Experimental Validation of a Globally Stabilizing Feedback Controller for a Quadrotor Aircraft with Wind Disturbance Rejection

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Abstract—This paper addresses the design and experimental evaluation of a nonlinearly stabilizing feedback controller to steer a quadrotor vehicle along a predefined path. The proposed solution consists of a nonlinear adaptive state feedback controller for thrust and angular velocity actuation that i) guarantees convergence of the closed-loop path following error to zero in the presence of constant force disturbances and ii) ensures that the actuation does not grow unbounded as a function of the position error. A prototyping and testing architecture, developed to streamline the implementation and the tuning of the controller, is also described. Experimental results, where the quadrotor is subject to external wind disturbances, are presented to demonstrate the performance and robustness of the proposed controller.

I. INTRODUCTION

Motion control of underactuated vehicles is an active topic of research, which raises new and challenging problems when measured against motion control of its fully actuated counterpart. The quadrotor, in particular, is a typical example of an underactuated vehicle ideally suited for the development and test of new control strategies due to its simplicity and maneuverability. In recent years, several approaches to the problem of helicopter and quadrotor motion control have been proposed, ranging from PID control [1], [2] to nonlinear methods such as feedback linearization [3], [4], dynamic inversion [5], high gain and nested saturation control [6], and backstepping [7], [8], [9].

Robustness to external disturbances is crucial for aerial vehicles as they are subject to wind and unknown payload distributions, that make the vehicles deviate from their nominal model. Several approaches have been proposed for aerial vehicles to deal with disturbances. The control approach proposed in [10] is based on linearization and piecewise affine approximations and the controlled output is attitude and not position. In the experimental setting, the wind disturbance is generated by a set of electrical fans, whose airflow is passed through a pipe-system, rendering the flow affecting the quadrotor laminar and less turbulent. The works [11] and [12] propose controllers for attitude robust to external disturbances. In both cases experimental results are presented but the considered disturbances include only unknown parameters or payloads. Trajectory tracking controllers that render aerial vehicles robust to external disturbances are considered in [9], [13], and [14], but these works present only simulation results.

Backstepping [15] is a well known technique extensively used for control of nonlinear systems that can be applied for quadrotor control. The backstepping procedure can be complemented by other methodologies to obtain additional characteristics for the control law. The use of integral control to achieve zero steady-state error or equivalently rejection of constant disturbances in a closed-loop regulation system is standard in control literature and can be combined with the backstepping technique as discussed in [16]. The control methodology known as adaptive backstepping was first proposed in [15] to address control problems in the presence of unknown parameters. The procedure complements standard backstepping with adaptive controllers to estimate the unknown parameters and, for particular choices of parameters, achieves the disturbance rejection effect of integral control.

The main contribution of this work is the design and experimental validation of a global control law for path following that is not subject to restrictions on the thrust magnitude or the desired thrust direction. The proposed controller is based on the one presented in [17], modified to be applicable to vehicles controlled in angular velocity, and augmented by bounding of the control action with respect to the position error and adding robustness to external constant disturbances, by means of integral action.

II. NOTATION

We use the prime $f'(x)$ to denote the partial derivative of the function $f$ with respect to $x$, $f'(x) = \frac{\partial f}{\partial x}(x)$, and the upper dot $\dot{f}(x(t)) = f'(x(t))\dot{x}(t)$ to denote the total time derivative of the function. Vectors are represented by boldface characters and $e_1$, $e_2$, and $e_3$ denote the unit vectors co-directional with the $x$, $y$, and $z$ axes, respectively. When designing an estimator for the unknown quantity $x$, we use $\hat{x}$ to denote the estimate and $\dot{x} = x - \hat{x}$ to denote the estimation error. A function $\sigma(s) : \mathbb{R} \rightarrow \mathbb{R}$ is a saturation function if it is differentiable and verifies, for positive $M$ and $\sigma_{\text{max}}$, $0 < \sigma'(s) < M$, for all $s$, $\sigma(s) = -\sigma(s)$, for all $s$, $\sigma(s) > 0$, for all $s \neq 0$, $\sigma(0) = 0$, $\lim_{s \rightarrow \pm \infty} \sigma(s) = \pm \sigma_{\text{max}}$. Examples of smooth saturation functions are $\sigma_1(s) = s\sqrt{1 + s^2}$ and $\sigma_2(s) = \arctan(s)$. The map $S(.) : \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}$ yields a skew symmetric matrix that verifies $S(x)y = x \times y$, for $x$ and $y \in \mathbb{R}^3$. 

