Abstract—In this paper, we consider a composite independent and coherent decode-forward (DF) relaying scheme and analytically evaluate its outage performance over Rayleigh fading channels. We examine the link-state regimes in which the optimal strategy is either direct transmission, independent DF (IDF), or coherent DF (CDF), depending on the relation among the links. Outage probabilities at both the relay and destination are determined in order to analytically derive the overall outage performance. With full channel state information (CSI), the relay can save power without affecting the overall outage performance. Specifically, by using only a portion of its maximum power for transmission, the relay conserves power in the IDF regime while still achieving the maximum transmission rate. Even with just statistical CSI, the relay can employ these link regime results to reduce transmit power significantly while maintaining the same outage performance as that of using full power.

I. INTRODUCTION

Relay transmission is an important technique in wireless communication as it improves the throughput and reliability of communication networks. For these reasons, relaying techniques are being introduced in future cellular standards (LTE-A, 5G) [1]. In wireless systems, the transmit power for each node is often limited. Minimizing the transmit power from a node also helps to reduce interference to other nodes. Hence, a relaying scheme that conserves transmission power while maintaining good outage performance is of interest.

Various relaying techniques were first developed in [2], [3] including decode-forward (DF), compress-forward and amplify-forward. This paper focuses on the DF relaying strategy in which the effects of noise are removed completely at the relay by decoding the message, either fully or partially, before re-encoding it to transmit to the destination. Outage performance has been analyzed for a number of half-duplex schemes, including full and partial DF relaying [4]–[13]. Recent results demonstrate that full-duplex transmission can also be used in future wireless networks by using self-interference cancellation circuits [14], [15]. In full-duplex transmission, partial and full DF relaying have the same performance for the single antenna relay channel [3]. In this paper, we consider a full-duplex full DF relaying scheme.

The schemes in [5]–[10], [16], [17] employ direct transmission (DT) and either coherent or independent DF relaying, depending on the decoding ability of the relay or the order between the source-to-relay and direct link strengths. Coherent DF relaying outperforms independent DF relaying when the source-to-relay link is much stronger than the direct link. However, when the source-to-relay link is slightly stronger than the direct link, it is not necessary to employ coherent DF relaying since independent DF relaying achieves the same rate using only partial relay transmit power [18]. Moreover, independent DF relaying does not require channel phase knowledge at the transmitters as in coherent DF relaying. Therefore, we are interested in studying the outage performance for composite independent and coherent DF relaying as it achieves the same transmission rate as coherent DF relaying with less required CSI at the transmitters and less relay transmit power.

In this paper, we analyze the outage performance of the composite independent and coherent DF relaying scheme [18]. Further, we compare the overall power savings with that of the coherent DF scheme in [2]. The proposed scheme can be categorized into three optimal transmission link-state regimes. Depending on the relation among link amplitudes, the link-state regimes describe the optimal transmission technique, either DT, independent DF or coherent DF.

This work is different from our previous work in which we analyzed the outage performance for a relaying scheme which employs either DT or coherent DF relaying with full relay power based on the link order between the source-to-relay and direct links [8]. In this work, the relay can also transmit using independent coding and only part of its power without reducing the achievable rate [18]. We provide analytical expressions for the outage probability of this composite DF relaying scheme by separately considering the outage probability at the relay and destination. We also show that the scheme achieves a full diversity order of 2 by considering the asymptotic outage behavior at high SNR. Even with only statistical transmit CSI, the relay can utilize the link regime results to save part of its transmission power with only a marginal impact on the outage performance.

