Application of fuzzy multi-objective linear programming to aggregate production planning

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Abstract

This study develops a fuzzy multi-objective linear programming (FMOLP) model for solving the multi-product aggregate production planning (APP) decision problem in a fuzzy environment. The proposed model attempts to minimize total production costs, carrying and backordering costs and rates of changes in labor levels considering inventory level, labor levels, capacity, warehouse space and the time value of money. A numerical example demonstrates the feasibility of applying the proposed model to APP problem. Its advantages are also discussed. The proposed model yields a compromise solution and the decision maker’s overall levels of satisfaction. In particular, in contrast to other APP models, several significant characteristics of the proposed model are presented. © 2003 Elsevier Ltd. All rights reserved.

Keywords: Aggregate production planning; Multi-objective linear programming; Fuzzy multi-objective linear programming; Decision maker

1. Introduction

Aggregate production planning (APP) deals with matching capacity to demand of forecasted, varying customer orders over the medium term, often from 3 to 18 months in advance. APP aims to (1) to set overall production levels for each product category to meet fluctuating or uncertain demand in the near future, and (2) to set decisions and policies concerning hiring, layoffs, overtime, backorders, subcontracting and inventory level, and thus determining appropriate resources to be used (Lai & Hwang, 1992). APP has attracted considerable attention from both practitioners and academia (Shi & Haase, 1996). In the field of planning, it falls between the broad decisions of long-range planning and...
the highly specific and detailed short-range planning decisions. APP is one of the most important functions in production and operations management. Other forms of family disaggregation planning involve a master production schedule, capacity requirements planning, material requirements planning, which all depend on APP in a hierarchical way.

Since Holt, Modigliani, and Simon (1955) proposed the HMMS rule in 1955, researchers have developed numerous models to help to solve the APP problem, each with their own pros and cons. According to Saad (1982), all traditional models of APP problems may be classified into six categories—(1) linear programming (LP) (Charnes & Cooper, 1961; Singhal & Adlakha, 1989), (2) linear decision rule (LDR) (Holt et al., 1955), (3) transportation method (Bowman, 1956), (4) management coefficient approach (Bowman, 1963), (5) search decision rule (SDR) (Taubert, 1968), and (6) simulation (Jones, 1967). When using any of the APP models, the goals and model inputs (resources and demand) are generally assumed to be deterministic/crisp and only APP problems with the single objective of minimizing cost over the planning period can be solved. The best APP balances the cost of building and taking inventory with the cost of the adjusting activity levels to meet fluctuating demand.

However, in real-world APP problems, the input data or parameters, such as demand, resources, cost and the objective function are often imprecise/fuzzy because some information is incomplete or unobtainable. Conventional mathematical programming schemes clearly cannot solve all fuzzy programming problems. The current APP model represents information in a fuzzy environment where the objective function and parameters are incompletely defined and cannot be accurately measured. In 1976, Zimmermann (1976) first introduced fuzzy set theory into conventional LP problems. That study considered LP problems with a fuzzy goal and fuzzy constraints. Following the fuzzy decision-making method proposed by Bellman and Zadeh (1970) and using linear membership functions, that same study confirmed that there exists an equivalent LP problem. Thereafter, fuzzy linear programming (FLP) has been developed into a number of fuzzy optimization methods for solving the APP problem. Hintz and Zimmermann (1989) presented an approach based primarily on FLP and approximate reasoning to solve APP, releasing of parts and machine scheduling problems in flexible manufacturing systems. Additional references on the use of FLP to solve APP problems include Lee (1990), Masud and Hwang (1980), Rinks (1982), Tang, Wang, and Fung (2000), and Wang and Fang (2000, 2001).

However, in practical production planning systems, the many functional areas in an organization that yield an input to the aggregate plan normally have conflicting objectives governing the use of the organization’s resources. These objectives minimize costs/maximize profits, inventory investment, customer service, changes in production rates, changes in work-force levels and utilization of plant and equipment (Krajewski & Ritzman, 1999). Moreover, the solution of fuzzy multi-objective optimization problems benefits from considering the imprecision of the decision maker’s (DM’s) judgments such as, ‘the objective function of the annual total production costs should be substantially less than or equal to 5 millions’, or ‘the changes in labor levels should be substantially less than or equal to 200 man-hours’. Especially, these conflicting objectives are required to be optimized simultaneously by the DM in the framework of fuzzy aspiration levels.

In 1978, Zimmermann (1978) first extended his FLP approach (Zimmermann, 1976) to a conventional multi-objective linear programming (MOLP) problem. For each of the objective functions of this problem, assume that the DM has a fuzzy goal such as ‘the objective functions should be essentially less than or equal to some value’. Then, the corresponding linear membership function is defined and the minimum operator proposed by Bellman and Zadeh (1970) is applied to combine all objective functions. By introducing the auxiliary variable, this problem can be transformed into the equivalent conventional
LP problem and can be easily solved by the simplex method of LP. Subsequent works on fuzzy goal programming (FGP) included Hannan (1981), Leberling (1981), Luhandjula (1982), and Sakawa (1988). Therefore, the aim of this study is to develop a fuzzy multi-objective linear programming (FMOLP) model for solving the multi-product APP decision problem in a fuzzy environment. First, a MOLP model of a multi-product APP decision problem is constructed. The model attempts to minimize total production costs, carrying and backordering costs, and rates of changes in labor levels with reference to inventory level, labor levels, capacity, warehouse space and the time value of money. Furthermore, this model is converted into an FMOLP model by integrating fuzzy sets and objective programming approaches.

The rest of this paper is organized as follows. Section 2 describes the problem, details the assumptions and develops the MOLP and FMOLP mathematical models of the APP decision problem. Section 3 presents a numerical example to demonstrate the application of the proposed model. Based on this numerical example, the proposed model is implemented using seven scenarios. Section 4 discusses the advantages of the model over other APP models. The final section draws conclusions and makes relevant recommendations.

2. Model development

2.1. Problem description, assumptions and notation

The multi-product APP problem can be described as follows. Assume that a company manufactures $N$ types of products to satisfy the market demand over a planning horizon $T$. The problem involves determining the most effective means of satisfying forecasted demand by adjusting output rates, hiring and layoffs, inventory levels, overtime work, subcontracting, back orders and other controllable variables. The objective functions of this APP decision problem are to minimize total production costs, carrying and backordering costs, and rates of changes in labor with reference to inventory level, labor levels, capacity, warehouse space, and the time value of money under each of the cost categories.

Based on the above characteristics of the considered APP problem, the mathematical model herein is developed on the following assumptions.

