

# Equity Crowdfunding: Harnessing the Wisdom of the Crowd\*

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## Abstract

We study the efficiency of securities-based crowdfunding platforms. Crowdfunding uniquely attracts both sophisticated investors and naïve investors who behave more like consumers. Naïve investors, possessing weak but on average correct signals, are required for efficient financing. Sophisticated investors, who are better informed and anticipate other investors' actions, cannot by themselves use their information to improve financing efficiency. Furthermore, increasing sophisticated investors' information or population decreases financing efficiency. Our results have important implications for the design of crowdfunding platforms and for regulators concerned with protecting investors and maintaining healthy markets for startup capital.

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# 1 Introduction

These investor protections are not just important to the college student, to the grandmother, and to the working mom who jump on the Internet wanting to experiment with crowdfunding. They also protect the small businesses that want a reliable market to raise capital. If investors don't feel the market is safe for them to invest, there won't be much capital raising going on there.

– SEC Commissioner Stein, Statement on the Adoption of Regulation Crowdfunding, 10/31/2015

Crowdfunding is a young form of financing allowing entrepreneurs to solicit small contributions from many individuals via internet platforms. On nearly all platforms, an entrepreneur will post her intended fundraising goal and deadline. If the entrepreneur meets or surpasses the fundraising goal by the deadline she receives financing, otherwise she receives nothing, i.e., financing is *all-or-nothing*. To date, most crowdfunding has relied on non-pecuniary rewards (e.g., Kickstarter and GoFundMe), attracting investors who enjoy private benefits from investing (Boudreau, Jeppesen, Reichstein and Rullani 2015). However, not all entrepreneurs can provide investors with private benefits. Securities-based crowdfunding (e.g., the sale of equity) allows entrepreneurs to provide investors with the promise of future returns. While securities-based crowdfunding represents less than 5% of crowdfunding investment worldwide (Massolution 2013), the JOBS Act and the SEC's adoption of Regulation Crowdfunding have set the stage for the growth of securities-based crowdfunding in the US.<sup>1</sup>

Securities-based crowdfunding is unique in that it will likely attract both sophisticated, return-focused investors and naïve, consumer-like investors. Our paper focuses on crowdfunding platform efficiency given that these two distinct, heterogeneously-skilled investor groups will jointly fund opportunities. Understanding platform efficiency is vital as securities-based crowdfunding enters its infancy for two reasons. First, naïve investors will participate in campaigns without properly evaluating the financial risks. As such, the welfare of naïve investors is directly linked to the efficiency of securities-based crowdfunding platforms. Second, platform efficiency and the legitimacy of securities-based crowdfunding are inherently linked — entrepreneurs' participation requires that platforms provide legitimate capital formation and investors' participation requires legitimate investment opportunities. Given that our paper is the first to analyze the effects of sophisticated and naïve investors' interactions in securities-based crowdfunding, we provide valuable insights to regulators

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<sup>1</sup>Stemler (2013) details the JOBS Act, specifically Title III (the CROWDFUND Act) which enables entrepreneurs to sell small amounts of securities to a large number of investors via the Internet.

who care about protecting investors and maintaining healthy financial markets.<sup>2</sup>

Our main result is that naïve investors, not sophisticated investors, are required for efficient crowdfunding. Having weak but on average correct signals about projects' qualities, naïve investors who myopically follow their signals will collectively invest more in good projects than bad projects. Platforms can use this positive correlation between project quality and naïve investor participation via the fundraising goals. Consider a population of naïve investors with \$1,000,000 of collective capital. If the naïve investors are correct 51% of the time, a good project attracts \$510,000 while a bad project attracts \$490,000. When the fundraising goal lies between these two amounts (e.g., \$500,000), the good project receives sufficient capital and proceeds, while the bad project fails to meet the fundraising goal and the capital is returned to investors. This hedging-effect, due to the all-or-nothing fundraising rule, results in first-best financing by harnessing the collective wisdom of imprecisely informed, naïve investors.

Despite having individual informational advantages over naïve investors, a crowd of sophisticated investors cannot use that advantage, and instead, financing breaks down. If sophisticated investors could commit to follow their signals (as naïve investors do), there also would be a funding gap between good and bad projects. The gap would allow only good projects to attract sufficient capital, creating a hedge against bad projects and a risk-free investment opportunity. Were such an opportunity to exist, sophisticated investors would be willing to invest regardless of their signals. Bad-signal, sophisticated investors would deviate and invest, so sophisticated investors cannot commit to following their signals. As such, for there to be any hope of first-best financing, the crowd must include a measurable fraction of naïve investors.

Turning to the details of the model, our key assumptions are based on fundamental features of crowdfunding. First, we consider a mass of infinitesimal investors, reflecting the small contributions common in crowdfunding.<sup>3</sup> As a result, individuals do not internalize their actions and the model is tractable. In an extension, we show our main results are robust to considering crowdfunding from a discrete population of investors. Second, projects are subject to all-or-nothing financing, also known as a “provision point mechanism” (Bagnoli and Lipman 1989), in which projects not receiving sufficient capital are canceled. All-or-nothing financing mechanisms have been applied by almost all non-equity crowdfunding platforms

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<sup>2</sup>Agrawal, Catalini and Goldfarb (2014) and Xu (2015) consider further benefits of crowdfunding, including increased trade between investors and entrepreneurs, innovation spill-overs, better geographic dispersion of capital and feedback from investors to entrepreneurs.

<sup>3</sup>Mollick (2014) documents an average investment size of \$64 and proposed SEC rules set maximums on individual investments. See <https://www.congress.gov/bill/112th-congress/senate-bill/2190/text>.

(Agrawal, Catalini and Goldfarb 2014) and such a mechanism is mandated by Regulation Crowdfunding.<sup>4</sup> Third, the mass of investors is comprised of sophisticated and naïve investors. Sophisticated investors, whose signals are weakly more precise than those of naïve investors, are fully rational and properly anticipate the actions of other investors. Naïve investors, due to limited financial literacy or alternative motivations for investing, myopically act on their signals. Finally, we assume that the securities being sold are equity and oversubscribed offerings are allocated on a pro rata basis.

When combined, the mix of sophisticated and naïve investors, and their information precisions, determine the financing efficiency of different scales of projects. For example, naïve investors will provide capital to every project, with bad projects receiving less than good projects. As a result, there is a boundary, below which all projects are financed regardless of quality. While naïve investors enable all small-scale projects to be financed, the capital gap between good and bad projects also enables efficient financing. When the project scale is very big (large-scale), the project can only be financed if all sophisticated investors participate alongside a majority of naïve investors. Naïve investors bring more capital for good projects, so only good projects meet their fundraising goals, creating the hedging effect and financing is efficient, i.e., first-best.

For projects that are too big for naïve investors alone to finance, and not big enough to benefit from the large-scale-project hedging effect, sophisticated investors hinder financing. The intuition is similar to the case with only sophisticated investors. Unless promised returns are very high, sophisticated investors who receive bad signals (which are likely correct) cannot earn positive expected returns. As a result, all sophisticated investors cannot participate in equilibrium. However, if only good-signal sophisticated investors participated, a hedging effect would create a risk-free investment opportunity. This cannot be an equilibrium – bad-signal sophisticated investors would optimally deviate from their non-participation strategies. However, if bad-signal sophisticated investors participate, all projects would be financed and the hedging effect disappears. Only zero-participation by sophisticated investors can be supported in equilibrium. As a result, projects in this region fail to meet their fundraising goals.

Platforms can improve efficiency by influencing the mix and characteristics of their investors. Most striking, as sophisticated investors become more informed or more prevalent, financing becomes less efficient. Increasing either the precision of their information or their proportion expands the region of projects where financing breaks down. Alternatively, increasing the proportion of naïve investors shrinks this region, and

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<sup>4</sup>See Rule 201(g) of Regulation Crowdfunding.

more projects can be financed. In fact, when at least a portion of investors are naïve, the platform can be designed to deliver first-best financing on a project-by-project basis. By adjusting the maximum capital contributed per investor, platforms can ensure a project is in the large-scale region, and thus achieves first-best financing. In the case of incomplete information, such precise project-by-project control may not be possible, and in such cases platforms will face a trade off between efficiency and feasibility.

Proponents of securities-based crowdfunding envision it as a new source of startup capital for entrepreneurs. While we are agnostic regarding the social welfare implications of allowing securities-based crowdfunding, a natural question is what is an equivalent substitute for a properly designed crowdfunding platform? To that end, we consider what level of signal precision a single, financially unconstrained, venture capitalist (VC) would require to perform equally, both in ex-ante and ex-post outcomes, as a properly-designed platform. We coin this measure “VC equivalence.” In our base model, first-best financing is possible implying that an equivalent substitute for the platform would be a perfectly-informed VC. We consider an extension of our base model in which first-best is unattainable because there is noise in the fundraising process (either because there are a finite number of investors or due to exogenously imposed noise). We show that properly-designed platforms still exploit the all-or-nothing thresholds and perform well relative to a VC. For example, a platform comprised of thirty naïve investors with individual signal precision equal to 65% will perform as well as a single VC with signal precision of 95%. The analysis optimistically suggests that properly-designed crowdfunding platforms comprised of weakly informed investors may substitute for a well-informed VC (or an alternative start-up capital provider, such as an angel investor).

Considering a multi-period setting shows the robustness of our main results and also provides several additional insights relevant to platform efficiency. First, the multi-period setting introduces learning, allowing sophisticated investors to learn from early investors and improve financing efficiency. However, the desire to wait for more information can lead to financing failures. In response, fundraising time limits arise to spur sophisticated investors to action. Alternatively, first-come, first-served allocations can encourage early movement by sophisticated investors. Unfortunately, first-come, first-served allocations prevent waiting in equilibrium, suggesting pro rata allocations better encourage efficient financing.<sup>5</sup>

Our work relates to a nascent theoretical literature on crowdfunding. Belleflamme, Lambert and Schwienbacher (2014) analyzes an entrepreneur’s choice between crowdfunding via pre-orders and selling equity

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<sup>5</sup>Rule 201(h) of Regulation Crowdfunding leaves the ownership allocation decision on an oversubscribed offering to the discretion of the issuer.

claims. They emphasize that private benefits lead to different financing outcomes that are unique to crowdfunding — without those benefits, crowdfunding would not be different from bank financing. Similarly, we assume naïve investors are not perfectly rational, which is critical to our results. Hakenes and Schlegel (2014) analyzes a model in which a finite number of households endogenously produce information and then invest using debt sourced via crowdfunding. As in our model, their model highlights the winner’s curse and the natural hedge that comes about from not financing bad projects. However, their focus is on endogenous information production, while ours is on the interactions of investors. Cumming, Leboeuf and Schwiendbacher (2015) compares keep-it-all versus all-or-nothing financing, and shows that keep-it-all mechanisms are better for small, scalable projects. We only consider all-or-nothing financing, given its prominence in practice and requirement for US security-based crowdfunding. Grüner and Siemroth (2015) considers crowdfunding as a mechanism in which consumers signal future product market demand via investment. In our model, some naïve investor participation is motivated by product market demand, that is, naïve investors’ participation is positively correlated with an opportunity’s quality because naïve investors may ultimately consume the opportunity’s good or service. Our result that sophisticated investors alone cannot efficiently finance projects is related to Axelson and Makarov (2014), which shows that an auction’s informational efficiency can be destroyed when the information is used to make an investment decision. In their setting, reducing the number of investors improves efficiency, while efficiency in our setting depends on the presence of naïve investors.

Our analysis contributes to understanding the efficiency of crowdfunding platforms, a topic neglected by existing literature. In particular, to our knowledge, ours is the first paper to study the interaction of sophisticated and naïve investors in equity crowdfunding. We show the importance of naïve investors for efficient financing and highlight several tensions governing the efficiency of equity crowdfunding platforms. On the eve of security-based crowdfunding becoming available to all US investors, understanding the tensions facing investors and platforms is critical to properly matching capital to entrepreneurs and delivering the myriad benefits of crowdfunding.

