number and positions of the passive joints, must be avoided. The robot configurations where $R_p(q)$ is not full rank are discussed in [4]. Examples with cooperative systems formed by two planar manipulators and by two Puma robots indicate that the regions where $R_p(q)$ is not full rank are limited.

The Jacobian matrix $Q(q)$ has important kinematic properties and can be used to compute the manipulability of cooperative system with passive joints. The analysis of the manipulability of cooperative system with passive joints, which was studied in [1], can be utilized to choose which joints should be passive, when this is a desired characteristic [3].

A. Motion Control

It is possible to partition (2) as

$$\ddot{\mathbf{M}}\ddot{\mathbf{q}} + \dot{\mathbf{C}}\dot{\mathbf{q}} + \mathbf{g} = \mathbf{\tau} - \ddot{\mathbf{J}}\mathbf{h} \quad (18)$$

where the terms in parenthesis were not written for simplicity and

$$\dot{\mathbf{q}} = \begin{bmatrix} q_a^T & q_p^T \end{bmatrix}^T, \quad \mathbf{g} = \begin{bmatrix} g_a^T & g_p^T \end{bmatrix}^T,$$

$$\ddot{\mathbf{J}} = \begin{bmatrix} \mathbf{J}_a & \mathbf{J}_p \end{bmatrix}^T, \quad \dot{\mathbf{r}} = \begin{bmatrix} \tau_a^T & 0 \end{bmatrix}^T, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_a^T & \mathbf{C}_p^T \end{bmatrix}^T,$$

and

$$\ddot{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ap} \\ \mathbf{M}_{pa} & \mathbf{M}_{pp} \end{bmatrix}.$$

In the controller developed here, the matrix $Q(q)$ is used to design motion forces proportional to the position and velocity errors in the actuated joints space. The gravity forces and torques in the load and in the passive joints are compensated by the actuated joints using the Jacobian matrices $R_p(q), R_a(q),$ and $Q(q)$. In this way, the following motion control law is proposed for the actuated joints

$$\mathbf{\tau}_{mg} = \mathbf{\tau}_m + \mathbf{\tau}_g \quad (19)$$

where the motion component is given by

$$\mathbf{\tau}_m = \mathbf{Q}^T(\mathbf{T}^{-T}\mathbf{K}_p\Delta\mathbf{x}_a + \mathbf{K}_v\Delta\mathbf{v}_a) \quad (20)$$

where $\Delta\mathbf{x}_a = (\mathbf{x}_{od} - \mathbf{x}_a)$ is the load position error, $\mathbf{x}_{od}$ is the desired position of the load, the diagonal matrices $\mathbf{K}_p$ and $\mathbf{K}_v$ are positive definite, $\Delta\mathbf{v}_a = (\mathbf{v}_{od} - \mathbf{v}_a)$ is the load velocity error, $\mathbf{v}_{od}$ is the desired velocity of the load, and the singularity of $\mathbf{T}$ depends on the choice of the minimal representation of orientation.

The compensation for the gravitational torques is given by

$$\mathbf{\tau}_g = \mathbf{g}_a - \mathbf{A}_o^T\mathbf{g}_p + (\mathbf{J}_a^T - \mathbf{A}_o^T\mathbf{J}_p^T)f_o \quad (21)$$

where

$$\mathbf{A}_o = \begin{cases} \mathbf{R}_p \# \mathbf{R}_a & \text{if } m \text{ is even} \\ \mathbf{R}_p \# (\mathbf{R}_a - \mathbf{Q}) & \text{if } m \text{ is odd} \end{cases}$$

and $f_o$ is an $mk$-dimensional vector chosen to satisfy

$$\mathbf{J}_o^Tf_o = b_o. \quad (22)$$
Based on this motion control law, the following theorem is proposed

**Theorem 1**: Assume that Assumptions 1 and 2 are satisfied and the desired trajectories belong to $S = \{v_{od}(t) \text{ and } \dot{v}_{od}(t) \in L_2([0,\infty)) : v_{od}(t) \text{ and } \dot{v}_{od}(t) \text{ are uniformly continuous}\}$. Let the control law be given by (19)-(21), then

a) The cooperative system is asymptotically stable, i.e., the velocities of the load are convergent to zero as $t \to \infty$;

b) The position error $\Delta x_o$ is convergent to the manifold given by

$$Q^T T^{-T} K_p \Delta x_o + (J_a^T - A_o^T J_p^T) J_{ooq}^{-T} h_{sc} = 0.$$  

**Proof**: Consider firstly a class of desired trajectories with desired velocities equal to zero (set point control problem) and the following Lyapunov function candidate

$$V = \frac{1}{2} v_{od}^T M_o v_{od} + \frac{1}{2} \dot{\hat{q}}^T \hat{M} \hat{q} + \frac{1}{2} \Delta x_o^T K_p \Delta x_o$$  

(23)

where the sum of the two first terms are equal to the kinetic energy of the system.