II. CHANNEL MODEL

In this paper, we consider a full-duplex relay channel in which the source (S) wishes to communicate with the destination (D) with the help of a relay (R) as in Fig. 1. The discrete-time channel model is given as follows:

\[ Y_r = h_{rs}X_s + Z_r, \quad Y_d = h_{ds}X_s + h_{dr}X_r + Z_d, \]  

where \( Y_r \) (\( Y_d \)) is the signal received by \( R \) (\( D \)) and \( Z_r \) and \( Z_d \) are i.i.d complex Gaussian noises (\( \mathcal{C}\mathcal{N}(0,1) \)). \( X_s \) and \( X_r \) are the signals transmitted from \( S \) and \( R \), respectively. Each link is affected by Rayleigh fading and pathloss as follows:

\[ h_k = \tilde{h}_k/(d_k^{\alpha_k/2}) = g_k e^{j\theta_k}, \quad k \in \{rs, ds, dr\} \]
where $\tilde{h}_k$ is the small scale fading component distributed according to $\mathcal{CN}(0,1)$. The large scale fading component is captured by a pathloss model where $d_k$ is the distance between two nodes and $\gamma_k$ is the attenuation factor. Let $g_k$ and $\theta_k$ be the amplitude and phase of a link coefficient; then $g_k = |\tilde{h}_k|/d_k^{\eta/2}$ has Rayleigh distribution while $\theta_k$ has uniform distribution between $[0, 2\pi]$. We assume full CSI at the receivers and partial CSI at the transmitters, which can be obtained via feedback from the destination. Specifically, we assume $S$ and $R$ each know the phase of its respective forward link to $D$ for coherent transmission, and they both know the order between $g_{rs}^2$, $g_{ds}^2$ and $g_{ds}^2 + \frac{P_r}{P_s}g_{dr}^2$. The phase knowledge at the transmitters is only necessary for coherent transmission and the knowledge of amplitude order among $g_{rs}^2$, $g_{ds}^2$ and $g_{ds}^2 + \frac{P_r}{P_s}g_{dr}^2$ determines the optimal transmission scheme as specified in Section III-B.

III. A COMPOSITE DF SCHEME

The composite DF scheme contains both coherent block Markov coding and independent coding, meaning that the relay forwards source information both coherently and independently. These two distinct signal structures have different impacts on the transmit power and outage.

In each transmission block $i$, the source encodes new information ($m_i$) by superposing it onto the old information ($m_{i-1}$) as in block Markov coding. The source transmits the signal that conveys this superposed codeword for $m_i$ and $m_{i-1}$. Next, the relay decodes $m_i$ and then performs both block Markov coding and independent coding. Specifically, the relay performs random binning for the decoded message $m_i$ to generate a codeword for its bin index $l_i$ and superposes this onto the codeword for $m_i$. It transmits the signal of this superposed codeword (for $m_i$ and $l_i$) in block $i+1$.

The destination employs backward decoding: in block $i$, it decodes $m_{i-1}$ given that it knows $m_i$ from the decoding in block $i+1$. The destination can also employ sliding window decoding over two consecutive blocks and achieve the same transmission rate.

Using Gaussian signaling, the source and relay construct their respective transmit signals $X_s$ and $X_r$ in block $i$ as follows:

$$X_{s,i} = \sqrt{\alpha_s} W_s(m_{i-1}) + \sqrt{\beta_s} U_s(m_i),$$

$$X_{r,i} = \sqrt{k_s \alpha_s} W_s(m_{i-1}) + \sqrt{k_s \beta_s} U_r(l_{i-1}),$$

where $W_s$, $U_s$ and $U_r$ are i.i.d Gaussian signals ($\mathcal{N}(0,1)$), and $U_s$ is superposed onto $W_s$. Here, $\beta_s$ and $\beta_r$ are the transmit powers for signals $U_s$ and $U_r$, respectively; $\alpha_s$ and $k_s \alpha_s$ are the transmit powers for signal $W_s$ by the source and relay, respectively. $k_s$ is a scaling factor that relates the power allocated to transmit the same message at the source and relay in the block Markov signal structure. These power allocation parameters satisfy the following constraints:

$$\alpha_s + \beta_s \leq P_s, \quad k_s \alpha_s + \beta_r \leq P_r.$$  \hspace{1cm} (4)

where $P_s$ and $P_r$ are the transmit powers of the source and relay respectively.

Using maximum likelihood decoding, the relay utilizes the received signal $Y_r(i)$ in (1) to decode $m_i$ at the end of block $i$. The destination employs backward decoding where in block $i$, it utilizes $Y_d(i)$ to decode $m_{i-1}$ given that it knows $m_i$ form the decoding in block $i+1$.