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III. QUADROTOR MODEL

The quadrotor vehicle is modeled as a rigid body, that can generate a thrust force along the body $z$ axis and is attitude controlled through angular velocity. Consider a fixed inertial frame $\{I\}$ and a frame $\{B\}$ attached to the vehicle’s center of mass. The configuration of the body frame $\{B\}$ with respect to $\{I\}$ can be viewed as an element of the Special Euclidean group, $(R,p) = (\frac{1}{2}R, \frac{1}{2}p_b) \in \SE(3)$, where $p \in \mathbb{R}^3$ is the position and $R \in \SO(3)$ the rotation matrix. The kinematic and dynamic equations of motion for the rigid body can be written as

$$\dot{p} = R\dot{v},$$
$$\dot{v} = -S(\omega)v + \frac{1}{m}f + R^Tb,$$
$$\dot{R} = RS(\omega),$$

where the angular velocity $\omega \in \mathbb{R}^3$, linear velocity $v \in \mathbb{R}^3$, and the external force $f$ are expressed in the body frame $\{B\}$. The unknown external disturbance $b \in \mathbb{R}^3$ is constant and expressed in the inertial frame $\{I\}$ and the scalar $m$ represents the quadrotor’s mass. The force disturbance can model exogenous inputs, such as constant wind, and also model uncertainties, such as variations in the vehicle mass or an unknown gain in the thrust input.

$$F_1 \quad F_2 \quad F_3 \quad F_4$$

A sketch of the quadrotor platform is presented in Figure 1, together with illustrations of reference frames, the force generated by each motor $F_i$ and the direction of rotation for each propeller. The quadrotor platform used in this work is equipped with an inner-loop controller circuit, responsible for generating the forces $F_i$ applied to each motor, so that the total thrust force $T$ and the angular velocity $\omega$ can be considered as inputs for the path following controller design.

The external force in body coordinates is given by

$$f = -Te_3 + mgR^Te_3$$

where $e_3 = [0 \ 0 \ 1]^T$ and $g$ is the gravitational acceleration. Throughout the remainder of the paper, the time dependence of variables is often omitted to lighten notation.

Due to the underactuated nature of the vehicle, the desired attitude cannot be arbitrarily selected. From (2) and (4), it is easy to observe that the equilibrium for path following satisfies

$$T_d R_d e_3 = mge_3 - m\ddot{p_d} + mb.$$ Consequently, the desired rotation matrix $R_d$ is automatically prescribed up to a rotation about the body $z$ axis $(T_d R_d R_z(\psi)) = mge_3 - m\ddot{p_d} + mb$, with $\psi \in \mathbb{R}$ which we regard as an additional degree of freedom.

We consider the full state of the vehicle to be available for feedback. In our setup, the state measurements are obtained through a high speed motion tracking system, based on external cameras tracking reflective markers on the vehicle, as described in section VI.

V. CONTROLLER DESIGN

To devise a control law that steers the aerial vehicle along the path $p_d(\gamma)$ we consider a two stage process. First, a virtual controller, bounded with respect to the position system, is designed for the translational subsystem, which can be modelled as three independent double integrators. Once this controller is designed, the input error resulting from the underactuation is backstepped through the rotational subsystem until a control law for the angular velocity actuation is reached. The controller for the translational subsystem, composed by three independent double integrators, is presented in the following Proposition, for a single double integrator (adaptation from [18]).

