A Control Performance Metric for Real-Time Timing Constraints

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Abstract

We study how to characterize different values for control task timing constraints according to the performance of closed-loop systems. We present a Quality-of-Control (QoC) performance metric, in terms of the closed-loop system error, that can be associated with each timing constraint value. Finally we discuss the QoC metric impact/potential for real-time task timing constraints and scheduling.

1. Introduction

For closed-loop systems, designed using classical discrete-time control theory [AST97], it is standard practice [ARZ00] to model control activities with periodic tasks characterized by *fixed timing constraints* such as *periods* and *deadlines*. Control task period and deadline are given by *sampling period* and *time delay* used in the controller design stage. Several settings for period and deadline fulfill the control performance specifications, providing different degrees of control performance. However, only one setting can be selected [FOH97], thus loosing the control performance information that each feasible setting incorporates.

In this paper we study how to characterize different values for control task timing constraints according to the performance of closed-loop systems. We present a *Quality-of-Control* (QoC) performance metric in terms of the closed-loop system error that can be associated with each timing constraint value. Finally we discuss the QoC metric impact/potential for real-time task timing constraints and scheduling.

We first review several properties that are used to evaluate the performance of closed-loop systems and discuss performance criteria and their relation to control task timing constraints, such as periods and deadlines. We argue and show experimentally that different values for the control task period have more influence on the control performance than different values for the control task deadline.

Then, focusing on the task period, we show that from a control quantitative measure (closed-loop system error) we obtain a quantitative control performance metric, QoC, that can be associated with each period value. Consequently, the period constraint for control tasks not only implies temporal requirements (as it traditionally does) but it also provides control information in terms of control performance.

Finally, we discuss the potential for real-time scheduling that the QoC metric has, depending on the type of constraints that characterize the timing of control tasks (see [RAM96] for a review on the origin of timing constraints).

2. Performance of control systems

In classical feedback control theory, several properties are used to evaluate the performance of closed-loop systems. The primary evaluation concerns itself with meeting the closed-loop system *response characteristics* (such as *transient response* and *steady-state accuracy*) and *stability* [AST97].

Beyond these requirements, looking at the closed-loop system response, controller designs attempt to minimize the *system error* for certain anticipated inputs or perturbations. The closed-loop system error is defined as the difference between the desired response and the actual response of the controlled system. We illustrate these concepts in Figure 1.

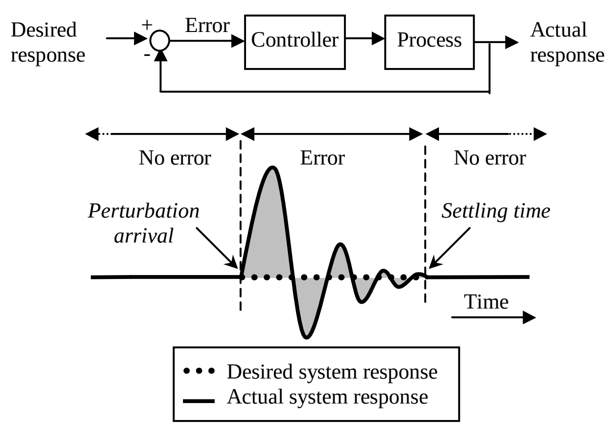

Figure 1. Top - Closed-loop system. Bottom - System error (shaded area)

Two criteria, IAE and ITAE, are generally used to evaluate control system design and performance. IAE (1) is the Integral of the Absolute value of the Error and ITAE (2) is the Integral of the Time-weighted Absolute value of the Error [DOR95]:

$$IAE = \int_{t_0}^{t_f} \left| y_{des}(t) - y_{act}(t) \right| dt \tag{1}$$

$$ITAE = \int_{t_0}^{t_f} t \cdot \left| y_{des}(t) - y_{act}(t) \right| dt \tag{2}$$

where y_{des} is the desired system response, y_{act} is the actual system response and t_0 and t_f are the initial and final times of the evaluation period. ITAE weights late-errors heavier and discounts the transient response, whereas IAE weights all errors equally.

3. Impact of timing constraints on control performance

In this section we discuss the impact of periods and deadlines in the controlled system performance, in terms of the system error.

3.1. Design parameters

In control design, the desired controlled system performance is met by specifying the *closed loop poles* location [AST97]. Care must be taken of the fact that sampling periods affect the location of the closed loop poles, thus giving different degrees of performance. On the other hand, time delays only affects the controlled system performance in terms of delaying the response. Figure 2 illustrates these concepts. In Figure 2 (left) we show five responses of a generic controlled system that is controlled by a task with five different periods. In Figure 2 (right) we show the five system responses if the task has five different deadlines.

