Design of Experiments for nonlinear
dynamic system identification

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Abstract: In this contribution different methods to excite dynamic systems are presented. Model-based/model-free and offline/online design plans are discussed and methods to optimize design plans with respect to the intended use are introduced. The presented design plans are appropriate to identify the dynamics of combustion engines and are evaluated by simulation and practical examples.

Keywords: DoE, Fisher Information, D-optimal design, Latin Hypercube design, Query by Committee, online Design of Experiment, APRBS

1. INTRODUCTION

Parametric models are often used to model dynamic systems whose parameters are estimated on the basis of measured data. The input-signal defines how the system is excited and thus which data set is available for modeling and parameter estimation. The Design of Experiments (DoE), which determines the excitation signal, is therefore of vital importance for the quality of the identified model. Only a data set, which contains enough information about the system, allows an accurate estimation of the parameters and the identification of a good model. Since the excitation signal is the only possibility to influence the information content of the data set it defines the base quality of the model. This attribute is independent of the chosen model architecture and structure. The target of the DoE is to excite the system so that with every measurement a maximum of information can be achieved. In doing so both the kind and the parameterization of the excitation signal must be determined.

Common excitation signals for nonlinear dynamic system identification are multi-valued PRBS (pseudo random binary signals), amplitude-modulated PRBS (APRBS), sinus signals and filtered Gaussian noise. If the actuation costs are high, the APRBS and the multi-valued PRBS are the most practicable signals because sinus signals and filtered Gaussian noise require an actuation at every sampling instance. This should be avoided in industrial identification tasks to spare the actuators and the testee (Rivera et al. (2003)). The multi-valued PRBS was developed for linear system identification and hence covers only a small number of amplitude levels causing a poorly covered input space. To identify nonlinear systems it is by contrast important to cover a wide amplitude range to capture the nonlinearities. Thus, the APRBS is a very popular excitation signal which is often used in system identification at engine test beds (see Hafner (2003) and Zimmerschied et al. (2005)).

The target of this contribution is the determination of optimal amplitude levels and amplitude sequence. The second section contains the presentation of parametric excitation signals, popular methods of offline DoE and some aspects of their deficits.

The third section contains the discussion of model based online experimental designs and methods to optimize the parametric excitation signal in reference to the later application of the models.

The last section contains the evaluation of the presented experimental designs by simulations and at the combustion engine test bed.

2. EXCITATION OF DYNAMIC SYSTEMS

The choice of the excitation signal is very important for the achieved model quality. The system should be excited in such a way that all interesting areas of the input space and all relevant frequencies are covered. This aspect is satisfied by the APRBS, which is presented in the following and is a special case of the parametric excitation signals used in this contribution.

2.1 Parametric excitation signals

The APRBS is an amplitude modulated PRBS which is a periodic deterministic signal with properties like the Gaussian noise (see Tan and Godfrey (2002)). Since the values of the amplitudes are free design parameters in the input space in the following they are called design points in respect to the terminology of DoE. According to this the APRBS can be understood as a sequence of step functions fixed by the N design points $d_a \in [u_{\text{min}}, u_{\text{max}}]$. With the design points $d_a$ and dwell time $T_{h,s}$ the APRBS is completely described (see figure 1). Beside the input range the signal length and the minimum dwell time are selectable design parameters.

Problematical in industrial use are the jumps of the APRBS in the input space. On the one hand the actuators are not arbitrary fast and on the other hand certain input
The singular ramp functions $U_s$ consist in a time-discrete input sequences, which are defined by the sequence time $T_s$ and the sampling time $h$ (see Deflorian et al. (2010)):

$$U_s(d_s, d_{s-1}, T_{h,s}) = \{u_{k+i}\}_{i=1}^{T_r/h}$$

in which $k = \frac{1}{h} \sum_{j=1}^{r-1} T_j$ and

$$u_{k+i} = \begin{cases} d_s, & T_r/s 
\leq i < T_s/h \\
g \cdot i + d_{s-1}, & 1 \leq i \leq T_r/s \end{cases}$$