## 2 Crowdfunding Model

A project hosted on a crowdfunding platform has a business plan to produce and sell a product (a good or service). The project is financially valued according to its financial soundness  $F \in \{G, B\}$ , its upfront cost

$c$  (which is equivalent to its scale), and its promised net rate of return  $\delta > 0$ :

$$V = \mathbb{1}_G(\delta + 1)c - c. \tag{1}$$

where  $\mathbb{1}_G$  equals one when  $F = G$ . The project’s financial soundness is not observable, but with equally probability, will be financially sound (good),  $F = G$ , or not financially sound (bad),  $F = B$ , i.e.,

$$\Pr(F = G) = \Pr(F = B) = \frac{1}{2}. \tag{2}$$

The project’s upfront scale and fundraising goal on the crowdfunding platform is  $c \in \mathbb{R}^{++}$ . To finance the project, initial owners sell 100% of the project’s cash flows via equity claims. The project only receives funding if it meets its goal of raising  $c$ , i.e., it is *all-or-nothing*. If the project fails to meet its fundraising goal, committed capital is returned to investors. In the event that the project raises more capital than it needs, ownership is divided on a pro rata basis.<sup>6,7</sup>

A unit continuum of risk neutral investors participates on the crowdfunding platform. Each investor on the platform can invest at most  $M$  dollars into the project. A fraction  $\gamma$  of investors are fully rational. These investors are referred to as *sophisticated*. Each sophisticated investor receives a private signal about the project’s financial soundness,

$$\hat{F} \in \{G, B\}, \tag{3}$$

where,

$$\Pr(\hat{F} = F) = \alpha > \frac{1}{2}, \tag{4}$$

and signals are i.i.d. Sophisticated investors optimally choose whether to invest; they consider their private signals and the anticipated actions of other participants on the platform. The remaining  $1 - \gamma$  fraction of investors are not sophisticated. These investors are referred to as *naïve*. Naïve investors’ participation is

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<sup>6</sup>In a world without agency conflicts, it is equivalent to assume that the project’s entrepreneur stores excess cash in an account that earns investors their opportunity cost of capital.

<sup>7</sup>In Section B.3, we consider an extension to a multi-period setting in which ownership is allocated on a first-come, first-served basis rather than a pro rata basis.

positively correlated with the project’s financial soundness. If the project is good, a fraction,

$$\beta > \frac{1}{2} \tag{5}$$

of naïve investors will participate. If the project is bad, only a fraction  $1 - \beta$  participate. Naïve investors may be consumers that invest because they think the project is chic or amateur investors that do not correctly assess the opportunity (perhaps due to overconfidence) and invest myopically based on a signal with precision  $\beta$ . Nevertheless, the investment decisions of naïve investors are positively correlated with the project’s success. We further assume that  $\beta \leq \alpha$ , however the majority of our analysis does not rely on that assumption.<sup>8</sup> The presence of naïve investors captures two paramount features of crowdfunding: (i) investors of heterogeneous skill jointly fund projects, and (ii) the investment process may include investors that will eventually consume the project’s good or service, implying that investment activity provides feedback about the project’s potential success.

After receiving their signals, naïve and sophisticated investors simultaneously participate on the crowdfunding platform. We model financing on the platform as a one-shot game to focus on the interactions between naïve and sophisticated investors. Appendix B analyzes a multi-period setting, showing that our main results hold in an environment with learning. In the one-shot game, naïve investors myopically act on their signals and are infinitesimal, so by the strong law of large numbers, naïve investors’ contributed capital is,

$$\bar{\kappa} \equiv (1 - \gamma)\beta M \text{ if } F = G \tag{6}$$

$$\underline{\kappa} \equiv (1 - \gamma)(1 - \beta)M \text{ if } F = B. \tag{7}$$

Recalling that  $\beta > \frac{1}{2}$ , comparing (6) and (7) reveals that  $\bar{\kappa} > \underline{\kappa}$ , i.e., good projects attract more naïve investors’ capital than bad projects. The positive correlation between project quality and raised capital is a valuable trait we refer to as the *crowd signal*.

Sophisticated investors play mixed strategies with regards to whether or not they participate. Define the

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<sup>8</sup>If  $\beta > \alpha$ , the participation strategies of sophisticated investors are deterministic. Furthermore, in characterizing sophisticated investors’ participation criteria, the signs on a few comparative statics change. The change in signs is due to a stronger winner’s curse (discussed shortly) among naïve investors relative to sophisticated investors.



mixing strategies for sophisticated investors to be,

$$\pi_{\hat{F}} \in [0, 1], \hat{F} \in \{G, B\}. \quad (8)$$

Sophisticated investors' observing  $\hat{F} = G$  participate with probability  $\pi_G$ , while those observing  $\hat{F} = B$  invest with probability  $\pi_B$ . If all sophisticated investors choose to participate regardless of their signals, that is,  $\pi_G = 1$  and  $\pi_B = 1$ , total contributed capital (including naïve investors' capital) is,

$$\overline{K} \equiv \gamma M + \overline{\kappa} \quad (9)$$

$$\underline{K} \equiv \gamma M + \underline{\kappa}. \quad (10)$$

$\overline{K}$  is the largest-scale project that can be financed on the platform, which will only be achieved for good projects.  $\underline{K}$  is the largest-scale bad project that can be financed on the platform.

The preceding capital boundaries  $(\underline{\kappa}, \overline{\kappa}, \underline{K}, \overline{K})$  are useful in characterizing the sophisticated investors' optimal strategies and the resulting financing efficiency. To ease exposition, we define small-scale and large-scale projects using these boundaries.

**Definition 1. *Small-scale projects***, having  $c \in (0, \underline{\kappa})$ , will be financed by naïve investors regardless of sophisticated investors' strategies or the quality of the project.

**Definition 2. *Large-scale projects***, having  $c \in (\underline{K}, \overline{K}]$ , will be financed if all sophisticated investors choose to participate.

The following propositions divide the characterization of the simultaneous-move Nash equilibrium strategies according to these small-scale and large-scale boundaries, highlighting that different tensions arise according to the scale of the project. The all-or-nothing financing rule introduces discontinuities to investors' total contributed capital, adding complexity to the associated algebra. As a result, the algebraic details of our analysis are in the proofs in Appendix A. We first consider small-scale projects.

**Proposition 1.** *Define*

$$\bar{\delta} \equiv \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\gamma + (1-\gamma)\beta}{\gamma + (1-\gamma)(1-\beta)} \right) \quad (11)$$

$$\underline{\delta} \equiv \frac{(1-\alpha)\beta}{\alpha(1-\beta)} \quad (12)$$

For small-scale projects and parameter sets in which  $\bar{\delta} > \underline{\delta}$ , sophisticated investors' equilibrium strategies depend on a project's promised return:

$$\left\{ \begin{array}{ll} \text{All sophisticated investors participate if} & \delta \in [\bar{\delta}, \infty) \\ \pi_1 \text{ of sophisticated investors that observed } \hat{F} = G \text{ participate if} & \delta \in [\underline{\delta}, \bar{\delta}) \\ \text{No sophisticated investors participate if} & \delta \in [0, \underline{\delta}). \end{array} \right. \quad (13)$$

**Corollary 1.1.** *The threshold  $\bar{\delta}$  is increasing in  $\alpha$  and  $\beta$  and decreasing in  $\gamma$ . The threshold  $\underline{\delta}$  is decreasing in  $\alpha$  and increasing in  $\beta$ .*

The existence of the thresholds  $\bar{\delta}$  and  $\underline{\delta}$  is intuitive. All investors participate when promised returns are very high, no investors participate when promised returns are very low, and some investors participate when promised returns are somewhere in between. To develop economic intuition, we focus on the thresholds' comparative statics.

The upper threshold,  $\bar{\delta}$ , is the promised return that is sufficiently large to outweigh investors' bad signals and the crowding-out effect from naïve investors. First,  $\bar{\delta}$  is increasing in  $\alpha$ . As  $\alpha$  increases, investors that observe a bad signal are more confident that the project is not financially sound and are less willing to invest, i.e., they require a higher promised return. Second,  $\bar{\delta}$  is increasing in  $\beta$ . This result is due to naïve investors' crowding-out sophisticated investors in good projects. Because naïve investors are on average correct, good projects receive more naïve investors' capital than bad projects, so sophisticated investors receive larger pro rata shares of bad projects. As a result of asymmetric exposure to bad projects, i.e., a winner's curse, sophisticated investors require higher promised returns.<sup>9</sup> As  $\beta$  increases, the asymmetry between good and bad projects increases, raising the required return. Third,  $\bar{\delta}$  is decreasing in  $\gamma$ . As  $\gamma$  increases, sophisticated investors grow relative to the population of naïve investors, reducing the asymmetric

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<sup>9</sup>The asymmetry introduced by naïve investors is similar to the winner's curse identified in initial public offerings (Rock 1986).

exposure to bad projects, decreasing the required return.

Turning to the lower threshold,  $\underline{\delta}$  is simple to characterize: it is decreasing in  $\alpha$  and increasing in  $\beta$ . Sophisticated investors require less promised return when they have greater confidence in their signals and when they are less exposed to the winner's curse from naïve investors.

Proposition 1 only applies to projects in which  $\bar{\delta} > \underline{\delta}$ . Under a relatively small set of parameters, in which  $\alpha$  and  $\beta$  are close to  $\frac{1}{2}$ ,  $\bar{\delta} < \underline{\delta}$  and sophisticated investors' strategies change. Either all sophisticated investors or no sophisticated investors participate. Over this parameter set, the comparative statics are uninteresting and details of the analysis are relegated to Appendix A.

The preceding strategies of sophisticated investor apply only to small-scale projects, where all projects will be financed regardless of quality. As a result, small-scale-project financing is uninformed and inefficient. We next consider the other end of the spectrum: large-scale projects that are financed when all sophisticated investors participate.

**Proposition 2.** *For large-scale projects, all sophisticated investors participate and only good projects are financed.*

All large-scale projects are bigger than  $\underline{K}$ , the highest amount of capital that a bad project can attract (which occurs when  $1 - \beta$  of naïve investors and all sophisticated investors participate). Sophisticated investors therefore know that bad, large-scale projects will not receive sufficient capital from naïve investors, and due to the all-or-nothing financing feature, their capital will be returned. The all-or-nothing feature creates a hedging effect – good projects are financed and bad projects are not.

Naïve investors' information allows the platform to achieve first-best financing. The hedging effect allows sophisticated investors to safely ignore their signals and always participate in large-scale projects. Naïve investors, however, myopically participate based on their signals, and only the good projects are financed. It is important to note that this result relies on  $\beta > \frac{1}{2}$ . If naïve investors were collectively uninformed, there would be no difference in their capital contributions and no hedging effect.

While the all-or-nothing feature and associated hedging-effect allow for first-best financing for large-scale projects, they also lead to a parameter space of moderate-scale projects in which no equilibrium strategies exist.

**Proposition 3.** *No non-zero equilibrium participation strategies of sophisticated investors exist for projects*

with a promised net return

$$\delta < \bar{\delta}, \tag{14}$$

and scale

$$c \in (\underline{\kappa} + \pi_G \gamma (1 - \alpha) M, \underline{K}], \tag{15}$$

with  $\pi_G \in [0, 1]$ . We refer to the region of projects that satisfy these criteria as the indeterminant region. If  $c > \bar{\kappa}$ , the only equilibrium features zero sophisticated investor participation. If  $c \leq \bar{\kappa}$ , no equilibrium exists.

**Corollary 3.1.** *If  $\delta \geq \bar{\delta}$  and*

$$c \in (\underline{\kappa} + \pi_G \gamma (1 - \alpha) M, \underline{K}], \tag{16}$$

*all sophisticated investors participate.*

Projects in the indeterminant region are of moderate scale and have sufficiently small promised returns. They are too big to be financed by naïve investors regardless of quality, and they are too small to be financed only when good. The promised return is also small enough that sophisticated investors who observe a bad signal are not sufficiently compensated to ignore their signals, similar to Proposition 1. As a result, the indeterminant region cannot support an equilibrium in which all sophisticated investors participate. Importantly, the existence of the indeterminant region relies on both the project scale and project return, as Corollary 3.1 shows that no indeterminant region exists if  $\delta$  is sufficiently large.

The indeterminant region also cannot support an equilibrium in which only a portion of sophisticated investors participate. Suppose sophisticated investors who observe a good signal do participate in equilibrium. Because they are right on average, more capital is contributed to good projects than bad projects. In fact, for the range of moderate-scale projects, the financing gap is large enough that only good projects receive sufficient capital. This would create the same hedging effect that gives rise to first-best financing for large-scale projects. However, the hedging effect cannot occur in equilibrium. Sophisticated investors who observe bad signals realize that participation would be riskless due to the hedging effect, so these investors would deviate from their no-participation strategies.<sup>10</sup> Therefore, there cannot be an equilibrium in which only good-signal sophisticated investors participate. Similar logic applies to any potential equilibrium in which a

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<sup>10</sup>In a setting in which there are a discrete number of sophisticated investors, an equilibrium could be supported in which the marginal investor fully internalizes his decision to deviate from his strategy of not participating. With a continuum of agents, each investor is of zero measure and cannot internalize his decision to deviate.

fraction of sophisticated investors participate.