*Proposition 1:* Consider the double integrator system

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = u$$

driven by input $u \in \mathbb{R}$ and let $\sigma$ be a saturation function. Then, for positive gain $k_2$, the control law

$$u(x_1, x_2) = -k_2(x_2 + \sigma(x_1)) - \sigma(x_1) - \sigma'(x_1)x_2,$$ (5)

in closed-loop with the double integrator system renders the origin globally asymptotically stable.

The following Lyapunov function for the double integrator system in closed loop with the controller from Proposition 1

$$V_{DI}(x_1, x_2) = \phi(x_1) + \frac{1}{2}(x_2 + \sigma(x_1))^2,$$

with $\phi(s) = \int_0^s \sigma(\tau)d\tau$, has closed-loop negative-definite time derivative

$$\dot{V}_{DI}(x_1, x_2) = -\sigma(x_1)^2 - k_2(x_2 + \sigma(x_1))^2.$$ In order to accomplish the path following objective, we now perform the following change of variables, where the new variables correspond to the inertial position and velocity errors

$$z_p = p - p_d(\gamma),$$
$$z_v = \dot{p} - \dot{p}_d,$$

with derivatives given by

$$\dot{z}_p = z_v,$$
$$\dot{z}_v = -\frac{T}{m}R e_3 + ge_3 - \ddot{p}_d + b.$$
This subsystem can be regarded as three independent double integrators, each one driven by one of the entries of the vector \(-\frac{L}{m} \mathbf{e}_3 + g \mathbf{e}_3 - p_3 + \mathbf{b}\). As such, we can design a stabilizing control action based on (5),
\[
\mathbf{u}^* = \begin{bmatrix}
 u(z_{p1}, z_{e1}) \\
u(z_{p2}, z_{e2}) \\
u(z_{p3}, z_{e3})
\end{bmatrix}.
\]

In addition, we use a term to mitigate the effect of the unknown disturbance on a first level. This results in a tentative Lyapunov function for the entire quadrotor system, composed by the sum of the Lyapunov functions for each of the double integrators and additional terms related to the disturbance estimation error,
\[
V_1 = \phi(z_p) + \frac{1}{2} (z_v + \sigma(z_p))^T (z_v + \sigma(z_p)) + \int_0^{||z_l||} \sigma(\tau)d\tau + \int_0^{||\mathbf{b}||} \sigma^{-1}(\tau)d\tau - \mathbf{b}^T z_l,
\]
where \(z_l\) is an integral state given by \(z_l = \int_0^{t} \dot{z}(\tau) + \sigma(z_p(\tau))d\tau\). The addition of the integral terms to the Lyapunov function draws inspiration from [9] and is used to cancel out the external disturbance in the Lyapunov derivative and add a bounded integral term to the desired thrust direction. Computing the time derivative of \(V_1\), one gets
\[
\dot{V}_1 = -\sigma(z_p)^T \sigma(z_p) - k_2 (z_v + \sigma(z_p))^T (z_v + \sigma(z_p)) + (z_v + \sigma(z_p))^T (-\frac{T}{m} \mathbf{e}_3 - \mathbf{u}^* + g \mathbf{e}_3 - \hat{p}_d + \sigma(||z_l||) \frac{z_l}{||z_l||}).
\]
Partitioning the rotation matrix as \(R = [\mathbf{r}_1 \ \mathbf{r}_2 \ \mathbf{r}_3]\) and defining the thrust control law as
\[
\mathbf{T} = \mathbf{T}_d \mathbf{r}_3^T \mathbf{r}_3,
\]
where the desired thrust \(T_d\) and desired thrust direction \(\mathbf{r}_{3d}\) are given by
\[
T_d = m ||\mathbf{u}^* + g \mathbf{e}_3 - \hat{p}_d + \sigma(||z_l||) \frac{z_l}{||z_l||}||,
\]
\[
\mathbf{r}_{3d} = \frac{\mathbf{u}^* + g \mathbf{e}_3 - \hat{p}_d + \sigma(||z_l||) \frac{z_l}{||z_l||}}{||\mathbf{u}^* + g \mathbf{e}_3 - \hat{p}_d + \sigma(||z_l||) \frac{z_l}{||z_l||}||},
\]
we can rewrite the closed-loop derivative (8) as
\[
\dot{V}_1 = -W_1(z_p, z_v) - \frac{T_d}{m} (z_v + \sigma(z_p))^T S(\mathbf{r}_3)^2 \mathbf{r}_{3d},
\]
where \(W_1(z_p, z_v) = \sigma(z_p)^T \sigma(z_p) + k_2 (z_v + \sigma(z_p))^T (z_v + \sigma(z_p))\) is a positive definite function.

