Confinement studies in lattice QCD [1]

Richard W. Haymaker [a]

[a]*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803 USA*

Abstract

We describe the current search for confinement mechanisms in lattice QCD. We report on a recent derivation of a lattice Ehrenfest-Maxwell relation for the Abelian projection of SU(2) lattice gauge theory. This gives a precise lattice definition of field strength and electric current due to static sources, charged dynamical fields, gauge fixing and ghosts. In the maximal Abelian gauge the electric charge is anti-screened analogously to the non-Abelian charge.

1 Introduction

Quenched lattice QCD calculations of the static quark anti-quark potential have firmly established a linearly rising behavior over all distances obtainable in state-of-the-art simulations(1). However this situation satisfies no one, least of all a Dick Slansky. A 'black box' calculation at limited quark separation gives supporting evidence that QCD confines quarks, but it offers no explanation nor reveals a principle governing the phenomenon.

Lattice QCD is more than just an algorithm to calculate quantities at strong coupling. The lattice is a regulator for QCD, parametrized by the lattice spacing a. Gauge symmetry is preserved at all costs. This regulation scheme is perhaps the only one that gives a completely self consistent cutoff model in its

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[1] Talk given at the Symposium in Honor of Dick Slansky, Los Alamos, May 20-21 1998

own right. The dynamical variables are elements of the symmetry group rather than the Lie algebra. As a consequence, many of the topological features that are 'likely suspects' in the physics of confinement have natural definitions on the lattice. These include conserved U(1), Z(N), SU(N)/Z(N) monopole loops, Dirac sheets, Z(N) and SU(N) vortex sheets and other features.

Lattice work(2; 3; 4; 5; 6) is coming into its own in sorting through some of the seminal ideas on confinement(7; 8; 9; 10; 11; 12) proposed in the '70's. I venture to say that Dick Slansky could easily have been drawn into the fray because it has all the elements that typically captured his imagination. The central questions are fundamental. They involve interesting issues in group theory, topology, and duality. Candidate mechanisms are proliferting and fundamental questions remain unsettled [2].

Confinement is a consequence of a disordered state characterized by the expectation value of a Wilson loop suppressed to an area law rather than a perimeter law for an ordered system. Contributions from topological objects e.g. monopoles and vortices can accomplish this. On the lattice these objects are not singular. They occur abundantly in SU(N) lattice theories. It is only in the continuum limit of zero lattice spacing where these approach singular structures. In lattice simulations, one can tamper with these objects and see if suppression is accompanied by the loss of an area law. In this way one has a laboratory to study candidate disordering mechanisms.

It is not realistic to try to give a proper review of the many competing views in this brief article. Further, the studies are possibly diverging more rapidly than at any point in the past considering the variety of papers on the subject over the past year. However I would like to touch on them and further to argue that all the studies are describing genuine properties of QCD, seen through the eyes of subsets of the full dynamical variables. The controversies have to do with which scenario will lead to the most compelling explanation of confinement.

Although these lattice studies reveal relevant confinement physics, the goal is not completely clear. Most would agree however that if we knew the dual form of QCD it would give a definitive description of the physics of confinement.

Consider the example of superconductivity. By identifying the carriers of the persistent current one can discover an instability in the normal vacuum. The Ginzburg-Landau effective theory describes the consequence of the instability which of course elucidates the fundamental principle underlying the phenomenon which is the spontaneous breaking of the U(1) gauge symmetry(13). We might imagine that an understanding of the topological structures might in

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[2] At Lattice '98, the XVI International Symposium on Lattice Field Theory in Boulder, there were 52 contributed papers on color confinement, up from a dozen or so in the early '90's.

the same way lead to a discovery of a spontaneous gauge symmetry breaking. The problem is that a definitive scenario still seems quite distant.

2   Lattice confinement studies - a brief survey

In this section I wish to call attention to some of the approaches in lattice studies of confinement that have been active over the past 12 months. This is principally to give a flavor of the work and a list of recent references, and rely in the references therin for a more complete bibliography.

Our most recent work, which we describe in Sec. 5 is based on the Abelian projection.

We will restrict our attention to SU(2) theories throughout this article, believing that the essential issues will be revealed in this simplest case.

2.1   Abelian projection, Abelian dominance

In the Georgi-Glashow model with a gauge field coupled to the adjoint Higgs one can define a gauge invariant Abelian field strength(14). This is the construction used to identify the magnetic field of a 'tHooft-Polyakov monopole. Further, the Higgs field can be used to define an Abelian projected theory. This is accomplished by gauge transforming the Higgs field into the 3 direction. The resulting partially gauge fixed theory is still invariant under U(1) gauge transformations – rotations about the 3 axis.

Pure gauge SU(2) has no adjoint Higgs field and so there is no straightforward way to define Abelian variables. It is still possible to define an Abelian Projection(15; 16; 18).

One can find a "collective" Higgs field transforming under the adjoint representation which we denote as an "adjoint field". Consider an arbitrary Wilson line starting and ending at a particular site. This can be parametrized by $W(x) = \cos\chi + i\boldsymbol{\phi}\cdot\boldsymbol{\sigma}\sin\chi$ where $\boldsymbol{\phi}$ is a normalized adjoint field. The Wilson line could be (i) an open (no trace) Poliakov loop, (ii) an open plaquette in e.g. the $(1, 2)$ plane, or (iii) an arbitrary sum of such lines then normalized to construct an SU(2) element. One can generalize to (iv) define the adjoint field self consistently by requiring that the adjoint field at one site be equal to the normalized sum of the eight neighboring adjoint fields after parallel transport to that site.

In these four examples, one can further fix the gauge by rotating the adjoint

field into the third direction. This then fixes (i) to the Polyakov gauge, (ii) to a plaquette gauge (iv) e.g. a clover gauge, and (iv) in the maximal Abelian gauge. The "collective" adjoint field itself is gauge covariant, it does not necessitate a gauge fixing. Different adjoint fields correspond to different Abelian projections.

The maximal Abelian gauge appears to be the most promising choice. The condition, equivalent to the one given just above(17), maximizes $\sum(U_4^2 + U_3^2 - U_2^2 - U_1^2)$, where $U = U_4 + i\boldsymbol{\sigma} \cdot \boldsymbol{U}$, $U_4^2 + \boldsymbol{U}^2 = 1$. The maximization brings the links as close as possible to the values $U_1 = U_2 = 0$ and $U_4^2 + U_3^2 = 1$, leaving a U(1) invariance.

A new and interesting alternative has been proposed recently by Van der Sijs(19). This gauge fixing is based on the lowest eigenstate of the covariant Laplacian operator: i.e. sum the adjoint fields at the eight neighboring sites - parallel transported to a given site - and subtract $8\times$ the field at the site. This defines a gauge with no lattice Gribov copies. And it identifies the positions of monopoles as singularities (zeros) of the adjoint field.

'Abelian dominance' asserts further that operators built out of the U(1) links defined by the 'Abelian projection' will dominate the calculation of string tension and other observables. There are a large number of numerical tests which support Abelian dominance. However the successful tests are for the most part restricted to (a) certain specific quantites e.g. string tension in the fundamental representation, and (b) for the gauge choice of the maximal Abelian gauge. In other gauges, e.g. plaquette gauge, and/or for other quantities such as string tension in other representations Abelian dominance is not well supported and in some cases strongly violated.

On the other hand in a series of papers Di Giacomo et. al.(20; 21; 22; 23; 24) have studied the finite temperature confining phase transition and shown that the disorder parameter is very insensitive to the choice of Abelian projection. This is the foundation of the dual superconductor scenario. This is not in conflict with the above results because it does not require the Abelian dominance ansatz.

Using the Abelian projection and Abelian dominance, the problem is reduced to a U(1) gauge theory albeit with a complicated action coming from the effects of gauge fixing. As in a pure U(1) gauge theory, Dirac monopoles can be identified as the charge carriers of the persistent currents in the dual superconductor scenario(26; 18; 27; 28; 29).

Bakker, Chernodub and Polikarpov(30) have shown that Abelian monopole currents, defined in the maximal Abelian gauge are physical objects: there is a strong local correlation between monopoles and enhancements in the SU(2) gauge invariant action.

Other recent results of Polikarpov et al(31; 32; 33) include: Abelian monopoles in the maximal Abelian gauge are dyons. There are strong correlations between magnetic charge, electric charge and topological charge density. These same connections have been reported independently by Ilgenfritz et al(34).

In order to understand the disordering effect of monopoles on large Wilson loops, Hart and Teper(35) have studied the clustering properties of monopole loops and found two classes of clusters: A single cluster permeates the whole lattice volume, the remaining are small localized clusters.

In a series of papers, Ichie and Suganuma(36; 37; 38; 39) have sought a deeper understanding of Abelian dominance. They look at the residual degrees of freedom after Abelian projection, i.e. the coset fields and argue that in a random variable approximation that Abelian dominance is exact.

Ogilvie(40) has argued in the context of the Abelian projection that gauge fixing is in principle unnecessary, that results are the same whether the gauge is fixed or not. This is at odds with many simulations, leaving much to be sorted out.

Grady(41; 42) has given evidence that casts doubt on whether Abelian monopoles confinement mechanism carries over to the continuum limit.

