Effects of Noise Power Estimation on Energy Detection

for Cognitive Radio Applications

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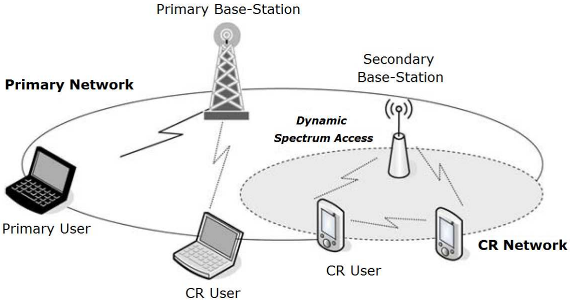
Cognitive radio scenario

- Opportunistic use of the spectrum resources $\Longrightarrow$ spectrum sensing
- Adaptation to the environment, optimization, learning

Cognitive Radio concept



Signal Processing Algorithms for Cognitive Radio Networks

Study of spectrum sensing algorithms

- Analysis of the principal spectrum sensing algorithms proposed in literature;
- Energy detection with estimated noise power: existence of the SNR wall;
- Wideband techniques using Information Theoretic Criteria (ITC);
- FP7-EUWB European project: WP2 Cognitive UWB Radio and Coexistence;
- EDA-B CORASMA project: WP3 Sensing;
- COST Action ICT - IC0902: Cognitive Radio and Networking for Cooperative Coexistence of Heterogeneous Wireless Networks
- Test of spectrum sensing algorithms using USRP2 platforms;

Motivation

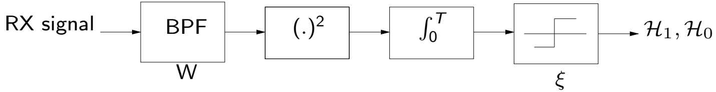


Figure: Energy Detector.

Noise uncertainty ⇒ How to set the threshold under noise uncertainty?
⇒ SNR wall

- Analytical performance of the energy detector with noise power estimation (ENP-ED).
- We provide the conditions in which the SNR wall phenomenon occurs and the asymptotical analysis of the ENP-ED

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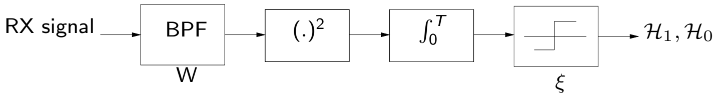

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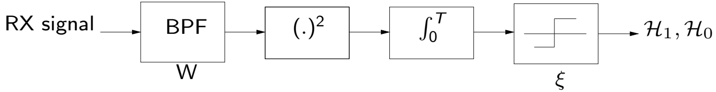

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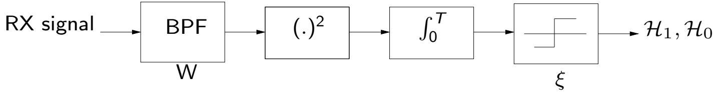

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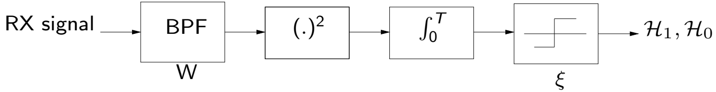

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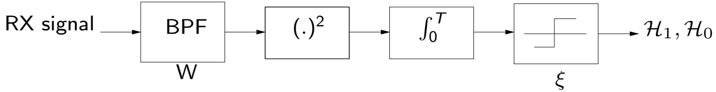

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Outline

- Background
  - Design curves and SNR wall
  - Ideal ED (with known σ^2)
  - Noise uncertainty
- Existence of the SNR wall for the ENP-ED
  - Example: ENP-ED with ML noise power estimator
- Asymptotical behavior of the design curve for the ENP-ED
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Design curve

Detectors must be designed in order to guarantee

$$P_D \geq P_D^{\mathrm{DES}}$$
$$P_{FA} \leq P_{FA}^{\mathrm{DES}}$$

where $\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}\right)$ is the target performance pair.