The indeterminant region occurs whenever sophisticated investors who receive good signals would participate and generate a hedging effect. Sophisticated investors' rationality dictates that they all participate if such a hedging effect exists, but if they all participate the hedging effect would disappear and all projects would be financed. However, full participation cannot be an equilibrium, as promised returns are too small for those with bad signals to participate. Sophisticated investors' own rationality prevents their financing projects in the indeterminant region.

A subset of projects in the indeterminant region will be financed by naïve investors. If the project scale is appropriate ( $\underline{c} < c \leq \bar{c}$ ), naïve investors will contribute sufficient capital for good projects but not bad projects, so good projects in this subset will be financed. However, no equilibrium exists for sophisticated investors' participation, so the anticipated pro rata allocations are ambiguous. For projects in the indeterminant region with  $c > \bar{c}$ , no financing occurs in equilibrium. Naïve investors alone cannot supply sufficient capital, and the only equilibrium strategy for sophisticated investors is to not participate.

While the intuition for the indeterminant region is straight-forward, the lower boundary on the region is more complicated. Because the boundary depends on sophisticated investors' mixing strategies,  $\pi_G$ , and those mixing strategies depend on promised returns, the boundary is a function of  $\delta$ . When  $\delta$  is high,  $\pi_G = 1$  and the boundary is as high as possible. All sophisticated investors' receiving good signals participate, the amount received by bad projects is higher, and the hedging effect begins at a larger scale. When  $\delta$  is low,  $\pi_G = 0$  and the boundary is as low as possible. The hedging effect only comes from naïve investors, so the boundary corresponds with  $\underline{c}$ . When  $0 < \pi_G < 1$ , sophisticated investors' mixing determines whether or not a hedging effect occurs. At the boundary, sophisticated investors break-even. Increasing the fraction of sophisticated investors would either be unprofitable, or results in a potential hedging effect, pushing the project into the indeterminant region.

The indeterminant region harms financing efficiency in two ways. First, it prevents good, moderate-scale projects from being financed. If sophisticated investors could commit to myopically follow their signals, only good projects in the indeterminant region would be financed. Second, it prevents any moderate-scale projects from being financed. If sophisticated and naïve investors were uninformed, all moderate-scale projects with  $\delta \geq 1$  would be financed. While not most efficient (as both good and bad projects would be financed), these projects have sufficient ex ante returns to justify investment. Therefore, the presence of information and

the rationality of sophisticated investors prevents the financing of a significant portion of moderate-scale projects.

The negative effects of the indeterminant region grow with the population and information of sophisticated investors. As sophisticated investors become more prominent, i.e.,  $\gamma$  increases, the small-scale project and large-scale project boundaries move outwards. In fact, when all investors are sophisticated ( $\gamma = 1$ ), only projects with very large expected returns ( $\delta \geq \bar{\delta}$ ) can be financed; all other projects are in the indeterminant region. As sophisticated investors become more informed, i.e.,  $\alpha$  increases, the promised return required for sophisticated investors to always participate increases, increasing the indeterminant region. Additionally, the lower boundary on the indeterminant region moves lower, further increasing its size. Overall, increasing the prominence of sophisticated investors harms financing efficiency.

Increasing the information of naïve investors ambiguously affects financing efficiency. Increasing  $\beta$  increases the return threshold for sophisticated investors similarly to  $\alpha$ , so the indeterminant region includes more projects with higher promised returns. On the other hand, increasing  $\beta$  narrows the project-scale boundaries of the indeterminant region. When naïve investors are more informed they contribute more to good projects and less to bad projects, thereby shrinking the region. Due to the ambiguous effects, platforms should encourage or discourage more informed naïve investors based on the population of projects attracted.

The indeterminant region poses a challenge to crowdfunding platforms by limiting the projects that are likely to be financed. However, the indeterminant region is accompanied by a first-best region, opening the possibility that a principal could strategically choose  $M$  such that only good projects are financed and bad projects fail to meet their fundraising goal.

**Proposition 4.** *If naïve investors participate on the crowdfunding platform, any contribution level*

$$M \in \left[ \frac{c}{\gamma + (1 - \gamma)\beta}, \frac{c}{\gamma + (1 - \gamma)(1 - \beta)} \right) \quad (17)$$

*achieves first-best.*

By setting  $M$  on a project-by-project basis, the principal influences investors' capital inflows such that  $\underline{K} < c < \bar{K}$ , and all projects are classified as large-scale. The first inequality ensures that a bad project never meets its goal, even if all sophisticated investors participate. The second inequality ensures that a good project is fully funded. Importantly, the ability to strategically use  $M$  to achieve first-best relies on

there being naïve investors. If all investors were sophisticated ( $\gamma = 1$ ), no equilibrium could be supported in which first-best is attained. Conversely, if all investors are naïve ( $\gamma = 0$ ), first-best is achievable. These investors, while right on average, do not unravel the correlation between crowd size and project quality.

In reality, imperfect information may prevent crowdfunding platforms from setting  $M$  on a project-by-project basis, so they must trade off financing efficiency and feasibility. By lowering the maximum contribution per investor, more investors must participate to fund a project. As a result, bad projects are less likely to be financed and financing efficiency improves. However, lowering the maximum contribution also restricts the size of project that can be financed on a platform, reducing financing feasibility. This efficiency versus feasibility trade off suggests crowdfunding platforms will specialize, tailoring investment maximums based on the scale of projects they attract.

Crowdfunding platforms may also improve financing efficiency by influencing the characteristics of their investor populations. By conducting due diligence or screening the projects accepted, platforms could influence investors' information precision. For example, publishing information on the financial soundness of a project may reduce the cost of information acquisition, closing the gap between naïve and sophisticated investors. While the direct effects of publishing information may be to increase  $\beta$  and/or decrease  $\alpha$ , it could also influence the relative proportions of naïve and sophisticated investors attracted to a platform. Given the diversity of projects that are expected to be crowdfunded, we expect platforms will use these and other methods to best match their population of investors to the population of projects they host.

### 3 Applications

The result of Proposition 4 suggests that it is possible for a crowdfunding platform, comprised of imperfectly informed investors, to perform as well as a perfectly informed agent. To emphasize this point, we introduce a measure of platform efficiency coined *venture capital (VC) equivalence*. Specifically, we are concerned with what signal precision,  $\alpha_{VC}$ , a single VC would need to perform equally to the crowdfunding platform, both in ex ante and ex post terms.<sup>11</sup> With the platform we have explored thus far,  $M$  can be chosen such that 50% of projects are chosen ex ante and ex post 100% of those selected are financially sound. Thus, the platform can perform as well as a VC with signal precision  $\alpha_{VC} = 1$ .

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<sup>11</sup>One could also consider other start-up capital providers like angel investors.

The strength of the preceding result relies heavily on the strong law of large numbers. We now depart slightly from our model to show that the crowd signal may still be harnessed even when project quality is not perfectly revealed through funding levels. We show that  $M$  can be strategically chosen to enhance platform efficiency when: (i) the platform has an all-or-nothing funding feature, and (ii) the participant crowd size positively correlates with the project's quality.

Suppose investor contribution levels are noisy and that the total number of investors financing a project is a random variable  $x$ . The source of the noise could be due to frictions in the investment process, unsophisticated investors (akin to noise traders), or the random arrival of sophisticated and naïve investors. Define a continuous density function  $\underline{h}(x)$  as the distribution of crowd sizes for non-financially sound, bad projects. Similarly, define  $\bar{h}(x)$  as the distribution of crowd sizes for projects that are financially sound, i.e., good projects.  $\underline{H}(\cdot)$  and  $\bar{H}(\cdot)$  represent the corresponding cumulative distribution functions of  $\underline{h}(\cdot)$  and  $\bar{h}(\cdot)$ . Define the support of  $\underline{h}(x)$  as  $[\underline{B}, \bar{B}]$  and the support of  $\bar{h}(x)$  as  $[\underline{H}, \bar{H}]$ .

We assume  $\underline{h}(x)$  is first-order stochastically dominated by  $\bar{h}(x)$  such that higher realizations of  $x$  are more suggestive of a good project than a bad project. This implies that the crowd size positively correlates with the project's quality. As such,

$$E[x|\underline{h}(\cdot)] < E[x|\bar{h}(\cdot)], \quad (18)$$

and

$$\bar{H}(x) \leq \underline{H}(x) \quad (19)$$

for all permissible  $x$  and with strict inequality at some  $x$ . VC equivalence requires that 50% of projects are chosen ex ante and that a fraction  $\alpha_{VC}$  of funded projects are financially sound.

**Proposition 5.** *A crowdfunding platform is equivalent to a single VC with signal precision  $\alpha_{VC}$  if the platform participants are limited to an investment of  $M^*$ , where the set  $\{\alpha_{VC}, M^*\}$  is implicitly defined by the system of equations,*

$$\alpha_{VC} = 1 - \bar{H}\left(\frac{c}{M^*}\right), \quad (20)$$

$$1 - \underline{H}\left(\frac{c}{M^*}\right) = \bar{H}\left(\frac{c}{M^*}\right). \quad (21)$$

*If  $\underline{H}(x)$  and  $\bar{H}(x)$  are strictly monotonic in  $x$ , the set  $\{\alpha_{VC}, M^*\}$  is unique.*



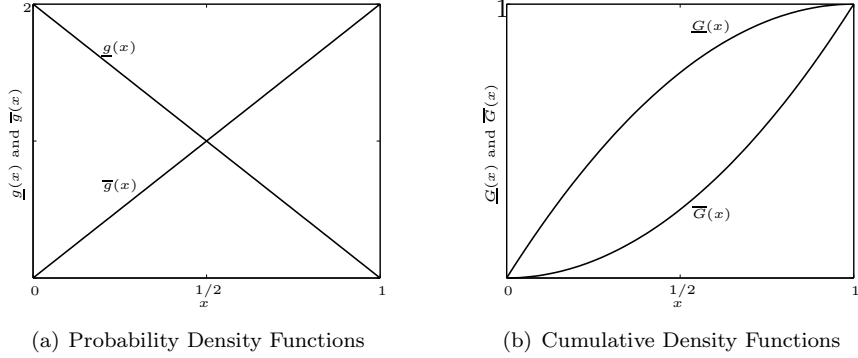


Figure 1: The triangular probability density functions  $\underline{h}(x)$  and  $\bar{h}(x)$  and their respective cumulative density functions.

**Corollary 5.1.** *If the supports for  $\underline{h}$  and  $\bar{h}$  do not overlap,  $M^*$  can be chosen such that the crowdfunding platform provides financing equivalent to a perfectly informed VC.*

According to Corollary 5.1, first-best is achievable if the supports of the two distributions of investors do not overlap. Indeed, the base model we have explored in our paper is an example of such a setting. However, Proposition 5 provides a general setup for determining  $\alpha_{VC}$ . In the following application we consider a setting where the distributions do overlap.

### 3.1 Application of VC-Equivalent Precision

Here we consider a simple application of the preceding analysis and solve for the VC-equivalent precision  $\alpha_{VC}$  when the distribution supports overlap. Consider a project with scale  $c$  and suppose that the distributions  $\underline{h}(x)$  and  $\bar{h}(x)$  are both on the support  $[0, 1]$  and are triangular distributions of the form,

$$\underline{h}(x) = 2 - 2x \tag{22}$$

$$\bar{h}(x) = 2x. \tag{23}$$

The corresponding CDFs are given by,

$$\underline{H}(x) = 2x - x^2 \tag{24}$$

$$\bar{H}(x) = x^2. \tag{25}$$

The two probability density functions are depicted in the first panel of Figure 1 and their cumulative distributions are in the second panel. From Proposition 5, the system of equations that pins down  $\alpha_{VC}$  is,

$$\alpha_{VC} = 1 - \left(\frac{c}{M^*}\right)^2, \quad (26)$$

$$1 = 2 \left(\frac{c}{M^*}\right) - \left(\frac{c}{M^*}\right)^2 + \left(\frac{c}{M^*}\right)^2. \quad (27)$$

Solving the system of equations yields,

$$\alpha_{VC} = 75\%, \quad (28)$$

$$M^* = 2c. \quad (29)$$

Therefore, it is possible for a principal to design the platform to be as efficient as a VC with a signal precision of 75%.

