INCREASING RETURNS, INDUSTRIALIZATION, AND INDETERMINACY OF EQUILIBRIUM*

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This paper asks whether adjustment processes over real time help to "select" the long-run outcome in a model of industrialization, where multiple stationary states exist because of increasing returns in the manufacturing sector. "History" alone cannot in general determine where the economy will end up. Self-fulfilling expectations often make the escape from the state of preindustrialization (the takeoff) possible. The global bifurcation technique is used to determine when an underdevelopment trap exists and when a takeoff path exists. The role of government policy and agricultural productivity in industrialization are then considered.

I. INTRODUCTION

Recently there has been growing interest in the analysis of market economies in the presence of externalities. The traditional literature examined Marshallian external economies in the production process, while recent studies are also concerned with market size and the improved matching between potential buyers and sellers due to transaction externalities or the aggregate demand spillover. These studies show how there may exist multiple Pareto-ranked equilibria.[1] Facing the problem of equilibrium selection, numerous authors turn to historical factors. For example, Krugman and Obstfeld [1987, p. 130] discussed in the context of international trade, "In interpreting the real world implications of the (indeterminacy) result, however, the right way to think of it is to say that initial advantages can cumulate over time, so that history and accidental factors—which we do not capture with our

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1. International trade has a long tradition of the analysis of Marshallian externalities and multiple equilibria in the static framework; see Helpman [1984] for a survey. Lucas [1988] and Romer [1986] studied Marshallian externalities in the dynamic framework, but their main concern is endogenous growth, not multiple equilibria. See Mortensen [1988] for the recent literature on transaction externalities and Kiyotaki [1988] and Murphy, Shleifer, and Vishny [1989a] for aggregate demand spillover externalities.

simple model—can have a persistent effect on the pattern of international trade." Similarly, Blanchard and Summers [1988, p. 184] introduced the concept of "fragile equilibria," in the context of labor markets, to refer to situations "where outcomes are very sensitive to shocks and may be history dependent" and argued that, "Research on multiple equilibria and on hysteresis suggest mechanisms that may generate unemployment rates that depend sensitively on the shocks an economy has experienced." Some predict that the idea of history dictating the choice of equilibrium survives a formalization and that the explicit analysis of dynamic adjustment resolves the issue of multiplicity. Helpman [1984, p. 341] argued, for example, that "There remain, however, open questions which have to be answered before the relevance of this possibility can be evaluated. These have to do with dynamic adjustment processes which should help determine both autarky and trading equilibria."

Ethier [1982] and Panagariya [1986] recently introduced dynamics in two-sector models with externalities and showed how history, captured by initial allocations of factors, affects the long-run outcomes. They postulated the adjustment processes in which the reallocation of factors takes place at a rate determined by the difference between the *current* returns in the two sectors. This type of behavioral relation is in general inconsistent with perfect foresight and can best be understood as a tatonnement process à la Marshall. When one cannot move factors instantaneously, the owner's decision to commit his factors should be considered as an investment decision and thus be based on the present discounted values of *future* returns. The perfect foresight assumption would be more natural when analyzing adjustment processes over real time.[2]

Once the assumption of perfect foresight is made, however, it is not at all clear whether history matters in selecting a long-run position of the economy. It is conceivable that if everybody believes that the economy will end up in state 1, then it will; and that if everybody instead believes that it will end up in state 2, then it will.

2. Blanchard and Summers [1988] also seem to have the Marshallian tatonnement process in mind. In order to motivate the concept of fragile equilibria and the importance of history dependence (hysteresis), they use the analogy between an economy with multiple equilibria and a ball moving on a surface containing at least two valleys. The problem with this analogy is, of course, that a ball cannot anticipate, while economic agents can anticipate.

The possibility of self-fulfilling expectations cannot be ruled out.[3] In particular, the economy may be able to escape from a "bad" state only if expectations of agents are somehow coordinated. "History" alone may not be enough to dictate the long-run behavior of the economy with externalities. The question is then under what circumstances the initial condition, or history, matters and when expectations could play a role.

Addressing this question in a general setting is a difficult task. First of all, the nature of the problem considered requires one to solve the *global* perfect foresight dynamics. Knowing the local dynamics is not enough, because, for example, demonstrating the uniqueness of a perfect foresight path in a neighborhood of a stationary state does not necessarily rule out the existence of other perfect foresight paths in the large. One also needs to pay careful attention to the boundary conditions. Second, a model with multiple stationary states due to externalities generally needs to be highly *nonlinear.* It is well-known that the global analysis of nonlinear differential equations is still far from complete.

The goal of this paper is thus modest and limited. It addresses the above problem in the context of industrialization using a version of sectoral adjustment models developed in Matsuyama [1988a]. The economy has two sectors: agriculture and manufacturing. The manufacturing sector is subject to increasing returns, producing multiple stationary states. One stationary state, with zero employment in manufacturing, can be considered as the state of preindustrialization. This economy is inhabited by overlapping workers, and each worker's career decision (choice of sector) is irreversible. Sectoral labor movement takes place due to the demographic change. This model provides a convenient framework in which to address the above question. First, the career decision by agents based on perfect foresight is treated explicitly. Second, the dynamics of employment is described by the relatively simple nonlinear differential equations on a *plane,* for which some mathematical results are available.

The history versus expectations distinction seems of particular importance in the context of development. The diversity of per capita income levels across countries suggests the presence of some sort of multiplicity. The idea of history determining the long-run

3. Another way of stating this is that making a model dynamic also increases the dimensionality of the commodity space, and thus does not necessarily imply a tighter restriction on the possible outcomes.

position of the economy then implies that many countries may be in underdevelopment traps.[4] A corollary view would be the advocacy of active national development planning; the active state intervention may be called for in order to break a vicious cycle of poverty. On the other hand, if coordination failure of agents' expectations is the cause of the problem, the state's role in initiating and sustaining the development process should be limited to promoting the optimism and the entrepreneurial spirits in the private sector or to preaching "the Economics of Euphoria."

Mention should be made of the recent studies by Howitt and McAfee [1988] and Krugman [1991]. Howitt and McAfee consider the possibility of a locally indeterminate stationary state in the context of transaction externalities in the labor market. Their discussion is mainly limited to the local dynamics, although they anticipated the possibility of some of the global dynamics discovered below (see, in particular, their Figure III). Krugman [1991] is more related to the present analysis. He considers a model of sectoral adjustment similar to Mussa [1978]. His model is linear, and thus it does not possess multiple stationary states in the interior of the system. And some of his results crucially rest on the linearity of the model. Of course, these comments should not be viewed as a criticism of their studies. The present analysis immensely benefits from theirs.

In the first half of Section II, the static version of the model is developed, and it is shown how multiple equilibria arise in the presence of externalities. The second half makes the model dynamic in such a way that the stationary states in the dynamic economy coincide with the equilibria in the static economy. This exercise also makes it possible to describe industrialization as a continuous, self-sustaining process of structural transformation (or unbalanced growth), along which the economy traverses between two stationary states. Section III performs the global analysis. It first analyzes the case of zero rate of time preference. In this case, the dynamics can be described as a Hamiltonian system, whose global information is easily obtained. Then, by using a

4. By an underdevelopment trap, I mean a state of a lower level of industrialization from which the economy cannot escape under laissez-faire, which is different from the usage of Murphy, Schleifer, and Vishny [1989a]. They simply refer to a state of a lower level of industrialization, which coexists with a higher level one. In their essentially static framework, the possibility of takeoff from underdevelopment cannot be addressed.

perturbation method, it is shown that, if the rate of time preference is sufficiently close to zero, there exist generally multiple perfect foresight paths leading to different stationary states. History, as captured by the initial manufacturing employment, cannot necessarily select the long-run outcome. In particular, there is a case in which an industrialization path exists for the economy whose initial manufacturing employment is zero. A takeoff is possible in such a case. However, there are also situations where history determines the outcomes. For example, there is a case in which, if the initial employment in the manufacturing sector is below some threshold level, the equilibrium is unique, and the economy always converges to the zero level stationary state. The economy will be trapped into the state of preindustrialization. The global bifurcation technique, which is new in economics, is used to find the exact condition under which the takeoff path exists.

Section IV considers two applications. First, the role of government policy is discussed. When the zero level stationary state is a trap under laissez-faire, the government can make escape from this stationary state (a takeoff) possible, by subsidizing the production of the manufacturing good. The subsidy also eliminates the equilibrium path leading to the zero stationary state, and thus the possibility of deindustrialization when initial manufacturing employment is large. In the second application the effect of agricultural productivity in industrialization is considered. Contrary to the conventional wisdom, but with some supporting evidence, it is shown that the existence of a takeoff path is more likely when the economy's agriculture is less productive.

Section V speculates on the result for the case of a large rate of time preference by working through a piecewise linear version of the model. History seems to play a more important role with heavy discounting. Section VI provides some concluding remarks.

II. The Framework

Industrialization is a very complex, multifaceted process. Any attempt to formalize it inevitably forces one to highlight a particular aspect. One may wish to describe it as a process of capital accumulation, or adoption of a new technology. Alternatively, one could view it as the shift of resources from agriculture to manufacturing.[5] The latter approach is taken in this paper. For this purpose, I construct two sector models in this section; the static economy model in the first subsection and the dynamic economy model in the second.