A. Achievable Rate

The rate constraints that ensure reliable decoding at the relay and destination lead to the following achievable rate:

**Theorem 1.** [18] The achievable rate ($R$) of the considered independent and coherent DF scheme for each Gaussian channel realization satisfies the following constraints:

$$R \leq \log (1 + g_{rs}^2 \beta_s) = J_1,$$

$$R \leq \log \left(1 + g_{ds}^2 P_s + g_{dr}^2 (\alpha_s k_s + \beta_r) + 2 g_{ds} g_{dr} \sqrt{k_s \alpha_s}\right) = J_2,$$

for power allocation parameters satisfying (4) and $g_s$ as amplitudes of link coefficients.

The above scheme achieves a transmission rate of at least that of classical coherent block Markov DF [2], [19] and can optimally adapt to the link state as discussed next.

B. Optimal Transmission by Link Regime

By solving the optimal power allocation via an optimization problem for the source’s rate, the composite DF transmission scheme is adapted to the link state of the relay channel [18]. Here we summarize the link-state regimes ($R0$, $R1$, and $R2$) and the corresponding rate-optimal transmission technique for a given set of link gains.

- **$R0$ ($g_{ds}^2 \leq g_{dr}^2$): Direct transmission (DT).** This case is identical to the classical point-to-point communication in which the source sends its information directly to the destination without using the relay. The DT achievable rate is denoted by $J_0 = \log(1 + g_{ds}^2 P_s)$.

- **$R1$ ($g_{ds}^2 < g_{dr}^2 \leq g_{ds}^2 + \frac{P_r}{P_s} g_{dr}^2$): IDelaying.** Here the source only sends new information in each block and there is no power splitting between new and old information (i.e. no block Markovity). The achievable rate is obtained from (5) by setting $\alpha_s = k_s = 0$ and $\beta_s = P_s$. $\beta_r$ is positive but less than $P_r$ because in this regime, the relay need not use full power to achieve the maximum rate as later discussed in Section V.

- **$R2$ ($g_{ds}^2 > g_{dr}^2 + \frac{P_r}{P_s} g_{ds}^2$): CDF relaying.** In this link-state regime, the source coherently forwards old information $m_{i-1}$ with the relay, in addition to new information $m_i$. Hence, the achievable rate is obtained from (5) by setting $\beta_r = 0$. The block Markov power allocation parameters $\alpha_s$ and $k_s$ can be solved for in closed form as in [18].
IV. Outage Analysis

Outage probability is an important performance measure for wireless systems that require a sustained minimum rate. For a particular decoding block, we assume that no outage occurred in the previous decoding blocks in the backward decoding process. Then for a particular fading realization, outage occurs if the rate supported by the channel of the current block is below the target rate \( R \). In this section, we first derive the probability of each link-state regime in Rayleigh fading, then derive the outage probability in closed form, and finally examine the diversity order achievable by this composite DF scheme.

A. Probability of Each Link-State Regime

First, we analytically derive the probability of each link-state regime for a given distance configuration among source, destination, and relay nodes. From the channel model in Section II, \( g_2^2 = |h|^2/d_k^2 \) where \( h \) is the Rayleigh fading component. As such, the squared amplitude of each link coefficient is an exponential random variable with mean \( \mu_{ij} = \frac{P}{d_i^2} \). For \( g_2^2 \), note that we look at the distribution of \( g_2^2 \), because this is the term that appears in the link-state regime boundaries in Section III-B. Using the Rayleigh distributions of \( g_{rs}, g_{ds} \) and \( g_{dr} \), the probabilities of each link-state regime can be written in closed form as shown in the following theorem.