### 3.2 Application With Discrete Number of Agents

VC equivalence also applies to settings with a discrete number of investors. For simplicity, consider a setting in which there is a set of  $N$  investors on the platform and they are all naïve and have precision  $\alpha > \frac{1}{2}$ . A principal may choose  $M$  in such a way that the platform is equivalent to a VC. Instead of the specific level of precision  $\alpha_{VC}$  explored in the previous application with a continuum of investors, there is an upper and lower bound on the equivalent precision with a discrete number of agents.

With a discrete number of agents, the two distributions are binomial,

$$\underline{h}(x) = \binom{N}{x} (1 - \alpha)^N (\alpha)^{N-x} \quad (30)$$

$$\bar{h}(x) = \binom{N}{x} (\alpha)^N (1 - \alpha)^{N-x}. \quad (31)$$

We consider three settings in which there are at most 30 investors on the platform. In the first setting, the investors' precision is equal to 55%, in the second it equals 65% and in the third it equals 75%. In Figure 2 we depict each of the three scenarios. In each graph, the number of participants is depicted on the horizontal axis and precision levels are depicted on the vertical axis. The investors' individual level of

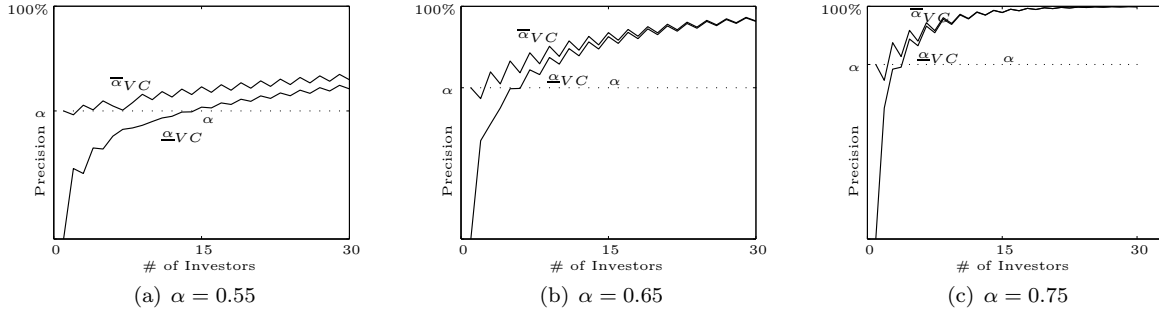


Figure 2: Venture-capital equivalence approaches first-best in a discrete setting with relatively few investors.

precision is depicted as a dashed horizontal line. By strategically choosing  $M$ , a principal makes the platform as efficient as a VC with precision  $\alpha_{VC}$  that is contained anywhere in the interval between the lines labeled  $\underline{\alpha}_{VC}$  and  $\bar{\alpha}_{VC}$ . The power of the crowd signal result is demonstrated in the three panels. At a precision of 55% with 30 participating investors, the crowd funding platform can be as efficient as a venture capitalist with precision  $\alpha_{VC} \in [64\%, 68\%]$ . More striking is how few participants are needed to approach  $\alpha_{VC} \approx 98\%$  when the participants' level of precision is equal to 65% and 75%.

## 4 Concluding Discussion

Our analysis suggests that naïve investors will play a critical role in equity crowdfunding. By acting on their information, naïve investors communicate the wisdom of the crowd and improve financing efficiency. Furthermore, platforms can set maximum investment thresholds to best utilize the crowd's information. Platform design influences the populations of investors and projects attracted, and can be used to improve financing efficiency.

While our results are most clearly seen in a single-period setting, a multi-period setting produces similar results and incorporates investors' learning over time, which has been shown to be an important feature of crowdfunding (Zhang and Liu 2012, Li 2015, Vismara 2015).<sup>12</sup> Appendix B contains the details of our multi-period setting, and the supporting analysis shows the robustness of our results. Considering multiple periods allows us to incorporate sophisticated investors' learning from early-moving investors. When sophisticated investors do not participate in the first period, they are able to learn from naïve investors. Because

<sup>12</sup>Entrepreneurs can also learn from crowdfunding platforms, as in Xu (2015).

naïve investors are individually infinitesimal and collectively informed, sophisticated investors perfectly infer project quality and only good projects are subsequently financed. By waiting, sophisticated investors improve financing efficiency, contrary to their ill-effects on efficiency in a one-period setting.

Our multi-period analysis provides several additional insights relevant to platform efficiency. Given the positive effects of learning, we show that encouraging slower financing improves efficiency. By encouraging sophisticated investors to not participate at the first opportunity, they learn from naïve investors and only good projects are financed. However, investors' desires to learn more information can lead to indefinite waiting. We show that exogenous time limits are needed on crowdfunding platforms, consistent with actual rules on nearly all existing platforms. By imposing a deadline, sophisticated investors are spurred to participate in good projects and bad projects are removed from the platform. An alternative to encourage earlier investment is the use of first-come, first-served allocations rather than pro rata. By rewarding early action with first-come, first-served allocations, sophisticated investors either move in the first period, or not at all. As a result, the efficiency gains from learning are destroyed, highlighting an advantage of pro rata allocations.

Our analysis emphasizes efficiency because equity crowdfunding platforms may attract projects and entrepreneurs that cannot find more traditional early-stage financing. The highest quality projects and most-connected entrepreneurs likely obtain alternative financing from friends and family, venture capitalists or angel investors. If this is the case, equity crowdfunding is exposed to a selection bias – lower-quality projects and less-connected entrepreneurs are forced to turn to the crowd. For equity crowdfunding to be successful, platforms must be structured to identify and finance only the best projects of those that cannot receive traditional financing – they must find the diamonds in the rough. By focusing on financing efficiency, equity crowdfunding platforms can create environments that are safe for naïve investors and effective for entrepreneurs.

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## Appendix A

We establish several useful results before proving the manuscripts' lemmas and propositions.

**Lemma A1.** *Conditional on sophisticated investors' equilibrium participation strategies  $\{\pi_G, \pi_B\}$ , a sophisticated investor uses an exposure weight  $\phi(\pi_G, \pi_B | \hat{F})$  in place of his subjective state probability in choosing whether or not to invest.*

**Proof of Lemma A1:** Sophisticated investors' expectations of the project's value depend on their signals:

$$E[V|\hat{F}] = \begin{cases} (\alpha(\delta + 1) - 1)c & \hat{F} = G \\ ((1 - \alpha)(\delta + 1) - 1)c & \hat{F} = B \end{cases} \quad (\text{A1})$$

Sophisticated investors do not necessarily invest when  $E[V|\hat{F}] \geq 0$  due to their asymmetric exposure to project outcomes. Define the share of an investors' committed capital deployed to the project as,

$$\sigma \equiv \frac{c}{\ell_S M + \ell_N M}, \quad (\text{A2})$$

where  $\ell_S$  is the measure of participating sophisticated investors and  $\ell_N$  is the measure of participating naïve investors. The remaining  $1 - \sigma$  is returned to investors implying that each investor's investment in the risky project is,

$$\sigma M. \quad (\text{A3})$$

Conditional on the equilibrium participation strategies of sophisticated investors, an explicit form for  $\sigma$  as a function of  $F$ ,  $\pi_G$ , and  $\pi_B$  is given by,

$$\sigma(F, \pi_G, \pi_B) = \begin{cases} \frac{c}{M(\gamma(\pi_G \alpha + \pi_B(1 - \alpha)) + (1 - \gamma)\beta)} & F = G \\ \frac{c}{M(\gamma(\pi_G(1 - \alpha) + \pi_B \alpha) + (1 - \gamma)(1 - \beta))} & F = B. \end{cases} \quad (\text{A4})$$

Now, define

$$\mathcal{S}(F, \pi_G, \pi_B) \quad (\text{A5})$$

as the set of projects funded in state  $F$ . The set  $\mathcal{S}(B, \pi_G, \pi_B)$  contains all projects with scale

$$c \leq M(\gamma(\pi_G(1 - \alpha) + \pi_B \alpha) + (1 - \gamma)(1 - \beta)),$$

and  $\mathcal{S}(G, \pi_G, \pi_B)$  contains all projects with scale,

$$c \leq M(\gamma(\pi_G \alpha + \pi_B(1 - \alpha)) + (1 - \gamma)\beta).$$

Some intuition regarding the importance of these sets is in order. Consider the following example: a project with scale  $\hat{c} \notin \mathcal{S}(B, \pi_G, \pi_B)$  will not be funded if it is not financially sound. Because the project will not attract sufficient capital in the state  $F = B$ , investors receive an implicit hedge against that state because of the all-or-nothing feature of the crowdfunding platform.

Sophisticated investors will invest in a project if their exposure to the state  $F = G$  is weakly greater than their investment. Roughly speaking, define an exposure weight for sophisticated investors as,

$$\phi(\pi_G, \pi_B | \hat{F}) = \begin{cases} \frac{\alpha \sigma(G, \pi_G, \pi_B) \mathbb{1}_G}{\alpha \sigma(G, \pi_G, \pi_B) \mathbb{1}_G + (1-\alpha) \sigma(B, \pi_G, \pi_B) \mathbb{1}_B} & \hat{F} = G, \\ \frac{(1-\alpha) \sigma(G, \pi_G, \pi_B) \mathbb{1}_G}{(1-\alpha) \sigma(G, \pi_G, \pi_B) \mathbb{1}_G + \alpha \sigma(B, \pi_G, \pi_B) \mathbb{1}_B} & \hat{F} = B. \end{cases} \quad (\text{A6})$$

where  $\mathbb{1}_F$  is shorthand notation for the indicator function  $\mathbb{1}_{c \in \mathcal{S}(F, \pi_G, \pi_B)}$ . As such, a sophisticated investor that observes  $\hat{F}$  will invest if,

$$\phi(\pi_G, \pi_B | \hat{F})(\delta + 1) - 1 \geq 0. \quad (\text{A7})$$

■

**Lemma A2.** *The exposure weight  $\phi(\pi_G, \pi_B | B)$  is weakly increasing in  $\pi_B$  and the exposure weight  $\phi(\pi_G, \pi_B | G)$  is weakly decreasing in  $\pi_G$ .*

**Proof of Lemma A2:**

If  $\mathbb{1}_B = 0$ , the comparative statics of  $\phi(\pi_G, \pi_B | B)$  and  $\phi(\pi_G, \pi_B | G)$  are trivial: they all equal zero. For the remainder of the proof, assume  $\mathbb{1}_B = 1$ .

First, consider  $\phi(\pi_G, \pi_B | B)$ . The expression may be rewritten as,

$$\phi(\pi_G, \pi_B | B) = \frac{1}{1 + \eta_B}, \quad (\text{A8})$$

where,

$$\eta_B = \frac{\alpha (\gamma (\pi_G \alpha + \pi_B (1 - \alpha)) + (1 - \gamma) \beta)}{(1 - \alpha) (\gamma (\pi_G (1 - \alpha) + \pi_B \alpha) + (1 - \gamma) (1 - \beta))}. \quad (\text{A9})$$

Therefore, the comparative static of  $\phi(\pi_G, \pi_B | B)$  with respect to  $\pi_B$  is,

$$\frac{\partial \phi(\pi_G, \pi_B | B)}{\partial \pi_B} = -\frac{\partial \eta_B / \partial \pi_B}{(1 + \eta_B)^2}. \quad (\text{A10})$$

The partial derivative of  $\eta_B$  with respect to  $\pi_B$  is given by,

$$\frac{\partial \eta_B}{\partial \pi_B} = \frac{\alpha \gamma ((1 - \alpha - \beta)(1 - \gamma) + (1 - 2\alpha) \gamma \pi_G)}{(1 - \alpha) (\gamma (\pi_G (1 - \alpha) + \pi_B \alpha) + (1 - \gamma) (1 - \beta))^2}. \quad (\text{A11})$$



The sign on the preceding expression is determined by the sign on,

$$(1 - \alpha - \beta)(1 - \gamma) + (1 - 2\alpha)\gamma\pi_G, \quad (\text{A12})$$

which is negative because  $\alpha > \frac{1}{2}$ ,  $\beta > \frac{1}{2}$  and  $\gamma \in [0, 1]$ . Therefore  $\frac{\partial \eta_B}{\partial \pi_B} < 0$  implying that,

$$\frac{\partial \phi(\pi_G, \pi_B | B)}{\partial \pi_B} \geq 0. \quad (\text{A13})$$

