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Limited Feedback Based on Tree-structured Codebook for MIMO Systems over Time-varying Channels

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***Abstract*—In this paper, we propose a novel limited feedback approach for MIMO systems with transmit beamforming over time-varying channels, where the optimal beamforming vector of the current frame is selected from the son-codewords of the codewords neighboring to previous beamforming codeword. The proposed approach exploits the channel temporal correlation to improve the CSI quantized precise. The simulation results show that the proposed approach can efficiently improve the BER performance relative to the common feedback approach, and the feedback rate can be further reduced under some conditions. Moreover, the effect of Doppler frequency shift on the performance of the proposed approach is analyzed.**

***Index Terms*—MIMO, transmit beamforming, temporal correlation, tree-structured codebook**

I. Introduction

The exploitation of channel state information (CSI) at the transmitter in wireless systems has been a highly active research area, and the transmit CSI can significantly improve the system performance [1]. In multiple-input multiple-output (MIMO) systems, when the CSI is available to the transmitter, the array gain as well as the transmit diversity gain can be obtained utilizing transmit beamforming [2]. In time division duplexing (TDD) systems, the CSI can be faithfully estimated at the transmitter by exploiting the channel reciprocity. In frequency division duplexing (FDD) systems, however, the partial CSI can be obtained at the transmitter through the forward-link CSI from the receiver to the transmitter over a limited rate feedback channel.

In MIMO systems with transmit beamforming, the limited feedback method based on a pre-designed codebook by conveying the index of optimal beamforming vector can significantly reduce the feedback rate with maintaining good performance [3]. The feedback rate can be further reduced by exploiting the temporal correlation of the channel. In [4], the temporal correlation between the quantized beamformers is modeled as a first-order finite-state Markov chain. Based on the Markov model, no feedback is sent for quantized beamformer with the highest transition probability, and the remaining quantized beamformers are encoded using a fixed-length code. This approach is extended in [5] where the quantized beamformers with the least transition probability are ignored. Apparently, the feedback method is lossy, and the feedback reduction is obtained at the expense of a performance loss. In [6], a lossless feedback method is proposed, where the quantized beamformers are encoded using a variable-length code, i.e. Huffman code, according to the transition probability. Despite minimizing the average feedback rate, this approach may not suit practical applications where CSI feedback consists of fixed-length bit block [7].

Analogously, the channel temporal correlation can be exploited to improve the CSI quantized precision. Motivated by that, an efficient feedback method based on tree-structured codebook is proposed in this paper. The tree-structured codebook is generated by Lloyd algorithm, in which the son-codewords is the more precise partition of Voronoi region corresponding to father-codeword. Due to the temporal correlation of wireless channels, the optimal quantized beamformer corresponding to the adjacent frames should be within a small neighborhood. Known to the previous beamformer, a new codebook is designed, which consists of the son-codewords of the neighborhood of the previous beamformer. Then the better CSI quantized precision can be obtained, and the number of feedback bits can be further reduced under some conditions.

The rest of the paper is organized as follows. In section 2, we introduce transmit beamforming technique for MIMO system with receive combining. Then, in section 3, we describe tree-structured codebook which is constructed by Lloyd algorithm and the developed approach for the beamforming MIMO systems to improve system performance. Finally, we present simulation results to demonstrate performance improvement of the proposed approach in section 4.

II. System Model

A MIMO system with transmit beamforming and receive combining that uses M_t transmit antennas, M_r receive antennas is illustrated in Fig.1. At the transmitter, the transmitted data symbol s is multiplied by an M_t dimensional beamforming vector $\mathbf{w} = \left[w_1, w_2, \ldots, w_{M_t} \right]^T$. At the receiver, the received signal is multiplied by an M_r dimensional combining vector $\mathbf{z} = \left[z_1, z_2, \ldots, z_{M_r} \right]^T$, Then the combined signal can be expressed as

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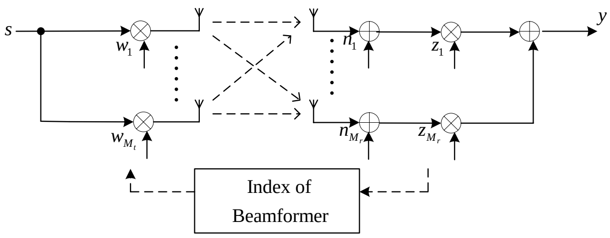

Figure 1. Block diagram of a MIMO-OFDM system with transmit beamforming and receive combining.

$$y = \mathbf{z}^H \left(\mathbf{H}\mathbf{w}s + \mathbf{n} \right) \quad (1)$$

Here the entries of $M_t * M_r$ dimensional matrix $\mathbf{H}$ represent the channel gains from the transmit antennas to the receive antennas.