The vector \(S(\mathbf{r}_3)^2 \mathbf{r}_{3d}\) in (11) belongs to the plane perpendicular to the thrust direction and can be regarded as an input error due to the under-actuation of the translational subsystem. When the thrust is aligned with the desired direction, i.e. \(\mathbf{r}_3 = \mathbf{r}_{3d}\), the input error is zero and regulation to zero of the path following error is achieved. The error between the desired and the actual thrust direction can be written in vector form as
\[
z_d = \mathbf{r}_3 - \mathbf{r}_{3d},
\]
or as an angular error \(z_\theta\) given by the relation
\[
\cos z_\theta = 1 - \frac{1}{2} z_d^T z_d = r_3^T r_{3d}.
\]

We now extend the tentative Lyapunov function to include the angular error as follows
\[
V_2 = V_1 + \frac{1}{2} z_{d1}^T z_d + \frac{1}{3 \kappa_3} \mathbf{b}^T \mathbf{b}
\]
\[
= V_1 + \frac{1}{\kappa_3} (1 - \cos z_\theta) + \frac{1}{3 \kappa_3} \mathbf{b}^T \mathbf{b},
\]
and compute its closed-loop derivative as
\[
\dot{V}_2 = -W_2(z_p, z_v, \sin z_\theta) + r_{3d}^T R S(e_3) \left( \frac{T_d}{m} (z_v + \sigma(z_p)) + k_3 r_{3d} \right)
\]
\[
\cdot \frac{\partial r_{3d}}{\partial z_v} S(\mathbf{r}_3)^2 r_{3d} + \frac{1}{\kappa_3} \mathbf{b}^T \mathbf{b},
\]
where \(k_\alpha, k_3\) are positive control gains, \(W_2\) is a positive definite function, given by \(W_2(z_p, z_v, \sin z_\theta) = W_1(z_p, z_v) + k_3 \sin^2 z_\theta\), and we used the relation
\[
-r_{3d}^T S(\mathbf{r}_3)^2 r_{3d} = \sin^2 z_\theta.
\]
The symbol \(\tilde{r}_{3d}\) represents the estimate of the time derivative of \(r_{3d}\) obtained by using the estimate \(\mathbf{b}\) of the external disturbance instead of \(\mathbf{b}\), when performing the necessary calculations. The estimation error is given by
\[
\tilde{r}_{3d} - r_{3d} = \frac{\partial r_{3d}}{\partial z_v} S(e_3) R^T r_{3d},
\]
We can now use the actuation \(\omega\) and the estimation law \(\dot{\mathbf{b}}\) to render \(\dot{V}\) negative semi-definite and achieve convergence of the path following error to zero. We state this formally in Lemma 2, assuming that the control law is well-defined.

**Lemma 2:** Let the quadrotor kinematics and dynamics be described by (1)-(3), let \(\mathbf{p}_d(\gamma(t)) \in C^3\) be the desired path, and consider the transformation to error coordinates \(z_p, z_v, z_\theta\) given by (6), (7), (12), respectively. For any bounded \(\omega_3(t) \in C^1\), the closed-loop system that results from applying the control laws (9),
\[
\omega = -S(e_3)^2 \left( R^T \tilde{r}_{3d} + \frac{T_d}{m \kappa_3} S(e_3) R^T (z_v + \sigma(z_p)) \right)
\]
\[
+ \omega_3(t) e_3,
\]
and the estimator law
\[
\dot{\mathbf{b}} = -k_3 \frac{\partial r_{3d}}{\partial z_v} S(e_3) R^T r_{3d},
\]
achieves global path following, by guaranteeing that the errors \(z_p\) and \(z_v\) converge to zero for any initial condition. In addition, the error vector \(z_p, z_v, z_\theta\) converges to the set \(\{(0, 0, 0), (0, 0, \pi)\}\) and the desired equilibrium point \((z_p, z_v, z_\theta) = (0, 0, 0)\) is uniformly asymptotically stable.