The slope $g$ is limited by the maximum allowable slope $g_{\text{max}}$ and is given by

$$g = \alpha \frac{g_{\text{max}}}{T_r/s}$$

with $T_r/s = \left[ \frac{1}{h} \max_j(|\alpha_j|) \right] h$ and $\alpha = \frac{d_s - d_{s-1}}{g_{\text{max}}}$. The ramp time $T_r/s$ and the dwell time $T_{h,s}$ are a multiple of the sampling time $h$ and each component of $g$ is limited by $g_j \leq g_{\text{max}} \forall j$. Thus, the length of the ramp function (1) depends on the ramp time, which depends on the design point and the dwell time $T_s = T_{h,s} + T_r/s$. In figure 2 a schematic illustration of a ramp function is shown.

The slope of the ramp function (1) affects the frequency characteristics of the signal. In case of steep slopes, i.e. using small ramp time $T_r$, the higher frequencies are excited stronger (see Isermann (1992)). The relative difference of the amplitude density spectrum of the excitation signal compared to a jump function can be estimated by $\leq 5\%$ respectively $\leq 1\%$, if the following condition is fulfilled (see Isermann (1992)):

$$T_r \leq T_{\text{min}} \frac{1.1}{2\pi} \quad \text{respectively} \quad T_r \leq T_{h,\text{min}} \frac{0.5}{2\pi}.$$

$T_{\text{min}}$ is the smallest interesting time constant of the process in this case. Since the minimal dwell time of the APRBS is chosen with respect to the wanted process dynamic, in practice the ramp time can be chosen in a way that it fulfills

$$T_r \leq T_{h,\text{min}} \frac{1.1}{2\pi} \quad \text{respectively} \quad T_r \leq T_{h,\text{min}} \frac{0.5}{2\pi}$$
at every time. For $T_r = 0$ this is the classic APRBS.

The question is how to choose the amplitudes independent of the choice of the ramp and dwell time. As discussed in Nelless (2001), a good covering of the input space can only be expected by a multitude of jumps and corresponding long excitation time.

To achieve a better distribution of the amplitudes different methods of DoE can be used which are presented in the following.

### 2.2 Offline DoE

The most popular methods of DoE are classified by model-free and model-based methods. The experimental designs which are discussed in this section can be calculated before starting the measurements and thus are called offline DoE.

#### Model-free DoE

Model-free DoE does not care about the used model structure and tries to distribute the input space as equally as possible (space filling designs). The most popular space filling DoE is based on the Latin Hypercube (LHC) distribution.

**Latin Hypercube distribution**

To calculate a LHC distribution the input space is divided into $N$ intervals. In every column and row only one design point $d_s$ is placed. A LHC design covers the input space well if the distance between the design points is maximal. In figure 3 a typical LHC distribution for 50 design points is displayed.

#### Model-based DoE

Model-based designs try to distribute the design points in the input space in a way that the estimation of the parameters is as insensitive as possible to the measurement noise. The Fisher Information (FI) can be used to estimate the variance of the parameter estimation of any parametric model. The FI describes the information content of the random variable $x \sim N(\mu(\theta), \sigma I)$ about the parameter vector $\theta$ and with $\mu$ : expectation, $\sigma$ : variance and $I$ as identity matrix. Thereby it is assumed that the random variable $x$ is distributed by the multivariate normal distri-

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Fig. 1. APRBS in time domain.

combinations lead to unsafe operating conditions. If ramps with a maximum allowable slope are used instead of jumps, the excitation signal can be described by a sequence of $N$ ramp functions, which are determined by the dwell time $T_{h,s}$ and the design points $d_{s-1}$ and $d_s$ at the time instances $s - 1$ and $s$:

$$U = \sum_{s=1}^{N} U_s(d_s, d_{s-1}, T_{h,s}).$$

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Fig. 2. Sequence of the input $u$: design point $d$ with associated slope $g$, ramp time $T_r$ and dwell time $T_h$. Preprints of the 18th IFAC World Congress Milano (Italy) August 28 - September 2, 2011
distribution and the Gaussian noise is uncorrelated. Under this conditions the FIM is given by Kay (1993):
\[
\mathcal{I}(\theta) = \frac{1}{\sigma^2} \frac{\partial \mu(\theta)}{\partial \theta} \frac{\partial \mu(\theta)}{\partial \theta}.
\] (2)