Differentiating (23), considering (3), (18), and antisymmetric ($\dot{M} = 2\bar{C}$), then

$$\dot{V} = -v_{od}^T b_o - \dot{\hat{q}}^T \bar{C} \bar{q}_a - \dot{\hat{q}}^T \bar{G} \bar{q}_p + \dot{\hat{q}}^T \tau_a + \Delta x_o^T K_p \Delta \dot{x}_o.$$  

(24)

Substituting (19) in (24)

$$\dot{V} = -v_{od}^T K_v v_o \leq 0$$  

(25)

which, by using the Invariance Principle of Lasalle, implies the asymptotic convergence of $v_{od}(t)$ to zero. Thus, the load always goes to the steady state under the control law given by (19).

In the following, the trajectory tracking control problem is addressed. The following Lyapunov function candidate is chosen for solving this problem

$$V = \frac{1}{2} \Delta v_{od}^T M_o \Delta v_{od} + \frac{1}{2} \Delta \dot{\hat{q}}^T \bar{M} \Delta \dot{\hat{q}} + \frac{1}{2} \Delta x_o^T K_p \Delta x_o$$  

(26)

where $\Delta \dot{\hat{q}} = (\dot{\hat{q}}_d - \dot{\hat{q}})$, being $\dot{\hat{q}}_d$ the projection of $v_{od}$ in the joint space obtained using (10), and $\bar{D} = [D_a \ D_p]$. It is important to observe that the errors $\Delta \dot{\hat{q}}$ and $\Delta \ddot{\hat{q}}$ are not present in the control law, and $\Delta \dot{\hat{q}}$ is used here only in the stability proof for the trajectory tracking control.

Differentiating (26), considering (3) and (18), and applying the control law given by (19-21), then

$$\dot{V} = v_{od}^T M_o \dot{v}_{od} - v_{od}^T M_o \dot{v}_{od} + \dot{\hat{q}}^T \bar{M} \ddot{\hat{q}}_d - \dot{\hat{q}}^T \bar{M} \ddot{\hat{q}}_d$$

$$+ \dot{\hat{q}}^T \left( C - \frac{1}{2} \bar{M} \right) \ddot{\hat{q}} + \frac{1}{2} \dot{\hat{q}}^T \bar{M} \dot{\hat{q}}_d - \frac{1}{2} \dot{\hat{q}}^T \bar{M} \dot{\hat{q}}_d$$

$$- v_{od}^T K_v v_{od} + 2v_{od}^T K_v v_{od} - v_{od}^T K_v v_{od}.$$  

(27)

As $K_v$ is symmetric positive definite, then $v_{od}^T K_v v_{od}$ and $v_{od}^T K_v v_{od}$ satisfy the following inequalities

$$v_{od}^T K_v v_{od} \geq k_v \| v_{od} \|$$  

(28)
\[ \mathbf{v}_{od}^T \mathbf{K}_v \mathbf{v}_{od} \geq k_v \| \mathbf{v}_{od} \| \] (29)

respectively, where \( k_v \) is the smallest eigenvalue of \( \mathbf{K}_v \). In this way, at instant \( t \)

\[ \dot{V}(t) \leq \mathbf{v}_{od}(t)^T \vartheta_1(t) + \mathbf{v}_o(t)^T \vartheta_2(t) \]

\[ - k_v \| \mathbf{v}_{od}(t) \|^2 - k_v \| \mathbf{v}_o(t) \|^2 \] (30)

where \( \vartheta_1(t) \) and \( \vartheta_2(t) \) are terms dependent on the model parameters and the desired trajectory. If desired trajectories that belong to \( S \) are used, then \( \vartheta_1(t) \) and \( \vartheta_2(t) \in L_2([0, \infty)) \). Integrating both sides of (30) from \( t_0 \) to \( t \), and considering that the inner product satisfies the Cauchy-Schwarz inequality, then

\[ V(t) - V(t_0) \leq \| \vartheta_1 \|_{L_2([t_0, t])} \| \mathbf{v}_{od} \|_{L_2([t_0, t])} \]

\[ - k_v \| \mathbf{v}_{od} \|_{L_2([t_0, t])}^2 - k_v \| \mathbf{v}_o \|_{L_2([t_0, t])}^2 \]

\[ + \| \vartheta_2 \|_{L_2([t_0, t])} \| \mathbf{v}_o \|_{L_2([t_0, t])} \] (31)