**Proof:** We assume that the control law is well defined, i.e. \(T_d \neq 0\). Starting with the positive definite Lyapunov function
\[
V = \phi(z_p) + \frac{1}{2} (z_v + \sigma(z_p))^T (z_v + \sigma(z_p)) + k_\alpha (1 - \cos z_\theta)
\]
\[
+ \phi(||z_l||) - \mathbf{b}^T z_l + \int_0^{||\mathbf{b}||} \sigma^{-1}(\tau)d\tau + \frac{1}{3 \kappa_3} \mathbf{b}^T \mathbf{b},
\]
and computing its time derivative we have that
\[
\dot{V} = -\sigma(z_p)^T \sigma(z_p) - k_2 (z_v + \sigma(z_p))^T (z_v + \sigma(z_p))
\]
\[
- k_3 \sin^2 z_\theta.
\]
is a negative semidefinite function. Since the quadrotor error dynamics are non-autonomous, we resort to Barbalat’s Lemma to prove convergence of $\dot{V}$ to zero.

From the unboundedness of $V$ with respect to $z_p$, $z_v$, and $\omega$, and observing that $\dot{V}$ is negative semi-definite, we conclude that the states $z_p$, $z_v$, and $\omega$ are bounded. The state $z_\theta$ evolves in a compact set. The external inputs $\omega_3(t)$ and $p^b_{\omega i}$ are bounded by assumption. We have thus that $\dot{V}$ is bounded and, consequently, $V$ is uniformly continuous. We can therefore apply Barbalat’s Lemma to prove convergence of $V$ to zero and, consequently, of the states $z_\theta$ to zero.

The closed-loop trajectories $(z_p(t), z_v(t), z_\theta(t))$ converge to the invariant set $\{(0,0,0),(0,0,\pi)\}$ or, equivalently, the closed-loop trajectories $(p_\omega,p_{\omega i},z_\theta)$ converge to the set $\{(0,0,0),(0,0,-2e_3)\}$. For initial conditions such that $V(0) < 2k_v$, the time derivative of $V$ is negative definite, meaning that the origin is uniformly asymptotically stable.

The rotational degree of freedom allowed by $\omega_3(t)$ in (13) is due to the axial symmetry exhibited by the quadrotor and is explored to control the heading of the vehicle independently of the path following law. Notice that both actuations $T$ and $\omega$ depend only on bounded functions of the position error. This is a desired property since the position error can be arbitrarily large, depending on the initial conditions for the quadrotor.

VI. EXPERIMENTAL RESULTS

In order to experimentally validate the proposed control algorithms we developed a rapid prototyping and testing architecture using a Matlab/Simulink environment to seamlessly integrate the sensors, the control algorithm and the communication with the vehicle. The vehicle used for the experiments is a radio controlled Blade mQX quadrotor [19], which can be seen in Figure 5. This aerial vehicle is very agile and maneuverable, readily available and inexpensive, making it the ideal platform for the present work. The quadrotor weighs 80 g with battery included and the arm length from the center of mass to each motor is 11 cm. The available commands are thrust force and angular velocity.

Due to the lack of support for on-board sensors, the state of the vehicle must be estimated through external sensors. In our setup we use a VICON Bonita motion capture system [20], comprising 12 cameras, together with markers attached to the quadrotor. The motion capture system is able to accurately locate and estimate the positions of the markers, from which it obtains position and orientation measurements for the aircraft. VICON Bonita is a high performance system, able to operate with sub-millimeter precision at up to 120 Hz. The performance of the motion capture system is such that the linear velocity can be well estimated from the position measurements by a simple backwards Euler difference, with relatively low noise level. For the experimental setup, the state measurements from the motion capture system are obtained at 50 Hz, allowing for improved accuracy.