1. The piecewise linear membership functions are specified for all fuzzy sets involved.
2. The minimum operator is used to aggregate fuzzy sets.
3. The values of all parameters are certain over the next $T$ planning horizon.
4. The escalating factors in each of the costs categories are certain over the next $T$ planning horizon.
5. Actual labor levels, machine capacity and warehouse space in each period cannot exceed their respective maximum levels.
6. The forecasted demand over a particular period can be either satisfied or backordered, but the backorder must be fulfilled in the next period.

Assumptions 1 and 2 are made to convert the original fuzzy MOLP problem into an equivalent LP problem that can be solved efficiently by the standard simplex method (Zimmermann, 1997). Assumption 1 follows from the fact that the DM could specify the degree of membership on distinct values for each of the objective functions, so piecewise linear functions may used to represent the fuzzy sets. Assumption 2 implies that the minimum operator suffices for aggregating the fuzzy sets.
Assumptions 3 and 4 imply that the certainty property must be technically satisfied to represent an optimization problem as a LP problem. Assumption 5 represents the limits on the maximum available labor, machine and warehouse capacity in a normal business operation. Assumption 6 concerns the portion of market demand that must be satisfied during any period, whereas the rest of the market demand can be backordered. However, backorders should not be carried over for more than one period in a practical situation.

The following notation is used.

- $D_{nt}$: forecasted demand for $n$th product in period $t$ (units)
- $a_{nt}$: regular time production cost per unit for $n$th product in period $t$ ($/unit)
- $Q_{nt}$: regular time production for $n$th product in period $t$ (units)
- $i_a$: escalating factor for regular time production cost (%)
- $b_{nt}$: overtime production cost per unit for $n$th product in period $t$ ($/unit)
- $O_{nt}$: overtime production for $n$th product in period $t$ (units)
- $i_b$: escalating factor for overtime production cost (%)
- $c_{nt}$: subcontracting cost per unit of $n$th product in period $t$ ($/unit)
- $S_{nt}$: subcontracting volume for $n$th product in period $t$ (units)
- $i_c$: escalating factor for subcontract cost (%)
- $d_{nt}$: inventory carrying cost per unit of $n$th product in period $t$ ($/unit)
- $I_{nt}$: inventory level in period $t$ for $n$th product (units)
- $i_d$: escalating factor for inventory carrying cost (%)
- $e_{nt}$: backorder cost per unit of $n$th product in period $t$ ($/unit)
- $B_{nt}$: backorder level for $n$th product in period $t$ (unit)
- $i_e$: escalating factor for backorder cost (%)
- $k_t$: cost to hire one worker in period $t$ ($/man-hour)
- $H_t$: worker hired in period $t$ (man-hour)
- $m_t$: cost to layoff one worker in period $t$ ($/man-hour)
- $F_t$: workers laid off in period $t$ (man-hour)
- $i_f$: escalating factor for hire and layoff cost (%)
- $n_{nt}$: hours of labor per unit of $n$th product in period $t$ (man-hour/unit)
- $r_{nt}$: hours of machine usage per unit of $n$th product in period $t$ (machine-hour/unit)
- $v_{nt}$: warehouse spaces per unit of $n$th product in period $t$ (ft$^2$/unit)
- $W_{tmax}$: maximum labor level available in period $t$ (man-hour)
- $M_{tmax}$: maximum machine capacity available in period $t$ (machine-hour)
- $V_{tmax}$: maximum warehouse space available in period $t$ (ft$^2$)
- $S_{ntmax}$: maximum subcontracted volume available for $n$th product in period $t$ (units)
- $I_{ntmin}$: minimum inventory level available of $n$th product in period $t$ (units)
- $B_{ntmax}$: maximum backorder level available of $n$th product in period $t$ (units)

### 2.2. Problem formulation

#### 2.2.1. Objective function

This study selected the multi-objective functions for solving the APP problem by reviewing the literature and considering practical situations. In their SEMOPS model, Masud and Hwang (1980)
specified three objective functions to minimize total production costs, carrying and backordering costs, and rates of change in labor levels. In their multi-product APP decision model, Tang et al. (2000) presented two objective functions that minimize total production and carrying costs. See also (Krajewski & Ritzman, 1999; Saad, 1982; Wang & Fang, 2001). However, most practical decisions involved in business APP problems must consider production costs, skills of workers, product life cycle, employment law, and other factors, to minimize total production costs and rate of change in labor levels. Therefore, three objective functions are simultaneously considered during the development of the proposed MOLP model as follows.

- Minimize total production costs

\[
\min \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ a_{nt}Q_{nt}(1 + i_d)^t + b_{nt}O_{nt}(1 + i_b)^t + c_{nt}S_{nt}(1 + i_c)^t + d_{nt}I_{nt}(1 + i_d)^t \\
+ e_{nt}B_{nt}(1 + i_e)^t \right] + \sum_{t=1}^{T} (k_iH_t + m_iF_t)(1 + i_j)^t
\]  

(1)

where the terms of

\[
\sum_{n=1}^{N} \sum_{t=1}^{T} \left[ a_{nt}Q_{nt}(1 + i_d)^t + b_{nt}O_{nt}(1 + i_b)^t + c_{nt}S_{nt}(1 + i_c)^t + d_{nt}I_{nt}(1 + i_d)^t + e_{nt}B_{nt}(1 + i_e)^t \right]
\]

are used to calculate production costs. The production costs include five components—regular time production, overtime, subcontracts, carrying inventory and backordering; \( \sum_{t=1}^{T} (k_iH_t + m_iF_t)(1 + i_j)^t \) specifies the costs of change in labor levels, including the costs of hiring and layoff workers. Escalating factors were also included for each of the cost categories.

- Minimize carrying and backordering costs

\[
\min \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ d_{nt}I_{nt}(1 + i_d)^t + e_{nt}B_{nt}(1 + i_e)^t \right]
\]  

(2)

- Minimize rate of change in labor levels

\[
\min \sum_{t=1}^{T} (H_t - F_t)
\]  

(3)

where the symbol \( \equiv \) is the fuzzified version of \( = \) and refers to the fuzzification of the aspiration levels. In real-world APP decision problems, the environmental coefficients and operation parameters are typically uncertain because some information is incomplete or unobtainable in a medium time horizon. Accordingly, Eqs. (1)–(3) are fuzzy with imprecise aspiration levels, and incorporate the variations in the DM’s judgments concerning the solutions of fuzzy multi-objective optimization problems. For each of the objective functions of the proposed MOLP model, this study assumes that the DM has such imprecise goals as, ‘the objective functions should be essentially equal to some
value’. These conflicting goals are required to be simultaneously optimized by the DM in the framework of fuzzy aspiration levels.