Now the Cramer–Rao inequation (Kay (1993))
\[
\text{cov}(\theta(x)) \geq \mathcal{I}(\theta, x)^{-1}
\] (3)
shows, that the variance of the parameter estimation is limited by the inverse of the Fisher Information matrix (FIM). Thereby a parameter estimation method is efficient, if the lower bound of the Cramer–Rao inequality is reached. To reduce the variance of the parameter estimation the FIM must be maximized. Is the relation \( \mu = f(\theta, u) \) known or approximated closely enough, the FIM can be calculated for the random variables \( y_{k+1} = f(\theta, u_k) + \epsilon_k \) and \( \epsilon \sim \mathcal{N}(0, \sigma^2I) \):
\[
\mathcal{I}_k(\theta, u_k) = \frac{1}{\sigma^2} \frac{\partial f(\theta, u_k)}{\partial \theta} \frac{\partial f(\theta, u_k)}{\partial \theta}.
\] (4)
The (mean) FIM for K pairs \( (y_k, u_k) \) is defined by
\[
\mathcal{I}(\theta, U) = \frac{1}{K} \sum_{k=1}^{K} \mathcal{I}_k(\theta, u_k).
\] (5)

Thereby the input sequence \( U = \{u_1, ..., u_K\} \) is a compact subset of \( \mathbb{R}^{N_k} \).

To maximize the information content and thus to minimize the variance of the estimator, the FIM (5) must be maximized. Since no unique criterion exists to maximize a matrix, various criteria \( c \) are used to optimize \( \mathcal{J}(U) = c(\mathcal{I}(\theta, U)) \), in which the D-optimal design
\[
\mathcal{J}(U) = \det \left( \frac{1}{K} \sum_{k=1}^{K} \frac{\partial f(\theta, u_k)}{\partial \theta} \frac{\partial f(\theta, u_k)}{\partial \theta} \right)
\] (6)
is the most popular. A discussion of various criteria can be found in Walter and Pronzato (1997).

D-optimal distribution
The FIM can only be calculated independently of the estimated parameters if models are linear in their parameters. Thus in practice the complexity of the approximation problem is often estimated and an experimental design based on an accordingly complex polynomial model is calculated. For a model \( \hat{y} = X\theta \), which is linear in its parameters, the D-optimal criterion is given by
\[
\mathcal{J}(X) = \det(X^\top X).
\] (7)

This is independent of the parameter \( \theta \) and so it is possible to determine the DoE completely without any measurement.

Figure 3 shows a typically D-optimal distribution with 50 design points based on a polynomial model of third order. In contrast to the LHC distribution mainly the boundary of the input space is covered well. Thus, no extrapolation appears in the boundary area.

2.3 Sequence of the design points
Both, the LHC and the D-optimal distribution provide no information about the optimal sequence of design points. If no physical knowledge can be integrated into the DoE the following criteria are suggested in Schreiber and Isermann (2009):
- Sorting by the minimal distance
- Sorting by the maximal distance
- Sorting by the mean distance

These heuristic criteria do not take into account the dynamic of the system. Although the effect of the inputs on the future calculation of the FIM must be considered. This can be achieved by the use of online methods which include the knowledge of the model in the DoE.