**Lemma 1.** The probability of each link-state regime in Rayleigh fading is as follows:

\[
\begin{align*}
\Pr(R0) &= \frac{\mu_{ds}}{\mu_{rs} + \mu_{ds}}, & \Pr(R1) &= \frac{\mu_{rs} \mu_{dr}}{(\mu_{rs} + \mu_{ds})(\mu_{rs} + \mu_{dr})}, \\
\Pr(R2) &= \frac{\mu_{rs}^2}{(\mu_{rs} + \mu_{ds})(\mu_{rs} + \mu_{dr})},
\end{align*}
\]

where \( \mu_{ij} = E[g_{ij}^2] \) for \( i \in \{d, r \} \) and \( j \in \{s, r \} \), \( \mu_{dr} = \frac{P}{d_r^2} \mu_{dr} \).

**Proof.** See Appendix A.

Simply by knowing inter-node distances, node transmit powers, and the path loss exponent in Rayleigh fading, the probability of each link-state regime can be calculated analytically using Lemma 1. This closed form link regime probability will help in computing the outage in the next section and in analyzing the relay power savings in Section V.

B. Outage Probability

Because our scheme employs backward decoding, outage events are limited to just one transmission block, even though information is sent over two consecutive blocks when block Markov coding is utilized in link regime \( R2 \). To determine the overall outage probability, lets first understand outage events in each link-state regime. In link-state regime \( R0 \), \( g_{rs} \leq g_{ds} \) and the outage probability is computed as in point-to-point transmission. In link regime \( R1 \), an outage occurs only when the relay is in outage. This is because if the relay can decode the source’s information, then so can the destination because of the stronger combined links from the source and relay. In link regime \( R2 \), outage can occur separately at the relay or destination. Only in this link regime can an outage can occur at the destination even when there is no outage at the relay. Next we formulate the outage probability.

**Lemma 2.** For a target rate \( R \), given the relay rate constraints in Theorem 1 and the direct transmission rate defined as \( \log(1 + g_{ds}^2P_s) \), the average outage probability \( \bar{P}_{out} \) of the composite DF scheme can be formulated as follows:

\[
\bar{P}_{out} = \bar{P}_{dt} + \bar{P}_{relay} + \bar{P}_{dest},
\]

with \( \bar{P}_{dt} = \mathbb{P}[R > J_0] \cap J_0], \)

\[
\bar{P}_{relay} = \mathbb{P}[R > J_1] \cap (R1 U R2)]
\]

\[
\bar{P}_{dest} = \mathbb{P}[J_2 < R \leq J_1] \cap R2]
\]

where \( \bar{P}_{dt} \) is the outage probability of direct transmission that occurs in link-state regime \( R0 \); \( \bar{P}_{relay} \) is the outage probability at the relay applicable in link-state regimes \( R1 \) and \( R2 \); and \( \bar{P}_{dest} \) is the outage probability at the destination applicable only in link-state regime \( R2 \).

**Proof.** See Appendix B.

Individual outage probabilities \( \bar{P}_{dt}, \bar{P}_{relay} \) and \( \bar{P}_{dest} \) are evaluated in closed form as in the following theorem.

**Theorem 2.** The average outage probability \( \bar{P}_{out} \) of the composite DF scheme in Rayleigh fading for a given power allocation can be evaluated as follows:

\[
\bar{P}_{out} = \bar{P}_{dt} + \bar{P}_{relay} + \bar{P}_{dest},
\]

\[
\bar{P}_{dt} = 1 - e^{-\frac{\eta_1^2}{\mu_{ds}}} - c_1 \left( 1 - e^{-\frac{\eta_1^2}{\mu_{ds}}} \right) \frac{\eta_1^2}{\mu_{ds}} \int_0^{\beta_1} \frac{2}{\mu_{ds}} g_{ds} e^{-\left( \frac{\eta_1^2 g_{ds}}{\mu_{ds}} + \frac{\eta_1^2}{\mu_{ds}} \right) g_{ds}} dg_{ds}
\]

\[
\bar{P}_{relay} = 1 - e^{-\frac{\eta_1^2}{\mu_{rs}}} - c_2 \left( 1 - e^{-\frac{\eta_1^2}{\mu_{rs}}} \right) \frac{\eta_1^2}{\mu_{rs}} \int_0^{\beta_1} \frac{2}{\mu_{ds}} g_{ds} e^{-\left( \frac{\eta_1^2 g_{ds}}{\mu_{ds}} + \frac{\eta_1^2}{\mu_{ds}} \right) g_{ds}} dg_{ds}
\]