Now, consider  $\phi(\pi_G, 0 | G)$ . The expression may be rewritten as,

$$\phi(\pi_G, 0 | G) = \frac{1}{1 + \eta_G}, \quad (\text{A14})$$

where,

$$\eta_G = \frac{(1 - \alpha)(\gamma\pi_G\alpha + (1 - \gamma)\beta)}{\alpha(\gamma\pi_G(1 - \alpha) + (1 - \gamma)(1 - \beta))}. \quad (\text{A15})$$

Therefore, the comparative static of  $\phi(\pi_G, \pi_B | G)$  with respect to  $\pi_G$  is,

$$\frac{\partial \phi(\pi_G, 0 | G)}{\partial \pi_G} = -\frac{\partial \eta_G / \partial \pi_G}{(1 + \eta_G)^2}. \quad (\text{A16})$$

The partial derivative of  $\eta_G$  with respect to  $\pi_G$  is given by,

$$\frac{(1 - \alpha)\gamma((\alpha - \beta)(1 - \gamma))}{\alpha(\gamma(\pi_G(1 - \alpha) + \pi_B\alpha) + (1 - \gamma)(1 - \beta))^2}. \quad (\text{A17})$$

The sign on the preceding inequality is determined by  $\alpha - \beta$  which is positive. Therefore,  $\frac{\partial \eta_G}{\partial \pi_G} > 0$ , implying,

$$\frac{\partial \phi(\pi_G, 0 | B)}{\partial \pi_G} \leq 0. \quad (\text{A18})$$

■

**Lemma A3.** *Sophisticated investors that observe  $\hat{F} = B$  adhere to a deterministic strategy,  $\pi_B \in \{0, 1\}$ . If,*

$$\delta \geq \bar{\delta}, \quad (\text{A19})$$

where,

$$\bar{\delta} \equiv \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))} \quad (\text{A20})$$

*all sophisticated investors, regardless of signal invest in the project.*

**Proof of Lemma A3:**

All sophisticated investors that observe  $\hat{F} = B$  will invest if and only if,

$$0 \leq (1 - \alpha) \frac{(\delta + 1)c - c}{c} M\sigma(G, 1, 1) \mathbb{1}_G - \alpha M\sigma(B, 1, 1) \mathbb{1}_B, \quad (\text{A21})$$

where  $\mathbb{1}_F$  is shorthand notation for  $\mathbb{1}_{c \in \mathcal{S}(F, 1, 1)}$ . The inequality simplifies to,

$$1 \leq (\delta + 1) \frac{(1 - \alpha)\sigma(G, 1, 1)}{(1 - \alpha)\sigma(G, 1, 1) + \alpha\sigma(B, 1, 1)\mathbb{1}_B} \equiv (\delta + 1)\phi(1, 1|B). \quad (\text{A22})$$

If the preceding inequality holds with equality, then for any return marginally smaller than  $\delta$  it must be the case that not all sophisticated investors participate. One might think that a mixing equilibrium exists in this case, however,

$$\phi(1, 1|B) > \phi(1, \pi_B|B) \quad (\text{A23})$$

for any  $\pi_B < 1$  by Lemma A2. Therefore, for any  $(\delta + 1)$  smaller than  $1/\phi(1, 1|B)$ , investors that observe  $\hat{F} = B$  will not participate. Thus, the investors' decision to participate is deterministic when they observe  $\hat{F} = B$ .

The threshold return that compels the set of sophisticated investor that observe  $\hat{F} = B$  to invest is implicitly defined by the following equality (using (A4) and (A6)),

$$(\delta + 1) \left( \frac{\frac{c(1-\alpha)}{M(\gamma+(1-\gamma)\beta)}}{\frac{c(1-\alpha)}{M(\gamma+(1-\gamma)\beta)} + \frac{c\alpha}{M(\gamma+(1-\gamma)(1-\beta))}} \right) = 1, \quad (\text{A24})$$

which simplifies to,

$$(\delta + 1) \left( \frac{1}{1 + \frac{\alpha(\gamma+(1-\gamma)\beta)}{(1-\alpha)(\gamma+(1-\gamma)(1-\beta))}} \right) = 1. \quad (\text{A25})$$

Therefore, for all investors to participate (regardless of signal) the promised return needs to be sufficiently large,

$$\delta \geq \bar{\delta} \equiv \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))}. \quad (\text{A26})$$

Now, consider when  $\delta < \bar{\delta}$  so sophisticated investors' observing  $\hat{F} = B$  do not participate. Furthermore, assume that  $\mathbb{1}_B = 1$  without the participation of sophisticated investors that observe  $\hat{F} = B$  (we consider the case in which  $\mathbb{1}_B$  switches between 1 and 0 depending on the participation of sophisticated investors in

Proposition 3). Sophisticated investors that observe  $\hat{F} = G$  will participate if and only if,

$$0 \leq \alpha \frac{(\delta + 1)c - c}{c} M\sigma(G, 1, 0) \mathbb{1}_G - (1 - \alpha) M\sigma(B, 1, 0) \mathbb{1}_B. \quad (\text{A27})$$

The inequality simplifies to,

$$1 \leq (\delta + 1) \frac{\alpha \sigma(G, 1, 0)}{\alpha \sigma(G, 1, 0) + (1 - \alpha) \sigma(B, 1, 0) \mathbb{1}_B} \equiv (\delta + 1) \phi(1, 0|G). \quad (\text{A28})$$

If the preceding inequality holds with equality, then for any marginally-smaller return it must be the case that only a fraction of sophisticated investors participate. This is because,

$$\phi(1, 0|G) < \phi(\pi_G, 0|G) \quad (\text{A29})$$

for any  $\pi_G < 1$  by Lemma A2. The mixing probability  $\pi_G$  is pinned down by the equality,

$$1 = (\delta + 1) \phi(\pi_G, 0|G). \quad (\text{A30})$$

A substitution of the explicit form of  $\phi(\pi_G, 0|G)$  into the preceding equality yields,

$$1 = (\delta + 1) \frac{\alpha \frac{1}{(\gamma \pi_G \alpha + (1 - \gamma) \beta)}}{\alpha \frac{1}{(\gamma \pi_G \alpha + (1 - \gamma) \beta)} + (1 - \alpha) \frac{1}{\gamma \pi_G (1 - \alpha) + (1 - \gamma) (1 - \beta)}}. \quad (\text{A31})$$

The preceding expression simplifies to,

$$\frac{1}{(\delta + 1)} = \frac{1}{1 + \frac{(1 - \alpha)}{\alpha} \frac{(\gamma \pi_G \alpha + (1 - \gamma) \beta)}{\gamma \pi_G (1 - \alpha) + (1 - \gamma) (1 - \beta)}}, \quad (\text{A32})$$

or

$$\delta = \frac{(1 - \alpha)}{\alpha} \frac{(\gamma \pi_G \alpha + (1 - \gamma) \beta)}{\gamma \pi_G (1 - \alpha) + (1 - \gamma) (1 - \beta)} \quad (\text{A33})$$

since  $\delta = (\delta + 1) - 1$ . A rearrangement yields the solution for  $\pi_G$ ,

$$\pi_G = \frac{(1 - \gamma)(\alpha(1 - \beta)\delta - (1 - \alpha)\beta)}{\gamma\alpha(1 - \alpha)(1 - \delta)}, \quad (\text{A34})$$

which is increasing in  $\delta$ . The expression  $\pi_G$  is greater than zero as long as,

$$\delta \geq \frac{(1 - \alpha)\beta}{\alpha(1 - \beta)} \equiv \underline{\delta}, \quad (\text{A35})$$

and note that,

$$\frac{(1-\alpha)\beta}{\alpha(1-\beta)} \in [0, 1]. \quad (\text{A36})$$

The expression  $\pi_G$  is less than one as long as,

$$\delta < \frac{(1-\alpha)(\beta + (\alpha - \beta)\gamma)}{\alpha(1-\beta - (\alpha - \beta)\gamma)} \equiv \bar{\delta}. \quad (\text{A37})$$

and note that,

$$\frac{(1-\alpha)(\beta + (\alpha - \beta)\gamma)}{\alpha(1-\beta - (\alpha - \beta)\gamma)} \in [0, 1]. \quad (\text{A38})$$

If

$$\phi(1, 1|B) \leq \phi(\pi_G, 0|G) \quad (\text{A39})$$

holds for all  $\pi_G \in [0, 1]$ , then we have completely characterized the system. However, we have not ruled out that there are parameter sets in which,

$$\phi(1, 1|B) > \phi(0, 0|G), \quad (\text{A40})$$

which is explicitly,

$$\frac{(1-\alpha)\frac{1}{(\gamma+(1-\gamma)\beta)}}{(1-\alpha)\frac{1}{(\gamma+(1-\gamma)\beta)} + \alpha\frac{1}{(\gamma+(1-\gamma)(1-\beta))}} > \frac{\alpha\frac{1}{(1-\gamma)\beta}}{\alpha\frac{1}{(1-\gamma)\beta} + (1-\alpha)\frac{1}{(1-\gamma)(1-\beta)}}. \quad (\text{A41})$$

or,

$$\frac{1}{1 + \frac{\alpha}{1-\alpha}\frac{(\gamma+(1-\gamma)\beta)}{(\gamma+(1-\gamma)(1-\beta))}} > \frac{1}{1 + \frac{(1-\alpha)}{\alpha}\frac{(1-\gamma)\beta}{(1-\gamma)(1-\beta)}}. \quad (\text{A42})$$

which is possible if,

$$\bar{\delta} = \frac{\alpha}{1-\alpha}\frac{(\gamma+(1-\gamma)\beta)}{(\gamma+(1-\gamma)(1-\beta))} < \frac{(1-\alpha)}{\alpha}\frac{\beta}{(1-\beta)} = \underline{\delta}. \quad (\text{A43})$$

Because  $\alpha \in [\frac{1}{2}, 1]$ ,  $\alpha/(1-\alpha)$  is monotonically increasing in  $\alpha$  and it has a range  $(1, \infty)$ . Similarly,  $(1-\alpha)/\alpha$  is monotonically decreasing in  $\alpha$  and has a range  $[0, 1)$ . As such, the inequality cannot hold for arbitrarily large values of  $\alpha$ . However, if  $\alpha$  is sufficiently close to  $1/2$ , the inequality can hold because  $\gamma > 0$  and  $(1-\gamma)\beta > (1-\gamma)(1-\beta)$ .

In the cases in which  $\underline{\delta} > \bar{\delta}$ , the participation strategies of all sophisticated investors are determined by the subset that observes  $\hat{F} = B$ . If those sophisticated investors that observe  $\hat{F} = B$  are willing to participate, i.e.  $\delta \geq \bar{\delta}$ , then all sophisticated investors participate. If  $\delta < \bar{\delta}$ , then no sophisticated investors participate. Therefore, any characterization of sophisticated investors' participation strategies conditional on observing  $\hat{F} = B$  applies to all sophisticated investors when  $\underline{\delta} > \bar{\delta}$ .

■

**Lemma A4.** *If*

$$\underline{\delta} \leq \delta < \bar{\delta}, \quad (\text{A44})$$

where

$$\underline{\delta} \equiv \frac{(1-\alpha)\beta}{\alpha(1-\beta)}, \quad (\text{A45})$$

then sophisticated investors that observe  $\hat{F} = B$  do not participate and a sophisticated investor that observes  $\hat{F} = G$  will invest with probability,

$$\pi_G = \min \left\{ \max \left\{ \frac{(1-\gamma)(\alpha(1-\beta)\delta - (1-\alpha)\beta)}{\gamma\alpha(1-\alpha)(1-\delta)}, 0 \right\}, 1 \right\}. \quad (\text{A46})$$

**Proof of Lemma A4:**

The proof comes from Lemma A3.

■

**Lemma A5.** *If*

$$\delta < \underline{\delta} \quad (\text{A47})$$

no sophisticated investors participate.

**Proof of Lemma A5:**

The proof comes from Lemma A3.

■

**Proof of Proposition 1 and Corollary 1.1:**

We now proceed with the proof of the proposition. Lemmas A3, A4 and A5 provide the participation ranges for sophisticated investors based on their signals which are summarized in Proposition 1. For Corollary 1.1, the proof begins in a similar manner as Lemma A2. If  $\mathbb{1}_B = 0$  then the comparative statics of  $\phi(\pi_G, \pi_B|B)$  and  $\phi(\pi_G, \pi_B|G)$  are trivial: they all equal zero. For the remainder of the proof, assume  $\mathbb{1}_B = 1$ .