2.2.2. Constraints.

- Constraints on carrying inventory

\[ I_{nt} - B_{nt} = I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - D_{nt} \quad \forall n, \forall t \quad (4) \]

\[ I_{nt} \geq I_{nt \min} \quad \forall n, \forall t \quad (5) \]

\[ B_{nt} \leq B_{nt \max} \quad \forall n, \forall t \quad (6) \]

- Constraints on labor levels

\[ \sum_{n=1}^{N} n_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t = \sum_{n=1}^{N} n_{nt}(Q_{nt} + O_{nt}) \quad \forall t \quad (7) \]

\[ \sum_{n=1}^{N} n_{nt}(Q_{nt} + O_{nt}) \leq W_{t \max} \quad \forall t \quad (8) \]

- Constraints on machine capacity and warehouse space

\[ S_{nt} \leq S_{nt \max} \quad \forall n, \forall t \quad (9) \]

\[ \sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq M_{t \max} \quad \forall t \quad (10) \]

\[ \sum_{n=1}^{N} v_{nt}I_{nt} \leq V_{t \max} \quad \forall t \quad (11) \]

- Non-negativity constraints on decision variables

\[ Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \geq 0 \quad \forall n, \forall t \quad (12) \]

References on the use of FLP to solve multi-objective APP problems with fuzzy constraint coefficients include Hintz and Zimmermann (1989), Wang and Fang (2001), and Zimmermann (1997).
are imprecise, owing to the uncertainty of the demand and supply of the labor forces market and the machine capacity. Accordingly, constraints (4), (8), and (10) are fuzzy in nature. On the other hand, constraints (5)–(7), (9) and (11), which represent the limiting minimum inventory levels, maximum backordering and subcontracting levels, and actual warehouse space are normally certain. This study addresses a practical application of a multiple fuzzy goals programming model to solve the multi-product APP decision problem. Therefore, the constraints (4)–(12) of the proposed MOLP model are assumed to be crisp.

2.3. Fuzzy multi-objective linear programming model

The original MOLP model for previous problems can be converted to the FMOLP model using the piecewise linear membership function of Hannan (1981) to represent the fuzzy goals of the DM in the MOLP model, together with the fuzzy decision-making of Bellman and Zadeh (1970). Generally, the piecewise linear membership function and the fuzzy decision-making of Bellman and Zadeh (1970) can be adopted to convert the problem to be solved into an ordinary LP problem. The algorithm includes the following steps.

2.3.1. Algorithm

Step 1: Specify the degree of membership for several values of each objective function $z_i (i = 1, 2, ..., k)$.

Step 2: Draw the piecewise linear membership function.

Step 3: Formulate the linear equations for each of the piecewise linear membership functions $f_i(z_i) (i = 1, 2, ..., k)$.

Step 4: Introduce the auxiliary variable $L$; the problem can be transformed into the equivalent conventional LP problem. The variable $L$ can be interpreted as representing an overall degree of satisfaction with the DM’s multiple fuzzy goals.

Step 5: Execute and modify the interactive decision process. If the DM is not satisfied with the initial solution, the model must be changed until a satisfactory solution is found.

Appendix A details the derivation. Fig. 1 presents the block diagram of the interactive FMOLP model development.

The complete FMOLP model can be formulated as follows.

Max $L$

s.t.

\[
L \leq \left( \frac{t_{i2} - t_{i1}}{2} \right) (d_{11}^- - d_{11}^+) + \left( \frac{t_{i3} - t_{i2}}{2} \right) (d_{12}^- - d_{12}^+) + \cdots + \left( \frac{t_{i,p+1} - t_{ip}}{2} \right) (d_{1p}^- - d_{1p}^+)
\]

\[+ \left( \frac{t_{i,p+1} + t_{i1}}{2} \right) \sum_{n=1}^{N} \sum_{t=1}^{T} [a_{nt}Q_n(1 + i_a)^t + b_{nt}O_n(1 + i_b)^t + c_{nt}S_n(1 + i_c)^t]
\]

\[+ d_i B_i(1 + i_d)^t + e_i B_i(1 + i_e)^t ] + \sum_{t=1}^{T} (k_i H_t + m_i F_t)(1 + i_f)^t\]

\[+ \frac{S_{i,p+1} + S_{i1}}{2}\]
$L \leq \left( \frac{t_{22} - t_{21}}{2} \right) (d_{21} - d_{21}^+) + \left( \frac{t_{23} - t_{22}}{2} \right) (d_{22} - d_{22}^+) + \cdots + \left( \frac{t_{2,p+1} - t_{2p}}{2} \right) (d_{2p} - d_{2p}^+)
+ \left( \frac{t_{3,1} + t_{21}}{2} \right) \left\{ \sum_{n=1}^{N} \sum_{i=1}^{T} [d_{n}^{m}(1 + i_d)^{+} + e_{m}^{n} B_{n}(1 + i_e)^{+}] \right\} + \frac{S_{2,1} + S_{21}}{2}
L \leq \left( \frac{t_{32} - t_{31}}{2} \right) (d_{31} - d_{31}^+) + \left( \frac{t_{33} - t_{32}}{2} \right) (d_{32} - d_{32}^+) + \cdots + \left( \frac{t_{3,p+1} - t_{3p}}{2} \right) (d_{3p} - d_{3p}^+)
+ \left( \frac{t_{3,1} + t_{31}}{2} \right) \left\{ \sum_{i=1}^{T} (H_i - F_i) \right\} + \frac{S_{3,1} + S_{31}}{2}
\sum_{n=1}^{N} \sum_{i=1}^{T} [a_{n}^{m} Q_{n}(1 + i_a)^{+} + b_{m}^{n} O_{n}(1 + i_b)^{+} + c_{n}^{m} S_{n}(1 + i_c)^{+} + d_{n}^{m} I_{m}(1 + i_i)^{+} + e_{m}^{n} B_{n}(1 + i_e)^{+}]
+ \sum_{i=1}^{T} (k_i H_i + m_i F_i)(1 + i_j)^{+} + d_{ij}^{+} - d_{ij}^{-} = X_{ij} \quad j = 1, 2, \ldots, P$
\[
\sum_{n=1}^{N} \sum_{t=1}^{T} [d_{nt} I_{nt}(1 + i_d)^t + e_{mt} B_{mt}(1 + i_e)^t] + d_{n2j}^{-} - d_{n2j}^{+} = X_{2j} \quad j = 1, 2, \ldots, P
\]

\[
\sum_{t=1}^{T} (H_t - F_t) + d_{3j}^{-} - d_{3j}^{+} = X_{3j} \quad j = 1, 2, \ldots, P
\]

\[
I_{nt} - B_{nt} = I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - D_{nt} \quad \forall n, \forall t
\]

\[
I_{nt} \geq I_{nt \min} \quad \forall n, \forall t
\]

\[
B_{nt} \leq B_{nt \max} \quad \forall n, \forall t
\]

\[
\sum_{n=1}^{N} n_{n-1}(Q_{n-1} + O_{n-1}) + H_{t} - F_{t} = \sum_{n=1}^{N} n_{n}(Q_{nt} + O_{nt}) \quad \forall t
\]

\[
\sum_{n=1}^{N} n_{n}(Q_{nt} + O_{nt}) \leq W_{t \max} \quad \forall t
\]

\[
S_{nt} \leq S_{nt \max} \quad \forall n, \forall t
\]

\[
\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq M_{t \max} \quad \forall t
\]

\[
\sum_{n=1}^{N} v_{nt} I_{nt} \leq V_{t \max} \quad \forall t
\]

\[
L, d_{1j}^{-}, d_{1j}^{+}, d_{2j}^{-}, d_{2j}^{+}, d_{3j}^{-}, d_{3j}^{+}, P_{nt}, O_{nt}, I_{nt}, B_{nt}, S_{nt}, H_{t}, F_{t} \geq 0 \quad \forall j, \forall n, \forall t
\]