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1M_t} \\ \vdots & \ddots & \vdots \\ h_{M_r 1} & \cdots & h_{M_r M_t} \end{bmatrix} \quad (2)$$

And the $\mathbf{n}=[n_1, n_2, ..., n_{M_r}]$ is the M_r dimensional noise vector with entries that have an identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance N_0. We allocate the equal power P to each symbol, and restrict $\|\mathbf{w}\| = 1$ to maintain the overall power constraint (where $\|(\cdot)\|$ denotes the l_2-norm of $(\cdot)$).

In the MIMO system with transmit beamforming and receive combining, $\mathbf{w}$ and $\mathbf{z}$ can be designed to maximize the signal-to-noise ratio (SNR). The SNR can be written as

$$\gamma = \frac{P}{N_0} \left| \mathbf{z}^H \mathbf{H}\mathbf{w} \right|^2 \quad (3)$$

Here $|\mathbf{zHw}|^2$ is the effective channel gain. Without loss of generality let $\|\mathbf{z}\| = 1$. For any $\|\mathbf{w}\|$, it is possible to show that the SNR maximizing solution uses maximum ratio combining (MRC) with

$$\mathbf{z} = \frac{\mathbf{H}\mathbf{w}}{\|\mathbf{H}\mathbf{w}\|} \quad (4)$$

If the channel matrix $\mathbf{H}$ is perfectly known by the transmitter as well as the receiver, considering maximum ratio transmission (MRT), the optimal beamforming vector $\mathbf{w}$ is the right singular vector of channel matrix $\mathbf{H}$ corresponding to the largest singular value of $\mathbf{H}$ [8]. But, in a realistic communication system, the channel matrix $\mathbf{H}$ cannot be perfectly known by the transmitter. It is assumed that there exists a low-rate, error-free and zero-delay feedback link from the receiver to the transmitter, i.e., the receiver informs the transmitter of beamforming vector $\mathbf{w}$ through the feedback channel. With the complete CSI at the receiver, the best beamforming vector can be found from the pre-designed codebook $\Omega = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_L\}$, which is known to both the transmitter and receiver, according to some optimization criteria. Then the index of the selected codeword is fed back to the transmitter with $B = \log_2 L$ feedback bits and the transmitter performs beamforming according to the received index.

III. LIMITED FEEDBACK

In this section, we describe a novel limited feedback approach based on tree-structured codebook for MIMO systems with transmit beamforming and receive combining over time-varying channels.

A. *Tree-structured Codebook*

The tree-structured codebook is constructed using the Lloyd algorithm [9]. The Lloyd algorithm is based on two conditions: i) optimum encoder (partition regions) for a fixed decoder (code vectors), and ii) optimum decoder for a fixed encoder. They are also called the nearest neighborhood condition (NNC) and the centroid condition (CC), respectively.

- NNC: For given code vectors $\{\mathbf{w}_i \,|\, i = 1, \ldots, N\}$, the optimum partition cells satisfy

$$R_i = \left\{ \mathbf{h} \in C^{M_t \times 1} : \left| \mathbf{h}^{\mathrm{H}} \mathbf{w}_i \right| \geq \left| \mathbf{h}^{\mathrm{H}} \mathbf{w}_j \right|, \forall i \neq j \right\} \quad (5)$$

for $i = 1, \ldots, N$, where R_i is the partition cell (Voronoi region) for the ith code vector $\mathbf{w}_i$.

- CC: For a given partition $\{R_i \,|\, i = 1, \ldots, N\}$, the optimum code vectors satisfy

$$\mathbf{w}_i = \arg \max_{\mathbf{w} \in R_i, \|\mathbf{w}\|=1} E\left[\left| \mathbf{h}^H \mathbf{w} \right|^2 \middle| \mathbf{h} \in R_i \right] \quad (6)$$

for $i = 1, \ldots, N$, Since

$$E\left[\left| \mathbf{h}^H \mathbf{w} \right|^2 \middle| \mathbf{h} \in R_i \right] = \mathbf{w}^H E\left[\mathbf{h}\mathbf{h}^H \middle| \mathbf{h} \in R_i \right] \mathbf{w} \quad (7)$$

The solution for the above optimization problem is principal eigenvector of $E\left[\mathbf{h}\mathbf{h}^H \middle| \mathbf{h} \in R_i \right]$.

The above two conditions are iterated until the optimum codebook converges. In practice, a quantizer is designed using a sufficiently large number of training samples (channel realizations). In that case, the statistical correlation matrix in (7) is estimated with an experimental expectation.