Completing the squares of (31)

\[ V(t) - V(t_0) \leq -k_v \left( \| \mathbf{v}_{od} \|_{L_2([t_0, t])} - \frac{\| \vartheta_1 \|_{L_2([t_0, t])}}{2k_v} \right)^2 \]

\[ - k_v \left( \| \mathbf{v}_o \|_{L_2([t_0, t])} - \frac{\| \vartheta_2 \|_{L_2([t_0, t])}}{2k_v} \right)^2 \]

\[ + \frac{\| \vartheta_2 \|_{L_2([t_0, t])}^2}{4k_v} + \frac{\| \mathbf{v}_o \|_{L_2([t_0, t])}^2}{4k_v} \] (32)

From the previous equation, \( V(t) \) is superiorly bounded by \( V(t_0) \) plus the third and forth terms on the right side of (32). As \( V(t) \geq 0 \) (26), \( \vartheta_1(t) \in L_2([0, \infty]) \), and \( \vartheta_2(t) \in L_2([0, \infty]) \), then (32) implies that \( V(t) \) is uniformly bounded for all \( t > 0 \), which implies that \( \Delta \mathbf{x}_o, \Delta \mathbf{v}_o, \Delta \dot{q} \) are all uniformly bounded (26). Still from (32), if \( V(t) \) is uniformly bounded, then \( \mathbf{v}_o \in L_2([0, \infty)) \), \( \mathbf{v}_o \) is uniformly continuous, and \( \mathbf{v}_o \) is convergent to zero as \( t \to \infty \). Thus, the load always goes to the desired state under the control law given by (19) following desired trajectories that belong to \( S \).

Consider now the item b). Substituting the second line in the first line of (18) for \( \dot{q} = \ddot{q} = 0 \), one obtains

\[ \mathbf{Q}^T \mathbf{T}^{-1} \mathbf{K}_p \Delta \mathbf{x}_o + (\mathbf{J}_a^T - \mathbf{A}_o^T \mathbf{J}_p^T) \mathbf{h} \]

\[ + (\mathbf{J}_a^T - \mathbf{A}_o^T \mathbf{J}_p^T) \mathbf{f}_o = 0. \] (33)

Substituting (22) in (3) for \( \mathbf{v}_o = \dot{\mathbf{v}}_o = 0 \) and choosing \( \mathbf{f}_o \) in order to result in a null squeeze at frame CM, then

\[ \mathbf{f}_o = -\mathbf{J}_{eq}^{-1} \mathbf{h}_{om}. \] (34)

Finally, substituting (34), (6), and (7) in (33), one obtains

\[ \mathbf{Q}^T \mathbf{T}^{-1} \mathbf{K}_p \Delta \mathbf{x}_o + (\mathbf{J}_a^T - \mathbf{A}_o^T \mathbf{J}_p^T) \mathbf{J}_{eq}^{-1} \mathbf{h}_{bc} = 0. \] (35)
B. Squeeze Force Control

An important property of (7) is that the component of the squeeze force affected by the motion \( h_{sm} \) is not affected by joint torques in the form \( D T h_{sc} \). Hence, \( h_{sm} \) can be interpreted as a disturbance.

Here, as \( h_{sc} \) is an \( mk \)-dimensional vector and \( \dim(X_s) = k(m - 1) \), the control law for the actuated joints can be written as

\[
\tau_s = -D_{sa}^T A_{e}^T \gamma_s \tag{36}
\]

where the image of \( A_{e}^T \) projects the null space of \( A^T \), i.e. \( \text{Im}(A_{e}^T) = X_s \), the \( k(m - 1) \)-dimensional vector \( \gamma_s \) is the effective squeeze control variable for the control of the effective squeeze variable \( \Gamma_s \) obtained from the measured squeeze forces, and

\[
D_{sa} = \begin{bmatrix} D_{a1} & 0 & \cdots & 0 \\ 0 & D_{a2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{am} \end{bmatrix},
\]

where \( D_{ai} \) relates velocities of the actuated joints of arm \( i \) and load velocities.