Online DoE for dynamic systems
In contrast to the offline methods discussed before, model based online methods allow to adapt the DoE under consideration of the gained information. With the use of adaptive DoE it is possible to improve models at certain areas of the input space which are relevant to the identification problem. An online method for static models proved in practice, is the adaptive DoE based on model committees, also called Query by Committee (QBC). The idea is to place design points in the input space where the models of the committee are most discordant. Here the used models are time-discrete input/output models in parallel configuration (see Nelles (2001)). The committee consists in \( l \) models:
\[
\hat{y}_k = \sum_{i=1}^{l} \lambda_i \hat{y}_k^{(i)} \quad \sum_{i=1}^{l} \lambda_i = 1
\] (8)
\[
\hat{y}_k^{(i)} = f(\theta^{(i)}, \varphi_k^{(i)}).
\] (9)

Thereby \( \theta^{(i)} \) is the parameter vector, \( \varphi_k^{(i)} \) the regressor and \( \hat{y}_k^{(i)} \) the output of the model. The regressor vector consists in \( n \) time-delayed model inputs and \( m \) time-delayed model outputs:
\[
\varphi_k = [\hat{y}_{k-1}^\top, ..., \hat{y}_{k-n}^\top, u_{k-1}^\top, ..., u_{k-n}^\top]^\top.
\]

For methods of the adaptive choice of the weighting coefficients \( \lambda_i \) of the models and for a discussion about the advantages of model committees it is referred to Valentini and Masulli (2002). In this section QBC and FIM based DoE adapted for model committees are developed. Since the sequence of the design points is of vital importance, the criteria of the DoE are defined as model predictive control under boundary conditions. The cost function to be minimized over the time horizon \( T_s \) and hence over \( N_s = T_s/T_s \) sampling points is defined by:
\[ V(\mathcal{U}_s, k, T_s) = \sum_{j=k}^{k+N_s} \sum_{i=1}^{l} \lambda_i l(\theta^{(i)}_k, \phi^{(i)}_j) \quad (10) \]

S.C. \[ \hat{y}_j = \sum_{i=1}^{l} \lambda_{i,j} \quad \sum_{i=1}^{l} \lambda_i = 1 \]
\[ \hat{y}^{(i)}_j = f(\hat{\theta}^{(i)}_k, \phi^{(i)}_j) \]
\[ u_j \in U \quad \hat{y}^{(i)}_j \in \mathcal{Y} \]

with model dependent costs \( l(\theta^{(i)}_k, \phi^{(i)}_j) \) and parameter estimation \( \hat{\theta}^{(i)}_k \) at time \( k \). \( U \) and \( \mathcal{Y} \) are the allowed input and output parameters. With the use of the parametric excitation signal (1), the number of the optimization parameters can be reduced:

\[ V(\mathcal{D}_s, d_{s-1}, k, T_h,s) = \frac{1}{N_s} \sum_{j=k}^{k+N_s} \sum_{i=1}^{l} \lambda_i l(\theta^{(i)}_k, \phi^{(i)}_j) \quad (11) \]

S.C. \[ \hat{y}_k = \sum_{i=1}^{l} \lambda_{i,k} \quad \sum_{i=1}^{l} \lambda_i = 1 \]
\[ \hat{y}^{(i)}_k = f(\hat{\theta}^{(i)}_k, \phi^{(i)}_k) \]
\[ u_{k+1} = \begin{cases} d_s, & \quad T_{r,s} \leq i < T_{h,s} + T_{r,s} \, t_s \\ g \cdot i + d_{s-1}, & \quad 1 \leq i \leq T_{r,s} \, t_s \end{cases} \]
\[ u_j \in U \quad \hat{y}^{(i)}_j \in \mathcal{Y} \]

with \( N_s = (T_{h,s} + T_{r,s})/t_s \) and \( g \) is calculated as in (1). Attention should be paid to the scaling of the costs because the length of the input sequence can change during the ramp period dependent on the design points. This formulation allows also to consider the dwell time \( T_{h,s} \in [T_{h,min}, T_{h,max}] \) as further optimization parameters, instead of determining it at the beginning. The optimal input sequence or design point is accordingly calculated by solving the following optimization problem:

\[ U^*_{Q} = \max_{u_s} V(\mathcal{U}_s, k, T_s) \quad d^*_{s} = \max_{d_s} V(\mathcal{D}_s, d_{s-1}, k, T_h,s). \]