3. Numerical example

3.1. Basic data for numerical example

Table 1 summarizes the basic data of the numerical example (Lin & Liang, 2002). Other relevant data are as follows (in units of US dollars).

1. Initial inventory in period 1 is 500 units of product 1 and 200 units of product 2.
2. End inventory in period 3 is 400 units of product 1 and 200 units of product 2.
3. The initial labor level is 225 man-hours.
4. The costs associated with hiring and layoff are $10 and $2.5 per worker per hour, respectively.
5. The expected escalating factor in each of the costs categories is 1%.
3.2. Formulate the FMOLP model

First, determine the initial solutions for each of the objective functions using the conventional LP model. The results are $z_1 = 229,155$, $z_2 = 971$ and $z_3 = 42.5$ man-hours. Then, formulate the FMOLP model using the initial solutions and the MOLP model presented in Section 2. Table 2 gives the piecewise linear membership functions of the proposed model. Figs. 2–4 illustrate the corresponding shapes of the piecewise linear membership functions.

Table 2
Membership functions

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$f_1(z_1)$</th>
<th>$z_2$</th>
<th>$f_2(z_2)$</th>
<th>$z_3$</th>
<th>$f_3(z_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 370,000</td>
<td>0</td>
<td>&gt; 4000</td>
<td>0</td>
<td>&gt; 90</td>
<td>0</td>
</tr>
<tr>
<td>370,000</td>
<td>0</td>
<td>4000</td>
<td>0</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>320,000</td>
<td>0.5</td>
<td>3000</td>
<td>0.4</td>
<td>70</td>
<td>0.4</td>
</tr>
<tr>
<td>270,000</td>
<td>0.8</td>
<td>2000</td>
<td>0.7</td>
<td>50</td>
<td>0.7</td>
</tr>
<tr>
<td>220,000</td>
<td>1.0</td>
<td>1000</td>
<td>1.0</td>
<td>30</td>
<td>1.0</td>
</tr>
<tr>
<td>&lt; 220,000</td>
<td>1.0</td>
<td>&lt; 1000</td>
<td>1.0</td>
<td>&lt; 30</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Fig. 2. Shape of membership function ($z_1, f_1(z_1)$).
The complete FMOLP model of numerical example is as follows.

Max $L$

s.t.

\[ L \leq -0.000002d_{11}^+ - 0.000002d_{11}^- - 0.000001d_{12}^+ - 0.000001d_{12}^- - 0.000007 \]

\[ \times \left\{ \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ a_m Q_m (1 + i_d)^t + b_m O_m (1 + i_b)^t + c_m S_m (1 + i_e)^t + d_m I_m (1 + i_d)^t + e_m B_m (1 + i_e)^t \right] + \sum_{t=1}^{T} (k_t H_t + m_t F_t) (1 + i_j)^t \right\} + 2.79 \]

\[ L \leq -0.00005d_{21}^- - 0.0000521^- - 0.00035 \left\{ \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ d_m I_m (1 + i_d)^t + e_m B_m (1 + i_e)^t \right] \right\} + 1.45 \]

\[ L \leq -0.0025d_{31}^- - 0.0025d_{31}^+ - 0.0175 \left\{ \sum_{t=1}^{T} (H_t - F_t) \right\} + 1.625 \]

\[ \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ a_m Q_m (1 + i_d)^t + b_m O_m (1 + i_b)^t + c_m S_m (1 + i_e)^t + d_m I_m (1 + i_d)^t + e_m B_m (1 + i_e)^t \right] \]

\[ + \sum_{t=1}^{T} (k_t H_t + m_t F_t) (1 + i_j)^t + d_{11}^- - d_{11}^+ = 320,000 \]
\[
\sum_{n=1}^{N} \sum_{t=1}^{T} [a_m Q_{nt} (1 + i_a) + b_m O_{nt} (1 + i_b) + c_m S_{nt} (1 + i_c) + d_m I_{nt} (1 + i_d) + e_m B_{nt} (1 + i_e)] \\
+ \sum_{t=1}^{T} (k_t H_t + m_t F_t) (1 + i_f) + d_{t2}^- - d_{t2}^+ = 270000 \\
\sum_{n=1}^{N} \sum_{t=1}^{T} [d_{nt}^m (1 + i_d) + e_m B_{nt} (1 + i_e)] + d_{t31}^- - d_{t31}^+ = 3000 \\
\sum_{t=1}^{T} (H_t - F_t) + d_{t31}^- - d_{t31}^+ = 70 \\
I_{nt} - B_{nt} = I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - D_{nt} \ \forall n, \forall t \\
I_{nt} \geq I_{nt \min} \ \forall n, \forall t \\
B_{nt} \leq B_{nt \max} \ \forall n, \forall t \\
\sum_{n=1}^{N} n_{nt-1} (Q_{nt-1} + O_{nt-1}) + H_t - F_t = \sum_{n=1}^{N} n_t (Q_{nt} + O_{nt}) \ \forall t \\
\sum_{n=1}^{N} n_t (Q_{nt} + O_{nt}) \leq W_{t \max} \ \forall t \\
S_{nt} \leq S_{nt \max} \ \forall n, \forall t \\
\sum_{n=1}^{N} r_{nt} (Q_{nt} + O_{nt}) \leq M_{t \max} \ \forall t \\
\sum_{n=1}^{N} v_{nt} I_{nt} \leq V_{t \max} \ \forall t \\
L, d_{11}^+, d_{11}^-, d_{12}^+, d_{12}^-, d_{21}^+, d_{21}^-, d_{33}^+, d_{33}^-, P_{nt}, O_{nt}, I_{nt}, B_{nt}, S_{nt}, H_t, F_t \geq 0 \ \forall n, \forall t
\]

3.3. Output solutions

LINDO computer package was used to run this FMOLP model which yielded the following results. \(z_1 = \$230090\), \(z_2 = \$1134\), \(z_3 = 33\) man-hours and the overall degree of satisfaction with the DM’s multiple fuzzy goals was 0.9598. Table 3 presents solutions for each decision variables.
4. Model implementation and results analysis

4.1. Implementation

This section discusses the actual implementation of the FMOLP model by manipulating different alternatives and analyzing the sensitivity of decision parameters to the variations of relevant conditions, based on the preceding numerical example. The implementation is adapted to the seven following scenarios.