A. The Static Economy

Consider a small open economy with two sectors: agriculture (A) and manufacturing (M). Each sector produces a homogeneous good, employing the labor service specific to the sector. Let X^i and L^i denote the output and the aggregate supply of labor (and employment) of sector i, measured in efficiency units. Agriculture operates under the constant returns to scale; $X^A = L^A$. Manufacturing is subject to economies of scale that are external to the firm but internal to the sector: $\chi^M = h(L^M)l^M$, where χ^M and l^M are the output and the employment of a firm.[6] The average (and marginal) productivity of labor is positively related to the size of the sector: $h'(L^M) > 0$.[7] The aggregate production function is given by $X^M = h(L^M)L^M$. Take the agricultural good as a numeraire, and let q denote the relative price of the manufacturing good, exogenously given in the world market. Then, perfect competition in the goods and labor markets ensures that $w^A = 1$ and $w^M = h(L^M)q$, where w^i is the wage rate in sector i, or

$$W = h(L^M)q, \tag{1}$$

where W is the relative wage in manufacturing.

The economy is populated by a continuum of agents, whose measure is normalized to one. An agent of type τ can provide $g^i(\tau)$ efficiency units of labor service inelastically if she works in sector i (and she cannot work in both sectors at the same time). The index

5. As documented by Clark [1940], Kuznets [1966], and Chenery and Syrquin [1975], the share of agriculture in a country's labor force and total output declines in both cross-section and time series as income per capita increase.

6. I doubt that external economies of scale of this type, usually attributed to Marshall [1920, Book IV, Chs. X, XI], are large enough to explain the huge diversity of economic performances across countries; other forms of externalities may be equally significant. And there are some conceptual problems about external economies; see Helpman and Krugman [1985, Ch. 2]. The results that the multiplicity of equilibria in the model is entirely due to the Marshallian externalities should not be taken literally. This formulation is adopted here because external economies of scale are a convenient way of making increasing returns consistent with perfect competition, and thus amenable to dynamic analysis. Implications of internal economies of scale in development are discussed in Murphy, Shleifer, and Vishny [1989a, 1989b].

7. For the sake of simplicity, we do not attempt to provide the sharpest results and instead assume that all functions are sufficiently "smooth."

of type τ is numbered so that $g^A(\tau)/g^M(\tau)$ is a strictly increasing, differentiable function of τ; an agent with high τ has comparative advantage in agriculture relative to an agent with low τ. Let $\Phi(\tau)$ be the distribution function of τ with $\Phi'(\tau) > 0$ on the support of Φ, $[\tau^-,\tau^+]$. Let T denote the inverse function of $g^A(\tau)/g^M(\tau)$. Clearly, $T' > 0$. Given the relative wage W, all agents whose types are greater than $T(W)$ work in agriculture, and all agents whose types are smaller than $T(W)$ work in manufacturing. That $T' > 0$ implies that, if $\tau^- < T(W) < \tau^+$, a higher relative wage in manufacturing attracts more agents to the sector. If $T(W) \geq \tau^+$, all agents work in manufacturing; and if $T(W) \leq \tau^-$, all agents work in agriculture. Thus, the labor supply schedule in agriculture is given by

$$(2)\quad L^A = Y(W) \equiv \begin{cases} \int_{\tau-}^{\tau+} g^A(\tau)\, d\Phi(\tau) & \text{for } T(W) \leq \tau^- \\ \int_{T(W)}^{\tau+} g^A(\tau)\, d\Phi(\tau) & \text{for } \tau^- \leq T(W) \leq \tau^+ \\ 0 & \text{for } T(W) \geq \tau^+. \end{cases}$$

Likewise, the labor supply in manufacturing is given by

$$(3)\quad L^M = Z(W) \equiv \begin{cases} 0 & \text{for } T(W) \leq \tau^- \\ \int_{\tau-}^{T(W)} g^M(\tau)\, d\Phi(\tau) & \text{for } \tau^- \leq T(W) \leq \tau^+ \\ \int_{\tau-}^{\tau+} g^M(\tau)\, d\Phi(\tau) & \text{for } T(W) \geq \tau^+. \end{cases}$$

Note that $Y'(W) = -g^A(T(W))\Phi'(T(W))T'(W) < 0$ and $Z'(W) = g^M(T(W))\Phi'(T(W))T'(W) = -Y'(W)/W > 0$, for $\tau^- < T(W) < \tau^+$.

An equilibrium of this economy must satisfy the labor market-clearing condition in manufacturing. Figure Ia shows how to find equilibria. The labor supply curve in manufacturing is given by (3), $Z(W) = L^M$, which is upward sloping if $\tau^- < T(W) < \tau^+$, and vertical otherwise. Labor supply increases with the wage rate in manufacturing. Labor demand in manufacturing is given by (1), $W = h(L^M)q$. This represents the wage rate that manufacturing firms can offer when the size of the sector is given by L^M. Because of increasing returns, $h' > 0$, it is upward sloping. Therefore, it is possible to have multiple intersections. Generically, there are an odd number of them. In Figure Ia there are three: S_0 (the zero level equilibrium), S_L (the low level one), and S_H (the high level one). Although the two curves need not intersect on the W-axis, this situation would be of particular interest since S_0 can be interpreted

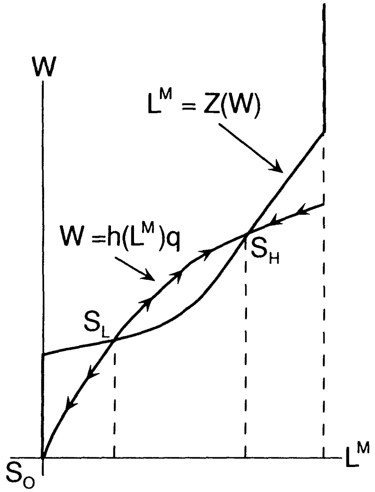

FIGURE IA
The Static Economy

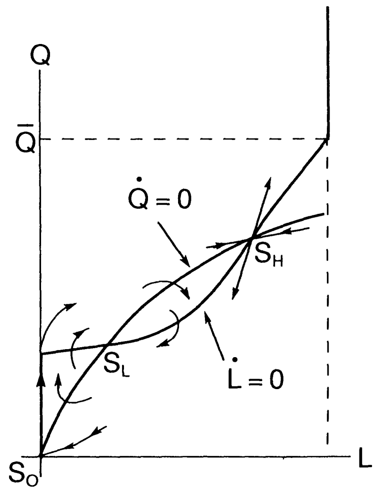

FIGURE IB
The Dynamic Economy

as the stage of preindustrialization. These three intersections are all equilibria, since the labor-market-clearing condition in agriculture is simply given by (2) and does not impose any additional restriction. (Labor demand in agriculture is perfectly elastic.)

Facing the problem of equilibrium selection, one often appeals to the following story, which is sometimes referred to as the Marshallian tatonnement process. In the short run the relative wage is determined by the demand condition. Suppose that initial employment in manufacturing is somewhere between S_L and S_H. Then, the relative wage in manufacturing is higher than the level required to keep the employment constant. In responding to a higher wage rate, more agents switch sectors, and this *gradually* increases the labor supply in manufacturing. This process continues until the economy converges to S_H. Similarly, if the initial labor supply is somewhere between S_0 and S_L, the economy converges to S_0, and if the economy is initially above S_H, it converges to S_H. Thus, S_H and S_0 are stable; S_L is unstable; and history can help us to select the equilibrium. In particular, when initial employment in the manufacturing sector is small, the sector will vanish, and the economy will specialize in agriculture. The economy will be trapped into the state of preindustrialization; the vicious cycle of poverty results. In order to take off and industrialize, some sorts of government intervention would be necessary. This story may be also used to illustrate some unequalizing process, or the doctrine of "uneven development." Imagine that there are two economies: one's initial employment is slightly below S_L; while the other's is slightly above S_L.[8] The small difference at the beginning would magnify over time, and eventually, the two economies follow completely different courses. History matters in selecting the long-run position of the economy.[9] In other words, "hysteresis exists."

The problem with this story is that it is not clear why sectoral adjustment of labor takes place gradually. What makes the economy incapable of, say, jumping from S_0 to S_H? It is often argued that there are some sorts of inertia or adjustment costs which make instantaneous movement of labor difficult. But, if it is hard to move across sectors, then the choice of a sector made by an agent becomes an investment decision, which should depend not only on the current relative wage but also on expected future relative wages as well. And these future wages depend on the decision of other agents because of externalities. Then, even when the economy starts to the left of S_L, or even near S_0, it may be possible to reach S_H, if every agent believed that the economy would industrialize and entered the manufacturing sector. Industrialization may occur, if expectations of agents are somehow coordinated. The self-fulfilling optimism could make a takeoff possible. Likewise, even when the economy starts to the right of S_L, it may be possible to reach S_0. Deindustrialization may occur due to self-fulfilling pessimism.

Of course, this is not to say that history does not matter. A little reflection suggests that, if every agent is myopic and discounts future returns completely, the economy would follow the course depicted by the adjustment process considered above. The question is under which circumstances history matters and when self-fulfilling expectations play a role. To address this question, it is necessary to state exactly the dynamic adjustment process over real time and then to analyze it explicitly, instead of appealing to a Marshallian story in the pure static model.

8. Implicit here is the assumptions that labor is immobile across regions and that economies of scale are internal to each region.