**Query by Committee**

The QBC criterion with dynamic models looks for the input sequence which causes the biggest disagreement of the models within the considered time horizon \( T_s \) (Deflorian et al. (2010)):

\[ I_Q(\theta^{(i)}_k, \phi^{(i)}_j) = \left\| f(\theta^{(i)}_k, \phi^{(i)}_j) - \sum_{r=1}^{l} \lambda_r f(\theta^{(r)}_k, \phi^{(r)}_j) \right\|_2^2 \quad (12) \]

The optimal input sequence \( U^*_{Q} \) or design point \( d^*_{s} \) leads to the biggest variance of the discrete model outputs, dependent on the committee output. The outputs of the discrete models can be weighted differently by replacing the squared two norm with a weighted sum of the squared differences of the discrete model outputs of the model committee.

**Sequential FIM based DoE**

To find an optimal input sequence the FIM of the discrete models must be maximized over the considered time horizon:

\[ \sum_{j=k}^{k+N_s} l_{FIM}(\theta^{(i)}_k, \phi^{(i)}_j) = c \left( \sum_{j=k}^{k+N_s} \mathcal{I}(\theta^{(i)}_k, \phi^{(i)}_j) + \mathcal{I}_0^{(i)}(\theta^{(i)}_k) \right) \]

\[ \mathcal{I}_0^{(i)}(\theta^{(i)}_k) = \frac{\partial f(\theta^{(i)}_k, \phi^{(i)}_j)}{\partial \theta^{(i)}_k} \right) \quad \mathcal{I}_0^{(i)}(\theta^{(i)}_k) \]

where

\[ \mathcal{I}_0^{(i)}(\theta^{(i)}_k) = \sum_{j=1}^{k-1} \mathcal{I}(\theta^{(i)}_k, \phi^{(i)}_j) \]

is the FIM which depends on the previous training data and the actual parameter estimation \( \hat{\theta}^{(i)}_k \). The most popular criterion \( c(\cdot) \) for the maximization of the FIM is the D-optimal (see Walter and Pronzato (1997)). A discussion of the D-optimal criterion of the DoE for dynamical models is found in Deflorian and Klöpper (2009) and Deflorian et al. (2010).

**Task-related modification**

Dynamic models are often used to avoid time intensive measurements and the prediction of steady state values. Therefore the quality of the steady state behaviour is essential. Another application area is the model based optimization in which either the steady state value or the transient state of the system must be optimized.

**Improvement of the steady state behaviour**

To improve the steady state behaviour of the model committee the uncertainty of the model in the steady state

\[ V_S(d_s) = \sum_{i=1}^{m} \lambda_i \left\| f(\phi^{(i)}_s, d_s) - \sum_{i=1}^{l} \lambda_i f(\phi^{(i)}_s, d_s) \right\|_2^2 \quad (14) \]

can be added to the criterion (11) and can accordingly be weighted using \( \mu \in [0, 1] \):

\[ d^*_{s} = \max_{d_s} (\mu V(\mathcal{D}_s, d_{s-1}, k, T_h,s) + (1 - \mu) V_s(d_s)). \]

Thereby \( \phi^{(i)}_s \) stands for the steady state regressor of the model \( i \) at the design point \( d_s \). An individual weighting of the model committee outputs can be achieved by using a weighted sum.

**Optimization**

The synchronous optimization of the model output and the parameter estimation is a big challenge to the DoE. At the one hand the system must be excited heavily to estimate the parameters well. On the other hand the DoE should concentrate on such areas where the supposed optima are arranged because only here a high model quality is necessary. Since these are contradictory challenges for the DoE it must be guaranteed first that the model quality is sufficient enough to implement a model based system optimization. Hence the optimization criterion must be changed in that way that the main focus moves from optimal system excitation to system optimization. Depending on whether the steady state or the transient system behaviour must be optimized the first criterion or the second one should be used:

\[ d^*_{s} = \max_{d_s} (\mu V(\mathcal{D}_s, d_{s-1}, k, T_h,s) + (1 - \mu) V_{opt}(\mathcal{U}_s, k, T_s)) \]
\[ U^*_{s} = \max_{u_s} (\mu V(\mathcal{U}_s, k, T_s) + (1 - \mu) V_{opt}(\mathcal{U}_s, k, T_s)). \]

Thereby the weighting factor \( \mu \in [0, 1] \) is very important for the achieved optimization results and is currently
subject of research (see Pronzato (2009)).