The *design curve* is the relation between the minimum SNR required for guarantee $\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}\right)$, and the number of samples collected N

$$\mathrm{SNR}_{\mathrm{min}} = \psi\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}, N, \dots\right)$$

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Design curve: typical behavior and SNR wall

Given $\left(P_{FA}^{\mathrm{DES}}, P_{D}^{\mathrm{DES}}\right)$

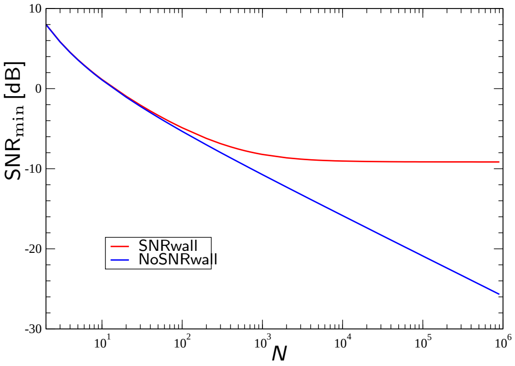

Figure: Typical behavior of the design curves

Ideal ED: P_{FA} and P_D

$$y_i \sim \begin{cases} \mathcal{CN}\left(0, 2\sigma^2\right) \; \mathcal{H}_0 \\ \mathcal{CN}\left(0, 2\sigma^2(1+SNR)\right) \; \mathcal{H}_1 \end{cases}$$

Ideal ED metric $\Rightarrow \sigma^2$ is perfectly known

$$\Lambda(\mathbf{y}) \triangleq \frac{1}{2\sigma^2} \cdot \frac{1}{N} \sum_{i=0}^{N-1} |y_i|^2 \sim \chi^2_{2N}$$

$$P_{FA} = \widetilde{\Gamma}\left(N, N\xi\right)$$

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$\widetilde{\Gamma}\left(a, x\right) = \frac{1}{\Gamma(a)} \int_x^{+\infty} x^{a-1} e^{-x} dx$ is the (upper) *gamma regularized function.*

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In practice, the SU has no perfect knowledge of the noise power, but its value will maintain a certain degree of uncertainty.

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BWB: worst case approach under uncertain noise power

If the noise power is not perfectly known....

Common model for noise power uncertainty [Tandra, Sahai, 2006]:

$$\widetilde{\sigma}^2 \in \left[\sigma^2_{\min}, \sigma^2_{\max}\right]$$

Typically P_{FA} and P_D are computed in the "worst cases", i.e.,

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We will refer to this one as the "bounded worse-behavior", BWB.

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BWB: design curve

BWB approach

$$\mathsf{SNR}_{\min,\,\mathrm{BWB}} = \frac{\sigma^2_{\max}}{\sigma^2_{\min}} \cdot \frac{\mathrm{Inv}\widetilde{\Gamma}\left(N, P_{FA}^{\mathrm{DES}}\right)}{\mathrm{Inv}\widetilde{\Gamma}\left(N, P_{D}^{\mathrm{DES}}\right)} - 1$$

It can be demonstrated that

$$\lim_{N\to\infty} \mathsf{SNR}_{\min,\,\mathrm{BWB}} = \frac{\sigma^2_{\max}}{\sigma^2_{\min}} - 1 > 0$$

BWB approach always give rise to the SNR wall phenomenon.

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BWB approach implications

An idea that is growing into the spectrum sensing community is that the SNR wall is an unavoidable problem;
even if the noise variance is estimated there is always some degree of uncertainty that originates the SNR wall.

ENP-ED

$$\Lambda_{\mathrm{g}}(\mathbf{y}) \triangleq \frac{1}{2\widehat{\sigma}^2} \cdot \frac{1}{N} \sum_{i=0}^{N-1} |y_i|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \xi$$

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Theorem I: Existence of the SNR wall

Hp: $\widehat{\sigma}^2$ is an unbiased asymptotically Gaussian estimator of σ^2.

Given $(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}})$,

if $var\left(\widehat{\sigma}^2\right)$, is $o\left(1\right)$ for $N \to \infty$

$$\mathrm{SNR}_{\min}^{(\infty)} = \lim_{N\to\infty} \mathrm{SNR}_{\min} = 0$$

No SNR wall;

if $var\left(\widehat{\sigma}^2\right)$, is $\Theta\left(1\right)$ for $N \to \infty$

SNR wall:

$$\mathrm{SNR}_{\min}^{(\infty)} = \frac{1-\delta\sqrt{\phi}}{1-\alpha\sqrt{\phi}} - 1$$

where $\alpha = Q^{-1}\left(P_{FA}^{\mathrm{DES}}\right)$, $\delta = Q^{-1}\left(P_D^{\mathrm{DES}}\right)$ and $\phi = \lim_{N\to\infty} var\left(\widehat{\sigma}^2/\sigma^2\right)$.