A number of authors, including our group have reported that a well defined electric vortex forms between static sources(43; 17; 44). The vortex is very well described by a dual Ginzburg-Landau effective theory. In effect we are able to show that there is a local relation between the electric flux and the curl of the monopole current, defined by Abelian projection of full SU(2) field configurations that gives a damping of fields as they penetrate the dual superconductor. This directly accounts for confinement. In Sec. 5 we derive a precise definition of the electric field which tightens up the definition of vortices(45).

2.2   Vortices in Z(N) × SU(N)/Z(N) formulation

In the early 80's there was an effort by Mack and Petkova(46), Yaffe(47), Tomboulis (48), Yoneya(49), Cornwall(50), and Halliday and Schwimmer(51), to understand confinement in terms of SU(N) vortices. These are singular gauge configurations characterized by their topological properties. They are multiple valued in SU(N) but single valued in SU(N)/Z(N).

There are more recent developments by Tomboulis and Kovacs(52; 53; 54) [3] .

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[3] A thorough review of the topological issues in this approach can be found in

Their study is based on a decomposition of the SU(2) partition function into variables defined over the center Z(2) and variables defined over the quotient group SU(2)/Z(2) (= SO(3))(48). Whereas the SU(2) manifold has a trivial topology, SO(3) does not: $\pi_1(SO(3)) = Z(2)$.

To accomplish this reformulation, one appends new Z(2) valued variables σ_p, which live on plaquettes, to the usual set of SU(2) links. This allows one to write the standard SU(2) Wilson action partition function in terms of these Z(2) variables, σ_p and link variables, $\{\hat{U}_l\}$, defined over the manifold of SO(3) rather than SU(2). The latter distinction is manifest in the reformulated partition function in that the SU(2) links $\{U_l\}$ occur only in Z(2) invariant combinations i.e. invariant under the transformation $U_l \rightarrow \pm U_l$.

Consider a single (1, 2) plaquette with $\sigma_p = -1$, all others $= +1$. A Z(2) monopole occurs in each of the two (1,2,3) cubes adjacent to the face of the negative plaquette and similarly in the two (1,2,4) cubes.

On the dual lattice, each cube becomes a dual link orthogonal to the cube and this forms a conserved Z(2) monopole current loop (the smallest such loop) on the dual lattice. The negative plaquettes become dual plaquettes on the dual lattice. They form a surface bounded by the monopole loop. This surface is called a "thin vortex" sheet.

One can build larger monopole current loops and vortices by laying out stacks of negative σ plaquettes.

There are also the usual plaquettes built from the trace of 4 SU(2) link variables. Again consider a configuration in which all such plaquettes are positive except for a single (1, 2) plaquette which is negative. An SO(3) monopole occurs in each of the two (1,2,3) cubes adjacent to the negative plaquette and similarly in the two (1,2,4) cubes. This is an SO(3) object because every link involved in its construction occurs quadratically and hence is invariant under a Z(2) transformation of each link.

Analogously there is an SO(3) monopole current loop on the dual lattice. The surface bounded by the loop is a a Dirac sheet which differs from the "thin vortex" sheet in that it can be moved arbitrarily by multiplying the SU(2) links by elements of the center $\pm$ without costing any action.

The equivalence of the two forms of the partition function requires that every Z(2) monopole be coincident with an SO(3) monopole and vice versa. Hence we can visualize the vortex structure as two currents on coincident loops, bounded by two surfaces. This is called a 'hybrid' vortex. We can also shrink the monopole loop to zero, leaving either a pure "thin vortex" sheet, or in the

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other case a pure Dirac sheet.

The SO(3) monopoles also have associated non-Abelian (non-integer valued) flux which costs action. These configurations are denoted "thick vortices", The Dirac sheet is the return flux required by the topological group $\pi_1(SO(3)) = Z(2)$.

In this picture, it has been shown that thin vortices are exponentially suppressed at weak coupling and can not account for confinement. However SO(3) thick vortices can occur at weak coupling in which all terms in the action $|trU_p| \approx 1$. In recent papers Kovacs and Tomboulis(52; 53) have shown that the static quark potential can be reproduced by contributions from SO(3) thick vortices linking the Wilson loop in SU(2); and in SU(3); and that exclusion of vortices results in a perimeter law.

See Gavai and Mathur(55) for a study of Z(2) monopoles and the deconfinement phase transition. Also see Grady(56) for variation on the SO(3) - Z(2) monopole construction.

2.3 Center Projection Vortices

There are other closely related approaches that also focus on the center of the group as the key to understanding confinement. In a number of recent papers, Greensite, Del Debbio, Faber and Olejnik(57) have introduced the 'Center projection' as a way of identifying Z(2) vortices for SU(2). which they denote as "Projection vortices". They differ from the thick and thin vortices of Tomboulis. They are defined in the the maximal center gauge. In this gauge, one maximizes $\sum U_4^2$. Hence U_4 will be as close as possible to ± 1. The Z(2) links are then defined $Z = sign(U_4)$. This gives a Z(2) gauge theory that has "P vortex" excitations which are somewhat analogous to the Tomboulis 'thin' vortices. They also define "center vortices" which can be much larger than one lattice spacing and are analogous to the Tomboulis "thick vortices".

Greensite et. al.(58; 59; 60) have reported a growing list of encouraging results. Among the results are: P vortices are well correlated with SU(N) vortices; No P vortices no confinement; P vortices account for the full string tension; P vortex density scales; Center vortices thicken as the lattice cools; P-vortex locations are correlated among Gribov Copies; Preliminary successful generalizations to SU(3); and center vortices are compatible with Casimir scaling.

See also Langfeld et al(61; 62) and Stephenson(63) for further results on center vortices.

A variation in the center projection procedure is to first fix to the maximal

Abelian gauge, and then maximize $\sum \cos \chi$ where χ is the U(1) link angle. This is denoted the indirect maximal center gauge, as opposed to the above procedure denoted the direct maximal center gauge. In this gauge, a sheet consisting of monopole loops alternating with anti-monopole loops coincides with the P vortex sheet(58; 60) indicating a possible overlap between this and other scenarios.

2.4   Dual variables

One expects that the disorder regime in QCD would be described by an ordered regime in dual QCD. A definitive form of dual QCD would probably take us a long way toward an understanding of confinement.

We would like to call attention to recent lattice work on dual variables by Cheluvaraja(64), and continuum work by Majumdar and Sharatchandra(65), Sharatchandra et al(66), Mathur(67), Faddeev and Niemi(68), and Chan and Tsun(69).

The work by Majumdar and Sharatchandra supports the dual QCD ansatz by Baker, Ball and Zachariasen(70).

Seiberg and Witten(71) exploited duality to establish confinement for supersymmetric QCD.

2.5   Instantons

An important aspect of confinement studies is to identify what objects can not account for confinement. Instantons were likely suspects at one time. Smoothing techniques are very important in the identification of instantons. Unlike monopoles, small instantons can fall through the holes in the lattice and further can be swamped by short distance fluctuations. New smoothing techniques which overcome these difficulties, denoted 'renormalization group mapping', have been applied to configurations by DeGrand, Hasenfratz and Kovacs(72) to elucidate the role of instantons in the QCD vacuum. The come to the strong conclusion that instantons alone do not confine.

See also other recent works by Narayan and Neuberger(73); Narayan and Vranas(74); Ph. de Forcrand, M. Garcia Perez and I.-O. Stamatescu(75); and B. Alles, M. D'Elia and A. Di Giacomo(76).

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Even though the possible explanations for confinement seem to be diverging at this point, I would like to re-emphasize that different descriptions can in some cases be describing the same underlying physics:

Abelian monopoles in the maximal Abelian gauge correlate with

- SU(2) enhancements in the gauge invariant action(30),
- gauge invariant topological charge(31; 32; 33; 34),
- P vortices in the maximal center gauge(59),
- Instantons(77; 78).

Therefore the monopoles can not be thought of as merely gauge artifacts.

3   U(1) gauge theories and superconductivity

The onset of superconductivity is governed by the spontaneous breaking of the U(1) gauge symmetry via a non-zero vacuum expectation value of a charged field (13). An immediate consequence of this is the generation of a photon mass and, for type II superconductors, the formation of magnetic vortices which confine magnetic flux to narrow tubes(79) as revealed by the Ginzburg-Landau effective theory. Lattice studies of dual superconductivity in SU(N) gauge theories seek to exploit this connection in establishing the underlying principle governing color confinement.

In U(1) lattice pure gauge theory (no Higgs field), this same connection is seen to be present, not in the defining variables, but rather in the dual variables. More specifically:

(1) A field with non-zero magnetic monopole charge, Φ, has been constructed(80). It is a composite 4-form living on hypercubes constructed from gauge fields. There are also monopole current 3-forms. On the dual lattice this monopole operator is a 0-form living on dual sites. The monopole currents are 1-forms living on dual links. These currents either form closed loops or end at monopole operators. The monopole operator has a non-zero vacuum expectation value in the dual superconducting phase, $\langle \Phi \rangle \neq 0$, thereby signaling the spontaneous breaking of the U(1) gauge symmetry.