However, \( n_p \) constraints are imposed by the passive joints in \( \gamma_s \), i.e.

\[
0_{n_p} = -D_{sp}^T A_{e}^T \gamma_s \tag{37}
\]

where \( D_{sp} \) relates velocities of the passive joints and load velocities.

In the full-actuated cooperative system, \( k \) inputs are needed to control the \( k \) components of the motion of the load and \( k(m - 1) \) inputs are utilized to control the squeeze forces.

For the cooperative system with passive joints, if the arms are not kinematically redundant, it is not possible to independently control all components of \( \Gamma_s \). As \( n_p \) constraints are imposed by (37), the number of components of the \( k(m - 1) \)-dimensional vector \( \Gamma_s \) that can be independently controlled is

\[
n_s = \begin{cases} \frac{k(m - 1)}{2} - n_p = n_a - k & \text{if } n_a > k \\ 0 & \text{otherwise} \end{cases}
\]

If \( n_a > k \), \( n_s \) components of \( \Gamma_s \) can be independently controlled. In this case, the vector \( \gamma_s \) can be partitioned by a permutation matrix \( P_{sd} \), i.e.

\[
P_{sd} \gamma_s = \begin{bmatrix} \gamma_{sc} \\ \gamma_{sn} \end{bmatrix} \tag{38}
\]

where \( \gamma_{sc} \) is the \( n_a \)-dimensional vector of the independently controlled components and \( \gamma_{sn} \) has its \( n_p \) components computed using (37) and (38), i.e., \( \gamma_{sn} = f(\gamma_{sc}) \) where \( f(\cdot) \) is a function obtained using (37) and (38).

To compute the vector \( \gamma_{sc} \), it is recalled that, if an asymptotically stable motion control law is utilized, the \( h_{sm} \) goes to zero as \( t \to \infty \). As the transient performance and convergence rate of \( h_{os} \) are influenced by \( h_{sm} \) in a
feedback control approach, [13] suggests a pre-processing of the effective squeeze variable $\Gamma_s$ (obtained from the measured squeeze forces) by a strictly proper linear filter, as an integrator. Then, $\gamma_{sc}$ is given at time $t$ by

$$
\gamma_{sc}(t) = \Gamma_{sed}(t) + K_t \int_{s=t_0}^{s=t} (\Gamma_{sed}(s) - \Gamma_{sc}(s)) \, ds
$$

(39)

where $K_t$ is a positive diagonal matrix, $\Gamma_{sc}(s)$ is the vector formed by the independently controlled components of $\Gamma_s(s)$, and $\Gamma_{sed}(s)$ is the vector of their desired values. In this way, the squeeze control is given by (36) with $\gamma_{sc}$ computed in (37-39).

IV. DYNAMIC LOAD-CARRYING CAPACITY OF COOPERATIVE MANIPULATORS WITH PASSIVE JOINTS

The DLCC of cooperative manipulators, that is a key factor in the trajectory planning, was only investigated by few researchers [8], [9]. Here, the DLCC of the system with passive joints is obtained based on the algorithm presented in [9]. For this case, a new linear programming problem is defined taking into account torque constraints. It is important to observe that the DLCC is obtained based on desired trajectory and known parameters of the load.

From (3), it can be written that

$$
h_{ro} = \begin{bmatrix} m_o 1 & 0 \\
0 & I_o \end{bmatrix} \begin{bmatrix} \ddot{p}_o \\
\dot{\omega}_o \end{bmatrix} + \begin{bmatrix} m_o g \\
\omega_o \times (I_o \omega_o) \end{bmatrix}
$$

(40)

where $h_{ro} = J_o(x_o)^T h$ is the resulting force vector at the frame CM attached to the load, the terms in the left side of (3) were expanded, $I_o$ is the inertia matrix of the load, $m_o$ is the mass of the load, and $g$ is the gravity vector. Considering loads with inertia matrix equal to $I_o = I_{oc} m_o$, where $I_{oc}$ is constant, (40) can be written as

$$
h_{ro} = \begin{bmatrix} g + \ddot{p}_o \\
I_{oc} \dot{\omega}_o + \omega_o \times (I_{oc} \omega_o) \end{bmatrix} m_o.
$$

(41)