- **Scenario 1**: Removing $z_3$ (rate of change in labor levels), consider only $z_1$ (total production costs) and $z_2$ (carrying and backordering costs) simultaneously. Table 4 presents the membership function of Scenario 1.
- **Scenario 2**: Removing $z_2$ (carrying and backordering costs), consider only $z_1$ (total production costs) and $z_3$ (rate of change in labor levels) simultaneously. Table 5 presents the membership function of Scenario 2.
- **Scenario 3**: Setting $(z_2,f_2(z_2))$ and $(z_3,f_3(z_3))$ to their original values in the numerical example, vary only $(z_1,f_1(z_1))$. Table 6 presents the data for the implementation of Scenario 3.
- **Scenario 4**: Analyze the sensitivity by changing the escalating factors under each of the costs categories and the decision conditions of the preceding numerical example. For simplicity, consider only the escalating factor of regular time production costs.

### Table 3
**FMOLP model solutions**

<table>
<thead>
<tr>
<th>Product</th>
<th>Period</th>
<th>$Q_{st}$ (units)</th>
<th>$I_{st}$ (units)</th>
<th>$O_{st}$ (units)</th>
<th>$S_{st}$ (units)</th>
<th>$B_{st}$ (units)</th>
<th>$H_t$ (man-hours)</th>
<th>$F_t$ (man-hours)</th>
<th>Labor levels (man-hours)</th>
<th>Capacity (machine-hours)</th>
<th>Warehouse space (ft$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>213</td>
<td>265</td>
<td>5674</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2693</td>
<td>1891</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3895</td>
<td>895</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>76</td>
<td>0</td>
<td>289</td>
<td>497</td>
<td>10000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1345</td>
<td>1736</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4505</td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>31</td>
<td>258</td>
<td>488</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>464</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$z_1 = \$230090; z_2 = \$1134; z_3 = 33$ man-hours; $L = 0.9598$.

### Table 4
**Membership function of Scenario 1**

<table>
<thead>
<tr>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 370 000</td>
<td>&gt; 4000</td>
<td>&gt; 33</td>
</tr>
<tr>
<td>$f_1(z_1)$</td>
<td>$f_2(z_2)$</td>
<td>$f_3(z_3)$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>370 000</td>
<td>4000</td>
<td>3000</td>
</tr>
<tr>
<td>320 000</td>
<td>3000</td>
<td>2000</td>
</tr>
<tr>
<td>270 000</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>220 000</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>&lt; 220 000</td>
<td>&lt; 1000</td>
<td>&lt; 1000</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Scenario 5: Analyze the sensitivity by changing the regular time production costs per unit under the decision conditions of the preceding numerical example. For simplicity, consider only the regular time production costs per unit of product 1.

Scenario 6: Analyze the sensitivity by changing the carrying costs per unit under the decision conditions of the preceding numerical example. For simplicity, consider only the carrying costs per unit of product 1.

Scenario 7: Analyze the sensitivity by changing the hiring and layoffs costs per unit under the decision conditions of the preceding numerical example. For simplicity, consider only the hiring and layoffs costs per unit of product 1.

Table 7 presents the implementation data of Scenarios 4–7.

4.2. Analysis of results

Table 8 summarizes the results of implementing the previous seven scenarios.

Several significant management implications that emerged when practically applying the proposed model are as follows.
Comparing Scenarios 1 and 2 with the numerical example (Run 3 in Scenario 3), reveals the trade-offs and conflicts among dependent objective functions. Consequently, the proposed model can satisfy the requirement for the practical application because it aims to minimize total production costs, carrying and backordering costs, and the rates of change in labor levels.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Item</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 4</td>
<td>$i_u$ (%)</td>
<td>0.5</td>
<td>1.0</td>
<td>5.0</td>
<td>10.1</td>
<td>20.0</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>$a_{it}$ ($/unit)</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Scenario 6</td>
<td>$d_{it}$ ($/unit)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Scenario 7</td>
<td>$k_r$ ($/man-hour)</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 7
Data of Scenarios 4–7

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Objective</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>$L$</td>
<td>0.9616</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>229,624</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>999</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>$L$</td>
<td>0.9619</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>229,565</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>33</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>$L$</td>
<td>0.4064</td>
<td>0.7424</td>
<td>0.9598</td>
<td>0.9604</td>
<td>0.9604</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>229,544</td>
<td>229,610</td>
<td>230,090</td>
<td>230,141</td>
<td>230,141</td>
</tr>
<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>1071</td>
<td>956</td>
<td>1134</td>
<td>1132</td>
<td>1132</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>35</td>
<td>45</td>
<td>33</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>$L$</td>
<td>0.9615</td>
<td>0.9598</td>
<td>0.8786</td>
<td>0.7623</td>
<td>0.4432</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>227,555</td>
<td>230,090</td>
<td>250,262</td>
<td>241,299</td>
<td>325,690</td>
</tr>
<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>1128</td>
<td>1134</td>
<td>1105</td>
<td>1792</td>
<td>2856</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>33</td>
<td>33</td>
<td>31</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>$L$</td>
<td>0.9604</td>
<td>0.9604</td>
<td>0.9598</td>
<td>0.7691</td>
<td>0.4929</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>138,945</td>
<td>184,543</td>
<td>230,090</td>
<td>275,150</td>
<td>320,697</td>
</tr>
<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>1122</td>
<td>1132</td>
<td>1134</td>
<td>1417</td>
<td>1429</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>Scenario 6</td>
<td>$L$</td>
<td>0.9634</td>
<td>0.9625</td>
<td>0.9598</td>
<td>0.9484</td>
<td>0.9391</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>229,183</td>
<td>229,400</td>
<td>230,090</td>
<td>230,463</td>
<td>230,530</td>
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<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>947</td>
<td>1125</td>
<td>1134</td>
<td>1172</td>
<td>1203</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>32</td>
<td>31</td>
<td>33</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td>Scenario 7</td>
<td>$L$</td>
<td>0.9604</td>
<td>0.9604</td>
<td>0.9598</td>
<td>0.9579</td>
<td>0.9541</td>
</tr>
<tr>
<td></td>
<td>$z_1$ ($)</td>
<td>203,141</td>
<td>203,141</td>
<td>230,090</td>
<td>229,925</td>
<td>229,925</td>
</tr>
<tr>
<td></td>
<td>$z_2$ ($)</td>
<td>1132</td>
<td>1132</td>
<td>1134</td>
<td>1140</td>
<td>1153</td>
</tr>
<tr>
<td></td>
<td>$z_3$ (man-hours)</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>33</td>
</tr>
</tbody>
</table>

1. Comparing Scenarios 1 and 2 with the numerical example (Run 3 in Scenario 3), reveals the trade-offs and conflicts among dependent objective functions. Consequently, the proposed model can satisfy the requirement for the practical application because it aims to minimize total production costs, carrying and backordering costs, and the rates of change in labor levels.
2. The results of Scenario 3 show that the specific degree of membership for each of the objective functions strongly affects the overall level of satisfaction and output solutions for each decision variable. This fact has two significant implications. First, the most important task of DM is to specify the rational degree of membership for each objective function; second, the DM may flexibly revise the range of value of the degree of membership to yield satisfactory solutions. Fig. 5 depicts the changes for the objective and $L$ values of Scenario 3.