Existence of the SNR wall: comment

The SNR wall phenomenon is not caused
by the presence of an uncertainty itself,
but is rather due to the insufficient refinement ($var(\widehat{\sigma}^2)$)
of the noise power estimate with respect to the observation time (N).

Example: ENP-ED with ML noise power estimation

$$\Lambda_{\mathrm{g}}(\mathbf{y}) \triangleq \frac{1}{2\widehat{\sigma}^2_{\mathrm{ML}}} \cdot \frac{1}{N} \sum_{i=0}^{N-1} |y_i|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \xi$$

The maximum-likelihood (ML) noise power estimator is based on the observation of the M *noise only samples* $n_{-1}, \ldots, n_{-M}$

$$\widehat{\sigma}^2_{\mathrm{ML}} = \frac{1}{2\,M} \sum_{i=1}^{M} |n_{-i}|^2$$

$$var\left(\widehat{\sigma}^2_{\mathrm{ML}}\right) = \frac{\sigma^4}{M}$$

Two-step sensing scheme

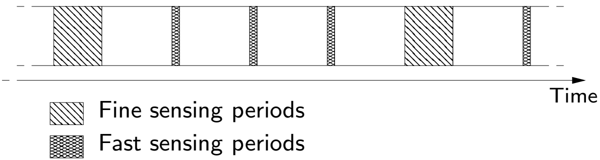

Figure: Two step sensing scenario

- Feature detector $\Rightarrow$ Less frequent fine-sensing periods
- Energy detector $\Rightarrow$ More frequent fast-sensing periods

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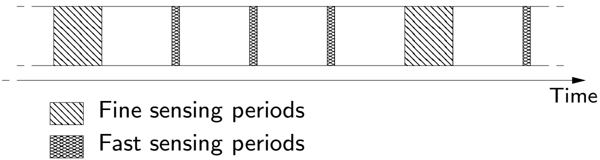

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Two-step sensing schemes are supported by
the CR standards IEEE 802.22 (draft) and ECMA 392.

ENP-ED in two-step sensing schemes

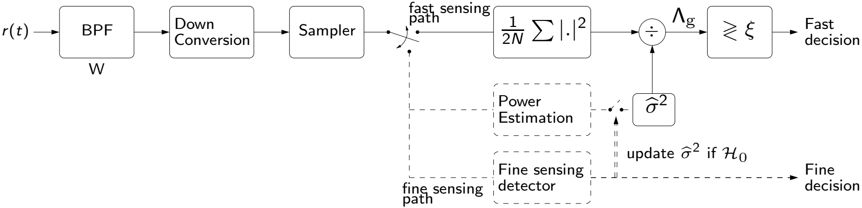
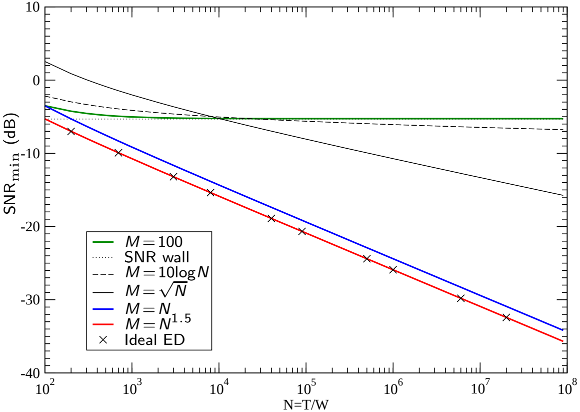


Figure: Two-step detection scheme supporting ED with noise power estimation for fast sensing.