Optimization of the steady state values. The optimization of the steady state behavior looks for the design points which lead to the minimal model output:

$$V_{opt}(d_s) = - \sum_{j=1}^{N_s} \left( \sum_{i=1}^{l} \alpha_i f(\theta_k^{(i)}, \varphi_s^{(j)}) \right)_{j}$$

where \( \varphi_s^{(j)} \) is the steady state regressor of the model \( i \) at design point \( d_s \).

Optimization of the transient state. To optimize the transient state find the input sequence \( \mathbf{U}_s \) which leads over the considered time horizon \( N_s \) to the lowest costs:

$$V_{opt}(\mathbf{U}_s, k, T_s) = - \sum_{j=1}^{N_s} \left( \sum_{i=k}^{k+N_s} \sum_{j=1}^{l} \alpha_i f(\theta_k^{(i)}, \varphi_s^{(j)}) \right)_{j}$$

(16)

Usually extra boundary conditions on pieces of the input vector are defined, e.g. the optimization of the valve timing in reference to the consumption at a given driving cycle.

3. EXPERIMENTS

To check the considered DoE criteria these are evaluated by simulations and with the help of measured data.

In all experiments a model committee is used which consists of 4 multilayer perceptron neural networks in parallel configuration with one hidden layer and different number of neurons (8 – 12) and identical weighting \( \alpha_i = \frac{1}{m} \).

The inputs and outputs are scaled to \( u \in [-1, 1] \) and \( y \in [-1, 1] \) and are delayed once or twice to describe differences of first or second order.

As an amount of error for the evaluation of the transient behavior the normalized mean squared error (NMSE) is used and for the evaluation of the steady state behavior the normalized absolute error (NAE) is taken:

$$NMSE = \frac{\| e \|_2}{K(\text{var}(y))} \quad NAE = \frac{\sum |e_k|}{K(\text{y}_{\text{max}} - \text{y}_{\text{min}})}$$

where \( e \) is the difference between system output and model output and \( \text{var}(y) \) is the variance of the measurement signal.

3.1 Simulative Evaluation

To evaluate the different DoE criteria with the help of comprehensive statistics, these are analyzed by simulation. Many measured variables which are relevant for the behavior of a combustion engine (e.g. the exhaust gas temperature), feature a PT1 behavior with a time constant depending on the operating point. The proposed modifications to the DoE are also targeted on an improved steady state behavior or a high model quality next to the static optima. To achieve this a static nonlinear system followed by a PT1 element with a time constant dependent nonlinear on the output of the static system. This allows to check easily the static accuracy of the dynamic model globally and close to the assumed optima. The RadCos function is used as static nonlinearity with input level \( u \in [0, 1]^l \) (see Poland (2002)).

$$x = \cos \left( 9\sqrt{u_1^2 + u_2^2 + 2} + 0.5 \cos(11u_1 + 2) + 15 \left( (u_1 - 0.4)^2 + (u_2 - 0.4)^2 \right)^2 \right).$$

The used PT1 element is described by the following differential equation:

$$Ty_1 - Kx = (2.4\cos(10x + 4) - 0.5y + 4)y + y - x$$

for which an anti derivative exists and which is discretised by \( h = 0.5s \). An illustration of the scaled transfer function of the system is shown in figure 4.

![Fig. 4. Composed dynamic test system](image)

At first different strategies for the distribution of the design points are compared. 50 different APRBS are constructed with random, LHC and D-optimal (polynomial of third order) distribution of the design points with \( T_{\text{h, max}} = 6s, T = 614s \) and \( T = 0 \). The online methods with the costs (12) and (13) need trained models. In doing so LHC experimental designs with \( T = 410s \) are constructed at first. During the next 128s the design points are chosen accordant the used criteria. The model committees are always adapted online. The optimization is always done for one design point and the D-optimality is used to maximize the FIM. The output of the test system is corrupted by noise with a signal-to-noise ratio (SNR) of \( SNR = 25 \). All data sets which are not used for the training of the model committee are used as validation data set for the transient model behavior. To validate the steady state behavior for 1000 LHC distributed points the arithmetic mean and the standard deviation of the NAE are calculated. The simulation results are listed in table 1.