The Lloyd algorithm can be used to generate the tree-structured codebook. We consider the Lloyd algorithm that proceeds iteratively by levels in the codebook design, according to the following steps:

1) Compute the optimum codebook with two code vectors by the Lloyd algorithm from the training samples.
2) Compute the optimum codebook with two code vectors by the Lloyd algorithm from the training samples.
3) Split the training samples into two subsets, where each subset collects all the channel vector of the

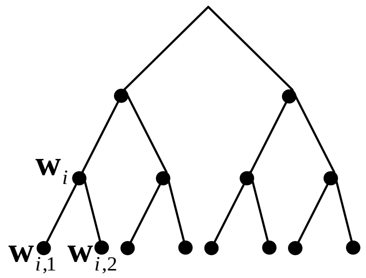

Figure 2. Illustration of tree-structured codebook

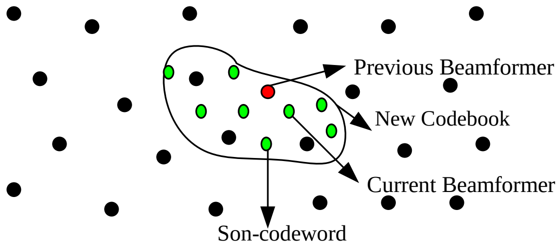

Figure 3. Illustration of the novel feedback approach

training samples at minimum chordal distance from the corresponding code vector.

4) Recursively iterate steps 1 and 2 to each of the subsets of training samples.

This binary construction procedure can be represented by a binary tree, having at level i the codewords of the optimal codebook with 2^i elements. Figure 2 is illustration of tree-structured codebook, $\mathbf{w}_{i,1}$ and $\mathbf{w}_{i,2}$ is the son-codewords of the father-codeword $\mathbf{w}_i$, and the son-codewords is the more precise partition of Voronoi region corresponding to father-codewords.

B. Feedback Approach

Due to the temporal correlation of channel responses across adjacent frames, the optimal beamforming vectors corresponding to the adjacent frames should be within a small neighborhood. Based on this observation, we develop an effective feedback approaches. In the proposed schemes, the new codebook for current frame consists of the son-codewords of the codewords neighboring to the previous beamformer, which is illustrated in Figure 3. Assuming that the number of codewords within the original codebook Ω is L and the number of codewords within the new codebook Ω' is L'. The detailed steps are as following:

1) The optimal beamforming vector corresponding to the first frame is selected from the original codebook Ω.

$$\mathbf{w}_1{}^* = \arg\max_{\mathbf{w}\in\Omega} \left\|\mathbf{H}_1\mathbf{w}\right\|^2 \tag{8}$$

The required feedback amount is $B = Log_2 L$.

2) Assuming neighborhood of $\mathbf{w}_1{}^*$ is defined as $N\left(\mathbf{w}_1{}^*\right)$, which consists of $L'/2$ codewords neighboring to $\mathbf{w}_1{}^*$ in the codebook Ω and includes $\mathbf{w}_1{}^*$ itself, with descending order according to the distance between the codeword and $\mathbf{w}_1{}^*$.

$$N\left(\mathbf{w}_1{}^*\right) = \left\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{L'/2}\right\} \tag{9}$$

$$\left|\mathbf{w}_1^H \mathbf{w}_1{}^*\right|^2 > \left|\mathbf{w}_2^H \mathbf{w}_1{}^*\right|^2 > \cdots > \left|\mathbf{w}_{L'/2}^H \mathbf{w}_1{}^*\right|^2 \tag{10}$$

Construct the new codebook Ω', which comprises the son-codewords of $N\left(\mathbf{w}_1{}^*\right)$.

$$\Omega' = \left\{\mathbf{w}_{1,1}, \mathbf{w}_{1,2}, \mathbf{w}_{2,1}, \mathbf{w}_{2,2}, \ldots, \mathbf{w}_{L'/2,1}, \mathbf{w}_{L'/2,2}\right\} \tag{11}$$

The optimal beamforming vector corresponding to the second frame is selected from the new codebook Ω'.

$$\mathbf{w}_2{}^* = \arg\max_{\mathbf{w}\in\Omega'} \left\|\mathbf{H}_2\mathbf{w}\right\|^2 \tag{12}$$

The required feedback amount is $B' = Log_2 L'$. The reduced feedback amount is $B - B'$.