ENP-ED: design curve behavior



$var\left(\widehat{\sigma}^2\right)$ is $\Theta(1)$ for $N \to \infty \Rightarrow M$ constant (independent on N)

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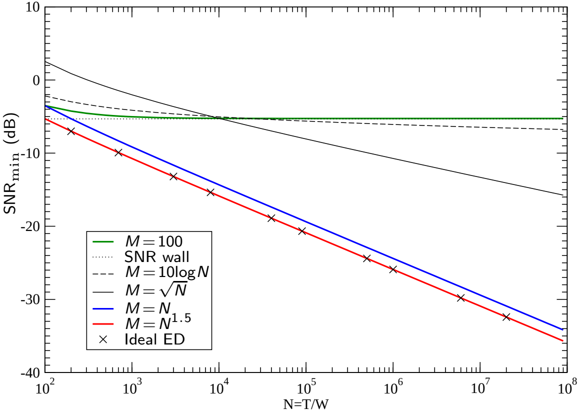

$var\left(\widehat{\sigma}^2\right)$ is $o(1)$ for $N \to \infty \Rightarrow M$ is an increasing function of N

Asymptotical behavior of the design curves

If the SNR wall does not occur,
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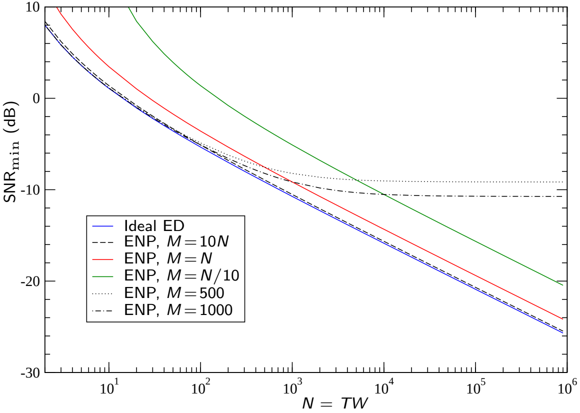
If the SNR wall does not occur,
design curve tends asymptotically to a straight line.

$var\left(\widehat{\sigma}^2\right)$ determines

- the slope of the asymptote
- the distance from the ideal ED design curve

ENP-ED with ML estimator: design curve - two step



$var\left(\widehat{\sigma}^2\right) \approx 1/(\lambda N)$ for $N \rightarrow \infty \Rightarrow M = \lambda N \qquad \lambda > 0$

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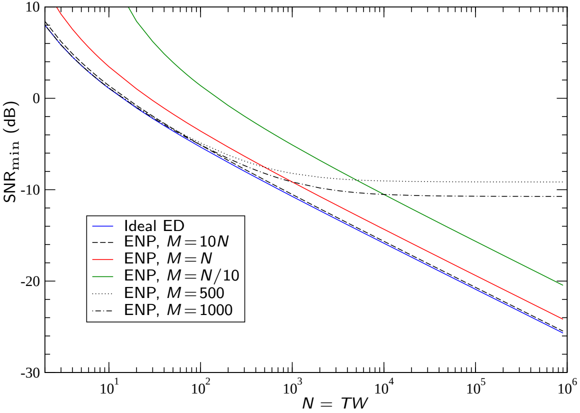

$$var\left(\widehat{\sigma}^2\right) \approx 1/(\lambda N) \text{ for } N \to \infty \Rightarrow M = \lambda N \qquad \lambda > 0$$

$$\mathrm{SNR}_{\min}^{(\infty)}(dB) \approx -5\log_{10} N - 5\log_{10}\left(\frac{1+\lambda}{\lambda}\right) + 10\log_{10}(\alpha - \delta)$$

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- These results are confirmed by the theoretical analysis and simulations of the performances of the ENP-ED with ML noise power estimation
- The ENP-ED can be adopted for *fast sensing* in realistic scenarios supported by current CR standards (two-step sensing schemes)

References

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Thank you!!!

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Design curve calculation

Design curve: calculation

In general we can always write:

$$P_{FA} = f_{FA}\left(\xi; N, \dots\right)$$
$$P_D = f_D\left(\xi; N, \mathrm{SNR}, \dots\right)$$

where $f_{FA}(\dots)$ and $f_D(\dots)$ are given by the specific implementation of the detector considered.

With the inversion of the P_{FA} and P_D formulas we can find the threshold ξ and the minimum SNR, $\mathrm{SNR}_{\min}$ fulfilling the specifications.

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Noise uncertainty causes

Noise power uncertainty

Noise power uncertainty is mainly caused by[1]

- temperature variation;
- change in low noise amplifier gain due to thermal fluctuations;
- initial calibration error;
- presence of interferers.

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- change in low noise amplifier gain due to thermal fluctuations;
- initial calibration error;

} Noise estimation

- presence of interferers. ⇒ Quiet periods

[1] S. Shellhammer and G. Chouinard, "Spectrum sensing requirement summary," IEEE P802.22-06/0089r1, Tech. Rep., Jun. 2006.