3. The results of Scenario 4 indicate that the escalating factor for each cost category affects the objective and $L$ values. In particular, increasing the escalating factor increases objective values including the total production costs and the carrying and the backordering costs, while sharply decreasing the $L$ value. This finding implies that the DM must consider the time value of money in a practical APP problem. Additionally, the DM must also increase the efficiency of internal management and seek to reduce the cost of capital to reduce the escalating factor. Fig. 6 depicts the changes for the objective and $L$ values of Scenario 4.

4. The sensitivity analysis of production cost per unit, carrying cost per unit and hiring and layoff costs of Scenarios 5–7 reveals that the change in each cost category influences the objective functions, $L$ values and other output solutions, implying that the DM should improve production, material, and human resources to reduce effectively the production cost, carrying cost, backordering and hiring and layoff costs. Figs. 7–9 presents the changes of the objective and $L$ values of Scenario 5–7, respectively.
4.3. Models comparisons

Table 9 compares the FMOLP model presented in this study to the LP (Charnes & Cooper, 1961; Singhal & Adlakha, 1989) and FLP (Lee, 1990) models. Several significant characteristics distinguish the proposed model from the other models.

1. The proposed model includes fuzzy multi-objective functions. In practice, the DM usually faces a fuzzy multi-objective planning problem when making an APP decision. The proposed model can satisfy the requirement for practical application because it minimizes total production costs, carrying
and backordering costs, and the rates of changes in labor levels, and can determine the DM’s overall degree of satisfaction.

2. The proposed model provides the most flexible decision-making and adjustment processes. For instance, if the DM does not accept the initial overall degree of satisfaction of 0.9598 as in the numerical example, then he may try to adjust this $L$ value by taking account of relevant information to seek a set of rational output solutions for APP decision-making.

3. The proposed model considers the time value of money in relation to relevant cost categories. Generally, satisfactory decision performance and objective values are affected by the interest factor. The DM computes the value in each cost category by considering the time value of money in the proposed model, which is appropriate for practical application to the APP problem.

4. The proposed model outputs more wide-ranging decision information than other models. This FMOLP model focuses on the multi-periods and multi-products (product family) problem in APP decision-making. It can also provide information on alternative strategies for overtime, subcontracting, inventory carrying, backorders, hiring and layoffs, but not regular time production, in response to variations in forecasted demand.

The proposed FMOLP model is constructed using the piecewise linear membership function of Hannan (1981) to represent the fuzzy goals of the DM in the MOLP model, together with the minimum operator of the fuzzy decision-making of Bellman and Zadeh (1970). Moreover, the original fuzzy MOLP problem can be converted into an equivalent crisp LP problem and is easily solved by the standard simplex method. Table 10 compares Hannan’s approach to the representative FGP models,
including those of Leberling (1981), Sakawa (1988), and Zimmermann (1978). The important differences among these models result from the types of membership functions and aggregation operators they apply. In general, aggregation operators can be roughly classified into three categories: intersection, union, and averaging operators (Zimmermann, 1996). Table 11 compares the common aggregation operators.

The minimum operator used in this study is preferable when the DM wishes to make the optimal membership function values approximately equal or when the DM feels that the minimum operator is an approximate representation. However, for some practical situations, the application of the aggregation operator to draw maps above the maximum operator and below the minimum operator is important. Alternatively, as shown in Table 11, averaging operators consider the relative importance of fuzzy sets and have the compensative property so that the result of combination will be medium. The $\gamma$-operator (Zimmermann & Zysno, 1980), which yields an acceptable compromise between empirical fit and computational efficiency, seems to be the convex combination of the minimum and maximum operators (Zimmermann, 1997). Zimmermann (1996) pointed out that the following eight important criteria must be applied selecting an adequate aggregation operator: axiomatic strength, empirical fit, adaptability, numerical efficiency, compensation, range of compensation, aggregating behavior, and required scale level of membership function.

---

Table 10: Comparisons of major fuzzy goal programming models

<table>
<thead>
<tr>
<th>Model</th>
<th>Membership function</th>
<th>Aggregation operator</th>
<th>Brief description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zimmermann (1978)</td>
<td>Linear</td>
<td>Minimum (maximize the minimum membership function value)</td>
<td>The rate of increased membership satisfaction is considered to be constant. The fuzzy MOLP problem can be converted into the equivalent crisp LP problem. Has higher computational efficiency.</td>
</tr>
<tr>
<td>Hannan (1981)</td>
<td>Piecewise linear and continuous</td>
<td>Minimum (as Zimmermann, 1978) Minimize weighted sum of goals deviations Maximize priorities sum of goals deviations</td>
<td>Enables the nonlinear membership function to approximate piecewise by linear functions. The fuzzy MOLP problem can be converted into the equivalent crisp LP problem. Has higher computational efficiency.</td>
</tr>
<tr>
<td>Sakawa (1988)</td>
<td>Linear</td>
<td>Minimum (as Zimmermann, 1978)</td>
<td>The resulting problem with the five types of membership functions is a nonlinear programming problem. Combines the bisection and LP methods. The computational efficiency is reduced.</td>
</tr>
</tbody>
</table>

---

5. Conclusions

APP deals with matching supply and demand of forecasted, varying customer orders over the medium term. The aim of APP decision-making is to set overall production levels for each product category to meet fluctuating or uncertain demands in the near future, such that APP also determines the appropriate resources to be used. This study develops a FMOLP model of the multi-product APP decision problem in a fuzzy environment. The proposed model aims to minimize total production costs, carrying and backordering costs, and the rates of changes in labor levels with reference to inventory level, labor levels, capacity, warehouse space and the time value of money.

The proposed model yields a compromise solution and the DM’s overall levels of satisfaction, given these solved fuzzy multi-objective values. Moreover, the proposed model provides a systematic framework that facilitates the decision-making process, enabling a DM interactively to modify the membership functions of the objectives until a satisfactory solution is obtained. Consequently, the proposed model is the most practically applicable for making APP decisions.