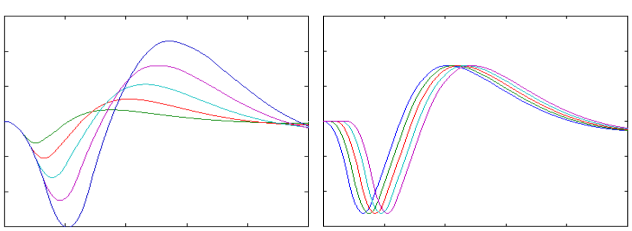

Figure 2. Effects of periods (left) and deadlines (right) on control performance (system error)

Even having different responses depending on different periods or deadlines, it can be seen in Figure 2 that the errors of the system response for different values for the task period follow a different tendency than for the deadline.

Consequently, we define the QoC metric in terms of sampling periods (periods) because its impact on the control performance is stronger than time delays (deadlines). To confirm this argumentation, we evaluate separately the influence of different values for the task period and different values for the task deadline on the controlled system error.

3.2. Experimental evaluation

For this evaluation, the IAE index would give the same evaluation (looking at the system error) for closed loops designed with or without time delays (see Figure 2). Therefore, we use the ITAE index, which penalizes delayed responses.

To proceed with this evaluation, we use an inverted pendulum as a controlled process [MAR01] and we introduce a perturbation that unbalances the pendulum. The goal of our controller is to maintain the desired vertical position of the inverted pendulum at all the times. The performance specification is to recover from a perturbation in less than two seconds (settling time of 2s). After the control analysis, we obtain a discrete-time controller that, while guaranteeing stability and fulfilling the control response performance specifications, supports several values for the sampling period (from 30 to 150ms, with a granularity of 10ms) and time delay (20 to 80ms, with a granularity of 10ms). Figure 3 shows the evaluation of the system error using the ITAE criterion (from the perturbation arrival to the settling time) when the inverted pendulum, in the presence of a perturbation, is controlled by a control task executing with a

- period, ranging from 30 to 150 ms (Figure 3 top) and
- deadline, ranging from 20 to 80ms (Figure 3 bottom).

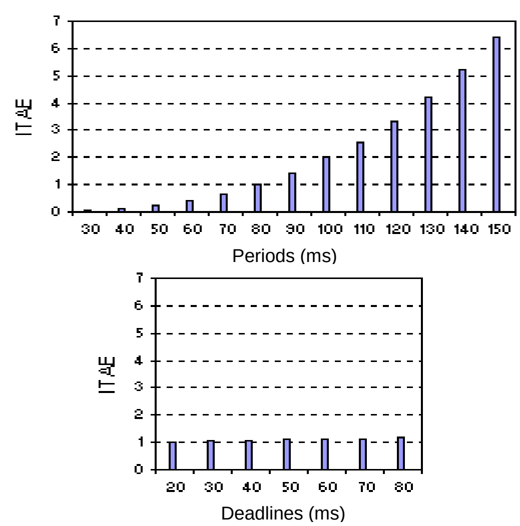

Figure 3. ITAE criterion depending on different periods (top) and deadlines (bottom)

From Figure 3 we confirm our argumentation: *periods have stronger effects on the system error than deadlines.* Although ITAE weights late-errors heavier, thus penalizing longer deadlines, periods still have more influence on the system error.

This conclusion leads us to decide to focus only on the relation between the values for the period and the system error for the QoC metric definition.

4. QoC metric definition

We define the QoC metric for each control task period value in terms of the closed-loop system error. Here, we use the IAE criterion because now we are interested on weighting all the errors equally. Note that we are defining an absolute metric for measuring the quality of the controlled system response given a control strategy (a specific task period). Therefore, by weighting all the errors equally, the measured values will not be time-dependent, thus separating the error magnitude from the time it happens.

Since the aim of controllers is to minimize the error, we define that better QoC will correspond to smaller errors. That it, the relation between the IAE index and the QoC is inversely proportional. For that reason, we define the QoC metric in terms of the controlled system response error as follows:

$$QoC(y_{act}:h) = \frac{\dfrac{1}{IAE(y_{act}:h)} - \dfrac{1}{IAE(y_{act}:h_{\max})}}{\dfrac{1}{IAE(y_{act}:h_{\min})} - \dfrac{1}{IAE(y_{act}:h_{\max})}} \qquad (3)$$

where:

- *IAE error evaluation time interval* is the time elapsed from the time of occurrence of the perturbation (t_0) to the settling time (t_f). Note that, due to the control analysis done at the design stage, the closed-loop performance specifications are met by all period values. Consequently, the settling time is the same for all of them.