First, consider  $\phi(1, 1|B)$ . The expression may be rewritten as,

$$\phi(1, 1|B) = \frac{1}{1 + \eta_B}, \quad (\text{A48})$$

where,

$$\eta_B = \frac{\alpha(\gamma + (1-\gamma)\beta)}{(1-\alpha)(\gamma + (1-\gamma)(1-\beta))}. \quad (\text{A49})$$

Therefore, the comparative static of  $\phi(1, 1|B)$  with respect to a generic parameter  $\psi$  is,

$$\frac{\partial \phi(1, 1|B)}{\partial \psi} = -\frac{\partial \eta_B / \partial \psi}{(1 + \eta_B)^2}, \quad (\text{A50})$$

implying the sign of the comparative static is determined by,

$$-\frac{\partial \eta_B}{\partial \psi}. \quad (\text{A51})$$

The relevant parameters for the comparative static calculations are  $\{\alpha, \beta, \gamma\}$ ,

$$\frac{\partial \eta_B}{\partial \alpha} = \left( \frac{1}{(1 - \alpha)^2} \right) \left( \frac{\gamma + (1 - \gamma)\beta}{(\gamma + (1 - \gamma)(1 - \beta))} \right) \quad (\text{A52})$$

$$\geq 0, \quad (\text{A53})$$

$$\frac{\partial \eta_B}{\partial \beta} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - \gamma^2}{(\gamma + (1 - \gamma)(1 - \beta))^2} \right) \quad (\text{A54})$$

$$\geq 0, \quad (\text{A55})$$

$$\frac{\partial \eta_B}{\partial \gamma} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - 2\beta}{(\gamma + (1 - \gamma)(1 - \beta))^2} \right) \quad (\text{A56})$$

$$\leq 0. \quad (\text{A57})$$

Therefore,

$$\frac{\partial \phi(1, 1|B)}{\partial \alpha} \leq 0 \quad (\text{A58})$$

$$\frac{\partial \phi(1, 1|B)}{\partial \beta} \leq 0 \quad (\text{A59})$$

$$\frac{\partial \phi(1, 1|B)}{\partial \gamma} \geq 0. \quad (\text{A60})$$

Now, consider  $\phi(\pi_G, 0|G)$ . The expression may be rewritten as,

$$\phi(\pi_G, 0|G) = \frac{1}{1 + \eta_G}, \quad (\text{A61})$$

where,

$$\eta_G = \begin{cases} \frac{(1 - \alpha)(\gamma\alpha + (1 - \gamma)\beta)}{\alpha(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))} & \text{if } \pi_G = 1 \\ \delta & \text{if } \pi_G \in [0, 1), \end{cases} \quad (\text{A62})$$

where  $\eta_G$  evaluated at  $\pi_G \in [0, 1)$  utilizes the explicit form of  $\pi_G$  in (A46). Without loss of generality, we focus on the cases in which  $\pi_G = 1$  for the comparative statics. As such, the comparative static of  $\phi(1, 0|G)$

with respect to a generic parameter  $\psi$  is,

$$\frac{\partial\phi(1, 0|G)}{\partial\psi} = -\frac{\partial\eta_G/\partial\psi}{(1 + \eta_G)^2}, \quad (\text{A63})$$

implying the sign of the comparative static is determined by,

$$-\frac{\partial\eta_G}{\partial\psi}. \quad (\text{A64})$$

The relevant parameters for the comparative static calculations are again  $\{\alpha, \beta, \gamma\}$ ,

$$\frac{\partial\eta_G}{\partial\alpha} = -\frac{(1 - \gamma)(\alpha(\alpha - \beta)\gamma + \beta(1 - \beta)(1 - \gamma) + \beta\gamma(1 - \alpha))}{\alpha^2(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2} \quad (\text{A65})$$

$$\leq 0, \quad (\text{A66})$$

$$\frac{\partial\eta_G}{\partial\beta} = \left(\frac{1 - \alpha}{\alpha}\right) \left(\frac{(1 - \gamma)}{(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2}\right) \quad (\text{A67})$$

$$\geq 0, \quad (\text{A68})$$

$$\frac{\partial\eta_G}{\partial\gamma} = \left(\frac{1 - \alpha}{\alpha}\right) \left(\frac{(\alpha - \beta)}{(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2}\right) \quad (\text{A69})$$

$$\geq 0. \quad (\text{A70})$$

Therefore,

$$\frac{\partial\phi(\pi_G, 0|G)}{\partial\alpha} \geq 0 \quad (\text{A71})$$

$$\frac{\partial\phi(\pi_G, 0|G)}{\partial\beta} \leq 0 \quad (\text{A72})$$

$$\frac{\partial\phi(\pi_G, 0|G)}{\partial\gamma} \leq 0. \quad (\text{A73})$$

■

**Proof of Proposition 2:**

Conditional on all sophisticated investors participating, regardless of their signals, large scale projects are only financed if  $F = G$ . Therefore, projects for which  $F = G$  are financed with probability one and projects for which  $F = B$  are never financed.

■

**Proof of Proposition 3:** Suppose sophisticated investors' observing  $\hat{F} = B$  are unwilling to participate, i.e.,  $\delta < \bar{\delta}$ , and only a fraction  $\pi_G$  of sophisticated investors that observe  $\hat{F} = G$  participate, where  $\pi_G$  is

defined in (A46). This means that the quantity of funds raised for a financially sound project is,

$$M(\pi_G \gamma \alpha + (1 - \gamma)\beta) \tag{A74}$$

$$= \bar{\kappa} + \pi_G \gamma \alpha M, \tag{A75}$$

and the quantity of funds raised for a non financially sound project is,

$$M(\pi_G \gamma (1 - \alpha) + (1 - \gamma)(1 - \beta)) \tag{A76}$$

$$= \underline{\kappa} + \pi_G \gamma (1 - \alpha) M. \tag{A77}$$

Projects with scale  $c > \underline{\kappa} + \pi_G \gamma (1 - \alpha) M$  will not be financed if they are not financially sound. For these projects, investors are not exposed to the state  $F = B$ . Consider a single sophisticated investor that is not participating, either because he saw  $\hat{F} = B$  or because he is in the fraction  $1 - \pi_G$  of those that saw  $\hat{F} = G$  but do not participate. This investor is strictly better off by deviating from his strategy since he would obtain a strictly positive net return on investment,

$$\frac{c}{M(\alpha \pi_G \gamma + (1 - \gamma)\beta)} \delta > 0. \tag{A78}$$

This is true for all non-participating investors. However, it cannot be an equilibrium for all non-participating investors to deviate and participate since  $\delta < \bar{\delta}$ . Furthermore, there does not exist a mixing strategy for non-participating investors that makes them indifferent between participation and non-participation. This is because investor payoffs are not continuous with the number of participants: if the indicator function  $\mathbb{1}_B$  switches from one to zero there is a discrete jump in investor payoffs.

Thus, for projects with returns  $\delta < \bar{\delta}$  and scale  $c > \underline{\kappa} + \pi_G \gamma (1 - \alpha) M$  no equilibrium participation strategies exist for sophisticated investors.

■

**Proof of Proposition 4:**

According to Proposition 2, if the project's scale  $c \in (\underline{K}, \bar{K}]$ , first-best financing efficiency is achieved. Explicit forms of  $\underline{K}$  and  $\bar{K}$  yield the following two inequalities for first-best to occur,

$$c > \gamma M + (1 - \beta)(1 - \gamma)M \tag{A79}$$

$$c \leq \gamma M + \beta(1 - \gamma)M. \tag{A80}$$



The inequalities may be rewritten as,

$$\frac{c}{\gamma + (1 - \beta)(1 - \gamma)} > M \quad (\text{A81})$$

$$\frac{c}{\gamma + \beta(1 - \gamma)} \leq M, \quad (\text{A82})$$

which yield the desired result.

■

**Proof of Proposition 5:**

A threshold required crowd size  $\tau$  can be chosen by choosing  $M$ , i.e.,

$$\tau = c/M \quad (\text{A83})$$

Without loss of generality, consider only the values of  $\tau \geq \min\{\underline{B}, \underline{G}\}$  and the values of  $\tau \leq \max\{\overline{B}, \overline{G}\}$ .

For any value of  $\tau$ , the probability that a bad project is funded is,

$$1 - \underline{H}(\tau). \quad (\text{A84})$$

Similarly, the probability that a good project is funded is,

$$1 - \overline{H}(\tau). \quad (\text{A85})$$

The probability that a funded project is good  $G$  is,

$$\Pr(G|\text{funded}, \tau) = \frac{1 - \overline{H}(\tau)}{1 - \underline{H}(\tau) + 1 - \overline{H}(\tau)}, \quad (\text{A86})$$

Now consider a venture capitalist that observes a signal whether or not the project should be funded. The venture capitalist's signal is accurate with probability  $\alpha_{VC} \geq \frac{1}{2}$ . As such, for the venture capitalist, denoted  $VC$ , the probability a project is funded is equal to the ex ante probability that a project is good,

$$\Pr(\text{funded}|VC) = \frac{1}{2}, \quad (\text{A87})$$

and the probability that a chosen project is good is equal to,

$$\Pr(G|\text{funded}, VC) = \alpha_{VC}. \quad (\text{A88})$$

Therefore, a crowdfunding platform's venture capitalist equivalent precision,  $\alpha_{VC}$  is determined by the

following set of equations,

$$\Pr(G|\text{funded}, VC) = \Pr(G|\text{funded}, \tau), \quad (\text{A89})$$

$$\Pr(\text{funded}|VC) = \Pr(\text{funded}|\tau). \quad (\text{A90})$$

The equations simplify to,

$$\alpha_{VC} = \frac{1 - \overline{H}(\tau)}{1 - \underline{H}(\tau) + 1 - \overline{H}(\tau)}, \quad (\text{A91})$$

$$\frac{1}{2} = \frac{1}{2} (1 - \underline{H}(\tau) + 1 - \overline{H}(\tau)). \quad (\text{A92})$$

Therefore, at  $\tau$  it is equivalent to a VC with signal precision

$$\alpha_{VC} = 1 - \overline{H}(\tau), \quad (\text{A93})$$

where the set of permissible  $\tau$  values are pinned down by,

$$1 - \underline{H}(\tau) = \overline{H}(\tau). \quad (\text{A94})$$

Notice that the left-hand side is decreasing in  $\tau$  and the right-hand side is increasing in  $\tau$ . Therefore, if the CDFs are strictly monotonic, the VC equivalent value of  $\tau$  is unique. The principal may map this value of  $\tau$  into a contribution level according to,

$$M = \frac{c}{\tau}. \quad (\text{A95})$$

■

**Proof of Corollary 5.1:**

The proof is obvious from the proof of Proposition 5.

■

# Appendix B

## B.1 Multi-period Financing

In this section we consider an extension in which the funding process is dynamic. As in Section 2, a project on the platform is valued according to the expression in (1), the project’s fundraising goal is  $c$ , the goal is accomplished by selling equity claims to investors, an investor’s contribution is capped at  $M$ , and the platform exhibits an “all-or-nothing” financing feature.

In this multi-period setup, the project is on the platform until it gets funded or pulled. However, as will be shown shortly, an exogenous deadline is necessary when no endogenous stopping time exists in equilibrium. Denote the first period that the project is launched on the platform as  $t = 0$ . The discount rate between periods is normalized to zero and all investors can borrow or lend at the discount rate (i.e., investors do not face capital constraints, other than the exogenously given maximum project investment  $M$ ). It is assumed that each investor can participate only once per project and investors that do participate are not replaced.