As there exist $k$ components of the motion of the load, the first $k$ components (partition $K$) of the joint space (with $n$ joints) are chosen as generalized coordinates. Then the dynamics of the joints in partition $K$ is given by

$$
\tau_k + J_k^{n-k}(q)^T \tau_{n-k} + D_k^o(q)^T h_{ro}
$$

$$
- J_k^o(q)^T (M(q) \ddot{q} + g(q) + C(q, \dot{q}) \dot{q}) = 0
$$

(42)

where $\tau_k$ is the vector of torques of the joints in partition $K$, $\tau_{n-k}$ is the vector of torques of the joints that do not belong to partition $K$, $J_k^{n-k}(q)$ is the Jacobian matrix that relates the velocities of the joints in partition $K$ to the velocities of the joints that do not belong to partition $K$, $D_k^o(q)$ is the Jacobian matrix that relates the velocities of the joints in partition $K$ to the velocities of the load, and $J_k^o(q)$ is the Jacobian matrix that relates the velocities of the joints in partition $K$ to the velocities of all joints of the system.

Substituting (41) in (42), one obtains

$$
A_t \tau + a_o m_o - J_k^o(q)^T (M(q) \ddot{q} + g(q) + C(q, \dot{q}) \dot{q}) = 0
$$

(43)

where $\tau = [\tau_k^T \tau_{n-k}^T]^T$, $A_t = \begin{bmatrix} I & J_k^{n-k}(q)^T \end{bmatrix}$, and $a_o = D_k^o(q)^T \begin{bmatrix} g + \ddot{p}_o \\
I_{oc} \dot{\omega}_o + \omega_o \times (I_{oc} \omega_o) \end{bmatrix}$.
It is possible to write (43) as

$$Ax = b$$  \hspace{1cm} (44)$$

where $A = [A_t \ a_o]$, $x = [\tau^T \ m_o]^T$, and $b = J_k^n(q)^T(M(q)\ddot{q} + g(q) + C(q, \dot{q})\dot{q})$.

As the number of constraints in $x$ is greater than the number of equations, (44) resembles a linear system with equality constraints on the load mass and joint torques. The constraints on $x$ are

$$m_o > 0$$  \hspace{1cm} (45)$$

and

$$|\tau_j| \begin{cases} \leq \tau_{max,j} & \text{if joint } j \text{ is actuated} \\ = 0 & \text{if joint } j \text{ is passive} \end{cases}, \ j = 1, \ldots, n$$  \hspace{1cm} (46)$$

where $\tau_{max,j}$ is the maximum torque applied to the joint $j$. One can observe that the problem of passive joints is addressed only in the constraints given by (46), which is the main difference of this method to that one proposed in [9] for the full-actuated system.

Eqs. (44-46) impose constraints in the linear programming problem to be solved. As the maximum load mass should be found for each desired trajectory, the objective function of the linear programming problem is

$$f(x) = c^T x$$  \hspace{1cm} (47)$$

where $c^T = [0_{n \times 1} \ 1]$.

The procedure to find the DLCC in a desired trajectory can be summarized as follows:

a) the desired trajectory is defined;

b) the linear programming problem is solved for each sampling time of the desired trajectory in order to obtain the optimal torques in the joints and the load mass that maximize (47) subject to the constraints imposed by (44-46);

c) the mass of the load obtained in each sampling time is stored;

d) the DLCC in the desired trajectory is the minimum value of the mass stored in all samples.

V. Results

The control system developed in Section III is firstly applied in a simulation study where a planar cooperative system was positioned in a vertical plane in order to verify the gravity effects. Then, the control system is applied in an actual planar cooperative system. To conclude this section, the results of the method developed in Section IV to analyze the DLCC of the simulated system are also presented.

A. Simulations of Two 3-joint Arms

The results presented in Figure 2 are based on the control of two three-revolute-joint planar cooperative arms with passive joints manipulating an object with mass equal to 2.5 kg in an x-y plane. In order to verify the effect of the gravity terms, the gravity was considered parallel to the y-axis (the x-axis passes through the bases of the
The robot links with, respectively, 0.85, 0.85, and 0.625 kg were considered. The length of the links was defined as 0.203 m, the load length (between the contact points) as 0.1 m, and the moment of inertia of the load as 0.0022 kg m$^2$.

The simulations were performed in the Cooperative Manipulators Control Environment (CMCE), developed in our laboratory, which allows to perform simulations or to control the cooperative system described below in the same Graphical User Interface. The sampling period adopted was 0.008 s and measurement noise with normal distribution was added to joint positions, joint velocities, and end-effector forces.