- y_{act}:*h* denotes that the actual system response has been obtained with a task executing with a specific period (given by the corresponding sampling period h value) value. Note that y_{act}: h_{min} and y_{act}: h_{min} denotes the actual system response if the task is executing with the shortest or longest period (from all the feasible period values obtained in the offline control analysis).

Note that given different values for the period of the control task, the resulting QoC values will fall in the range of *[0,1]* (due to the normalization obtained considering the lowest - h_{max} – and highest – h_{min} – possible rates), where *zero* is equivalent to the lowest QoC and *one* is the best QoC. In Figure 4 we show, numerically and graphically, the different values expressing control performance in terms of the QoC metric that can be associated to each task period. The inverted pendulum, in the presence of a perturbation, is controlled by a control task executing with periods ranging from 60 to 100 ms and a deadline of 20ms (note that 60ms and 100ms corresponds to the h_{min} and h_{max} in (3), accordingly to the offline control analysis).

Periods (ms)	**IAE**	**QoC**
60	0.96	1.00
70	1.45	0.53
80	2.04	0.26
90	2.72	0.10
100	3.47	0.00

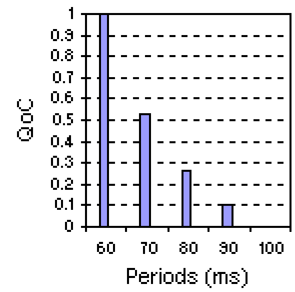

Figure 4. Periods vs. QoC

In Figure 4 it can be clearly seen that a control task running with a period of 60ms gives a better QoC than a task running with a period of 80ms. Therefore, the main conclusion we extract is that *the shorter period a control task is assigned, the smaller the system error* (corroborating the results of control theory [AST97]).

5. Discussion

In the previous sections we have argued and experimentally shown that the control performance of a real-time computer-controlled system depends on the timing of the controlling tasks. The timing of a control task in a real-time system depends on its timing constraints. From a control point of view, we have seen that at the design stage, several values for control task timing constraints fulfill the closed-loop performance specifications. Each of theses values, even fulfilling the closed-loop performance specifications, gives different degrees of control performance, which we express in terms of the system error with the QoC metric.

On the other hand, control task timing constraints are given by the timing requirements imposed by the control methods and models used in designing the controller. Different controller design methods impose different timing requirements on an implementation. This in turn leads to the derivation of different types of timing constraint for control tasks: *fixed timing constraints*, i.e., periods and deadlines, or *flexible timing constraints*, i.e., sets of feasible instance separations and response times [MAR01].

With the application of fixed timing constraints, the designer is faced with the need of selecting concrete values for the task period and deadline that will hold for all task instances, thus loosing the control performance information that each value implicitly has. Selecting a specific setting precludes the possibility of changing at run time the timing of the control task, thus excluding the opportunity of improving the performance of the controlled system.

With the application of flexible timing constraints, at the design stage, all these values are kept, giving to the scheduler a set of feasible values to choose from. At run time, at each control task instance execution, different settings (couples of <instance separation, response time>) will apply. This gives the opportunity of choosing specific setting at each instance execution in such a way that the performance of the controlled system is improved.

In [MAR01] we showed that flexible timing constraints offer the possibility of taking scheduling decisions accounting for the schedulability of other tasks. Accommodating the QoC metric to the instance separation constraint and associating each QoC metric value with each feasible instance separation (as we did with periods, section 4), we open the possibility of allowing taking scheduling decisions accounting for schedulability and control performance. That is, taking advantage of the control performance information given by the QoC metric, we will allow the possibility of taking scheduling decisions according to both temporal and control information. In this way, scheduling policies, although focusing on meeting timing constraints, will have the possibility of adequately meeting these constraints in such a way that the quality of the controlled system is also improved.

However, before looking at this new scheduling problem, few issues have still to be addressed. We have seen that for task periods, the shorter period a control task is assigned, the smaller the system error. With instance separations, we have to study the influence of different orderings of instance separation values (for a control task) on the system error. Notice that is not the same to have a constant instance separation value for all instances (as it is in the case of a value for the task period) than having different values at each instance execution (as flexible timing constraints permit).

6. Conclusions and future work

In this paper we have presented a Quality-of-Control metric that can be associated to different values of the control task period. It expresses control performance information in terms of the system error.

We have discussed control task timing constraints in relation with the QoC metric. We have argued that fixed timing constraints for control tasks cannot take advantage of the control information offered by the QoC metric. However, with the use of more flexible timing constraints for control tasks, the QoC metric could be used for taking scheduling decisions to account for task schedulability but also control performance improvement.

We are currently studying the relation of instance separations and QoC metric. We are also in the process of formulating a new scheduling problem in terms of the QoC metric associated to control task constraints and investigating possible scheduling solutions.

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