The major limitations of the proposed model concern the assumptions made for each of the decision parameters with reference to production costs, forecasted demand, maximum inventory and labor levels, maximum capacity and warehouse space available, and relevant production resources. Hence, the proposed model must be modified make it better suited to the practical application. Furthermore, future researchers may explore the fuzzy properties of decision variable, coefficients,
and relevant decision parameters in APP problems. The proposed FMOLP model is based on Hannan’s approach (1981), which implicitly assumes that the minimum operator is the proper representation of the human DM who combines fuzzy statements by ‘and’. Therefore, future research may also apply the averaging or other operators to solve APP decision problems in a fuzzy environment.

Appendix A

The FMOLP model is derived as follows.

Step 1: Specify the degree of membership \( f_i(z_i) \) \((i = 1, 2, 3)\) for several values for each of the objective functions \( z_i(i = 1, 2, 3)\). Table A1 presents the piecewise linear membership functions, \( f_1(z_1) \), \( f_2(z_2) \) and \( f_3(z_3) \).

Step 2: Draw the piecewise linear membership function.

Step 3-I: Convert the membership function \( f_i(z_i) \) \((i = 1, 2, 3)\) into the form

\[
f_i(z_i) = \sum_{j=1}^{P} \alpha_{ij} |z_i - X_{ij}| + \beta_i z_i + \gamma_i \forall i
\]

(A1)

where

\[
\alpha_{ij} = \frac{t_{ij+1} - t_{ij}}{2}, \quad \beta_i = \frac{t_{ip+1} + t_{il}}{2}, \quad \gamma_i = \frac{S_i}_{p+1} + \frac{S_i}{2}
\]

Assumed that \( f_i(z_i) = t_i z_i + S_i \) for each segment \( X_{ir-1} \leq z_i \leq X_{ir} \), where \( t_i \) is the slope and \( S_i \) is the \( y \)-intercept for the section of the line that begins at \( X_{ir-1} \) and ends at \( X_{ir} \).

Hence,

\[
f_i(z_i) = \left( \frac{t_{i2} - t_{i1}}{2} \right) |z_i - X_{i1}| + \left( \frac{t_{i3} - t_{i2}}{2} \right) |z_i - X_{i2}| + \cdots + \left( \frac{t_{ip+1} - t_{ip}}{2} \right) |z_i - X_{ip}|
\]

(A2)

\[
+ \left( \frac{t_{i1} + t_{i1}}{2} \right) z_i + \frac{S_{i,p+1} + S_{i1}}{2} \left( \frac{t_{ij+1} - t_{ij}}{2} \right) \neq 0 \quad j = 1, 2, \ldots, P
\]

| \( z_i \) | \( > X_{i0} \) | \( X_{i0} \) | \( X_{i1} \) | \( X_{i2} \) | \( \cdots \) | \( X_{ip} \) | \( X_{ip+1} \) | \( < X_{ip+1} \) | \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_i(z_i) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( q_{i1} )</td>
<td>( q_{i2} )</td>
<td>( \cdots )</td>
<td>( q_{ip} )</td>
</tr>
<tr>
<td>( \neq 0 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( q_{i1} )</td>
<td>( q_{i2} )</td>
<td>( \cdots )</td>
<td>( q_{ip} )</td>
</tr>
<tr>
<td>( f_i(z_i) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( \cdots )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

Table A1

Membership function \( f_i(z_i) \)

Note: \( 0 \leq q_{ij} \leq 1.0, q_{ij} = q_{ij+1} \ i = 1, 2, 3 \ j = 1, 2, \ldots, P \).
where

\[ t_{11} = \left( \frac{q_{11} - 0}{X_{11} - X_{10}} \right), \quad t_{12} = \left( \frac{q_{12} - q_{11}}{X_{12} - X_{11}} \right), \ldots, \quad t_{1,P+1} = \left( \frac{1.0 - q_{1P}}{X_{1,P+1} - X_{1P}} \right), \]

and \( S_{1,P+1} \) is the \( y \)-intercept of the section of the line that begins at \( X_{1P} \) and ends at \( X_{1,P+1} \), and can be derived using \( f_1(z_1) = t_{11}z_1 + S_{11} \).

\[
f_2(z_2) = \left( \frac{t_{21} - t_{11}}{2} \right)|z_2 - X_{21}| + \left( \frac{t_{22} - t_{12}}{2} \right)|z_2 - X_{22}| + \cdots + \left( \frac{t_{2,P+1} - t_{2P}}{2} \right)|z_2 - X_{2P}| \]

\[ + \left( \frac{t_{2,P+1} + t_{21}}{2} \right)z_2 + \frac{S_{2,P+1} + S_{21}}{2} \left( \frac{t_{2,P+1} + t_{21}}{2} \right) \neq 0 \quad j = 1, 2, \ldots, P \quad (A3)\]

where

\[ t_{21} = \left( \frac{q_{21} - 0}{X_{21} - X_{20}} \right), \quad t_{22} = \left( \frac{q_{22} - q_{21}}{X_{22} - X_{21}} \right), \ldots, \quad t_{2,P+1} = \left( \frac{1.0 - q_{2P}}{X_{2,P+1} - X_{2P}} \right) \]

and \( S_{2,P+1} \) is the \( y \)-intercept of the section of the line that begins at \( X_{2P} \) and ends at \( X_{2,P+1} \), and can be derived using \( f_2(z_2) = t_{21}z_2 + S_{21} \).

\[
f_3(z_3) = \left( \frac{t_{31} - t_{21}}{2} \right)|z_3 - X_{31}| + \left( \frac{t_{32} - t_{22}}{2} \right)|z_3 - X_{32}| + \cdots + \left( \frac{t_{3,P+1} - t_{3P}}{2} \right)|z_3 - X_{3P}| \]

\[ + \left( \frac{t_{3,P+1} + t_{31}}{2} \right)z_3 + \frac{S_{3,P+1} + S_{31}}{2} \left( \frac{t_{3,P+1} + t_{31}}{2} \right) \neq 0 \quad j = 1, 2, \ldots, P \quad (A4)\]

where

\[ t_{31} = \left( \frac{q_{31} - 0}{X_{31} - X_{30}} \right), \quad t_{32} = \left( \frac{q_{32} - q_{31}}{X_{32} - X_{31}} \right), \ldots, \quad t_{3,P+1} = \left( \frac{1.0 - q_{3P}}{X_{3,P+1} - X_{3P}} \right), \]

and \( S_{3,P+1} \) is the \( y \)-intercept for the section of the line that begins at \( X_{3P} \) and ended at \( X_{3,P+1} \), and can be derived using \( f_3(z_3) = t_{31}z_3 + S_{31} \).