(2) Dual Abrikosov vortices have been seen in simulations(27; 81). They are identified by the signature relationship between the electric field and the curl of the monopole current in the transverse profile of the

vortex. The dual coherence length, ξ_d measures the characteristic distance from a dual-normal-superconducting boundary over which the dual-superconductivity turns on. The dual London penetration length, λ_d measures the attenuation length of an external field penetrating the dual-superconductor. The dual photon mass $\sim 1/\lambda_d$ and the dual Higgs mass $\sim 1/\xi_d$.

A signal $\langle \Phi \rangle \neq 0$ without the consequent signal of a dual photon mass does not imply confinement. An observation of a dual photon mass, i.e. vortex formation, without $\langle \Phi \rangle \neq 0$ does not reveal the underlying principle governing the phenomenon.

3.1 Higgs effective theory

The Higgs theory, treated as an effective theory, i.e. limited to classical solutions, and considered in the dual sense, provides a model for interpreting simulations of the pure gauge theory that can reveal these important connections. Recalling the Higgs' current

$$J_\mu = -\frac{ie}{2} \left(\phi^* \left(\partial_\mu - ieA_\mu \right) \phi - \phi \left(\partial_\mu + ieA_\mu \right) \phi^* \right), \tag{1}$$

and spontaneous gauge symmetry breaking through a constraint Higgs potential

$$\theta = v e^{ie\omega(x)}, \quad v = constant, \tag{2}$$

leads to the London theory of a type II superconductor. Using Eqns.(1) and (2) we obtain

$$J_\mu = -e^2 v^2 \left(A_\mu - \partial_\mu \omega \right), \tag{3}$$

$$\left(\partial_\mu J_\nu - \partial_\nu J_\mu \right) + m_\gamma^2 \left(\partial_\mu J_\nu - \partial_\nu J_\mu \right) = 0,$$

$$\nabla \times \mathbf{J} + \frac{1}{\lambda^2} \mathbf{B} = 0,$$

where

$$e^2 v^2 = m_\gamma^2 = \frac{1}{\lambda^2}. \tag{4}$$

Using Ampere's law $\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{J}$, we obtain

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$$\nabla^2 \boldsymbol{B} = \boldsymbol{B}/\lambda^2,$$

identifying λ as the London penetration depth.

If the manifold is multiply connected, then the gauge term in Eqn.(3) can contribute, as long as $e\omega(x)$ is periodic, with period 2π on paths that surround a hole.

$$\int_S (\boldsymbol{B} + \lambda^2 \boldsymbol{\nabla} \times \boldsymbol{J}) \cdot \boldsymbol{n} da = \oint_C (\boldsymbol{A} + \lambda^2 \boldsymbol{J}) \cdot d\boldsymbol{l},$$

$$= \oint_C \boldsymbol{\nabla}\omega \cdot d\boldsymbol{l},$$

$$= N \frac{2\pi}{e} = N e_m = \Phi_m.$$

where N quanta of magnetic flux penetrates the hole in the manifold. In real superconductors, the hole is a consequence of the large magnetic field at the center which drives the material normal.

A cylindrically symmetric vortex solution is given by

$$\boldsymbol{B} + \lambda^2 \boldsymbol{\nabla} \times \boldsymbol{J} = \Phi_m \delta^2(\boldsymbol{r}_\perp)\mathbf{n}_z,$$

$$(1 - \lambda^2 \boldsymbol{\nabla}_\perp^2)B_z(r_\perp) = \Phi_m \delta^2(\boldsymbol{r}_\perp),$$

$$B_z(r_\perp) = \frac{\Phi_m}{2\pi\lambda^2} K_0(r_\perp/\lambda).$$

The delta function core of this vortex is normal, i.e. no spontaneous symmetry breaking, and the exponential tail of the vortex is a penetration depth effect at the superconducting-normal boundary. The key point is that the modulus of the Higgs field must be independent of position to get these idealized vortices. For a "Mexican hat" Higgs potential, there is a coherence length setting the length scale from a normal-superconducting boundary over which the vacuum expectation value of the Higgs field changes from zero to its asymptotic value.

On the lattice, the same phenomenon occurs. We can generate vortices from finite configurations. In the continuum these objects are singular. Since the lattice formulation is based on group elements, rather than the Lie algebra the periodic behavior of the compact manifold is manifest. This gives the $2\pi N$ ambiguity in the group angle leading to N units of quantized flux. To see how this works, consider the lattice Higgs action

$$S = \beta \sum_{x,\mu>\nu} (1 - \cos\theta_{\mu\nu}(x))$$
$$-\kappa \sum_{x,\mu} (\phi^*(x)e^{i\theta_\mu(x)}\phi(x+\mu) + H.c.) + \sum_x V_{Higgs}(|\phi(x)|^2),$$

where $\theta_{\mu\nu}(x)$ is the curl of the gauge field,

$$\theta_{\mu\nu}(x) = \Delta_\mu^+ \theta_\nu(x) - \Delta_\nu^+ \theta_\mu(x),$$

and where $\phi(x)$ is the Higgs field and $\phi(x+\mu)$ refers to the Higgs field at the neighboring site in the μ direction and Δ_ν^+ is the forward difference operator. The electric current is given by

$$\frac{a^3}{e\kappa} J_\mu^e(x) = Im(\phi^*(x)e^{i\theta_\mu(x)}\phi(x+\mu)),$$

where a is the lattice spacing. Let us choose a Higgs potential that constrains the Higgs field $|\phi(x)| = 1$. Then if

$$\sin[\theta + 2N\pi] \approx \theta,$$

we find a relation between the field strength tensor and the curl of the current

$$F_{\mu\nu} - \frac{a^2}{e^2\kappa} \frac{(\Delta_\mu^+ J_\nu^e(x) - \Delta_\nu^+ J_\mu^e(x))}{a} = \frac{2\pi N}{e} \frac{1}{a^2} = N e_m \frac{1}{a^2}, \qquad (5)$$

where

$$ea^2 F_{\mu\nu} = \sin[\theta_\mu(x) + \theta_\nu(x+\mu) - \theta_\mu(x+\nu) - \theta_\nu(x)].$$

If $N = 0$ then this is a London relation which implies a Meissner effect. For $N \neq 0$ there are N units of quantized flux penetrating that plaquette, indicating the presence of an Abrikosov vortex.

3.2   Pure U(1) gauge theory

In a pure U(1) lattice gauge simulation (without Higgs field), lattice averages over many configurations exhibit superconductivity in the dual variables. The superconducting current carriers are monopoles. They can be defined in a natural way on the lattice using the DeGrand Toussaint(26) construction. They arise in non-singular configurations again because of the $2\pi N$ ambiguity in

group elements. These are the magnetic charge carriers for dual superconductivity. For a review of the monopole construction and vortex operators in U(1) gauge theory see e.g. the 1995 Varenna Proceedings(79).

As a brief summary, consider the unit 3-volume on the lattice at fixed x_4. The link angle is compact, $-\pi < \theta_\mu \leq \pi$. The plaquette angle is also compact, $-4\pi < \theta_{\mu\nu} \leq 4\pi$ and defined

$$ea^2 F_{\mu\nu}(x) = \theta_{\mu\nu}(x) = \Delta_\mu^+ \theta_\nu(x) - \Delta_\nu^+ \theta_\mu(x),$$

where a is the lattice spacing. This measures the electromagnetic flux through the face. Consider a configuration in which the absolute value of the link angles, $|\theta_\mu|$, making up the cube are all small compared to $\pi/4$. Gauss' theorem applied to this cube then clearly gives zero total flux. Because of the 2π periodicity of the action we decompose the plaquette angle into two parts

$$\theta_{\mu\nu}(x) = \bar{\theta}_{\mu\nu}(x) + 2\pi n_{\mu\nu}(x). \tag{6}$$

where $-\pi < \bar{\theta}_{\mu\nu} \leq \pi$. If the four angles making up one of the six plaquette are adjusted so that e.g. $\theta_{\mu\nu} > \pi$ then there is a discontinuous change in $\bar{\theta}_{\mu\nu}$ by -2π and a compensating change in $n_{\mu\nu}$. We can clearly choose the configuration that leaves the plaquette angles on all the other faces safely away from a discontinuity. We then define a Dirac string $n_{\mu\nu}$ passing through this face (or better a Dirac sheet since the lattice is 4D). $\bar{\theta}_{\mu\nu}$ measures the electromagnetic flux through the face.

This construction gives the following definition of the magnetic monopole current.

$$\frac{a^3}{e_m} J_\mu^m(x) = \epsilon_{\mu\nu\sigma\tau} \Delta_\nu^+ \bar{\theta}_{\sigma\tau}(x). \tag{7}$$

This lives on dual links on the dual lattice. Although Eqn.(6) is not gauge invariant, Eqn.(7) is. Further it is a conserved current, satisfying the conservation law $\Delta_\mu^+ J_\mu^m(x) = 0$.

In simulations of a pure U(1) gauge theory we find that lattice averages give a relation similar to Eqn.(5), but in the dual variables

$$\langle {}^* F_{\mu\nu} \rangle - \lambda_d^2 \frac{\left\langle \Delta_\mu^- J_\nu^m(x) - \Delta_\nu^- J_\mu^m(x) \right\rangle}{a} = N e \frac{1}{a^2},$$

where ${}^* F_{\mu\nu}$ is dual of $F_{\mu\nu}$. This is the signal for the detection of dual vortices(79).