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Quiet periods have been proposed in CR standards, e.g. IEEE 802.22 draft and ECMA 392.

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Two-step sensing scheme

Two-step sensing scheme

- Feature detectors

Two-step sensing scheme

- Feature detectors

Long observation time decreases the efficiency of the SU communication

Two-step sensing scheme

- Feature detectors $\Rightarrow$ Less frequent fine-sensing periods

Two-step sensing scheme

- Feature detectors ⇒ Less frequent fine-sensing periods
- Energy detector ⇒ More frequent fast-sensing periods

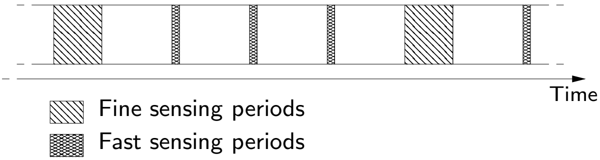

Figure: Two step sensing scenario

Two-step sensing scheme

- Feature detectors ⇒ Less frequent fine-sensing periods
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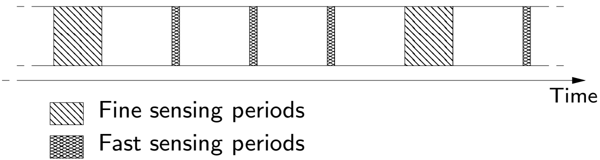


Figure: Two step sensing scenario

Two-step sensing schemes are supported by
the CR standards IEEE 802.22 (draft) and ECMA 392.

ENP-ED in two-step sensing schemes

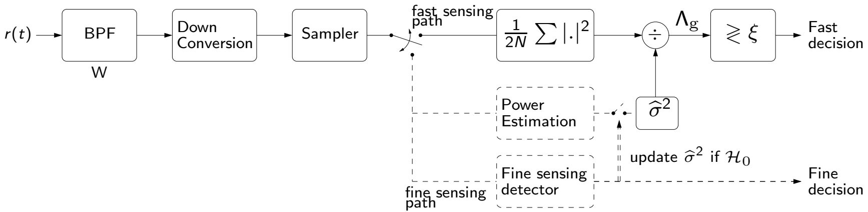


Figure: Two-step detection scheme supporting ED with noise power estimation for fast sensing.

Theorem I
Existence of the SNR wall

Theorem I: Existence of the SNR wall

Theorem (Existence of the SNR wall)

For the energy detection with noise power estimation, assuming that $\widehat{\sigma}^2$ is an unbiased asymptotically Gaussian estimator of the noise power σ^2, an arbitrary $(P_{FA}^{DES}, P_D^{DES})$ pair can be achieved by increasing the observation interval (i.e., increasing the number of collected samples N) for all values of SNR, if and only if the variance of the noise power estimator, $var\left(\widehat{\sigma}^2\right)$, is $o\left(1\right)$ for $N \to \infty$.
Moreover, if $var\left(\widehat{\sigma}^2\right)$ is $\Theta\left(1\right)$ for $N \to \infty$, there exists a minimum SNR (SNR wall), under which it is impossible to reach the desired $(P_{FA}^{DES}, P_D^{DES})$ pair; in this case, by defining $\alpha = Q^{-1}\left(P_{FA}^{DES}\right)$, $\delta = Q^{-1}\left(P_D^{DES}\right)$ and $\phi = var\left(\widehat{\sigma}^2/\sigma^2\right)$, the minimum SNR for $N \to \infty$ converges to

$$SNR_{min}^{(\infty)} = \frac{1-\delta\sqrt{\phi}}{1-\alpha\sqrt{\phi}} - 1.$$

Existence of the SNR wall: Proof of Theorem I

$$\Lambda_{\mathrm{g}}(\mathbf{y}) \triangleq \frac{1}{2\widehat{\sigma}^2} \cdot \frac{1}{N} \sum_{i=0}^{N-1} |y_i|^2 = \frac{\nu(\mathbf{y})}{\widehat{\sigma}^2}$$