9. See Buttrick [1958] and Nelson [1956] for this kind of argument in the context of neoclassical growth model and Krugman [1981] in the context of trade and development.

B. The Dynamic Economy

This subsection makes the previous model dynamic by introducing adjustment process over real time. This exercise is also useful in making the model capable of describing industrialization as a continuous, self-sustaining process of structural transformation (or unbalanced growth), along which the economy traverses between two stationary states. It should be emphasized that, in a dynamic model such as the one developed below, an equilibrium is an entire path of the economy and, when the economy stays still, then it is in a stationary state.

The model is similar to Matsuyama [1988a]. Time is continuous and starts from zero (the initial period) and extends indefinitely into the future. The production structure is identical to the static economy. Although the size of the population is constant over time and equal to one, there are overlapping agents. Every agent throughout her lifetime faces a constant instantaneous probability of death p. The risk of death is independent, and there exists no aggregate uncertainty. The constant population implies that a new cohort whose size is equal to p is born each moment. Skill distribution within a cohort, and thus skill distribution of those alive, are as in the static economy. The relative price of the manufacturing good is constant over time and equal to q.

The labor allocation in this economy is sluggish because of the irreversibility of the career decision. At the beginning of her life, every agent needs to decide in which sector to work, and once the career decision is made, she will be stuck in that sector for the rest of her life. (The idea is that, when you are young, you decide either to stay in the rural area and become a peasant or to go to the urban area and be an industrial worker. Once you have acquired your life-style, it is very difficult—impossible in the model—to change it.) The assumption of complete irreversibility is a strong one, but adopted to simplify the model. The cost of making this assumption seems small, given that one of the goals here is to put the idea of hysteresis into logical scrutiny, and that the irreversibility assumption clearly favors history over expectations.

Endowed with perfect foresight, an agent chooses her career in order to maximize her human wealth. That is, an agent born at time t goes to agriculture if and only if

$$\int_t^{\infty} g^A(\tau) w_s^A e^{-r(s-t)}\, ds \geq \int_t^{\infty} g^M(\tau) w_s^M e^{-r(s-t)}\, ds,$$

where $r > 0$ is the (constant) discount rate, exogenously given in the world capital market. Let $\theta = r - p$, which can be considered as a rate of time preference.[10] Using $w_s^A = 1$, $w_s^M = h(L_s^M)q$, and the definition of function T, this condition can be rewritten as

$$\tau \geq T(Q_t), \qquad \text{where } Q_t \equiv r \int_t^{\infty} h(L_s^M)\, qe^{-r(s-t)}\, ds.$$

That is, Q_t is the annuity value of $\{h(L_s^M)q\}_{s=t}^{\infty}$, a sequence of the relative wage rate in manufacturing. Thus, an agent's decision depends not only on current wages, but also on future wages because of the irreversibility. The externalities imply that future wages depend on manufacturing employment in the future, which in turn depends on the decisions of other agents.

Using the definition of function Y, the aggregate labor supply (and employment) in agriculture thus changes as

$$\dot{L}_t^A = p[Y(Q_t) - L_t^A]. \tag{4a}$$

The first term represents the flows of entry workers measured in efficiency units, and the second term represents the flows of retiring workers (due to death). Note that it is multiplied by p, since p is both the size of the new cohort and the probability of death. Similarly, using the definition of function Z,

$$\dot{L}_t^M = p[Z(Q_t) - L_t^M]. \tag{4b}$$

Matsuyama [1988a] demonstrates that, for any given path of $\{Q_t\}_{t=0}^{\infty}$ and any initial conditions $(L_0^A, L_0^M) \in \Sigma \equiv \{(L^A, L^M) | 0 \leq L^A \leq Y(Q),\ 0 \leq L^M \leq Z(Q)$ for some Q satisfying $\tau^- \leq T(Q) \leq \tau^+\}$, a path of (L_t^A, L_t^M) stays inside Σ, and they are on the frontier of Σ if and only if the economy is in a stationary state. Therefore, an equilibrium of this economy for given initial values, $(L_0^A, L_0^M) \in \Sigma$, is a path satisfying (4a), (4b), and

$$Q_t = r \int_t^{\infty} h(L_s^M) qe^{-r(s-t)} ds \leq \overline{Q} \tag{5}$$

10. To justify this interpretation, consider the world economy in which agents alive as of time t maximize

$$E_t \int_t^{\infty} [C^A + v(C^M)]e^{\theta(t-s)}\, ds = \int_t^{\infty} [C^A + v(C^M)]e^{(p+\theta)(t-s)}\, ds,$$

where C^i is consumption of good i and $\theta > -p$ is the rate of time preference. Then, the equilibrium discount rate is given by $r = \theta + p$.

for all $t \geq 0$, where $\overline{Q}$ is the upper bound of Q_t and equal to $h[\int_{\tau-}^{\tau+} g^M(\tau)d\Phi(\tau)]q$.

Two considerations simplify the problem of finding equilibria. First, the dynamics of $\{L^M,Q\}$ are independent of L^A, so that equation (4a) can be ignored for the purpose of this paper.[11] Second, differentiating equation (5) with respect to time shows that $\dot{Q}_t = r[Q_t - h(L_t^M)q]$ *and* $0 \leq Q_t \leq \overline{Q}$ for all $t \geq 0$ are equivalent to (5). Therefore, equilibria conditions now become, for a given $L_0^M \in [0,\int_{\tau-}^{\tau+} g^M(\tau)\mathrm{d}\Phi(\tau)]$:

$$\dot{L}_t^M = p[Z(Q_t) - L_t^M], \tag{6a}$$

$$\dot{Q}_t = r[Q_t - h(L_t^M)q], \tag{6b}$$

and $Q_t \in [0,\overline{Q}]$ for all $t \geq 0$. Equations (6a) and (6b) jointly define a planar dynamical system in (L^M,Q) on $[0,\int_{\tau-}^{\tau+} g^M(\tau)\, d\Phi(\tau)] \times [0,\overline{Q}]$, but the initial value for Q must be chosen to make a path consistent with these equilibrium conditions. The number of Q_0 satisfying the conditions is equal to the number of equilibria. In this model the notion of "history" is captured by the labor supply in the manufacturing sector. In what follows, superscript M will be dropped from L^M to simplify the notation. It should be kept in mind that L represents the labor supply (and employment) in manufacturing measured in efficiency units.

See Figure Ib. From equation (6a) the $\dot{L} = 0$ locus is given by $Z(Q) = L$. It is upward sloping, if $\tau^- < T(Q) < \tau^+$, and vertical otherwise. Above this locus, $\dot{L} > 0$, and $\dot{L} < 0$ below it. From equation (6b) the $\dot{Q} = 0$ locus is given by $Q = h(L)q$, and $\dot{Q} > 0$ above the locus, and $\dot{Q} < 0$ below it. Note that these loci are identical to the labor demand and supply curves of the static economy. The stationary states of this dynamic economy coincide with the equilibria of the static economy.Thus, if the uniqueness of an equilibrium path for a given L_0^M is demonstrated, then one can say that explicit analysis of the adjustment process solves the multiplicity problem of the static economy model.

Using the well-known technique, it can be shown from (6a) and (6b) that S_H is a saddle point; S_L is a source if $r - p = \theta > 0$, and a sink if $r - p = \theta < 0$. One can also show that S_0 is a saddle point; its stable manifold approaches it from northeast, while its unstable manifold coincides with the Q-axis below the intersection of the

11. The equilibrium path of L^A is uniquely determined once the path of Q has been solved for from (4b) and (5); see Matsuyama [1988a].

Q-axis and the $\dot{L} = 0$ locus. Unfortunately, the information on the local dynamics is not enough to address our questions; that is, in which stationary state the economy will find itself in the long run and whether it depends on the initial value of L. In particular, if the initial employment in manufacturing is small, will the economy necessarily be trapped in S_0? To answer the questions, we need to analyze (6a) and (6b) globally, which is the subject of the next section.

III. Global Dynamics: The Case of a Small Rate of Time Preference

This section is more technical than the rest of the paper. Those who are more interested in the applications are advised to see just how to read the figures below (Figure III in particular) and skip the technical details on a first reading.

A. Some Important Results

PROPOSITION 1. Suppose that $r - p \equiv \theta \neq 0$. Then, no solution curve of the dynamical system, (6a) and (6b), is a Jordan curve.[12]

Proof of Proposition 1. Note that (6a) and (6b) imply that $r[Q - h(L)q]dL = p[Z(Q) - L]dQ$. Therefore, if a Jordan curve Γ solves (6a) and (6b),

$$\oint_{\Gamma} \{p[Z(Q) - L]\,dQ - r[Q - h(L)q]\,dL\} = 0,$$

where the integral sign is a line integral. From Green's Theorem,

$$\int\int_{\Omega} (r - p)\,dL\,dQ = \int\int_{\Omega} \theta\,dL\,dQ = 0,$$

where the integral is a surface integral and Ω is the region bounded by Γ, or $\partial\Omega = \Gamma$. This equality holds if and only if $r - p = \theta = 0$, which contradicts the assumption.

Q.E.D.

12. A Jordan curve is $(x(t), y(t)) \in R^2$ for $t \in [a,b]$ $(-\infty \leq a < b \leq \infty)$, where x and y are piecewise smooth functions on the real line satisfying $(x(a) = x(b), y(a) = y(b)$ and that $x(t_1) = x(t_2), y(t_1) = y(t_2)$ for some $t_1, t_2 \in (a,b)$ implies that $t_1 = t_2$. Intuitively, it is a closed curve that does not intersect itself.