4 Non-Abelian theory

The link of these considerations to confinement in non-Abelian gauge theory is through the Abelian projection(15; 16). One first fixes to a gauge while preserving U(1) gauge invariance. The non-Abelian gauge fields can be parametrized in terms of a U(1) gauge field and charged coset fields. The working hypothesis is that operators constructed from the U(1) gauge field alone, i.e. Abelian plaquettes, Abelian Wilson loops, Abelian Polyakov lines and monopole currents, will exhibit the correct large distance correlations relevant for confinement.

In the continuum limit the maximal Abelian gauge condition is

$$\left(\partial_\mu \pm g A_\mu^3(x)\right) A_\mu^\pm(x) = 0.$$

This is achieved on the lattice by a gauge configuration that maximizes R, where

$$R[U] \equiv \sum_{n,\mu} \frac{1}{2} Tr \left(\sigma_3 U_\mu(n)\sigma_3 U_\mu^\dagger(n)\right),$$

and where $U_\mu(n)$ is the link starting a site n and extending in the μ direction.

$$U_\mu(n) = \begin{pmatrix} \cos(\phi_\mu(n))e^{i\theta_\mu(n)} & \sin(\phi_\mu(n))e^{i\chi_\mu(n)} \\ -\sin(\phi_\mu(n))e^{-i\chi_\mu(n)} & \cos(\phi_\mu(n))e^{-i\theta_\mu(n)} \end{pmatrix}.$$

After gauge fixing, the SU(2) link matrices may be decomposed in a 'left coset' form:

$$U_\mu(n) = \begin{pmatrix} \cos(\phi_\mu(n)) & \sin(\phi_\mu(n))e^{i\gamma_\mu(n)} \\ -\sin(\phi_\mu(n))e^{-i\gamma_\mu(n)} & \cos(\phi_\mu(n)) \end{pmatrix} \begin{pmatrix} e^{i\theta_\mu(n)} & 0 \\ 0 & e^{-i\theta_\mu(n)} \end{pmatrix}, \quad (8)$$

Under a U(1) gauge transformation, $\{g(n) = \exp[i\alpha(n)\sigma_3]\}$,

$$\theta_\mu(n) \to \theta_\mu(n) + \alpha(n) - \alpha(n+\hat{\mu}) \qquad \gamma_\mu(n) \to \gamma_\mu(n) + 2\alpha(n) \qquad (9)$$

In other words, the left coset field derived from the link $U_\mu(n)$ is a doubly charged matter field living on the site n and is invariant under U(1) gauge transformations at neighbouring sites.

The $c_\mu \equiv \cos(\phi_\mu)$ are real–valued fields which near the continuum $\sim 1 + O(a^2)$ where a is the lattice spacing. The off–diagonal $w_\mu \equiv \sin(\phi_\mu)e^{i\gamma_\mu}$ become the charged coset fields $gaW_\mu(x)$, and θ_μ the photon field $gaA_\mu^3(x)$. [The SU(2) coupling $\beta = \frac{4}{g^2}$ in 3+1 dimensions.]

The static potential constructed from Abelian links gives as definitive a signal of confinement as the gauge invariant static potential as found by Suzuki et.al.(18; 28), Stack et.al.(29) and Bali et. al.(82). Bali et.al. find the Abelian string tension calculation gives 0.92(4) times the full string tension for $\beta = 2.5115$. Whether this approaches 1.0 in the continuum limit remains to be seen.

Equation (4) gives the connection between the non-zero vacuum expectation value of an order parameter and the photon mass or equivalently the London penetration depth.

This connection between order parameter and penetration depth is the key to connecting spontaneous symmetry breaking to vortex formation and hence confinement. Dual superconductivity studies seek to establish a connection, of course calculated from the original variables.

Di Giacomo et.al.(20; 21; 22; 23; 24) have reported extensive studies of the behavior of the order parameter for the dual theory (denoted disorder parameter) at the confining transition. See also Polikarpov et. al.(83). Our group and others have reported dual vortex formulation between sources allowing the determination of the London penetration depth(43; 17; 44).

We can demonstrate a qualitative correspondence between these two indicators of dual superconductiviy in that they are both observed and both show the correct behavior on the two sides of the transition. However technical difficulties have eluded a direct comparison check of the dual form of Eqn.(4)

Figure 1 shows a plot of $\rho = 2\frac{d}{d\beta}\ln\langle\mu\rangle$ as a function of β near the transition. The spike indicates step behavior in $\langle\mu\rangle$ at the transition.

Electric dual vortices between sources are well established(43; 17; 44) in this gauge. The typical behavior is shown in Fig. 2. The London relation is seen in the confining case for transverse distances larger than about two lattice spacing. The dual coherence length $\xi_d \approx 2$, i.e. the onset of the violation of the London relation. The unconfined case is also shown where curl **J** is almost zero, and the electric field falls more gradually than in the confining case.

Bali et al(44) have done a large scale simulation of these vortices. Again fitting the electric field and the monopole current to the Ginzburg-Landau theory their results are shown in Fig 3 and 4. Their data is very well described by the two G-L parameters $\lambda = 1.84(24)$ and $\xi = 3.10(40)$ The ratio G-L parameter $\kappa = \lambda/\xi = 0.59(14)$. They find the total flux in the vortex, $\Phi = 1.10(2)$ in units of the quantized flux. They concluded tentatively that type I dual superconductivity is indicated.

5   Definition of electric field strength

Central to finding the effective theory is the definition of the field strength operator in the Abelian projected theory, entering not only in the vortex profiles but also in the formula for the monopole operator. All definitions should be equivalent in the continuum limit, but use of the appropriate lattice expression should lead to a minimization of discretization errors.

In a recent paper(45) we exploited lattice symmetries to derive such an operator that satisfies Ehrenfest relations; Maxwell's equations for ensemble averages irrespective of lattice artifacts. This gives a precise lattice definition of current and charge density independent of lattice size, and independent of the continuum limit.

In the Abelian projection SU(2) link variables are parametrized by U(1) links and charged coset fields. The latter are normally discarded in Abelian projection, as are the ghost fields arising from the gauge fixing procedure. Since the remainder of the SU(2) infrared physics must arise from these, an understanding of their role is central to completing the picture of full SU(2) confinement.

In the maximal Abelian gauge the supposedly unit charged Abelian Wilson loop has an upward renormalization of charge due to this current. A localised cloud of like polarity charge is induced in the vacuum in the vicinity of a source, producing an effect reminiscent of the antiscreening of charge in QCD. In other gauges studied, the analogous current is weaker, and acts to *screen* the source.

We show that this current can be quantitatively written as a sum of terms from the coset and ghost fields. The contribution of the ghost fields in the maximally Abelian gauge in this context is found to be small. The effect of the the Gribov ambiguity on these currents is argued to be slight.

5.1 U(1)

We first introduce and review the method due to Zach et al (86), in pure U(1) theories

Consider a shift in the U(1) link angles in the partition function containing a Wilson loop source term

$$Z_W(\{\theta^s_\mu\}) = \int [d(\theta_\mu + \theta^s_\mu)] \ \sin\theta_W \ e^{\beta \sum \cos\theta_{\mu\nu}}.$$

Since the Haar measure is invariant under this shift, Z_W is constant in these variables. Absorbing the shift into the integration variable and the taking the derivative

$$\frac{\partial}{\partial \theta^s_\nu(x_0)} Z_W = 0,$$

we get the relation

$$0 = \int [d\theta] \ (\cos\theta_W - \sin\theta_W \ \beta\Delta_\mu \sin\theta_{\mu\nu}) \ e^{\beta \sum \cos\theta_{\mu\nu}}.$$

This can be cast into the form

$$\langle \Delta_\mu F_{\mu\nu} \rangle_W \equiv \frac{\left\langle \sin\theta_W \ \Delta_\mu \frac{1}{e} \sin\theta_{\mu\nu} \right\rangle}{\langle \cos\theta_W \rangle} = e\delta_{x,x_W} = J_\nu.$$

This is a well known technique to generate exact relations between Green's functions that is useful in generating Ward identities, or Schwinger-Dyson equations, or in this case we denote as Maxwell-Ehrenfest relations. We use the term Ehrenfest because it is the expectation value of what is normally a classical extremum of the path integral - an Euler Lagrange equation.

5.2 SU(2) no gauge fixing

Before addressing the full problem we first generalize from U(1) to SU(2) without the complication of gauge fixing.

$$Z_W(\{U^s\}) = \int [d(UU^s)] \ W_3(U) \ e^{\beta S(U)},$$

The size of the source is irrelevant so we choose it to be the simplest case, i.e. a plaquette:

$$W_3 \equiv \frac{1}{2} Tr(U^\dagger U^\dagger U U i \sigma_3).$$

We choose the shift to be in the 3 direction

$$U_\mu^s(x_0) = \left(1 - \frac{i}{2} \epsilon_\mu(x_0) \sigma_3 \right).$$

The invariance

$$\frac{d}{d\epsilon_\mu(x_0)} Z_W = 0$$

gives the Ehrenfest relation

$$\beta \frac{\langle W_3 S_\mu \rangle}{\langle W \rangle} = \delta_{x,x_W}.$$

For $\beta = 2.5$, $\beta \langle W_3 S_\mu \rangle = 0.0815(2)$, and $\langle W \rangle = 0.0818(1)$, and the difference $= 0.0003(3)$.