<table>
<thead>
<tr>
<th>Table 1. transient and steady state accuracy</th>
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<tbody>
<tr>
<td>transient: NMSE</td>
</tr>
<tr>
<td>random</td>
</tr>
<tr>
<td>LHC</td>
</tr>
<tr>
<td>D-opt (offl.)</td>
</tr>
<tr>
<td>D-opt (onl.)</td>
</tr>
<tr>
<td>QBC</td>
</tr>
</tbody>
</table>

Appendantly the results the random distribution of the design points leads to the biggest error both at the steady state behavior and unsteady state one. The LHC distribution leads to a reduced error because of its uniform covering of the input space and the D-optimal distribution reduces this error further. In conclusion the model based experiment designs precede to the best results both at the steady state and transient error, thereby the D-optimal experimental design is the best.

To test the criteria (14) and (15) in combination with the D-optimal criterion (13) the steady state behavior of
the model committee is evaluated once global (1000 LHC distributed points) and once local around the three local minima. At the beginning of the online DoE the factor $\mu$ is adapted to 0 and at the end to 1. The transition is defined by a sigmoid function (see Poland (2002)). The results are shown in table 3.

Table 3. steady state validation data

<table>
<thead>
<tr>
<th></th>
<th>global (NAE)</th>
<th>local (NAE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$-opt (off.)</td>
<td>0.0578 ± 0.0089</td>
<td>0.1460 ± 0.0236</td>
</tr>
<tr>
<td>$D$-opt (14)</td>
<td>0.0245 ± 0.0034</td>
<td>0.0477 ± 0.0107</td>
</tr>
<tr>
<td>$D$-opt (15)</td>
<td>0.0251 ± 0.0070</td>
<td>0.0426 ± 0.0146</td>
</tr>
<tr>
<td>$D$-opt (14)+(15)</td>
<td>0.0236 ± 0.0060</td>
<td>0.0450 ± 0.0144</td>
</tr>
</tbody>
</table>

As shown in table 3 the criteria (14) and (15) lead to a global or local improved steady state behavior. Furthermore the steady state accuracy can be improved by concentrating on the area of potential optima. In conclusion a combination of both criteria is a compromise for both.

3.2 Measured data

A supercharged Otto engine is used for the evaluation with real measurement data. The engine is being excited in partial load. The inputs are the engine speed and the valve lift. The model factors are torque, exhaust temperature in front of the turbo charger (Temp. VT) and in front of the catalytic converter (Temp. VK). The slope of the parametric input signal is configured with a maximal ramp time of 2 seconds and a minimal dwell time of $T_{h,min} = 10$s. The results are presented in table 2.

To achieve an optimal model quality the dwell time must be chosen under consideration of the process dynamic and it must be chosen higher to get an better model of the temperatures. Apparently the model quality can be increased significantly by using model based experimental designs. This shows the application of the proposed criteria for the DoE in practice.

4. SUMMARY

In this contribution various methods are presented for the excitation of dynamic systems which are based on the APRBS. Model free/model based and offline/online experimental designs are discussed. With the help of simulation results and measured data at engine test benches it can be shown that the model based online methods which implicate the system dynamic in terms of model knowledge lead to better results than the offline methods. Those only consider the distribution at the input space without considering the system dynamic.

In addition methods are considered to optimize the DoE in respect to the later application of the models. Therefore optimization criteria are formulated, which allow to improve the steady state behavior of the models or to guarantee the model quality mainly in areas of optima. The presented DoE allow to identify more accurate dynamic models under consideration of their later application. This helps to consider the transient engine behavior in model based optimization tasks. Thus a reduction of emissions and fuel consumption can be reached using the presented methods in practice.

REFERENCES