Gaussian approximation for large N. $\phi \triangleq var\left(\widehat{\sigma}^2/\sigma^2\right)$

$$P_{FA} = Pr\{\nu(\mathbf{y}) - \xi\,\widehat{\sigma}^2 > 0|\mathcal{H}_0\} = Q\left(\frac{\xi - 1}{\sqrt{\frac{1}{N} + \xi^2\,\phi}}\right)$$

$$P_D = Pr\{\nu(\mathbf{y}) - \xi\,\widehat{\sigma}^2 > 0|\mathcal{H}_1\} = Q\left(\frac{\xi - (1+\mathrm{SNR})}{\sqrt{\frac{(1+\mathrm{SNR})^2}{N} + \xi^2\,\phi}}\right)$$

From these equations we can compute the minimum value of SNR required to reach the P_{FA}^{DES} and P_D^{DES} by solving, in the variables $(\xi_{\mathrm{asymp}}, \mathrm{SNR}_{\mathrm{min}})$, the second order system of equations

Existence of the SNR wall: Proof of Theorem I (2)

$$\begin{cases} \alpha\sqrt{\frac{1}{N}+\xi_{\mathrm{asymp}}^2\,\phi} = \xi_{\mathrm{asymp}}-1 \\ \delta\sqrt{\frac{(1+\mathsf{SNR}_{\mathrm{min}})^2}{N}+\xi_{\mathrm{asymp}}^2\,\phi} = \xi_{\mathrm{asymp}}-(1+\mathsf{SNR}_{\mathrm{min}})\,. \end{cases}$$

If $\phi = o(1)$ for $N \to \infty$ (necessary and sufficient condition),

$$\begin{cases} \xi_{\mathrm{asymp}}^{(\infty)} = 1 \\ \mathsf{SNR}_{\mathrm{min}}^{(\infty)} = 0. \end{cases}$$

SNR wall does not occur.

If $\phi = \Theta(1)$ for $N \to \infty$,

$$\begin{cases} \xi_{\mathrm{asymp}}^{(\infty)} = \frac{1}{1-\alpha\sqrt{\phi}} \\ \mathsf{SNR}_{\mathrm{min}}^{(\infty)} = \frac{1-\delta\sqrt{\phi}}{1-\alpha\sqrt{\phi}} - 1. \end{cases}$$

SNR wall occurs.

Theorem II
Asymptotical performance of the design curves

Theorem II: Asymptotical behavior of the design curves

Theorem (Asymptotical behavior of the design curves when there is no SNR wall)

For the energy detection test defined, assuming that $\widehat{\sigma}^2$ is an unbiased asymptotically Gaussian estimator of the noise power σ^2, let us consider the asymptotical behavior (for $N \to \infty$) of SNR_{min}, when there is no SNR wall.
If $var\left(\widehat{\sigma}^2\right)$ is $\Theta(N^{-\gamma})$ for $N \to \infty$ with $0 < \gamma \leq 1$, the design curve in log-log scale, i.e. $SNR_{min}(dB)$ as a function of $\log_{10} N$, tends asymptotically to a straight line with a slope of $-5\,\gamma$ dB/decade.

In particular, if, for large N, $\phi \approx N^{-\gamma}/\lambda$ with $0<\gamma<1$ and $\lambda>0$, we get

$$\mathrm{SNR}_{\min}(dB) \approx -5\,\gamma \log_{10} N - 5 \log_{10} \lambda + 10 \log_{10} (\alpha - \delta) \qquad (1)$$

while if $\phi \approx 1/(\lambda N)$, we have

$$\mathrm{SNR}_{\min}(dB) \approx -5 \log_{10} N + 5 \log_{10} \left(\frac{1+\lambda}{\lambda} \right) + 10 \log_{10} (\alpha - \delta). \qquad (2)$$

Moreover, if $var\left(\widehat{\sigma}^2\right)$ is $o\left(N^{-1}\right)$ for $N \to \infty$, the design curve has a slope of -5 dB/decade and we have

$$\mathrm{SNR}_{\min}(dB) \approx -5 \log_{10} N + 10 \log_{10} (\alpha - \delta). \qquad (3)$$

In addiction, since (3) is also valid for the ideal ED, any practical implementation of the ED where the estimator of the noise power has $var\left(\widehat{\sigma}^2\right)$ decreasing faster than $1/N$ has asymptotically the same performance of the ideal ED.

Theorem II: Asymptotical behavior of the design curves

Hp: $\widehat{\sigma}^2$ unbiased asymptotically Gaussian estimator of σ^2.