The notation S_μ denotes an ϵ derivative(45). The denominator is just the ordinary plaquette.

$$W \equiv \frac{1}{2} Tr(U^\dagger U^\dagger U U)$$

To cast this into the form of Maxwell's equation we decompose the link into diagonal D_μ and off-diagonal parts O_μ

$$U_\mu(x) = D_\mu(x) + O_\mu(x)$$

Further we simplify notation

$$\langle \cdots \rangle_W \equiv \frac{\langle W_3 \cdots \rangle}{\langle W \rangle}$$

We then group terms involving the diagonal part on the left and take all terms having at least one factor of the off-diagonal link to the right.

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$$\left[\beta \langle S_\mu\rangle_W\right]_{U=D} = \frac{1}{e} \langle \Delta_\mu F_{\mu\nu}\rangle_W .$$

Finally, note

$$\delta_{x,x_W} = \frac{1}{e} J_\nu^{static},$$

giving the final form of the Ehrenfest relation

$$\langle \Delta_\mu F_{\mu\nu}\rangle_W = \left\langle J_\nu^{dyn.}\right\rangle_W + J_\nu^{static}.$$

This then tells us how to choose a lattice definition of field strength that satisfies an Ehrenfest relation:

$$F_{\mu\nu} = \frac{1}{e}\frac{1}{2}Tr(D^\dagger D^\dagger D D i\sigma_3)_{\mu\nu}$$

5.3   Gauge fixed SU(2), U(1) preserved

The effect of gauge fixing gives

$$Z_W(\{U^s\}) = \int [d(UU^s)]\, W_3(U)\, \Delta_{FP}\, \delta[F]\, e^{\beta S(U)},$$

where we have introduced

$$1 = \Delta_{FP} \int \prod_{j,y} dg_j(y) \prod_{i,x} \delta[F_i(U^{\{g_j(y)\}}; x)],$$

and integrated out the g variables in the standard way.

In this case Z_W is *not invariant*. The shift is inconsistent with the gauge condition. However, it is invariant under an infinitessimal shift together with an infinitessimal 'corrective' gauge transformation that restores the gauge fixing

$$G(x) = \left(1 - \frac{i}{2}\boldsymbol{\eta}(x)\cdot\boldsymbol{\sigma}\right).$$

Use of the invariance of the measure under combination of a shift and a 'corrective' gauge transformation we obtain

$$\left[\frac{\partial}{\partial\epsilon_\mu(z_0)} + \sum_{k,z} \frac{\partial\eta_k(z)}{\partial\epsilon_\mu(z_0)}\frac{\partial}{\partial\eta_k(z)}\right] Z_W = 0.$$

In shorthand notation[4], the Ehrenfest relation reads

$$\left\langle (W_3)_\mu\bigg|_s + (W_3)_\mu\bigg|_g + W_3\left(\frac{(\Delta_{FP})_\mu}{\Delta_{FP}}\bigg|_s + \frac{(\Delta_{FP})_\mu}{\Delta_{FP}}\bigg|_g + \beta S_\mu\right)\right\rangle = 0. \qquad (10)$$

Gauge fixing has introduced three new terms:

- $(W_3)_\mu\bigg|_g$ comes from the corrective gauge transformation acting on the source which is U(1) invariant but not SU(2) invariant.

- $\frac{(\Delta_{FP})_\mu}{\Delta_{FP}}\bigg|_s$ is due to the shift of the Faddeev-Popov determinant.

- $\frac{(\Delta_{FP})_\mu}{\Delta_{FP}}\bigg|_g$ is due to the corrective gauge transformation of the Faddeev-Popov determinant.

The latter two derivatives are subtle. The key is to first consider the constraint up to first order in the shift and the corrective gauge transformations

$$F_i(x) + \frac{\partial F_i(x)}{\partial\epsilon(z_0)}d\epsilon(z_0) + \sum_{k,z} \frac{\partial F_i(x)}{\partial\eta_k(z)}d\eta_k(z) = 0,$$

and then define the Faddeev-Popov matrix as a derivative of the corrected constraint.

$$M_{ix;jy} + \delta M_{ix;jy} = \frac{\partial}{\partial\eta_j(y)}\left\{F_i(x) + \frac{\partial F_i(x)}{\partial\epsilon(z_0)}d\epsilon(z_0) + \sum_{k,z} \frac{\partial F_i(x)}{\partial\eta_k(z)}d\eta_k(z)\right\}$$

Finally we evaluate the derivative using

$$\frac{\Delta_\mu}{\Delta} = Tr[M^{-1}M_\mu]$$

A check of the Ehrenfest theorem is given in Table 1. Some of the individual terms on the right hand side require a $2N \times 2N$ matrix inversion, wherer N is the lattice volume. Hence to test the result numerically, we chose as small a

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[4] See Ref.(45)

lattice as possible. The result does not involve the size of the lattice which is 4^4 in table 1. The last column employees a different source. The links making up the plaquette are replaced by the diagonal parts only.

By separating the links into diagonal and off-diagonal parts we get the final form of the Ehrenfest-Maxwell relation.

$$\langle \Delta_\mu F_{\mu\nu} \rangle = \left\langle J_\nu^{dyn.} \right\rangle + J_\nu^{static}\bigg|_s + J_\nu^{static}\bigg|_g + \left\langle J_\nu^{FP}\bigg|_s \right\rangle + \left\langle J_\nu^{FP}\bigg|_g \right\rangle$$

The first term in the current comes from the excitation of the charged coset fields, the static term has an extra non-local contribution coming from the corrective gauge transformation, and the last two contributions are from the ghost fields.

These terms give a non vanishing charge density cloud around

a static source. The lefthand side can be used as a lattice

operator to measure the total charge density.

5.4 Abelian point charge has cloud of like charge

As a simple application we use this definition of flux to calculate div E on a source and the total flux away from the source. In Table 2, we see that the integrated flux on a plane between the charges plus the integrated flux on a back plane of the torus is larger than the div E on the source. The interpretation is that the bare charge is dressed with same polarity charge by the interactions and the neighborhood has a cloud of like charge. Hence there is anti-screening. This charge density has contributions from all terms in the Ehrenfest relation.

5.5 Summary

We have exploited symmetries of the lattice partition function to derive a set of exact, non–Abelian identities which define the Abelian field strength operator and a conserved electric current arising from the coset fields traditionally discarded in Abelian projection. The current has contributions from the action, the gauge fixing condition and the Faddeev–Popov operator. Numerical studies on small lattices verified the identity to within errors of a few per cent. We have found the Faddeev–Popov current in particular to be unusually sensitive

to systematic effects such as low numerical precision and poor random number generators, but the origin of any remaining, subtle biases, if they exist, is not clear; we have already considered all terms in the partition function.

In a pure U(1) theory the static quark potential may be measured using Wilson loops that correspond to unit charges moving in closed loops, as demonstrated by $|\langle \Delta_\nu^- F_{\nu\mu} \rangle| = \delta_W$. In Abelian projected SU(2) the same measurements in the maximally Abelian gauge yield an asymptotic area law decay and a string tension that is only slightly less than the full non–Abelian value. In other gauges it is not clear that an area law exists — certainly it is more troublesome to identify.

We have seen that in the context of the full theory the Abelian Wilson loop must be reinterpreted. The coset fields renormalize the charge of the loop as measured by $|\langle \Delta_\nu^- F_{\nu\mu} \rangle|$ and charge is also induced in the surrounding vacuum. Full SU(2) has antiscreening/asymptotic freedom of colour charge, and in the maximally Abelian gauge alone have we seen analogous behavior, in that the source charge is increased and induces charge of like polarity in the neighbouring vacuum. Whether this renormalisation of charge can account for the reduction of the string tension upon Abelian projection in this gauge is not clear. In other gauges, where Abelian dominance of the string tension is not seen, the coset fields appear to have a qualitatively different behavior, acting to suppress and screen the source charge.

In conclusion, the improved field strength expression defined by the Ehrenfest identity does not coincide with the lattice version of (17) of 't Hooft's proposed field strength operator (14). The Abelian and monopole dominance of the string tension invites a dual superconductor hypothesis for confinement. If this is to be demonstrated quantitatively such as by verification of a (dual) London equation then a a careful understanding of the field strength operator is required. The Ehrenfest identities may provide this (87).

6   Acknowledgments

References

[1]   G. S. Bali, C. Schlichter, and K. Schilling, Phys. Rev. **D 51** (1995) 5165

[2] M. Creutz <u>Quarks, Gluons and Lattices</u>, Cambridge University Press, Cambridge 1983

[3] I. Montvay and G. Münster <u>Quantum Fields on a Lattice</u>, Cambridge University Press, Cambridge 1994

[4] H. J. Rothe <u>Lattice Gauge Theories An Introduction</u>, Second Edition, World Scientific, Singapore 1997

[5] J. Kogut, Rev. Mod. Phys. **51** (1979) 659, ibid **55** (1983) 775

[6] Proceedings of the XVth International Symposium on Lattice Field Theory, Edinburgh Scotland 22-26 July 1997, Nucl Phys. B (Proc. Suppl) **63** (1998) April 1998. (Latest of the lattice conference proceedings dating back to 1982)

[7] H.B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[8] J. Kogut and L. Susskind, Phys. Rev. D **9**, 3501 (1974).