Figure 2 shows a simulation of this trajectory of this system with one passive joint where the proposed controller was utilized. As there was one passive joint, one component of the squeeze forces was not controlled in this simulation. One can observe that the positions and orientation were correctly controlled even with the presence of a passive joint. The same happened for the components of the squeeze forces in the x- and y-axes (the desired values for the squeeze forces are equal to zero in the results presented in this paper). Simulations indicated that, even with three passive joints, the load positions can be correctly controlled.

### B. Actual Cooperative System

The control system developed in this paper was applied in an actual cooperative system with two arms UARMII (Figure 3). Each UARMII is a three-revolute-joint, planar manipulator that floats on a thin air film on an “air table” (the base of each arm is fixed on the table). The two arms are equal and the axis of each joint is parallel to the gravity force. The cooperative system is controlled by a PC running Matlab. This is possible because the drivers for the UARMII servo board are written as Matlab mex-files. Each joint of the UARMII contains a brushless DC direct-drive motor, an encoder, and a pneumatic brake. The CMCE, which was used to simulate the cooperative system with three-joint arms, is used to control and to monitor the system. The robot parameters are the same of the simulated system and the sampling period considered is 0.04 s. The joint velocities are obtained by encoder readings utilizing an adaptive filter presented in [19].

In the trajectory presented in Figure 4, where the positions of the load is shown, a thin disc with 12 cm of diameter and 25 g of weight was manipulated by the cooperative system with three passive joints. As there are three passive joints, the squeeze forces were not controlled. One can observe that even with three passive joints, the positions of the load were correctly controlled.

In the results presented in Figure 5, the cooperative arms were fixed to a 4-component dynamometer Kistler 9272. The dynamometer with a mass equal to 4.2 kg (in this case it also works as load), was utilized to measure the squeeze forces. Figure 5 shows the positions of the load and the squeeze forces for a trajectory of the cooperative system with one passive joint. One can observe that even with this heavy load, the positions and the squeeze forces were correctly controlled.
C. DLCC of Two 3-joint Arms with Passive Joints

Here, the method presented in Section IV was used to analyze the DLCC in a desired trajectory of the simulated cooperative system formed by two planar three-joint robots, where the maximum torque in each actuated joint was equal to 25 Nm. Figure 6 shows the DLCC of the cooperative system in the desired trajectory in seven cases: the first with the full-actuated system and the others with one passive joint (one simulation for each joint). In Figure 6, one can observe that the DLCC is different for each case. As each joint is more or less actuated depending on the given trajectory, the DLCC for each simulation changes when a different joint is considered passive.

The DLCC can still be used to specify the trajectory of the load for the system with passive joints. Trajectories with the same initial and final positions can result in different DLCC as the terms of (2) changes with the position, velocity, and acceleration of the joints. In this way, a trajectory with smaller accelerations can result in a higher DLCC. As example, using the simulated cooperative systems with one passive joint (joint 1 of arm 1), the DLCC in two trajectories with the same initial and final load positions was computed. In the first trajectory, where the load was positioned in 0.3 s, the computed maximum load that can be manipulated was 31.60 kg. In the second, where the load was positioned in 3.0 s, the maximum load that can be manipulated was 41.35 kg. The computed maximum load that can be manipulated in the second case, where the accelerations and velocities of the joints were smaller, was almost 10 kg greater than that one for the first case.

VI. CONCLUSIONS

This work presents a stable control system for cooperative manipulators with passive joints. A hybrid control of motion and squeeze is utilized. For this purpose, the motion and squeeze control problems are decomposed and a Jacobian matrix relating the velocities of the load and the velocities of the actuated joints is calculated. The inertia matrices of the underactuated robots is not utilized, which, in general, reduces the modeling errors. Furthermore a method to compute the DLCC of the cooperative system with passive joints was presented.

REFERENCES

Fig. 2. Trajectory of the simulated system with one passive joint (joint 1 of arm 2). Left: positions and orientation of the load. The dashed lines show the desired trajectory (the dashed lines are coincident with the solid lines in this figure). Right: squeeze forces. The torque component of the squeeze forces was not controlled.

Fig. 3. Actual system.


Fig. 4. Positions and orientation of the load in a trajectory of the actual system with three passive joints (joints 2 and 3 of arm 1, and joint 1 of arm 2). The dashed lines show the desired trajectory.


Fig. 5. Trajectory of the actual system with one passive joint (joint 2 of arm 1). Left: positions and orientation of the load (the dashed lines show the desired trajectory). Right: squeeze forces.

Fig. 6. DLCC in a given trajectory for the full-actuated system and for the system with one passive joint.