Given $(P_{FA}^{\mathrm{DES}}, P_{D}^{\mathrm{DES}})$,

if $var(\widehat{\sigma}^2)$, is $\Theta\left(N^{-\gamma}\right)$ for $N \to \infty$ and $0 < \gamma \leq 1$

The design curve in log-log scale tends to a straight line with a slope of -5γ dB/decade

Theorem II: Asymptotical behavior of the design curves (2)

In particular, with $0 < \gamma < 1$ and $\lambda > 0$,

if $var\left(\widehat{\sigma}^2\right) \approx 1/(\lambda N^\gamma)$,

$$\text{SNR}_{\min}^{(\infty)}(dB) \approx -5\gamma \log_{10} N - 5 \log_{10} \lambda + 10 \log_{10} (\alpha - \delta)$$

if $var\left(\widehat{\sigma}^2\right) \approx 1/\left(\lambda N\right)$

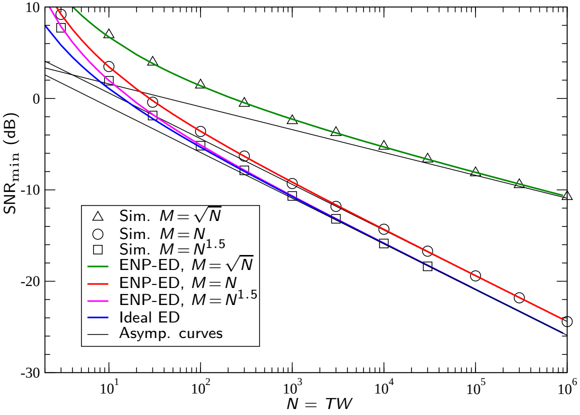
$$\text{SNR}_{\min}^{(\infty)}(dB) \approx -5 \log_{10} N - 5 \log_{10} \left(\frac{1+\lambda}{\lambda}\right) + 10 \log_{10} (\alpha - \delta)$$

if $var\left(\widehat{\sigma}^2\right)$ is $o\left(N^{-1}\right)$ for $N \to \infty$

$$\text{SNR}_{\min}^{(\infty)}(dB) \approx -5 \log_{10} N + 10 \log_{10} (\alpha - \delta)$$

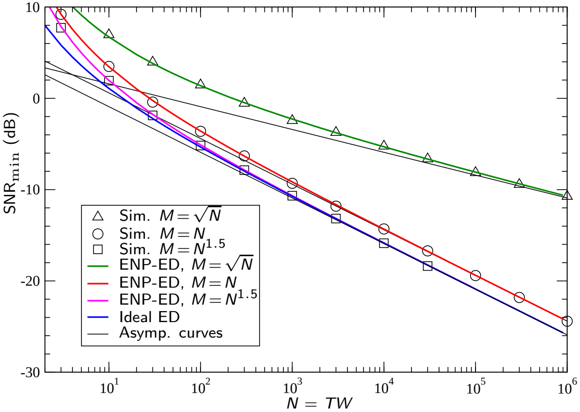
This is valid also for the ideal ED.

ENP-ED with ML estimator: asymp. design curve behavior



$var\left(\widehat{\sigma}^2\right)$ is $\Theta(1/N^{\gamma})$ for $N \to \infty \Rightarrow M = N^{\gamma}$

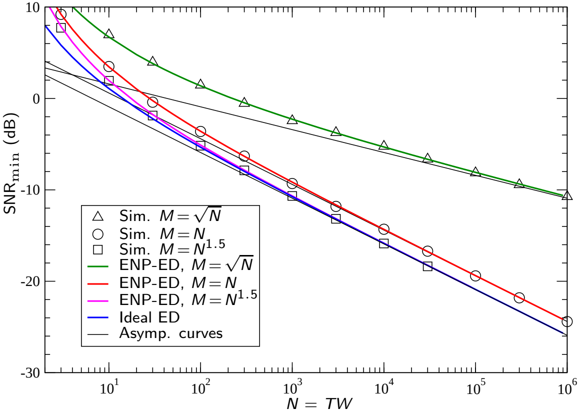
ENP-ED with ML estimator: asymp. design curve behavior



$var\left(\widehat{\sigma}^2\right)$ is $\Theta(1/N^\gamma)$ for $N \to \infty \Rightarrow M = N^\gamma \qquad 0 < \gamma \leq 1$

Slope $= -5\gamma$ dB/decade.