We maintain the information structure from Section 2 in which each sophisticated investor observes  $\hat{F}$  and his signal is accurate with probability  $\alpha > \frac{1}{2}$ . Each sophisticated investor receives his signal at  $t = 0$  and he chooses if and when to optimally participate. Naïve investors arrive at  $t = 0$  in a similar fashion as in the base model; if the project is financially sound, a fraction  $(1 - \gamma)\beta$  of naïve investors contribute  $M$  where  $\beta > \frac{1}{2}$ . Conversely, if the project is not financially sound, a fraction  $(1 - \gamma)(1 - \beta)$  contribute  $M$ .<sup>13</sup>

Define  $K_t \geq 0$  as the quantity of funds raised for the project at the beginning of period  $t$ . Initially, zero funds are pledged to the project,  $K_0 = 0$ . However, over time the project will accrue contributions, and the incremental contribution in period  $t$  is defined  $\kappa_t$ . Investment levels and flows provide signals of the project’s potential success which aids investors’ valuation of the project.<sup>14</sup> The set  $\{K_t, \kappa_t\}$  is publicly observable after the conclusion of period  $t$  and all participants have perfect recall for previous periods. Define  $\mathcal{I}_t$  as the project’s entire fundraising history up until period  $t$ . The model’s timing in every period  $t$  is as follows,

- (i) Project enters period  $t$  with raised capital  $K_t$ ,
- (ii) Participants observe  $K_t$  and update their beliefs,
- (iii) Participants contribute funds  $\kappa_t$ ,
- (iv) If  $K_t + \kappa_t \geq c$  the project is funded, the project is removed from the platform and its type is revealed,

---

<sup>13</sup>In previous versions of this manuscript, naïve investors that abstained from participating at  $t = 0$  were allowed to update their beliefs based on  $t = 0$  investment flows and participate. Under that assumption, our results were qualitatively unchanged.

<sup>14</sup>Zhang and Liu (2012) and Herzenstein, Dholakia and Andrews (2011) show evidence of rational herding by crowdfunding investors, in which lenders engage in active observational learning, inferring quality from prior capital commitments.

(v) If  $K_t + \kappa_t < c$  the project is not fully funded and period  $t + 1$  begins.

Before we proceed to solving the model, there is a matter of housekeeping to address. Under some parameter sets and model specifications, a set of investors will be indifferent between investing in period  $t$  or waiting until a later period. Both are equilibria. We, however, adopt a refinement that implies only the latter equilibrium of waiting survives. The following defines what we coin the “opportunity cost” equilibrium refinement,

**Definition B1.** *If two equilibrium strategies exist for a set of sophisticated investors with one equilibrium being to invest at time  $t$  and the other being to wait until some  $\hat{t} > t$ , and the investors’ payoffs are equal under the equilibria, then the **opportunity cost equilibrium refinement** dictates that the investors will choose the latter equilibrium, i.e., they will wait.*

The motivation for the opportunity cost equilibrium refinement is that each investor’s information set weakly improves the longer he waits and his payoff will not decrease if he waits. We will appeal to the opportunity cost equilibrium refinement in narrowing the set of equilibrium strategies studied hereafter.

Each sophisticated investor’s ability to accurately forecast the project’s value is weakly increasing with the time he waits; the incremental funding activity each period may contain information that allows him to refine his beliefs. Therefore,

**Lemma B1.** *It is never optimal for a sophisticated investor to invest at time  $t$  if the project would still need additional funding in a subsequent period.*

Lemma B1 is natural: a sophisticated investor that chooses to invest will not do so until he is compelled to. The investor’s information set weakly improves each period he waits. As such, if a project will need an additional period of financing, it cannot be an equilibrium action for the sophisticated investor to forfeit the opportunity to refine his information set further.<sup>15</sup>

**Lemma B2.** *If it is optimal for a sophisticated investor that observes  $\hat{F} = B$  to invest at time  $t$ , then it is optimal for an investor that observes  $\hat{F} = G$  to also invest at time  $t$ .*

The result of Lemma B2 is natural as well. If a sophisticated investor that observes  $\hat{F} = B$  finds it optimal to invest then it must be the case that sophisticated investors that observed  $\hat{F} = G$  also find it optimal to invest because their expectation about the project’s payoff is higher. We now proceed to solving the multi-period model and, to ease exposition, we use the  $t = 0$  investment flows from naïve investors to partition the universe of projects into distinct categories.

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<sup>15</sup>We assume sophisticated investors cannot coordinate across periods. Were they able to, sophisticated investors could reveal their information by a sufficiently small proportion investing at  $t = 0$ . However, such equilibria are not robust to non-pro-rata allocation mechanisms nor opportunity-cost equilibrium refinements, justifying our no-coordination assumption.

**Lemma B3.** *Define*

$$\bar{c} = M(\gamma + (1 - \gamma)\beta). \quad (\text{B1})$$

*Any project with scale  $c > \bar{c}$  cannot be funded on the platform. Define*

$$\underline{c} = (1 - \gamma)\beta M. \quad (\text{B2})$$

*Any financially sound project with scale  $c \in (0, \underline{c}]$  is funded with certainty at the conclusion of  $t = 0$ . These projects are referred to as “multi-period small-scale projects.” Any financially sound project with scale  $c \in (\underline{c}, \bar{c}]$  may require multiple periods to fund. These projects are referred to as “multi-period large-scale projects.”*

There are two different threshold levels of scale outlined in Lemma B3. The first scale size,  $\bar{c}$ , provides an upper bound on the projects that can be plausibly funded on the platform. The explicit form of  $\bar{c}$  is the maximum quantity of capital that can be raised on the platform for a financially sound project when the project’s type is known by all participants. If the project’s scale exceeds the maximum investment level from a perfectly informed crowd it cannot be feasibly financed on the platform even over multiple periods.

The second level of scale  $\underline{c}$  is an upper bound for a special set of projects. If a financially sound project has scale  $c \leq \underline{c}$  it can be entirely financed by naïve investors at  $t = 0$ . Thus, the projects with scale  $c \in (0, \underline{c}]$  are treated as a one-shot game similar to our base model in Section 2. The projects with scale  $c \in (\underline{c}, \bar{c}]$  cannot be entirely financed in the first period by naïve investors. As such, these projects *may* take multiple periods to finance.

**Remark 1.** *The equilibrium participation strategies provided and characterized in Section 2 apply to all multi-period small-scale projects  $c \in (0, \underline{c}]$ .*

Remark 1 states that our analysis of the static model in Section 2 carries over to a special set of projects in the multi-period setting. For the sake of brevity, we focus on the projects that may require multiple periods to finance hereafter.

Consider a project with scale  $c \in (\underline{c}, \bar{c}]$ . This project cannot be funded solely by naïve investors in the first period  $t = 0$ . It may, however, be funded at  $t = 0$  if sophisticated investors provide ample funds. In the case that sophisticated investors do not provide the funding to meet the project’s goal at  $t = 0$ , the project enters its second period on the platform.

Suppose that the project is not fully funded at  $t = 0$ . By the strong law of large numbers, the project’s type is signaled perfectly by the total funds raised from naïve investors. If  $F$  equals  $G$ , then  $K_1 = (1 - \gamma)\beta M$

and if  $F$  equals  $B$ , then  $K_1 = (1 - \gamma)(1 - \beta)M$ . All sophisticated investors update their beliefs fully,

$$E[V|\hat{F}, K_1] = \begin{cases} ((\delta + 1) - 1)c & K_1 = (1 - \gamma)\beta M \\ -c & K_1 = (1 - \gamma)(1 - \beta)M. \end{cases} \quad (\text{B3})$$

To be financed, the project requires the involvement of sophisticated investors. If the project is not financially sound,  $F = B$ , the project will not raise additional capital from sophisticated investors. Without an exogenous intervention, e.g., being pulled by the entrepreneur or the platform, the project becomes a zombie. We address this shortly. According to Lemma B1, sophisticated investors will invest only if they believe that no further periods of financing will be required. As such, there exist equilibria in which all investors invest at  $t = 1, 2, 3, \dots, \infty$ . In each of these equilibria, the sophisticated investors' payoffs are constant because their opportunity cost of capital is equal to zero. We now appeal to opportunity cost equilibrium refinement, which implies that investors will never stop waiting unless compelled to. The following proposition provides a mechanism to eliminate the possibility.

**Proposition B1.** *A fundraising deadline of  $t^* = 1$  is Pareto dominant (weakly). Any project that is not entirely financed at the conclusion of  $t^*$  is pulled from the platform.*

A deadline, as proposed by Proposition B1, provides two services. First, it guarantees that zombie projects, i.e., unfunded projects that are perfectly revealed as not financially sound, will be pulled. This ensures that naïve investors' capital committed at  $t = 0$  is returned.<sup>16</sup> Second, the deadline compels sophisticated investors to invest, rather than waiting indefinitely.<sup>17</sup> For the remainder of this section, we assume that the platform enforces a  $t^* = 1$  deadline.

If the project is not fully funded at  $t = 0$ , a financially-sound project will be fully financed at  $t = 1$  with the participation of all sophisticated investors. As such,

$$\kappa_1 = \gamma M, \quad (\text{B4})$$

and each investor's investment equals

$$\sigma_1(G, 1, 1)M, \quad (\text{B5})$$

where  $\sigma_1(G, 1, 1)$  is defined similar to (A4) but with the subscript 1 denoting  $t = 1$ ,

$$\sigma_1(G, 1, 1) = \frac{c}{M(\gamma + (1 - \gamma)\beta)}. \quad (\text{B6})$$

---

<sup>16</sup>Mollick (2014) finds that crowdfunding campaigns tend to either meet their goal or to miss it badly (unsuccessful projects only raise 10% of their goals), so deadlines force the removal of bad projects.

<sup>17</sup>Kuppuswamy and Bayus (2014) finds a u-shaped pattern in the timing of contributions, with capital surges in the first and last week of the project, consistent with a investors waiting in order to learn from others' actions.

The remaining funds invested,

$$(1 - \sigma_1(G, 1, 1))M, \tag{B7}$$

are returned to investors.

Although the project cannot be fully funded at  $t = 0$  by naïve investors, sophisticated investors may choose to also participate  $t = 0$  and fund the project. We call this *preemption*. By preempting, sophisticated investors avoid dilution from  $t = 1$  investors. Avoiding dilution via preemption is not costless, however. Investors forgo the ability to refine their beliefs based on the information contained in the  $t = 0$  investment flow.

Importantly, investors will only preempt if they believe the project will be fully funded at  $t = 0$ . Thus, since each investor is infinitesimal, multiple equilibria may exist. Before we examine the equilibria we define a sophisticated investor’s value of preempting relative to waiting.

**Lemma B4.** *For multi-period large-scale projects, the value of investing at  $t = 0$  (preemption) relative to waiting until  $t = 1$  to invest for an investor that observes  $\hat{F} = G$  is positive if,*

$$\delta \in \left[ \frac{(1 - \alpha)\sigma_0(B, 1, 0)\mathbb{1}_B}{\alpha(\sigma_0(G, 1, 0)\mathbb{1}_G - \sigma_1(G, 1, 1))}, \bar{\delta} \right). \tag{B8}$$

Sophisticated investors who receive good signals preempt to prevent dilution from sophisticated investors who receive bad signals. Importantly, sophisticated investors with good signals only uniquely preempt if the potential profitability of the project is sufficiently low, i.e., less than  $\bar{\delta}$ . If potential profitability is high (such that all investors would participate at  $t = 0$ , as in Proposition 1), good-signal sophisticated investors cannot avoid dilution from bad-signal sophisticated investors who would also be willing to invest at  $t = 0$ . In such high-profitability cases, equilibria exist where financing occurs at either  $t = 0$  or  $t = 1$ . However, only the more-efficient  $t = 1$  equilibrium satisfies the opportunity-cost equilibrium refinement. In summary, preemption only occurs for projects’ having high-enough potential to tempt good-signal investors into forgoing further information, while low-enough potential that bad-signal investors would not be similarly tempted.

Note that preemption can only be valuable when  $\delta$  is sufficiently large such that  $\pi_G = 1$ . If  $\pi_G < 1$ , investing at  $t = 0$  is break-even for sophisticated investors, so preemption provides no benefit. However, investing with less information is costly, so the value of preemption is strictly negative when  $\pi_G < 1$ .

If a multi-period large-scale project is financed at  $t = 0$ , it must be individually rational for each sophisticated investor that contributes capital. As such, the participation criteria outlined in Section 2 for the static model must be satisfied for large-scale projects as well,

**Remark 2.** *Any multi-period large-scale project may be financed entirely at  $t = 0$  in equilibrium if the participation criteria and conditions outlined in Section 2 are satisfied.*

While it is possible to support an equilibrium in which a multi-period large-scale project is entirely financed at  $t = 0$ , it is not necessarily the case that sophisticated investors prefer that equilibrium relative to one in which they fund the project at  $t = 1$  when more information is available. Despite their preferences, the equilibrium is sustainable because each investor is infinitesimal and an investor does not affect the equilibrium with their choice to deviate or not. If, however, sophisticated investors could coordinate, they would only invest at  $t = 0$  if the value of preemption was greater than the value of waiting.