PROPOSITION 2. Suppose that $r - p = \theta = 0$. Define the Hamiltonian by

$$H(L,Q) \equiv QL - \int_0^Q Z(s)\,ds - \int_0^L h(z)q\,dz.$$

Then all solutions of the planar dynamical system, (6a) and (6b), satisfy $H(L_t,Q_t) = $ constant.[13]

Proof of Proposition 2. Totally differentiating H yields $r(dH/dt) = r(H_L\dot{L} + H_Q\dot{Q}) = rH_L\dot{L} + pH_Q\dot{Q} = r[Q - h(L)q]\dot{L} + p[L - Z(Q)]\dot{Q} = \dot{Q}\cdot\dot{L} - \dot{L}\cdot\dot{Q} = 0$. Thus, H is constant along a solution path of (6a) and (6b).

Q.E.D.

Proposition 1 states that no perfect foresight path is a closed orbit or a *homoclonic orbit* or a *heteroclinic orbit.* Trajectories of a dynamical system are called homoclonic orbits if they connect a stationary point to itself and heteroclinic orbits if they connect distinct stationary points. This proposition is a simple application of Bendixson's theorem [Ye and others, 1986, p. 14]; Guckenheimer and Holmes, 1986, p. 44]. The essential idea is that the occurrence of closed trajectories requires *the divergence of the vector field,* which is equal to $r - p = \theta$ in the present model, to change its sign, or to be identically equal to zero.[14] Proposition 2 states that if the rate of time preference is zero, the dynamical system, (6a) and (6b), is a Hamiltonian system, and all solution curves can be found by solving the equations of the form $H(L_t,Q_t) =$ constant, or level curves of H. The Hamiltonian system has been analyzed thoroughly because of its central role in classical mechanics, in which the constancy of the Hamiltonian represents the Law of the Conservation of Energy. In a Hamiltonian system all stationary states are either saddle points or centers; no sinks or sources can exist. Many solution curves are Jordan curves. Thus, the two propositions jointly suggest that the properties of dynamics drastically change when θ changes its sign. The economy experi-

13. The converse is not true; all level curves of $H(L,Q)$ are not solutions of (6a) and (6b).

14. The divergence of the vector field, $\dot{x} = F(x,y)$, $\dot{y} = G(x,y)$, at (x,y) is $F_x(x,y) + G_y(x,y)$. It provides a measure of expansion of flows. Note that the divergence at a stationary point is equal to the trace of a linearized system associated with the stationary point. The crucial feature of the system (6a) and (6b) is that its divergence is constant in the entire region.

ences a global bifurcation at $\theta = 0$. The case of $\theta = 0$ is nongeneric and thus not very interesting in itself. However, it plays a very important role in the following analysis because the perfect foresight dynamics when θ is close to zero can be understood by perturbing the Hamiltonian system.

Remark 1. Bifurcations occur when dynamics are not structurally stable. A slight change in parameters alters the topological dynamics. The points of parameters at which it occurs are called bifurcation points. The type of bifurcations discussed here is *global* in nature since the global information of the system is required to analyze them. They contrast with *local* bifurcations whose analyses use only the local information. Saddle-node, transcritical, pitchfork, and Hopf are examples of local bifurcations [Guckenheimer and Holmes, 1986, Ch. 3]. The first three are concerned with changes of stationary states. Since loci $\dot{L} = 0$ and $\dot{Q} = 0$ and thus the stationary states are independent of r, p, and θ, their changes cannot produce these types of bifurcations. A saddle-node bifurcation occurs when parameters representing technology, and skill distribution change so as to shift the $\dot{Q} = 0$ locus down or to shift the $\dot{L} = 0$ locus up in Figure Ib, thereby eliminating the intersections. Hopf bifurcations are associated with the existence of closed orbits in the neighborhood of a stationary state, when a parameter is in a (one-side) neighborhood of the bifurcation point at which some roots of the stationary state are purely imaginary. One can conclude as a corollary of Proposition 1 that Hopf bifurcations cannot occur in the present model.

The next proposition is an immediate consequence of Proposition 2.

PROPOSITION 3. Suppose that $r - p = \theta = 0$, and let (L^*,Q^*) be a saddle point stationary state of (6a) and (6b).

(3.1) When the initial employment in the manufacturing sector is given by L_0, $H(L^*,Q^*) \notin H(L, [0,\overline{Q}])$ is a sufficient condition for the nonexistence of an equilibrium path converging to (L^*,Q^*). That $H(L^*,Q^*) \in H(L_0,[0,\overline{Q}])$ is necessary for the existence of an equilibrium path converging to (L^*,Q^*).

(3.2) Consider the case when the initial manufacturing employment is zero. If $H(L^*,Q^*) > 0$, there exists no equilibrium path converging to (L^*,Q^*). If there exists an equilibrium path converging to (L^*,Q^*), *and* $H(L^*,Q^*) < 0$, then it is unique, and the initial value for Q is given by $H(0,Q_0) = H(L^*,Q^*)$.

(3.3) Consider the case when initial manufacturing employment is zero, and (L^*,Q^*) is the unique saddle point stationary state of (6a) and (6b) with $L^* > 0$ (as is the case with S_H in Figure Ib). Then, there exists no equilibrium path converging to (L^*,Q^*) if $H(L^*,Q^*) > 0$. If $H(L^*,Q^*) \leq 0$, there exists an equilibrium path converging to (L^*,Q^*), and an initial value for Q is given by $H(0,Q_0) = H(L^*,Q^*)$. If $H(L^*,Q^*) < 0$, then it is unique.

Proof of Proposition 3. Part (3.1) is a consequence of Proposition 2 and the continuity of H. The first half of (3.2) follows from (3.1) and $H(0,Q) = -\int_0^Q Z(s)\,ds$, which is nonpositive since Z is a nonnegative function. To prove the second half, note that $H(0,Q) = -\int_0^Q Z(s)\,ds$ is strictly decreasing in Q, when $H(0,Q) < 0$, since Z is strictly positive in this range. Then, the uniqueness follows from $H(0,Q_0) = H(L^*,Q^*) < 0$. Part (3.3) can be proved by simply noting that the left stable manifold of (L^*,Q^*) is bounded by $Q = \overline{Q}$ from above and by the $\dot{L} = 0$ locus from below, and therefore, it must hit the Q-axis.

Q.E.D.

The perturbation method used below is rather intricate. In what follows, I try to present the essential ideas by focusing on the case where there exists the unique saddle point with positive employment, S_H, as depicted in Figure Ib. Proposition 3 suggests that, when $r - p = \theta = 0$, the qualitative nature of the dynamics crucially depends on whether $H(S_H) > 0$, called Case A below, or $H(S_H) < 0$, called Case B. The perturbation method below demonstrates that the distinction between these two cases remains important even when $r - p = \theta \neq 0$. Although the results obtained are general, I also work with the following example as a demonstration.

Example 1. Let $g^A(\tau) = \alpha k^2\tau + \beta k$, $g^M(\tau) = k$, $\Phi'(\tau) = 1$ on $[\tau^-,\tau^+] = [0,1]$, $q = 1$, and $h(L) = \{\alpha + \beta(\lambda + \lambda^{-1})\}L - \beta L^2$, where $k > \lambda > 1$, $k > \sqrt{3}$, and $\alpha > \beta(k - k^{-1})$.[15] Then, $T(Q) = (Q - \beta)/(\alpha k)$, and (6a) and (6b) become, by omitting the time subscripts,

15. The condition $k > \lambda > 1$ ensures two stationary states in the interior; $k > \sqrt{3}$ ensures the existence of Case II below; $\alpha > \beta(k - k^{-1})$ ensures that $h(L)$ is increasing in the entire range.

(7a) $$\dot{L} = p[Z(Q) - L],$$

(7b) $$\dot{Q} = r[Q - \{\alpha + \beta(\lambda + \lambda^{-1})\}L + \beta L^2],$$

with $0 \leq Q \leq \overline{Q} = h(k)$ and $0 \leq L \leq \int_{\tau-}^{\tau+} g^M(\tau)d\Phi(\tau) = k$, and $Z(Q)$ is given by $Z(Q) = 0$ for $Q \leq \beta$; $Z(Q) = (Q - \beta)/\alpha$ for $\beta \leq Q \leq \alpha k + \beta$; $Z(Q) = k$ for $\alpha k + \beta \leq Q$. Three stationary states are $S_H = (L,Q) = (\lambda, \alpha\lambda + \beta)$, $S_L = (\lambda^{-1}, \alpha\lambda^{-1} + \beta)$, and $S_0 = (0,0)$. The Hamiltonian is given by

(8) $$H(L,Q) = QL - \int_0^Q Z(s)\,ds - \{\alpha + \beta(\lambda + \lambda^{-1})\}\frac{L^2}{2} + \frac{\beta L^3}{3},$$

where $\int_0^Q Z(s)\,ds = 0$ for $Q \leq \beta$, $= (Q - \beta)^2/(2\alpha)$ for $\beta \leq Q \leq \alpha k + \beta$, $= k\{Q - \beta - (\alpha k)/2\}$ for $\alpha k + \beta \leq Q$. Some algebra shows that $H(S_H) = H(\lambda, \alpha\lambda + \beta) = \beta\lambda(3 - \lambda^2)/6$, $H(S_L) = H(\lambda^{-1}, \alpha\lambda^{-1} + \beta) = \beta\lambda^{-1}(3 - \lambda^{-2})/6 > 0$, and $H(S_0) = H(0,0) = 0$. Thus,

Case A:	$H(S_H) > H(S_0) = 0$	if $1 < \lambda < \sqrt{3}$,
The Borderline Case:	$H(S_H) = H(S_0) = 0$	if $\lambda = \sqrt{3}$,
Case B:	$H(S_H) < H(S_0) = 0$	if $\sqrt{3} < \lambda < k$.