ENP-ED with ML estimator: asymp. design curve behavior



$var\left(\widehat{\sigma}^2\right)$ is $\Theta(1/N^{\gamma})$ for $N \to \infty \Rightarrow M = N^{\gamma}$ $\gamma > 1$

$$\mathrm{SNR}_{\min}^{(\infty)}(dB) \approx -5\log_{10} N + 10\log_{10}(\alpha - \delta)$$

ENP-ED with ML estimator performances

ENP-ED: design curve

ENP detector

$$\mathsf{SNR}_{\mathrm{min,\,ENP\text{-}ED}} = \frac{1/\mathrm{Inv}\widetilde{B}\left(M, N, P_{FA}^{\mathrm{DES}}\right) - 1}{1/\mathrm{Inv}\widetilde{B}\left(M, N, P_{D}^{\mathrm{DES}}\right) - 1} - 1$$

and $\mathrm{Inv}\widetilde{B}\left(\cdot,\cdot,\cdot\right)$ is an inverse beta regularized function[2] (if $w = \widetilde{B}\left(a, b, z\right)$, then $z = \mathrm{Inv}\widetilde{B}\left(a, b, w\right)$).

For the ML estimator we have

$$var\left(\widehat{\sigma}^2_{\mathrm{ML}}\right) = \frac{\sigma^4}{M}$$

From Theorem I, the SNR wall is avoided if M increases with N.

[2] R. W. Abernathy and R. P. Smith, "Applying series expansion to the inverse beta distribution to find percentiles of the F-distribution," *ACM Trans. Math. Software*, vol.19, pp. 474-480, 1993.

ENP-ED: P_{FA} and P_D

$\Lambda_{\mathrm{g}}(\mathbf{y})$ is the ratio of two chi-squared distributions, then it follows an *$\mathcal{F}$-distribution*

$$\begin{aligned} \mathcal{H}_0 : \quad & \Lambda_{\mathrm{g}}(\mathbf{y}) \sim \mathcal{F}_{2N,2M}(0) \\ \mathcal{H}_1 : \quad & \Lambda_{\mathrm{g}}(\mathbf{y}) \frac{\sigma_{\mathrm{t}}^2}{\sigma^2} \sim \mathcal{F}_{2N,2M}(0) \end{aligned}$$

Then we get

$$\begin{aligned} P_{FA} &= \widetilde{B}\left(M, N, \frac{M}{M+N\xi}\right) \\ P_D &= \widetilde{B}\left(M, N, \frac{M(1+SNR)}{M(1+SNR)+N\xi}\right) \end{aligned}$$

where the the *incomplete beta function* is given by
$\widetilde{B}\left(a,b,z\right) = \frac{1}{B(a,b)} \int_0^z x^{a-1}(1-x)^{b-1} dx$ or
$\widetilde{B}\left(a,b,z\right) = z^{a+b-1} \sum_{i=0}^{b-1} \binom{a+b-1}{i} \left(\frac{1-z}{z}\right)^i$.

ENP-ED: BWB comparison

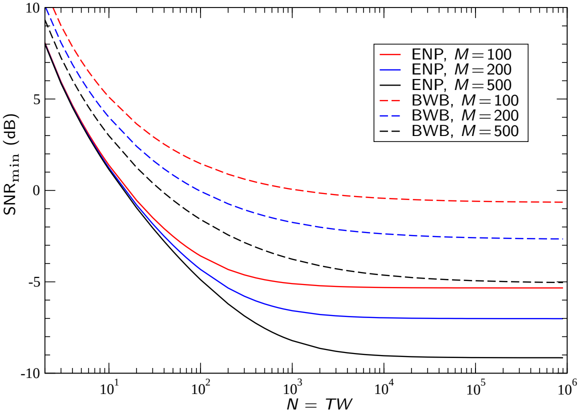


Figure: Design curve comparison between the ENP-ED with $M = 100, 200, 500$ and the corresponding BWB case, with $\sigma^2_{\min} = \sigma^2 - 3\sqrt{var\left(\widehat{\sigma}^2_{\mathrm{ML}}\right)}$ and $\sigma^2_{\max} = \sigma^2 + 3\sqrt{var\left(\widehat{\sigma}^2_{\mathrm{ML}}\right)}$. $P_{FA}^{\mathrm{DES}} = 1 - P_D^{\mathrm{DES}} = 10^{-1}$.