[9] Y. Nambu, Phys. Rev. D **10**, 4262 (1974).

[10] G. Parisi, Phys. Rev. **D11**, 970 (1975),

[11] S. Mandelstam, Phys. Rep. **23** 245,(1976).

[12] G. 'tHooft, High energy physics, ed. A. Zichichi (Editrice Compositori, Bologna, 1976).

[13] S. Weinberg, <u>The Quantum Theory of Fields</u> Vol II,Cambridge University Press, Cambridge 1996, See chapter 21.6.

[14] G. 't Hooft, Nucl. Phys. **B 79**, 276 (1974).

[15] G. 't Hooft, Nucl. Phys. **B 190** (1981) 455.

[16] A.S. Kronfeld, G. Schierholz and U.J. Wiese, Nucl. Phys. **B 293**, 461 (1987); A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.J. Wiese, Phys. Lett. **B 198**, 516 (1987); T. Suzuki and I. Yotsuyanagi, Phys. Rev. **D 42**, 4257 (1990).

[17] K. Bernstein, G. Di Cecio and R.W. Haymaker, Phys. Rev. D 55 (1997) 6730.

[18] For a review of monopoles and confinement, see e.g. T. Suzuki, Nucl.Phys. **B** Proc. Suppl. **30**, 176 (1993).

[19] A. J. van der Sijs hep-lat/9803001

[20] A. Di Giacomo, Preprint hep-lat/9802008

[21] A. Di Giacomo and G. Paffuti, Preprint hep-lat/9707003

[22] G. Boyd, B. Alles, M. D'Elia and A. Di Giacomo Preprint hep-lat/9711025

[23] A. Di Giacomo , B. Lucini, L. Montesi, G. Paffuti Preprint hep-lat/9709005

[24] L. Del Debbio, A. Di Giacomo, G. Paffuti and P. Pieri, Phys. Lett. **B 355** (1995) 255.

[25] M.N. Chernodub, M.I. Polikarpov, A.I. Veselov, Nucl. Phys. B (Proc. Suppl.) 49 (1996) 307, available as hep-lat/9512030.

[26] T. A. DeGrand and D. Toussaint, Phys. Rev. **D22** 2478,(1980).

[27] V. Singh, R.W. Haymaker, and D.A. Browne, Phys. Rev. **D47**, 1715 (1993).

[28] H. Shiba and T. Suzuki, Phys. Lett. **B 333**, 461 (1994).

[29] J. Stack and R. Wensley, Nucl. Phys. **D 22**, 597 (1992); J. Stack, S. Neiman and R. Wensley, Phys. Rev. **D 50**, 3399 (1994).

[30] B.L. G. Bakker, M. N. Chernodub, and M.I. Polikarpov , Preprint ITEP-TH-43/97, hep-lat/9709038

[31] M. N. Chernodub, F. V. Gubarev and M.I. Polikarpov, Preprint ITEP-TH-44/97, hep-lat/9709039; ITEP-TH-70/97, heplat/9801010

[32] M. N. Chernodub, F. V. Gubarev, M.I. Polikarpov and A. I. Veselov, Preprint ITEP-TH-14/98, hep-lat/9802036

[33] B.L. G. Bakker, M. N. Chernodub, F. V. Gubarev, M.I. Polikarpov and A. I. Veselov, Preprint ITEP-TH-1/97, hep-lat/9801011

[34] E.-M. Ilgenfritz, H. Markum, M. Müller-Preussker, and S. Thurner, Preprint HUB-EP 97/96, Kanazawa 97-23 hep-lat/9801040

[35] A. Hart and M. Teper hep-lat/9712003

[36] H Ichie and H. Suganuma, Preprint hep-lat/9802032, hep-lat/9807025

[37] H. Suganuma, H. Ichie A. Tanaka and K. Amemiya, Preprint hep-lat/9804027, hep-lat/9804025,

[38] A. Tanaka and H. Suganuma, Preprint hep-lat/9712027

[39] H. Ichie, A. Tanaka and H. Suganuma, Preprint hep-lat/9710004

[40] M. C. Ogilvie, Preprint hep-lat/9806018

[41] M. Grady, Preprint SUNY-FRE-98-01 hep-lat/9801016

[42] M. Grady, Preprint SUNY-FRE-98-02 hep-lat/9802035

[43] V. Singh, D.A. Browne, and R.W. Haymaker, Nucl. Phys. B (Proc. Suppl.) **30**, 658 (1993); Phys. Lett. **B306**, 115 (1993); Y. Matsubara, S. Ejiri, and T. Suzuki, Nucl. Phys. **B34** (Proc. Suppl.) 176 (19940; Y. Peng and R.W. Haymaker, Phys. Rev. **D 52** 3030, (1995).

[44] G.S. Bali, C. Schlichter, K. Schilling, hep-lat/9802005.

[45] G. Di Cecio, A. Hart, R.W. Haymaker, Nucl. Phys. B (Proc. Suppl.) 63 A-C (1998) 525; Preprint LSUHE No. 269-1998, hep-lat/9807001

[46] G. Mack and V. B. Petkova, Ann, Phys. (NY) **123** (1979) 442; ivis **125** (1980) 117; Z. Phys. **C 12** (1982) 177.

[47] L. G. Yaffe, Phys. Rev. **D 21** (1980) 1574.

[48] E. T. Tomboulis, Phys. Rev. **D 23** (1981) 2371.

[49] T. Yoneya, Nucl. Phys. **B 205** [FS5] (1982) 130

[50] J. M. Cornwall, Phys. Rev. **D 26** (1979) 1453.

[51] G. Halliday and A. Schwimmer Phys. Lett. **B101** (1981) 327, ibid Phys. Lett. **B102** (1981) 337.

[52] T. G. Kovacs and E. T. Tomboulis, Phys. Rev. **D57** (1998) 4054

[53] T. G. Kovacs and E. T. Tomboulis, Nucl. Phys. **B** (Proc. Suppl.) **63** (1998) 534; ibid **53** (1997) 509; Preprint hep-lat/9806030, hep-lat/9807027.

[54] E. T. Tomboulis, Phys. lett. **B 303** (1993) 103; Nucl. Phys. **B 34** (Proc. Suppl.) (1994) 192.

[55] R, V. Gavai and M. Manu, Preprint TIFR/TH/98-28, hep-lat/9807036

[56] M. Grady, Preprint SUNY-FRE-98-09 hep-lat/9806024

[57] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and S Olejnik, Preprint

hep-lat/9801027

[58] L. Del Debbio, M. Faber, J. Greensite, and S Olejnik, Preprints hep-lat/9802003, hep-lat/9708003, hep-lat/9709032, Phys. Rev. **D55** (1997) 2298, Nucl. Phys. Proc. Suppl. 5̲3 (1997) 141

[59] L. Del Debbio, M. Faber, J. Greensite, and S Olejnik, hep-lat/9708023

[60] M. Faber, J. Greensite, and S. Olejnik, Preprint hep-lat/9807008

[61] K. Langfeld, H. Reinhardt and O. Tennert, Preprint UNITU-THEP-19/97, hep-lat/9710068

[62] K. Langfeld, O. Tennert, M. Engelhardt and H. Reinhardt, Preprint UNITU-THEP-19/97, hep-lat/9801030, hep-lat/9805002

[63] P. W. Stephenson, IFUP-TH 27/98, hep-lat/9807019

[64] S. Cheluvaraja, Preprint hep-lat/9709021

[65] P. Majumdar and H. S. Sharatchandra, Preprint imsc/98/05/22, hep-lat/9805102; imsc/98/04/16, hep-lat/9804128; imsc/98/04/14, hep-lat/9804091

[66] H. S. Sharatchandra, Preprint imsc/98/06/30, hep-lat/9807284

[67] M. Mathur, Preprint SNB/98-07-06, hep-lat/9807167

[68] L. Faddeev and A J. Niemi, Preprint hep-lat/9807069

[69] H.-M. Chan and T. S. Tsun, Phys. Rev. **D 57** (1998) 2507; ibid **D56** (1997), 3634; Acta Physica Polonica **B 28** (1997) 3009, 3027, Some elementary Gauge Theory Concepts, World Scientific, Singapore 1993. H.-M. Chan J. Faridani, and T. S. Tsun; Phys. Rev. **D 51** (1995) 7040; ibid **D52** (1995) 6134; ibid **53** (1996) 7293.

[70] M. Baker, J. S. Ball and F. Zachariasen, Physics Reports, **209** No.3, 73 (1991); Phys. Rev. **51**, 1968 (1995), Further references in these papers.

[71] N. Seiberg and E. Witten, Nucl. Phys. **b426**, 12 (1994)

[72] T. DeGrand, A. Hasenfratz, and T. Kovacs, hep-lat/9801037; Nucl. Phys. **B478** (1997) 417; hep-lat/9711032; hep-lat/9710078

[73] R. Narayanan and H. Neuberger, Nucl. Phys. **B412** (1994) 574;

[74] R. Narayanan and P. Vranas Preprint UW-PT-97-04 hep-lat/9702005

[75] Ph. De forcrand, M. Garcia Perez and I.-O. Stamatescu, hep-lat/9701012

[76] B. Alles, M. D'Elia and A. Di Giacomo, heplat/9701012

[77] M. N. Chernobub, F.V. Gubarev, JETP Lett. **62** (1995) 100, hep-lat/9506026

[78] A. Hart and M. Teper, Phys. lett. **B 371** (1996) 261, hep-lat/9511016

[79] For a review and further references to superconductivity on the lattice see e.g. R.W. Haymaker, *"Dual Abrikosov vortices in U(1) and SU(2) lattice gauge theories"*, Proceedings of the international School of Physics "Enrico Fermi", Course CXXX, A. Di Giacomo and D. Diakonov (Eds.) IOS Press, Amsterdam 1996, hep-lat 9510035.