Since $\lambda > 1$, $\lambda + \lambda^{-1}$ increases with λ. Thus, the degree of increasing returns is relatively weak (strong) in Case A (B). One should thus expect that it is hard to escape from the zero (high) level stationary state in Case A (B). This intuition will be confirmed below.

B. The Analysis of the Hamiltonian Dynamical System

Figures IIa–IIc show what the phase portrait of (6a) and (6b), when $r - p = \theta = 0$, looks like for each case. Loci $\dot{L} = 0$ and $\dot{Q} = 0$ is *not* drawn in these figures. Some equilibrium paths for some L_0 are depicted by heavily barbed curves. In addition, there is a continuous family of equilibrium paths in each shaded area. Any other solutions of (6a) and (6b) would violate the condition $Q \in [0,\overline{Q}]$. In each case, S_L is a nondegenerate critical point of H (that is, H attains its strict local maximum at S_L), and therefore, it is a center. There exists no path converging to it. The more detailed discussion on each case will follow below.

Case A: $H(S_H) > 0$. From Proposition 3, $H(S_H) > 0$ implies that there exists no perfect foresight path converging to S_H when the initial employment is zero. As shown in Figure IIa, there exists a perfect foresight homoclonic orbit leaving S_H and returning to it.

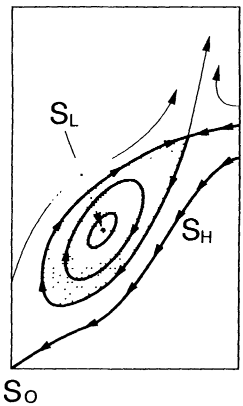

FIGURE IIA
The Hamiltonian Dynamical System ($\theta = 0$)—Case A: $H(S_H) > 0$

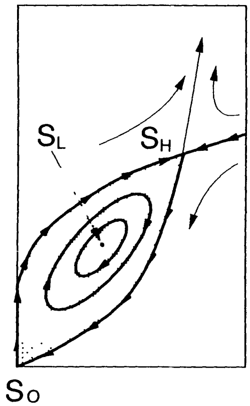

FIGURE IIC
The Hamiltonian Dynamical System ($\theta = 0$)—The Borderline Case C: $H(S_H) = 0$

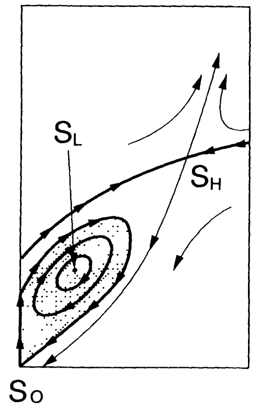

FIGURE IIB
The Hamiltonian Dynamical System ($\theta = 0$)—Case B: $H(S_H) < 0$

Along this homoclinic orbit, $H(L,Q) = H(S_H)$ holds. It may be considered as a cycle of infinite period. Inside this orbit there is a continuous family of (nesting) closed orbits as well as the stationary state S_L. The nearer a closed orbit to S_L (whose period length is zero), the shorter its period length is, and the nearer a closed orbit to the homoclinic orbit, the longer its period length is. Thus, there are cycles of any period! There are two other perfect foresight paths; one approaches S_H from the right, and the other leads to S_0. The zero level stationary state can be reached no matter where the economy starts. On the other hand, the high level stationary state can be reached only if initial manufacturing employment is sufficiently large. The threshold level of employment can be determined by the intersection of the homoclinic orbit and the $\dot{L} = 0$ locus. For example, 1 ($1 < \lambda < \sqrt{3}$), these conditions, $H(L,Q) = H(\lambda,\alpha\lambda + \beta)$ and $Z(Q) = (Q - \beta)/\alpha = L$, can be reduced, from (8), to $(L - \lambda)^2\{2\lambda L - (3 - \lambda^2)\} = 0$. Therefore, if

$$(3 - \lambda^2)/2\lambda \le L_0 \le k, \tag{9a}$$

both stationary states, S_0 and S_H, can be reached. Note that this threshold employment level is smaller than λ^{-1}, the employment level of S_L. Thus, unlike what the Marshallian adjustment process suggests, the economy can take off even when the economy is initially located to the left of S_L, if expectations of agents are coordinated on the equilibrium path converging to S_H. It should also be noted that starting from the right of S_L does not guarantee the convergence to S_H. No matter how high initial employment is, the economy may deindustrialize due to the self-fulfilling pessimism. If

$$\text{(9b)} \qquad 0 \leq L_0 < (3 - \lambda^2)/2\lambda,$$

the equilibrium is unique, and the economy converges to S_0. Thus, the economy will be trapped in the zero level stationary state. History helps to select the long-run position in this rather limited sense. The economy cannot take off under laissez-faire; active government intervention may be necessary to escape from the trap. (The role of government policy is considered in subsection IV.A.) This unique equilibrium path (the stable manifold of S_0) satisfies $H(L,Q) = H(S_0) = 0$. For Example 1 it can be shown from (8) that $Q = \{\alpha + \beta(\lambda + \lambda^{-1})\}L/2 - \beta L^2/3$ in the neighborhood of S_0.

Case B: $H(S_H) < 0$. See Figure IIb. From Proposition 3, $H(S_H) < 0$ implies that there exists a unique equilibrium path converging to S_H when initial employment is zero. Also, there is a homoclonic orbit leaving S_0 and returning to it. Along this orbit, $H(L,Q) = 0$. Again, one can find a cycle of any period inside the homoclonic orbit. As shown in Figure IIb, S_H can be reached no matter where the economy starts. On the other hand, S_0 cannot be reached when initial manufacturing employment is sufficiently large. The threshold level of employment can be determined by the intersection of the homoclinic orbit and the $\dot{L} = 0$ locus. For Example 1 ($\sqrt{3} < \lambda \leq k$), these conditions, $H(L,Q) = 0$ and $Z(Q) = (Q - \beta)/\alpha = L$, can be reduced to, from (8), $6L - 3(\lambda + \lambda^{-1})L^2 + 2L^3 = 0$. The threshold level is given by the middle root of this equation. Therefore, if

$$\text{(10a)} \qquad (3/4)[\lambda + \lambda^{-1} - \lambda^{-1}\{(\lambda^2 - 1/3)(\lambda^2 - 3)\}^{1/2}] < L_0 \leq k,$$

the equilibrium is unique, and the economy converges to S_H. In particular, if the economy is initially at S_H, there is no danger of deindustrialization. History determines the long-run position of the economy in this sense. If

$$\text{(10b)} \qquad 0 \leq L_0 \leq (3/4)[\lambda + \lambda^{-1} - \lambda^{-1}\{(\lambda^2 - 1/3)(\lambda^2 - 3)\}^{1/2}],$$

both S_0 and S_H can be reached. Note that this threshold employment level is larger than λ^{-1}, the employment level of S_L. Thus, unlike what the Marshallian adjustment process suggests, the economy may deindustrialize even when the economy is initially located to the right of S_L. It also implies that, even when the manufacturing employment is zero initially, there is an equilibrium path converging to S_H. The economy may industrialize due to the self-fulfilling optimism. From Proposition 3 the unique value of Q_0 that is consistent with convergence to S_H is given by $H(0,Q_0) = H(S_H)$, or, for Example 1, $Q_0 = \beta + [\alpha\beta\lambda(\lambda^2 - 3)/3]^{1/2}$. The economy can take off only if expectations of agents are coordinated. Active government intervention would not be necessary. What does matter is the confidence or the optimism among the private sector.

The Borderline Case: $H(S_H) = 0$. Before proceeding to the case of nonzero θ, brief mention should be made of the borderline case. See Figure IIc. In this case there is a pair of heteroclinic orbits. One of them is the unstable manifold of S_H *and* the stable manifold of S_0. The other is the stable manifold of S_H *and* the unstable manifold of S_0. Along the first the economy moves from S_H to S_0, and along the second it moves from S_0 to S_H. There is also a continuous family of closed orbits in the area bounded by the heteroclinic orbits. (Those familiar with mechanics would notice the similarity of Figure IIc with the phase diagram of the simple pendulum.) These paths are all perfect foresight paths. In addition, there is a perfect foresight path to S_H from the right. Thus, for Example 1 ($\lambda = \sqrt{3}$), the equilibrium is unique, and the economy converges to S_H if

$$\sqrt{3} \le L_0 \le k, \tag{11a}$$

and both S_H and S_0 can be reached if

$$0 \le L_0 < \sqrt{3}. \tag{11b}$$

(One may notice the discontinuities at $\lambda = \sqrt{3}$ by comparing the conditions from (9a) to (11b). This is due to the fact that the Hamiltonian dynamic system experiences a global bifurcation at $\lambda = \sqrt{3}$. A slight change in λ breaks heteroclinic orbits.)