The vehicles use a 2.4 GHz wideband Direct Sequence Spread Spectrum signal to generate a robust radio link with on-channel interference resistance. This radio technology also allows for the simultaneous use of several vehicles in a confined space, enabling formation flight. The commercial off-the-shelf quadrotor vehicles are designed to be human-piloted with remote controls but not directly from a computer. In order to be able to send commands to a quadrotor from a computer we identified the radio chip inside the remote control and connected the serial interface of the RF module to a computer serial port. To maintain the radio link, the radio transmitters must receive the control signals via serial port and send them to the vehicle once every 22.5 ms.

A graphical representation of the overall architecture is presented in Figure 2. We use two computer systems, one running the VICON motion tracking software and the Simulink model which generates the command signals sent to the other computer through Ethernet; and a second one that receives the command signals and sends them through serial port to the RF module at intervals of 22.5 ms. The decision to separate control and communications was made to avoid jitter in the transmission of the serial-port signals to the RF module, which occurred when running all the systems in the same computer, and lead to erratic communication with the vehicle.

![Quadrotor control architecture.](image)

The Matlab/Simulink interface (see Figure 3) enables a fast iteration between simulation and experimental testing of control algorithms. A VICON block handles the reception of estimates from the motion capture system and outputs the quadrotor state; computation of the control signals is performed based on measured or simulated vehicle state; and the actuation signals are ultimately relayed to the second computer for radio transmission to the quadrotor or to a simulator block.

![Simulink block diagram of the quadrotor controller featuring the alternative VICON sensor or quadrotor simulator, reference input, and output to the RF module.](image)
A. Identification

Identification of the platform was performed by applying different constant commands over several experiments and measuring the thrust force and angular velocity of the vehicle. The thrust force was measured by having weights attached to the quadrotor and finding the thrust command that balanced it. For the angular velocity a command was applied and the angular velocity measured directly with the motion capture system. The RC commands were verified to relate linearly with the vehicle inputs. The maximum thrust generated by the propellers is approximately 1.37 N (equivalent to 140 g) and varies slightly with the battery charge. The maximum angular velocity that can be commanded is 200 deg/s for the $x$ and $y$ axes and 300 deg/s for the $z$ axis. However, the commands issued to the quadrotor are not instantaneously followed. This delay nonlinearity can be well approximated by considering the motors as first order dynamic systems with a pole at 1.5 Hz.

B. Path following

For the first experimental evaluation of the proposed controller we selected for the desired path a lemniscate (figure eight) parameterized by $\gamma$ according to

$$
\mathbf{p}_d(\gamma) = \begin{bmatrix}
\cos(\pi/4) & \sin(\pi/4) & 0 \\
-\sin(\pi/4) & \cos(\pi/4) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\sin(\phi(\gamma))\cos(\phi(\gamma)) \\
\sin(\phi(\gamma))^2+1 \\
\sin(\phi(\gamma))^2+1
\end{bmatrix},
$$

where $\phi(\gamma)$ is defined as

$$
\phi(\gamma) = \int_0^\gamma \sqrt{1 + \sin^2(\tau)} d\tau.
$$

This parametrization results in a path derivative with unitary norm. The desired progression along the path is governed by $\dot{\gamma}_d = V_e \Psi(\|\mathbf{z}_p\|)$ where $\Psi$ is a bump-like function such that $\Psi(0) = 1$ and decreases to zero as the argument grows in module. For large position errors, $\dot{\gamma}_d$ is zero to ensure that the desired position waits for the vehicle to reach it. When the vehicle is close to the path the desired parameter derivative $\dot{\gamma}_d$ takes a constant value to enforce a vehicle velocity of constant norm $V_e$.