[80] J. Frölich and P. A. Marchetti, Euro. Phys. Lett. **2**, 933 (1986); Commun. Math. Phys. **112**, 343 (1987); **116**, 127 (1988); **121**, 177 (1989); Lett. Math. Phys **16**, 347 (1988); L. Polley and U.-J. Wiese, Nucl. Phys. **B356**, 621 (1991); M. I. Polikarpov, L. Polley and U.-J. Weise, Phys. Lett. **B253**, 212 (1991);. L. Del Debbio, A. Di Giacomo and G. Paffuti, Phys. Lett.

B349, 513, (1995); Nucl. Phys B (Proc. Suppl.) **42**, 231 (1995).

[81] M. Zach, M. Faber and P. Skala, Nucl. Phys. B (Proc. Suppl.) **63 A-C** (1998) 489.

[82] G. S. Bali, V. Bornyakov, M. Müller-Preussker and K. Schilling, Phys. Rev. **D 54** 2863 (1996).

[83] M.N. Chernodub, M.I. Polikarpov and A.I. Veselov, Nucl. Phys. B (Proc. Suppl.) **49** (1996) 307.

[84] N. Nakamura, V. Bornyakov, S. Ejiri, S. Kitahara, Y. Matsubara and T. Suzuki, Nucl. Phys.B (Proc. Suppl.)**53**, 512 (1997).

[85] M. Zach, M. Faber, W. Kainz and P. Skala, Phys. Lett. **B 358** (1995) 325; P. Skala, M. Faber and M. Zach, Nucl. Phys. **B 494** (1997) 293.

[86] M. Zach, M. Faber, W. Kainz and P. Skala, Phys. Lett. B 358 (1995) 325, available as hep-lat/9508017.

[87] A. Hart, R.W. Haymaker, Y. Sasai, Lattic98 Proceedings in preparation, and work in progress

7 Figure captions

Fig. 1. (figsu2.ps) (From ref. (20)) ρ vs. β for SU(2) gauge theory. The peak signals deconfining phase transition. here monopoles are defined by the Abelian projection on Polyakov line.

Fig. 2. (fig7.tex needs fig7 1.eps, fig7 2.eps and epsfig.sty) (From ref. (17)) Transverse profile of the electric field and curl of the monopole current in the mid plane between a static $q\bar{q}$ pair in the maximal Abelian gauge at finite temperature for a confining (left) and unconfining (right) phase.

Fig 3. (curlE.ps) (From (44)) Check of the dual Ampere law in the dual vortex profile. $\boldsymbol{E}$ is the electric field, $\boldsymbol{k}$ is the monopole current.

Fig 4. (fit3.ps) (From (44)) Fit of the electric field vortex profile $\boldsymbol{E}$ and the tangential component of the magnetic monopole current $\boldsymbol{k}_{\theta}$ to the Ginzburg Landau theory.

Source:	W_3	$W_3(U \to D)$
Ehrenfest term		
$\left\langle (W_3)_\mu \Big|_s \right\rangle$	0.65468(10)	0.63069(20)
$\left\langle (W_3)_\mu \Big|_g \right\rangle$	0.06095(7)	0.04463(4)
$\left\langle W_3 \frac{(\Delta_{FP})_\mu}{\Delta_{FP}} \Big|_g \right\rangle$	0.00127(21)	0.00132(50)
$\left\langle W_3 \frac{(\Delta_{FP})_\mu}{\Delta_{FP}} \Big|_s \right\rangle$	0.00529(3)	0.00564(3)
$\left\langle \beta(S)_\mu \Big|_s \right\rangle$	-0.72246(68)	-0.68275(50)
Zero	-0.00026(77)	-0.00045(64)

Table 1

Terms in the Ehrenfest relation, Eqn.(terms). The column labeled 'W_3' corresponds to the source described in the text. In the second column the links are replaced by the diagonal arts of the links in order to test a second example. The theorem gives zero for the sum. $\beta = 2.5$, 4^4 lattice
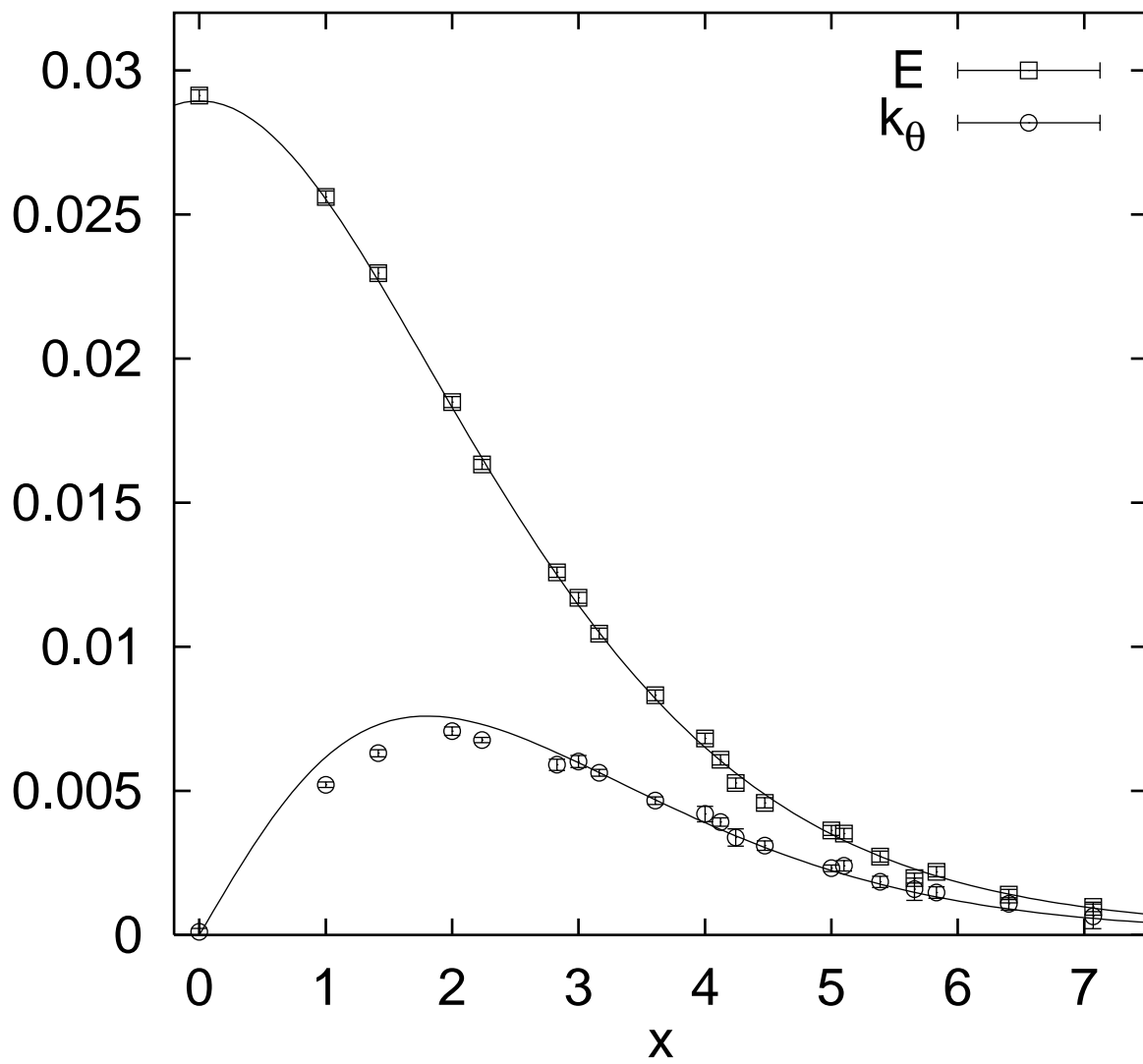
β	$divE$(cl.pt.charge) $= \frac{1}{\beta}$	$divE$ (on source)	total flux
10.0 (almost classical)	0.1	0.1042(1)	0.0910(8) (mid) 0.0148(8) (back) 0.1092(8) (total)
2.4	0.4166	0.5385(19)	0.7455(70) (mid) 0.0359(72) (back) 0.7815(95) (total)