C. Perturbations of the Hamiltonian Dynamical System

Once the cases with $\theta = 0$ are understood, the perfect foresight paths when θ is sufficiently close to zero can be analyzed by using the technique of perturbation; see, for example, Guckenheimer and Holmes [1986, Sections 4.5 and 4.6].

The Case of $r - p = \theta > 0$. The phase portraits of (6a) and (6b), when the rate of time preference is small and positive are shown in

Figures IIIa and IIIb.[16] The positive rate of time preference implies the positive divergence and thus the flows point outward compared with the case of the Hamiltonian system. Note that the change in θ causes a bifurcation by breaking the homoclinic orbit. Again, heavily barbed curves depict equilibrium paths for some L_0. As Proposition 1 suggests, there is no closed orbit or homoclonic orbit. The low level stationary state is a source, and thus it cannot be generally observed. There are perfect foresight paths leading to both the high and zero level stationary states. In Case A the economy will be trapped in the zero level stationary state if initial employment is small. Otherwise, both states can be reached. In Case B, when starting near the high level stationary state, the economy will approach it along the unique equilibrium path. Otherwise, both states can be reached. In particular, even when initial employment is small, the economy can escape from the zero level stationary state, if expectations are coordinated. Note that

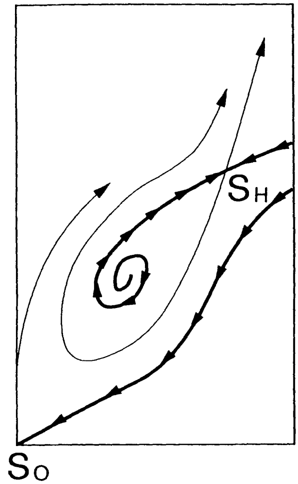

FIGURE IIIA
Perturbations of the Hamiltonian System ($\theta > 0$)—
Case A: $H(S_H) > 0$

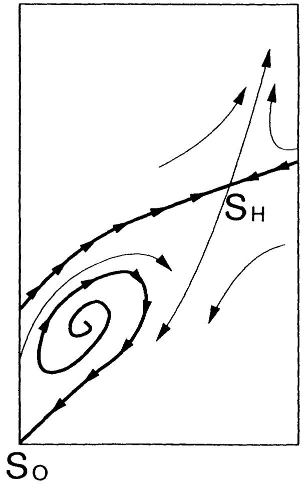

FIGURE IIIB
Perturbations of the Hamiltonian System ($\theta > 0$)—
Case B: $H(S_H) < 0$

16. See Chow and Hale [1982, p. 17] and Guckenheimer and Holmes [1986, p. 291] for similar phase diagrams. In fact, if one ignored the boundary condition and the regime switchings and let $Z(Q) = (Q - \beta)/\alpha$ everywhere, then it can be shown that their examples are topologically equivalent to (7a) and (7b).

despite the change in the topological property of the dynamics, the implication for history versus expectations remains unchanged. However, one can no more obtain analytical expressions for the threshold employment level or the initial value of Q consistent with the convergence to S_H, even if one specifies functional forms.

One of the important differences made by the bifurcation is that all perfect foresight paths are isolated if $\theta > 0$. Although the equilibrium is globally indeterminate in general, it is locally determinate (in the sense of Woodford [1984]). Thus, despite multiple equilibria, one can still perform a local comparative static exercise using this model. (This exercise is one of the subjects of Matsuyama [1988a].) The effect of an infinitesimal shock to the economy can be analyzed if one is willing to accept the somewhat ad hoc assumption that the economy jumps to the nearby perfect foresight path.[17]

The Case of $r - p = \theta < 0$. Perfect foresight paths are no longer isolated if the rate of time preference is negative ($-p < \theta < 0$). See Figures IVa and IVb. The divergence is negative and thus the flows point inward compared with the case of the Hamiltonian system. Again, the change in θ causes a bifurcation by breaking the homoclonic orbits. The negative divergence implies that the low level stationary state S_L becomes a sink, and thus there exists a continuous family of perfect foresight paths converging to it. It is locally indeterminate. (The possibility of a locally indeterminate stationary state is noted in the context of transaction externalities by Howitt and McAfee [1988].) Furthermore, because of the local indeterminancy, one may show that there are stationary sunspot equilibria in the neighborhood of S_L. With a negative rate of time preference, the indeterminacy of equilibria is more severe. In particular, one cannot hope to perform comparative static exercise in this case. However, the implication for history versus expectations remains the same. Initial employment alone matters only when it is sufficiently small in Case A or when it is sufficiently large in Case B. In Case A, S_0 is a trap. In Case B, it is not.

This ends the discussion of global dynamics in the case of a small rate of time preference, when there exists a unique saddle point stationary state with positive employment. It should be

17. Recall that we generally need this assumption even in a model in which the First Welfare Theorem holds, because it may also have isolated multiple equilibria; see Kehoe [1985].

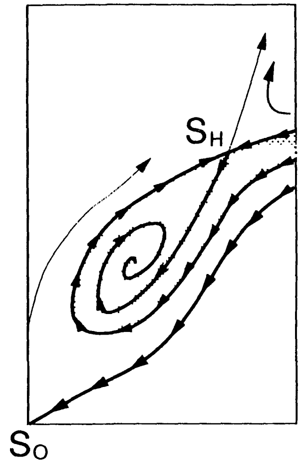

FIGURE IVA
Perturbations of the Hamiltonian System ($\theta < 0$)— Case A: $H(S_H) > 0$

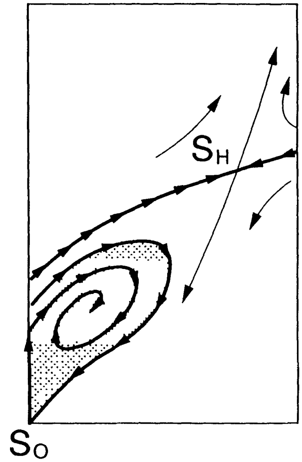

FIGURE IVB
Perturbations of the Hamiltonian System ($\theta < 0$)— Case B: $H(S_H) < 0$

emphasized that most of the results obtained above will carry over when there are multiple saddle points with positive employment. What is crucial is that, in the Hamiltonian system ($\theta = 0$), two "adjacent" or "consecutive" saddle points, if they have the same "energy" level, must be connected by heteroclinic orbits. Then a slight change in technology or skill distribution causes a bifurcation to generate the situations that resemble Figures IIa or IIb. Then, the perturbation method can be applied to analyze the cases with $\theta \neq 0$.

IV. APPLICATIONS

The previous section has shown how the bifurcation technique can be used to determine when a takeoff path exists, along which the economy moves from the state of preindustrialization to the high level stationary state. This section considers two applications of this technique.

A. The Role of Government Policy

The standard and traditional approach to the question of the government's role in the presence of externalities is to consider how the policy tools, particularly Pigovian taxes and subsidies, can be used to implement the optimal allocation as the equilibrium allocation by countervailing the existing distortions. This exercise is simple if the externalities are relatively weak so that the model has a unique equilibrium. In a model with multiple equilibria such as ours, the problem is more complicated. Of course, one can solve, at least in principle, the central planning problem of the economy, and efficient allocations may be unique. However, one cannot in general decentralize efficient allocations through simple linear tax and subsidy policies. This is because what one can best hope for by using these policy tools is to make the first-order conditions right. In a nonconvex economy one also has to take care of some global conditions in order to implement efficient allocations. Another way of stating this difficulty is that the standard Euler equation and the transversality condition are only necessary, but not sufficient, for the optimality when the value function is not concave.[18]

A model with multiple equilibria also poses a serious problem concerning the validity of policy analyses based on comparative statics methodology. Of course, one may be able to establish the local uniqueness of equilibrium, as in the case of $\theta > 0$. Then, all equilibria are isolated from each other, and small changes in policy parameters produce small and unique changes in each of these equilibria. Comparative statics exercises may be performed by limiting one's attention to the neighborhood of the original equilibrium. However, the validity of such a restriction would crucially depend on the purpose of the analysis.

An alternative way of addressing the role of government policy in a model with multiple equilibria is to see how the government can affect the set of equilibria. One may argue, in the spirit of the mechanism design literature, that a certain policy is desirable if, by affecting the set of equilibria, it could either create a "good" equilibrium or eliminate a "bad" one. In this section this approach will be adopted.

The case for government intervention in the process of industrialization would crucially depend on whether Case A or Case B prevails under laissez-faire. In Case B the problem when initial manufacturing employment is small is the possibility of coordination failure, and thus there may be an important role for

18. For the analysis of optimal control and regulation in dynamic nonconvex economies, see, for example, Skiba [1978], Dechert and Nishimura [1983], and Brock and Dechert [1985].

the government in promoting confidence or optimism within the private sector, but the role for active government intervention seems relatively small. Furthermore, there is no danger of returning to the zero-level or low-level stationary states, once manufacturing employment becomes sufficiently large. In Case A, however, the economy cannot take off under laissez-faire if initial employment is low. It is also possible to return to the zero-level stationary state even if initial employment is high. Therefore, the government could play a very important role in making the escape from the zero-level stationary state (industrialization) possible or in eliminating the possibility of returning to the zero-level stationary state (deindustrialization) if its intervention can make the dynamics of the economy look like one given in Case B, instead of one given in Case A.