Step 3-2: Introduce the following nonnegative deviational variables \( d_{ij}^+ \) and \( d_{ij}^- \)

\[
\sum_{n=1}^{N} \sum_{i=1}^{T} \left[ a_n O_m(1 + i_o)^j + b_n O_m(1 + i_b)^j + c_n S_n(1 + i_c)^j + d_n I_m(1 + i_d)^j + e_n B_m(1 + i_e)^j \right] \\
+ \sum_{i=1}^{T} \left( k H_i + m F_i \right)(1 + i_j)^j + d_{ij}^- - d_{ij}^+ = X_{ij} \quad j = 1, 2, \ldots, P \quad (A5)\]

\[
\sum_{n=1}^{N} \sum_{i=1}^{T} \left[ d_n I_m(1 + i_d)^j + e_n B_m(1 + i_e)^j \right] + d_{2j}^- - d_{2j}^+ = X_{2j} \quad j = 1, 2, \ldots, P \quad (A6)\]

\[
\sum_{i=1}^{T} \left( H_i - F_i \right) + d_{3j}^- - d_{3j}^+ = X_{3j} \quad j = 1, 2, \ldots, P \quad (A7)\]
where \(d_{ij}^+\) and \(d_{ij}^-\) denote the deviational variables at the \(j\)th point and \(X_{ij}\) represent the values \(z_i\) of the \(i\)th objective function at the \(j\)th point.

**Step 3-3:** Substituting Eqs. (A5)–(A7) into (A2)–(A4), respectively, yields

\[
\begin{align*}
\text{(A8)} \\
\frac{f_1(z_3)}{f_2(z_3)} &= \left(\frac{t_{12} - t_{11}}{2}\right)(d_{11}^- - d_{11}^+) + \left(\frac{t_{13} - t_{12}}{2}\right)(d_{12}^- - d_{12}^+) + \cdots + \left(\frac{t_{1, p+1} - t_{1p}}{2}\right)(d_{1p}^- - d_{1p}^+)
+ \left(\frac{t_{1, p+1} + t_{11}}{2}\right)\left\{\sum_{n=1}^N \sum_{i=1}^T \left[a_m Q_m(1 + i_a)' + b_m O_m(1 + i_b)' + c_m S_m(1 + i_c)'
+ d_m I_m(1 + i_d)' + e_m B_m(1 + i_e)'
+ \frac{k_i H_i + m_i F_i}{2}(1 + i_f)\right]\right\} + \frac{S_{1, p+1} + S_{11}}{2}
\end{align*}
\]

**Step 4:** Introduce the auxiliary variable \(L\), the problem can be transformed into the equivalent conventional LP problem. The variable \(L\) can be interpreted as representing an overall degree of satisfaction with the DM’s multiple fuzzy goals.

The complete FMOLP model is as follows.

Max \(L\)

s.t.

\[
\begin{align*}
L &\leq \left(\frac{t_{12} - t_{11}}{2}\right)(d_{11}^- - d_{11}^+) + \left(\frac{t_{13} - t_{12}}{2}\right)(d_{12}^- - d_{12}^+) + \cdots + \left(\frac{t_{1, p+1} - t_{1p}}{2}\right)(d_{1p}^- - d_{1p}^+)
+ \left(\frac{t_{1, p+1} + t_{11}}{2}\right)\left\{\sum_{n=1}^N \sum_{i=1}^T \left[a_m Q_m(1 + i_a)' + b_m O_m(1 + i_b)' + c_m S_m(1 + i_c)'
+ d_m I_m(1 + i_d)' + e_m B_m(1 + i_e)'
+ \frac{k_i H_i + m_i F_i}{2}(1 + i_f)\right]\right\} + \frac{S_{1, p+1} + S_{11}}{2}
\end{align*}
\]
\[ L \leq \left( \frac{t_{22} - t_{21}}{2} \right)(d_{21}^+ - d_{21}^-) + \left( \frac{t_{23} - t_{22}}{2} \right)(d_{22}^+ - d_{22}^-) + \cdots + \left( \frac{t_{2,p+1} - t_{2p}}{2} \right)(d_{2p}^+ - d_{2p}^-) \]

\[ + \left( \frac{t_{32} - t_{31}}{2} \right)(d_{31}^+ - d_{31}^-) + \left( \frac{t_{33} - t_{32}}{2} \right)(d_{32}^+ - d_{32}^-) + \cdots + \left( \frac{t_{3,p+1} - t_{3p}}{2} \right)(d_{3p}^+ - d_{3p}^-) \]

\[ + \left( \frac{t_{32} - t_{31}}{2} \right) \sum_{i=1}^{T} (H_i - F_i) \]

\[ \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ d_{nt}^+ I_{nt} (1 + i_d)^j + e_{nt} B_{nt} (1 + i_e)^j + c_{nt} S_{nt} (1 + i_c)^j + d_{nt} I_{nt} (1 + i_d)^j \right] \]

\[ + e_{nt} B_{nt} (1 + i_e)^j \] + \sum_{t=1}^{T} (k_i H_i + m_i F_i) (1 + i_j) + d_{ij} - d_{ij}^- = X_{ij} \quad j = 1, 2, \ldots, P

\[ \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ d_{nt} I_{nt} (1 + i_d)^j + e_{nt} B_{nt} (1 + i_e)^j \right] + d_{2j}^- - d_{2j}^+ = X_{2j} \quad j = 1, 2, \ldots, P

\[ \sum_{t=1}^{T} (H_i - F_i) + d_{3j}^- - d_{3j}^+ = X_{3j} \quad j = 1, 2, \ldots, P

I_{nt} - B_{nt} = I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - D_{nt} \quad \forall n, \forall t

I_{nt} \geq I_{nt \min} \quad \forall n, \forall t

B_{nt} \leq B_{nt \max} \quad \forall n, \forall t

\[ \sum_{n=1}^{N} n_{nt-1} (Q_{nt-1} + O_{nt-1}) + H_i - F_i = \sum_{n=1}^{N} n_{nt} (Q_{nt} + O_{nt}) \quad \forall t \]

\[ \sum_{n=1}^{N} n_{nt} (Q_{nt} + O_{nt}) \leq W_{t \max} \quad \forall t \]

S_{nt} \leq S_{nt \max} \quad \forall n, \forall t

\[ \sum_{n=1}^{N} r_{nt} (Q_{nt} + O_{nt}) \leq M_{t \max} \quad \forall t \]

\[ \sum_{n=1}^{N} v_{nt} I_{nt} \leq V_{t \max} \quad \forall t \]
\[ L, d_{ij}^1, d_{ij}^2, d_{ij}^3, d_{ij}^4, P_n, O_n, I_n, B_n, S_n, H, F_i \geq 0 \ \forall j, \forall n, \forall t \]

References