Table 2

$divE$ normalized to $\frac{1}{\beta}$ for a classical point charge. Source is a 3 Wilson loop. $divE$ measured on a source. Electric Flux measured on the midplane centered on the Wilson loop. Also included is the flux through a plane on the far side of the torus, and the sum being the total flux. 8^4 lattice, 3×3 Wilson loop.
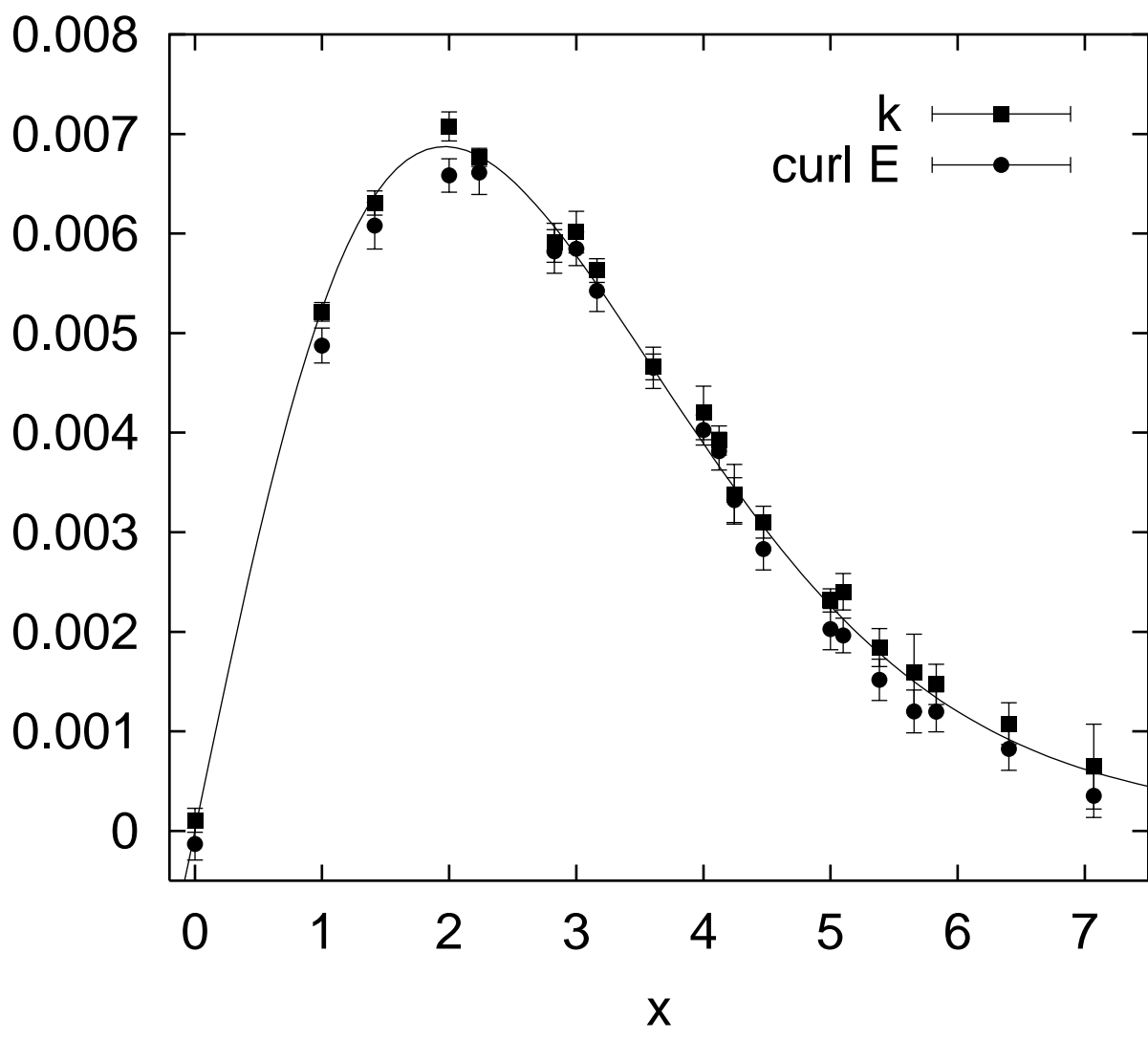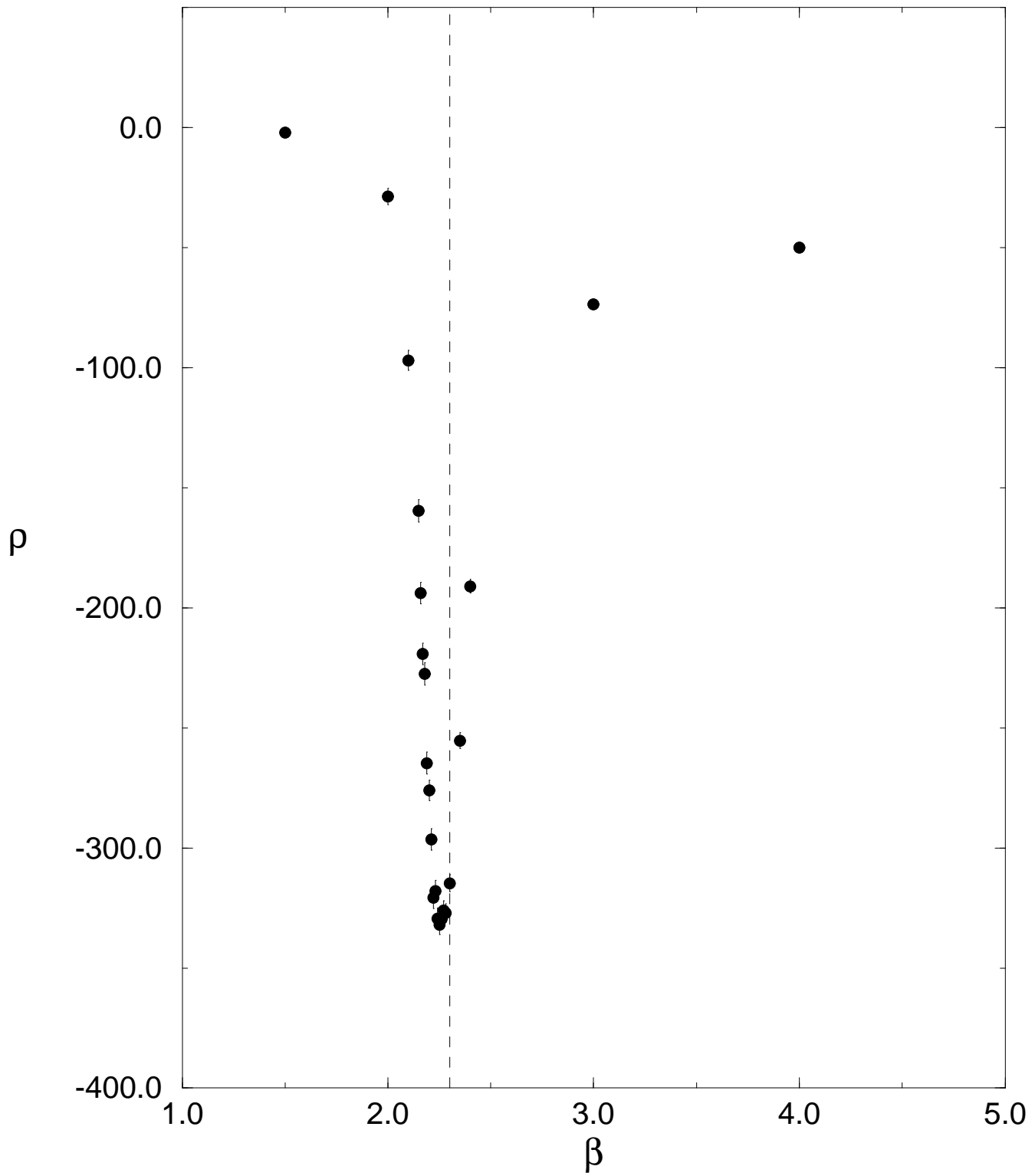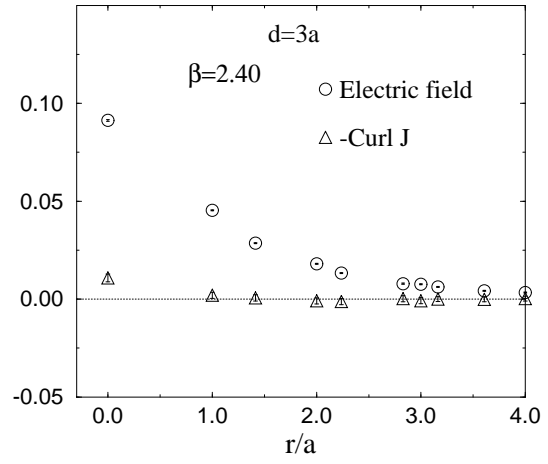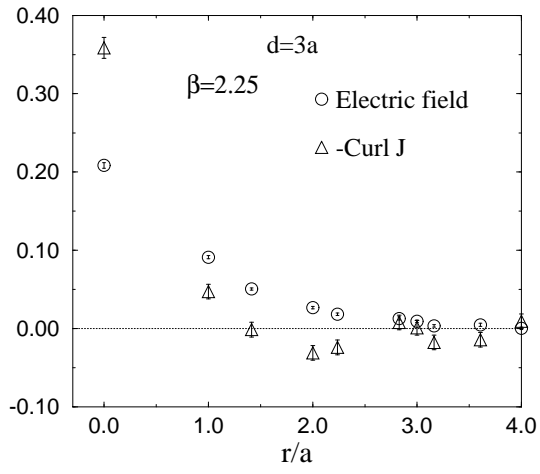
FIG. 7, K. Bernstein, Phys. Rev. D