## B.2 Platform Efficiency — Encouraging Slower Financing

Projects may live one or two periods when the fundraising deadline is set at  $t^* = 1$ . The projects that survive for two periods achieve first-best — a bad project never reaches its financing goal and a good project always does. This is because naïve investors participate at  $t = 0$  and reveal the projects' types and allow for sophisticated investors to provide the remaining capital if the project is good. Naturally, this implies that a principal may improve efficiency by increasing the likelihood that projects are not fully funded in the first period. One means to lengthen the platform life of a project is to decrease  $\underline{c}$  so that more projects are classified as multi-period large-scale rather than multi-period small-scale. Recall, the explicit form for  $\underline{c}$  is,

$$\underline{c} = (1 - \gamma)\beta M. \tag{B9}$$

As such, a principal has three means to decrease  $\underline{c}$ : (i) increase  $\gamma$ , (ii) lower  $M$ , and (iii) lower  $\beta$ . Consider the first option, increasing  $\gamma$ , i.e., increasing the proportion of sophisticated investors relative to naïve investors. One means to increase  $\gamma$  is to screen platform participants. If the principal is able to increase the fraction of sophisticated investors (who are prone to wait), less capital will flow to projects when the project is first launched. This allows the project to survive longer on the platform which allows subsequent investors to learn from investment flows and make first-best investment choices.

Next, consider the principal's second option of lowering  $M$ . By lowering  $M$ , investors will contribute less to a project, compelling it to take multiple periods. Obviously, lowering  $M$  is not costless: lowering  $M$  limits the upper bound on what projects can be feasibly financed on the platform. Thus, while efficiency is gained, feasibility is lost.

Finally, consider the principal's third option of lowering  $\beta$ . Lowering  $\beta$  is obtained by lowering the precision of naïve investors' signals. While taboo, lowering  $\beta$  may imply making project prospectuses complex or opaque.



### B.3 First-Come, First-Served

The platform we have explored grants ownership on a pro rata basis. Investors that participate at  $t = 0$  are treated equally with those that enter at  $t = 1$ . Dividing ownership on a pro rata basis may be common, but dividing ownership on a first-come, first-served basis (FCFS) is also natural. In this subsection, we consider how FCFS ownership affects the participation strategies of sophisticated investors.

FCFS ownership does not affect the participation strategies of investors for multi-period small-scale projects because they are one-shot games. For multi-period large-scale projects that require  $\bar{T}$  periods, ownership is allocated as follows: if an investor participates in period  $t < \bar{T}$ , he receives a full share

$$\sigma_{t,FCFS}(F, \pi_G, \pi_B) = 1, \quad (\text{B10})$$

for all  $F$ ,  $\pi_G$ , and  $\pi_B$ . Conversely, if an investor participates at period  $t = \bar{T}$  he receives a share,

$$\sigma_{T,FCFS}(G, \pi_G, \pi_B) = \frac{(c - K_{\bar{T}})}{M\kappa_{\bar{T}}}. \quad (\text{B11})$$

**Proposition B2.** *If ownership is allocated on a first-come, first-served basis, no equilibria exist in which a project survives more than a single period.*

According to Proposition B2, the only equilibrium in which a multi-period large-scale project is funded is one in which it is entirely funded by sophisticated and naïve investors at  $t = 0$ . To understand the result, suppose that only naïve investors invested at  $t = 0$  and sophisticated investors waited until  $t = 1$  to make their participation decisions. Naïve investors perfectly reveal the project's quality through their funding levels and sophisticated investors only participate if  $F = G$ . Financing efficiency is first-best and the naïve investors that invest at  $t = 0$  receive a share

$$\sigma_{0,FCFS}(\cdot) = 1$$

while sophisticated investors receive a share,

$$\sigma_{1,FCFS}(G, 1, 1) = \frac{(c - (1 - \gamma)\beta M)}{M\gamma},$$

which is less than 1. A single sophisticated investor strictly increases his payoff by deviating from his strategy and investing with naïve investors at  $t = 0$ : financing efficiency is still first-best when he deviates, but he receives a larger share of the project's payoff. Because each investor is infinitesimal, there is not a marginal sophisticated investor that internalizes the impact of his decision on the project outcome. Therefore, if an equilibrium exists with FCFS share allocations, it must be the case that the project is entirely financed at

$t = 0$ .

**Corollary B1.** *Any project that is financed in equilibrium at  $t = 0$  with first-come, first-served share allocations must satisfy the participation criteria and conditions of Proposition 1.*

The results of this section imply that FCFS share allocations may be welfare destroying because they encourage concentrated contributions rather than dispersed, staggered-across-time contributions. This prohibits investors from learning from investment flows and prevents multi-period large-scale, lower-return projects from being financed. Furthermore, there is a range of projects which will all be financed at  $t = 0$ , despite the value of preemption being negative. Overall, FCFS share allocations reduce financing efficiency and feasibility.

## B.4 Supporting Proofs for Multi-Period Extension

### Proof of Lemma B1:

The intuition is natural; because investment is irreversible, there is no incentive to invest early if there will still be an additional period of capital raising. Therefore, it is costless to wait until the next period.

■

### Proof of Lemma B2:

Because  $\alpha > 1/2$ , it can never be optimal for an investor that observes  $\hat{F} = B$  to invest if it is not optimal for an investor that observes  $\hat{F} = G$  to invest.

■

### Proof of Lemma B3:

The proof follows from the naïve investors' investment flows at  $t = 0$ .

■

### Proof of Proposition B1:

Both financially sound and unsound projects that require additional financing after  $t = 0$  may exist indefinitely on the platform without an exogenous deadline. Projects that are revealed to not be financially sound ( $F = B$ ), receive no additional capital with probability one. Without an exogenous intervention, these projects remain on the platform indefinitely. Projects that are revealed to be financially sound ( $F = G$ ) and have scale  $c > (1 - \gamma)M$  will not be fully financed according to Lemma B1.

An exogenous deadline of  $t^* = 1$  eliminates these two possibilities without making any investor worse off. Projects that are revealed to have  $F = B$  are pulled from the platform and the naïve investors that contributed  $(1 - \gamma)(1 - \beta)M$  at  $t = 0$  are returned their capital. Because the opportunity cost of capital is

equal to zero, these investors are no worse off and any slight perturbation to their opportunity cost of capital would make them strictly better off.

■

**Proof of Lemma B4:**

Consider sophisticated investors that observe  $\hat{F} = B$  when sophisticated investors are using mixing strategies  $\pi_B, \pi_G \in [0, 1]$ . The payoff to each of them if they invest at  $t = 0$  versus an investment at  $t = 1$ ,

$$\left( (1 - \alpha) \frac{(\delta + 1)c - c}{c} M\sigma_0(G, \pi_G, \pi_B) \mathbb{1}_G - \alpha M\sigma_0(B, \pi_G, \pi_B) \mathbb{1}_B \right) - \left( (1 - \alpha) \frac{(\delta + 1)c - c}{c} M\sigma_1(G, \pi_G, \pi_B) \right) \quad (\text{B12})$$

where the first large term in parenthesis is the expected value of investing at  $t = 0$  and the second large term is the expected value of waiting. Because sophisticated investors that observe  $\hat{F} = B$  play deterministic strategies at  $t = 0$  and by Lemma B2, the expression simplifies to,

$$\left( (1 - \alpha) \frac{(\delta + 1)c - c}{c} M\sigma_0(G, 1, 1) \mathbb{1}_G - \alpha M\sigma_0(B, 1, 1) \mathbb{1}_B \right) - \left( (1 - \alpha) \frac{(\delta + 1)c - c}{c} M\sigma_1(G, 1, 1) \right), \quad (\text{B13})$$

which is negative. Therefore, the value of preemption is negative for sophisticated investors that observe  $\hat{F} = B$ .

Now consider a sophisticated investor that observes  $\hat{F} = G$  when sophisticated investors are using mixing strategies  $\pi_B, \pi_G \in [0, 1]$ . The value of preemption is,

$$\left( \alpha \frac{(\delta + 1)c - c}{c} M\sigma_0(G, \pi_G, \pi_B) \mathbb{1}_G - (1 - \alpha) M\sigma_0(B, \pi_G, \pi_B) \mathbb{1}_B \right) - \left( \alpha \frac{(\delta + 1)c - c}{c} M\sigma_1(G, 1, 1) \right), \quad (\text{B14})$$

and the expression is positive if,

$$1 \leq (\delta + 1) \frac{\alpha(\sigma_0(G, \pi_G, \pi_B) \mathbb{1}_G - \sigma_1(G, 1, 1))}{\alpha(\sigma_0(G, \pi_G, \pi_B) \mathbb{1}_G - \sigma_1(G, 1, 1)) + (1 - \alpha)\sigma_0(B, \pi_G, \pi_B) \mathbb{1}_B}. \quad (\text{B15})$$

Preemption is only valuable if sophisticated investors that observed  $\hat{F} = B$  are unwilling to participate at  $t = 0$ . As such, preemption cannot be positive if  $\pi_B = 1$ . Furthermore, if  $\pi_G \in [0, 1)$ , then sophisticated investors mixing probability makes them indifferent between investing or not investing. This implies that the value of preemption is strictly negative if  $\pi_G \in [0, 1)$ . This implies that the preceding expression may be rewritten as,

$$1 \leq (\delta + 1) \frac{\alpha(\sigma_0(G, 1, 0) \mathbb{1}_G - \sigma_1(G, 1, 1))}{\alpha(\sigma_0(G, 1, 0) \mathbb{1}_G - \sigma_1(G, 1, 1)) + (1 - \alpha)\sigma_0(B, 1, 0) \mathbb{1}_B}, \quad (\text{B16})$$

which may be rewritten equivalently as,

$$\frac{(1 - \alpha)\sigma_0(B, 1, 0)\mathbb{1}_B}{\alpha(\sigma_0(G, 1, 0)\mathbb{1}_G - \sigma_1(G, 1, 1))} \leq \delta. \quad (\text{B17})$$

Furthermore, note that,

$$\frac{\alpha(\sigma_0(G, 1, 0)\mathbb{1}_G - \sigma_1(G, 1, 1))}{\alpha(\sigma_0(G, 1, 0)\mathbb{1}_G - \sigma_1(G, 1, 1)) + (1 - \alpha)\sigma_0(B, 1, 0)\mathbb{1}_B} < \frac{\alpha\sigma_0(G, 1, 0)\mathbb{1}_G}{\alpha\sigma_0(G, 1, 0)\mathbb{1}_G + (1 - \alpha)\sigma_0(B, 1, 0)\mathbb{1}_B} \quad (\text{B18})$$

$$= \phi_0(1, 0|G), \quad (\text{B19})$$

implying that if the value of preemption is positive then the criteria for sophisticated investors that observe  $\hat{F} = G$  to invest at  $t = 0$ ,

$$1 \leq (\delta + 1)\phi_0(1, 0|G), \quad (\text{B20})$$

is also satisfied.

■

#### **Proof of Proposition B2 and Corollary B1:**

Consider a proposed equilibrium in which only naïve investors invest at  $t = 0$  and sophisticated investors waited until  $t = 1$  to make their participation decisions. Naïve investors perfectly reveal the project's quality through their funding levels at  $t = 0$ . Sophisticated investors update their beliefs and all sophisticated investors participate if  $F = G$  and abstain otherwise. If  $F = G$ , naïve investors contribute a total capital quantity,

$$M(1 - \gamma)\beta, \quad (\text{B21})$$

and the naïve investors that invest at  $t = 0$  receive a share

$$\sigma_{0,FCFS}(\cdot) = 1, \quad (\text{B22})$$

because investment is based on a first-come, first-served basis.

Sophisticated investors that invest at  $t = 1$  contribute a total capital quantity,

$$M\gamma \quad (\text{B23})$$

and any investor that participates at  $t = 1$  receives a share,

$$\sigma_{1,FCFS}(G, 1, 1) = \frac{(c - (1 - \gamma)\beta M)}{M\gamma}, \quad (\text{B24})$$

which is less than 1.

A single sophisticated investor strictly increases his payoff by deviating from his proposed equilibrium strategy and investing with naïve investors at  $t = 0$ : financing efficiency is still first-best when he deviates, but he receives a larger share of the project's payoff. Because each investor is infinitesimal, there is not a marginal sophisticated investor that internalizes the impact of his decision on the project outcome. Therefore, the proposed equilibrium unravels.

By continuation, any proposed equilibrium in which some sophisticated investors delay their participation until  $t = 1$  unravels.

The only remaining equilibrium is one in which all sophisticated investors participate at  $t = 0$ .

■