The control law coefficients are $k_2 = 3$, $k_3 = 1$, $k_a = 20$, and $k_b = 1$. For the sigmoid function we use $\sigma(s) = 2s/\sqrt{1+s^2}$ and as timing law parameters and initial conditions we use $k_\gamma = 1$, $V_e = 1$, $\gamma(0) = 0$ and $\dot{\gamma}(0) = 1$. The time evolution of the actual quadrotor position and the reference for the lemniscate trajectory is shown in Figure 4, where the quadrotor follows closely the desired path, with a maximum error of 8 cm and a mean error of 4 cm. This small position error can be attributed to unmodeled dynamics of the plant and to the fact that the issued commands are not perfectly followed by the aircraft.

The main contributions to the unmodeled dynamics are threefold: i) there exist unmodeled cross-couplings between the angular velocity commands and lateral forces acting on the quadrotor, due to an uneven and not perfectly symmetric mass distribution of the vehicle; ii) the issued thrust and angular velocity commands are not perfectly followed due to motor inertia and incorrect thrust command gains; iii) there exists a non-constant wind disturbance affecting the vehicle. A video of the quadrotor takeoff and lemniscate path following is available at [21].

![Fig. 4. Time evolution of the position and reference signal.](image)

![Fig. 5. Setup of the quadrotor vehicle and the disturbance generator.](image)

C. Robustness to external disturbances

In order to attest the controller’s robustness to strong wind disturbances we devised a second experiment where the quadrotor is forced to hover in the slipstream of a mechanical fan, as pictured in Figure 5.

With the fan turned off, we initially hover the vehicle and then turn the fan on, creating an airflow that the quadrotor controller is designed to reject and achieve zero steady state error. The results of the experiment are shown in Figures 6 and 7, where we restricted the data to the airflow axis for better visualization. The effect of the airflow disturbance can be seen around the 10 s time instant, when the fan is turned on and the integral term $\sigma(\|\mathbf{z}_i\|) \dot{\mathbf{z}}_i$ in (10) and the estimator $\dot{\mathbf{b}}$ start adapting to the new environmental conditions. The position error grows from effectively zero to 15 cm, when the fan is turned on, but quickly returns back to zero due to the action of the integral terms. As a consequence of the integral terms and the new conditions, the quadrotor tilts against the airflow so that it can hover at a designated location and not be dragged by fan’s slipstream. This is evidenced by Figure 7, where the pitch angle goes from an average of $-1.5$ degrees, in a windless situation, to $7$ degrees, when the mechanical fan is in operation. Figure 7 also highlights the fact that the noise in the pitch angle is much larger when the disturbance is active, due to the turbulent airflow with narrow section that is generated by...
the mechanical fan. A video of the quadrotor hovering and resisting the wind disturbances is available at [21].

![Fig. 6. Time evolution of the disturbance estimation and related signals, restricted to the slipstream axis, when subject to a wind disturbance.](image)

![Fig. 7. Time evolution of the quadrotor pitch angle when subject to a wind disturbance.](image)

VII. CONCLUSIONS

This paper proposed a state feedback solution to the problem of steering a quadrotor vehicle along a predefined path and presented its experimental evaluation. The solution guarantees global convergence of the path following error to zero, for a large class of three-dimensional paths. The nonlinear controller, which was designed using Lyapunov-based backstepping techniques, ensures that the actuation does not grow unbounded as function of the position error and allows for zero thrust actuation to be applied when the vehicle is converging to the path. The proposed controller was designed to be robust to unknown constant force disturbances that arise from the presence of wind or imperfect knowledge of vehicle parameters. A rapid prototyping and testing architecture was developed to expedite the development process by creating an abstraction layer that integrates the sensors, controller, and communication with the vehicle. Experimental data for a path following trajectory was presented which evidenced the effects of the integral action. The robustness of the controller to non-ideal wind disturbances was experimentally demonstrated using a mechanical fan as a disturbance generator. Future work will focus on input saturation as the proposed controller generates inputs which are bounded with respect to the position error but can grow unbounded on the remaining errors.

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REFERENCES


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