Generally, this can be done simply by subsidizing the production of the manufacturing good. (In an open economy, a tariff on the manufacturing good has the same effect on labor allocation.) To see this, let $\nu - 1$ be the subsidy rate. Assume that the subsidy is financed by a lump sum taxation. The dynamics are then given by

$$\dot{L}_t = p[Z(Q_t) - L_t], \tag{12a}$$

$$\dot{Q}_t = r[Q_t - h(L_t)q\nu]. \tag{12b}$$

Consider the situation depicted in Figure Ib. The subsidy shifts the $\dot{Q} = 0$ locus upward. This slides S_H up and to the right along the $\dot{L}_t = 0$ locus, and S_L down and to the left, while S_0 stays still. From the Implicit Function Theorem, the values of L and Q at S_H can be expressed as a function of ν, $\{L(\nu),Q(\nu)\}$. Assume that $r - p = \theta = 0$. The Hamiltonian is given by

$$H(L,Q;\nu) \equiv QL - \int_0^Q Z(s)\,ds - \nu\Big[\int_0^L h(z)q dz\Big].$$

Note that the subsidy affects not only the stationary states, but also the Hamiltonian. Define $\Psi(\nu) = H(L(\nu),Q(\nu);\nu)$. By differentiating it,

$$\begin{aligned}\Psi'(\nu) &= H_L L'(\nu) + H_Q Q'(\nu) + H_\nu \\ &= H_\nu = -\int_0^{L(\tau)} h(z)q\,dz < 0,\end{aligned}$$

where use has been made of $H_L = H_Q = 0$ at a stationary state. Now, suppose that initial employment is equal to zero. From Proposition 3 an equilibrium path converging to $\{L(\nu),Q(\nu)\}$ exists if and only if $\Psi(\nu) \leq 0$. Thus, $\Psi'(\nu) < 0$ suggests that, if there exists $\nu^* > 1$ such that $\Psi(\nu^*) = 0$, then the state of preindustrialization is a trap under laissez-faire, and a sufficiently high subsidy rate $\nu - 1 \geq \nu^* - 1$ can make industrialization possible. This critical level of the subsidy rate can be determined once the loci of $\dot{Q} = 0$ and $\dot{L} = 0$ or their underlying functions are specified. One can also show that the same critical level of the subsidy rate can be applied for the elimination of the deindustrialization equilibrium when the economy starts at S_H. (This is because the bifurcation occurs at $\nu = \nu^*$ by breaking the heteroclinic orbits connecting S_H and S_0.)

The subsidy policy has a similar effect even for the case of $r \neq p$. This can be shown by using the perturbation method, although, with $r \neq p$, the critical level of the subsidy rate necessary to make industrialization possible when the economy starts at S_0 is different from what is necessary to eliminate the possibility of deindustrialization when the economy starts at S_H.

More generally, subsidizing the manufacturing good can be useful both in creating an industrialization equilibrium path when initial manufacturing employment is small and in eliminating a deindustrialization equilibrium path when initial manufacturing employment is large. It should be stressed, however, that this implication should be interpreted with caution. First, the above discussion focuses only on second-best policies that may bring about a higher level of industrialization. Second, the subsidy only makes industrialization possible; it does not guarantee it. To initiate a self-sustaining process of industrialization, voluntary coordinated responses of the private sector to opportunities are necessary. If excessive government intervention chokes off initiatives among private agents, then a takeoff may fail to materialize.[19]

B. Agriculture and Industrialization

The previous technique can be also used to analyze the effect of agricultural productivity on the possibility of industrialization. Suppose that technology in agriculture is now given by $\nu X^A = L^A$, where ν is the unit labor requirement. A low ν implies highly productive agriculture. Then, it is easy to see that the dynamics of the economy are described by equations (12a) and (12b). Thus, from the same argument in subsection IV.A, one can conclude that Case A prevails with low ν, and Case B prevails with high ν. That is, a takeoff is possible in an economy with less productive agriculture,

19. This point is the main difference between my argument and Graham's [1923] argument despite their apparent similarity.

while an economy with productive agriculture will be trapped into the state of preindustrialization.[20] This result, once stated, is quite intuitive. A low productivity in agriculture implies an abundant supply of "cheap labor" that the manufacturing sector can rely on. This is essentially the Principle of Comparative Advantage, enhanced by the presence of external economies of scale in manufacturing, which is responsible for the sudden creation of a takeoff path.

This result seems roughly consistent with recent experiences of successful industrialization in some countries, such as Hong Kong, Singapore, and South Korea, and much less satisfactory performances in India, Indonesia, and Thailand.[21] It explains why Belgium was the first to become the leading industrial country in continental Europe, while the Netherlands lagged behind and did not take off until the last decades of the nineteenth century.[22] It also explains why New England became the manufacturing center of the United States during the antebellum period.[23]

On the other hand, this result is in striking contrast to the conventional wisdom in the development literature, which asserted that "[e]veryone knows that the spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it [Nurkse, 1953, p. 52]," and that "revolutionary changes in agricultural productivity are an essential condition for successful takeoff [Rostow, 1960, p. 8]."

According to this conventional view, which partly comes from the experiences of the Industrial Revolution in England, there are positive links between agricultural productivity and industrialization. First, rising productivity in food production makes it possible to feed the growing population in the industrial sector. With more

20. I would like to acknowledge that this implication of the model was first pointed out to me by Professor Uzawa.

21. One might think that this evidence is inconsistent with the model because larger economies are more likely to industrialize in the presence of increasing returns. This is not necessarily the case; if external economies arise due to some local informational exchanges or the specialized infrastructure, what matters is the density, not the absolute size. Increasing returns come from agglomeration and geographic concentration of activities. The model here has no implication about industrialization and the size of the economy.

22. See Mokyr [1976] for a comparative study of industrialization in Belgium and the Netherlands.

23. Protection from British imports provided by the Embargo of 1807, the subsequent war, the tariffs of 1816, 1824, and 1828 were probably important for industrialization in the United States, which is consistent with the result in subsection IV.A. However, protection does not explain why industrialization, mostly in the cotton textile industry, started in New England, not in the South. See Field [1978] and Wright [1979].

food being produced with less labor, it releases labor for manufacturing employment. Second, high incomes generated in agriculture provide domestic demand for industrial products. Third, it increases the supply of domestic savings required to finance industrialization.

It should be noted that the logic behind the conventional view crucially rests on the implicit assumption that the economy under consideration is effectively a closed system. This assumption, which may be appropriate for England during the half century of the Seven Year War, the American Revolution, the French Revolution, and the Napoleonic Wars, should not be taken for granted when addressing the problems of underdeveloped countries today. Many economies that have successfully industrialized have heavily relied on foreign trade through importing agricultural products and raw materials and exporting manufacturing products. The result here should be at least taken as a caution when applying the lessons of early industrialization to the current problem of economic development.[24]

V. Global Dynamics: The Case of a Large Rate of Time Preference

In the previous sections I restricted the analysis to the cases of a small θ. The perturbation method can be useful only when the dynamics are close to the Hamiltonian system. This has clearly favored self-fulfilling expectations and biased against history. For a sufficiently large θ, history plays a decisive role in selecting the stationary state. This can be seen by letting θ be infinitely large. Then, (6b) becomes simply $Q_t = h(L_t^M)q$. In this case the dynamics are identical to those of the tatonnement process discussed at the end of subsection II.A, and so for any initial condition there exists a unique perfect foresight equilibrium. History alone can choose the long-run outcome, and there is no role for self-fulfilling expectations.

The question then is how large the rate of time preference should be for this to be the case and how this condition depends on the parameter representing the increasing returns. Unfortunately, solving (6a) and (6b) for a high θ is beyond our capability. In this

24. The results of Murphy, Shleifer, and Vishny [1989a, 1989b] crucially depend on the assumption that world trade is costly. They are careful enough to stress the importance of this assumption at length.

section we retreat to a (piecewise) linear model to speculate on the effect of a large rate of time preference. Assuming that linearity is not without cost. However, I believe that the following is illuminating enough.