3) Update $\mathbf{w}_1{}^*$ with the closest codeword to $\mathbf{w}_2{}^*$ in the original codebook Ω.

$$\mathbf{w}_1{}^* = \arg\max_{\mathbf{w}\in\Omega} \left\|\mathbf{w}_2{}^*\mathbf{w}\right\|^2 \tag{13}$$

4) Repeat step 2~5 for the remaining frames.

IV. Simulation Results

To illustrate the performance of the proposed approach, Monte Carlo BER simulations are performed for a system with $M_t = 4$, $M_r = 2$. The channels among different antenna pairs are generated independently. Assuming the receiver has perfect CSI, the feedback channel is error-free and zero-delay. The transmit power is equally allocated to all antennas, and MRC is used at the receiver. In all results, QPSK constellation is used and the duration of every data frame is 1ms. The SNR is defined as the ratio of the total transmit power to the total noise power at the receiver. Every point of the simulation results is obtained by averaging over more than 10000 independent realizations of the channel and the noise.

"Proposed" denotes the proposed feedback approach in this paper, and "Common" denotes the traditional feedback approach in block fading channel, in which temporal correlation cannot be exploited. In the experiment, initial feedback amount $B = 1, 2$ and new feedback amount $B' = 1, 2$ are considered. The neighborhood of $\mathbf{w}_1{}^*$ is itself when $B' = 1$, and the neighborhood consist of two closest codewords when $B' = 2$. Moreover, the performance comparison of different Doppler frequency shift is presented.

Figure 4 shows the un-coded BER performance for MIMO systems with different feedback amount and 30Hz Doppler frequency shift. From the figure, when $B = 1$ and $B' = 1$, the proposed approach has about 1dB

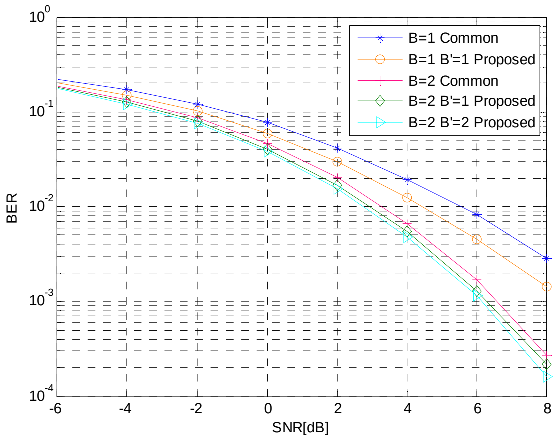

Figure 4. BER performance with 30Hz Doppler frequency shift

performance gain relative to common approach with invariable feedback bits. When $B = 2$ and $B' = 2$ the proposed approach also has about 0.3dB performance gain. For large feedback amount, the performance gain of the proposed approach is decreased compared to small feedback amount. When $B' < B$, the number of feedback bits can be further reduced by $B' - B$. Feedback reduction is obtained as well as performance improvement. The feedback amount is reduced to half when $B = 2$ and $B' = 1$.

Figure 5 shows the un-coded BER performance for MIMO systems with different Doppler frequency shift and 1bit feedback amount. From the figure, when Doppler frequency shift is 100Hz, the proposed approach has slight performance gain relative to the common feedback. For large Doppler frequency shift, Performance

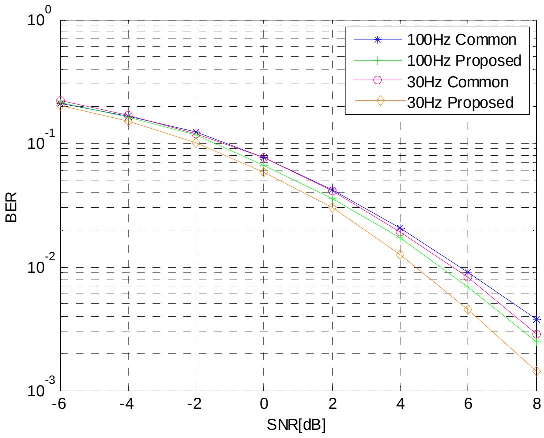

Figure 5. BER performance with 1 bit feedback

gain is degraded, that is because the temporal correlation becomes worse as Doppler frequency shift increases.

V. Conclusion

In this paper, we investigate limited feedback approach for MIMO beamforming systems with receive combining and a novel feedback approach is proposed. The approach exploited the temporal correlation to improve the CSI quantized precise, where the son-codewords of the neighborhood of the previous beamformer are considered as the new codebook for current frame. It is shown by the simulation that the proposed approach has better performance than common approach in terms of uncoded BER, especially small number of feedback bits and low Dopplor frequency shift. Moreover, the proposed methods can be easily extended to the precoding MIMO systems with special multiplexing.

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