Example 2. Let $g^A(\tau)$, $g^M(\tau)$, $\Phi(\tau)$, q be as in Example 1, and $h(L) = (\alpha + \beta\delta^{-1})L$, where $0 < \delta < k$. A low δ represents strong externalities. As before, $Z(Q) = 0$ if $0 \leq Q \leq \beta$; $= (Q - \beta)/\alpha$ if $\beta \leq Q \leq \alpha k + \beta$; $= k$ if $\alpha k + \beta \leq Q$. The three stationary states are $S_H = (L,Q) = (k, (\alpha + \beta\delta^{-1})k)$, $S_L = (\delta, \alpha\delta + \beta)$, and $S_0 = (0,0)$. The dynamics are given by

$$\text{(13a)} \quad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & 0 \\ -r(\alpha + \beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L \\ Q \end{bmatrix} \quad \text{for } 0 \leq Q \leq \beta,$$

$$\text{(13b)} \quad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & p/\alpha \\ -r(\alpha + \beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L - \delta \\ Q - \alpha\delta - \beta \end{bmatrix} \quad \text{for } \beta \leq Q \leq \alpha k + \beta,$$

$$\text{(13c)} \quad \begin{bmatrix} \dot{L} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -p & 0 \\ -r(\alpha + \beta\delta^{-1}) & r \end{bmatrix} \begin{bmatrix} L - k \\ Q - (\alpha + \beta\delta^{-1})k \end{bmatrix} \quad \text{for } \alpha k + \beta \leq Q.$$

Both S_0 and S_H are saddle points, and the slopes of the stable manifolds of these stationary points are equal to $r(\alpha + \beta\delta^{-1})/(r + p)$— thus less than the slope of the $\dot{Q} = 0$ locus. The matrix given in (13b) has two distinct, real eigenvalues if $\alpha\delta(r - p)^2 > 4\beta rp$ and a pair of imaginary eigenvalues if $\alpha\delta(r - p)^2 < 4\beta rp$. Using this information, one can show that there are three generic cases. Figure Va depicts the case where perfect foresight paths consist of a pair of intertwining, noncrossing spirals around S_L. This occurs when system (13b) has a pair of imaginary roots, or

$$\text{(14a)} \quad \alpha\delta\theta < 2p[\beta + (\beta^2 + \alpha\beta\delta)^{1/2}].$$

When $\alpha\delta\theta > 2p[\beta + (\beta^2 + \alpha\beta\delta)^{1/2}]$, there are two possibilities. Figure Vb depicts the case where perfect foresight dynamics are given by an S-shaped curve. In this case there are three initial values for Q consistent with equilibrium conditions if the initial employment is sufficiently close to δ. Figure Vc depicts the case where there exists a unique perfect foresight path, no matter where the economy starts. In this case history plays a decisive role

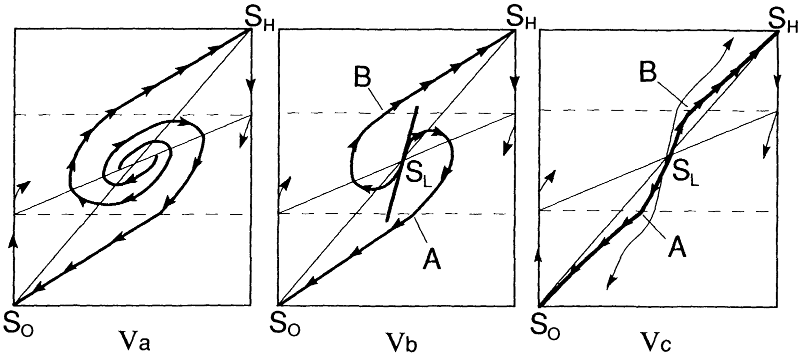

FIGURE V
The case of a Large Rate of Time Preference

in determining the long-run position of the economy. The condition that delineates these two cases can be found as follows. Because $\alpha\delta\theta > 2p[\beta + (\beta^2 + \alpha\beta\delta)^{1/2}]$, system (13b) has two real, positive, distinct roots. Let μ denote the larger eigenvalue, and let (x,y) be an eigenvector associated with μ. They satisfy $y/x = \alpha(\mu + p)/p$. The straight line given by $(\delta + x, \alpha\delta + \beta + y)$, which radiates from S_L and whose slope is $\alpha(\mu + p)/p$, is a solution path of (13b). The case depicted by Figure Vb is obtained if this straight line crosses the horizontal line $Q = \beta$ to the left of point A, the intersection of the stable manifold leading to S_0 and line $Q = \beta$, and if it crosses the horizontal line $Q = \alpha k + \beta$ to the right of point B, the intersection of the stable manifold leading to S_H and line $Q = \alpha k + \beta$. On the other hand, the case depicted by Figure Vc is obtained if it crosses line $Q = \beta$ to the right of or at point A and if it crosses line $Q = \alpha k + \beta$ to the left of or at point B. Some algebra shows that the line connecting points A and B passes through S_L and that its slope is equal to $\alpha\{1 - \beta(r + p)/(\alpha\delta + \beta)r\}^{-1}$. If it is negative or greater than $\alpha(\mu + p)/p$, then the case of Figure Vb occurs. This condition is rewritten as, after some algebra,

$$\text{(14b)} \qquad \beta(r + p)/r(\alpha\delta + \beta) > \mu/(\mu + p).$$

Likewise, the case of Figure Vc occurs if

$$\text{(14c)} \qquad \beta(r + p)/r(\alpha\delta + \beta) < \mu/(\mu + p).$$

Since $\mu = (1/2)[(r - p) + \{(r-p)^2 - 4rp\beta/(\alpha\delta)\}^{1/2}]$ and $r - p = \theta$, the left-hand side approaches $\beta/(\alpha\delta + \beta)$, and the right-hand side approaches one, as θ goes to infinity. Therefore, if the rate of time preference is sufficiently high, (14c) holds, and as shown in Figure

Vc, there is a unique perfect foresight path for any initial employment. If the economy starts to the right of S_L, the economy will always reach S_H; while if it starts to the left of S_L, then S_0 will be reached. History plays a decisive role in determining the long-run position of the economy. Hysteresis exists. The intuition should be clear. If the future is heavily discounted, agents will not care much about the future actions of other agents, and this will eliminate the power of self-fulfilling expectations.

One can also see that, given any positive rate of time preference, a sufficiently large δ, or small increasing returns eliminate the power of self-fulfilling expectations. (As δ goes to infinity, the left-hand side of (14c) approaches zero, while the right-hand side approaches to $\theta/(\theta + p)$.) This is because if externalities are small there will not be enough interdependence among decisions. The similar results are demonstrated in Krugman's [1991] linear model. However, as seen in Case A in the previous sections, the result that small increasing returns make history more important seem to rest crucially on the linearity of the model.[25]

VI. Concluding Remarks

The problems of poverty and stagnation among underdeveloped countries have generated and continue to generate keen interest and concern among economists. "Once one starts to think about them, it is hard to think about anything else," as Lucas [1988, p. 5] remarked. To many development economists, with the notable exception of Bauer [1971], the case for active state development planning has been almost axiomatic. The present analysis suggests that a history of industrial stagnation does not necessarily preclude the possibility of industrialization under laisser-faire; optimism and entrepreneurial spirit among the private sector may be more important in initiating and sustaining the development process. At the same time, however, the paper has shown that there are cases where underdevelopment traps exist and how government intervention could play an important role in such cases, although a takeoff may fail to materialize if excessive government intervention has a side effect of choking off private initiatives. The effect of agricultural productivity on industrializa-

25. Krugman argues that, in his linear model, whenever the roots around the middle stationary state are real, history alone determines the long-run outcome. As shown in Figure Vb, this criterion cannot be generalized even to a piecewise linear model.

tion has also been considered. Contrary to the conventional wisdom, but with some supporting evidence, an economy with less productive agriculture is more likely to take off. Needless to say, the model used is very simple and ignores many important aspects of the actual development process, such as capital accumulation, technology transfers, education, nutrition, negative externalities associated with urbanization, etc. It is hoped that the present analysis will turn out to be a useful guide for future study on more realistic models.

The problem of development is by no means the only issue in which the history versus expectations distinction is important. Many have pointed out the history dependence in the technology choice. For example, David [1985] emphasized the role of historical accidents when explaining how the economy may be locked into a bad technology. He pointed out that history dependence can be significant in the presence of three factors: technical interrelatedness, scale economies, and irreversibilities. Note that the dynamic economy discussed above includes all these elements in it. David [1985, p. 335] argued, "[i]ntuition suggests that if choices were made in a forward-looking way, rather than myopically . . . , the final outcome could be influenced by expectations." It is to be hoped that the present analysis will stimulate further research interest in the issue of history versus expectations.

To some economists a model with multiple equilibria may be unsettling in that the complete specification of the fundamentals cannot predict the unique outcome and that one need to rely on some extrinsic factors such as expectations.[26] In particular, it poses a serious problem concerning the validity of comparative statics. Of course, this does not justify making an assumption, such as weak externalities, in order to rule out multiple equilibria. The mere fact that certain parameter values ensure the uniqueness of equilibrium does not mean that they are more realistic than those implying multiplicity. Nor should one conclude that a model with multiple equilibria cannot yield useful predictions. First, the fact that the multiplicity results in certain cases and not in others itself allows one to make useful predictions. Second, one may argue, in the spirit of the mechanism design literature, that a certain policy is desirable if it could eliminate a "bad" equilibrium or generate

26. To some economists the multiplicity of equilibria is actually a virtue of the model in that it may create some room for various anthropological, cultural, psychological, or sociological factors, such as animal spirits, confidence, the Protestant Ethic, etc.

"good" one by affecting the set of equilibria. I hope that the two applications discussed in Section IV have shown the usefulness of this approach.

Finally, from the technical point of view, the analysis here may have demonstrated the importance of global analysis in nonlinear models. By restricting one's attention to the local dynamics, one often fails to notice the existence of many equilibria, some of which may have very different properties from those discovered by the local analysis. For example, the recent literature on the dynamic models of monetary economies demonstrated, in addition to the unique steady state equilibria, the existence of equilibria that exhibit endogenous, persistent fluctuations of the price level around the steady state; see, for example, Grandmont [1985], Woodford [1988], and Matsuyama [1988b]. In the present analysis the global analysis discovered equilibrium paths along which the economy traverses between two stationary states. The existence of equilibria of this kind seems universal in a perfect foresight model with multiple stationary states. Thus, the technique used in this paper may be quite useful in analyzing this class of models.

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