where \( c_1 = \frac{\mu_{ds}}{\mu_{rs} + \mu_{ds}}, \ c_2 = \frac{\mu_{rs}}{\mu_{rs} + \mu_{ds}}, \ \beta_1 = \sqrt{\frac{2R - 1}{P_s}}, \ \eta_1 = \sqrt{\frac{2R - 1}{\beta_s}}, \ \zeta_1 = -g_{ds} \sqrt{\alpha_s + \sqrt{g_{ds}^2(\alpha_s - P_s) + 2R - 1}}.

**Proof.** Using the Rayleigh distribution of \( g_{ds}, g_{rs} \) and \( g_{dr} \) as shown in Appendix B.

Theorem 2 provides the outage probabilities for a fixed power allocation. These outage expressions are necessary in deriving the diversity order and in studying the impact on outage performance when conserving relay power in the IDF link regime, \( R1 \), as will be shown in the following sections.

C. Diversity Analysis

The diversity order of a transmission scheme reveals how fast the error rate decreases at high SNR. For the composite DF relaying scheme, the diversity order is obtained by analyzing the outage probabilities in (8) at high SNR. However, from the formulation in (7), it is intuitively evident that the outage probability for each link regime is proportional to \( 1/\text{SNR}^2 \) as each regime requires at least two different links to be weak in order to have an outage at the relay or destination. For \( R0 \), outage occurs if \( g_{ds} \) is weak and \( g_{ds} \geq g_{rs} \). Similarly in \( R1 \), an outage occurs when both \( g_{ds} \) and \( g_{rs} \) are weak.
are weak. For $R_2$, an outage at the relay occurs if $g_{rs}$ is weak and $g_{rs} \geq \sqrt{\frac{a^2}{g_{ds}^2} + \frac{a^2}{g_{dr}^2}}$, hence, all three links must be weak, indicating that the relay outage probability in $R_2$ is proportional to $1/\text{SNR}^3$. If no outage occurs at the relay in $R_2$, then an outage at the destination occurs if $g_{ds}$ and $g_{dr}$ are weak while $g_{rs}$ is strong, implying that $P_{\text{dest}}$ is proportional to $1/\text{SNR}^2$. The next corollary shows the asymptotic outage probabilities at high SNR.

**Corollary 1.** The asymptotic outage probability of composite DF relaying at high SNR approaches the following values:

\[
\begin{align*}
    P_{dt} &= \frac{(2^R - 1)^2}{2\mu_ds\mu_{rs}P^2}, \\
    P_{\text{relay}} &= \frac{(2^R - 1)^2}{2\mu_{ds}\mu_{rs}bP^2}, \quad (9) \\
    P_{\text{dest}} &= \frac{(2^R - 1)^2}{2\mu_{ds}\mu_{dr}P^2} \left( \sqrt{\frac{a}{a-1}} \sinh^{-1}\left(\sqrt{a-1}\right) - 1 \right),
\end{align*}
\]

where $P = P_r = P_s$, $a = \alpha_s/P_s$, $b = \beta_s/P_s$ and $\sinh^{-1}$ is the inverse of the hyperbolic sine function. These expressions show that the diversity order is 2, the maximum diversity for the basic relay channel.

**Proof.** $P_{dt}$ and $P_{\text{relay}}$ in (9) are obtained directly using the second order Taylor series expansion for the exponential functions in (8). $P_{\text{dest}}$ is obtained by taking the second order Taylor series expansion for the exponential functions in (8) and evaluating the integral as shown in [20, pp. 64,68].

\[ V. \text{ POWER SAVINGS} \]

Power efficiency is an important design criteria as wireless devices operate with limited power. A wireless device that conserves power both extends its own battery life and creates less interference to other nodes in the network.

**A. Relay Power Savings in Link-State Regime $R_1$**

Recall that in regime $R_1$, the relay can conserve power while still achieving the maximum possible rate [18]. In $R_1$, the optimal relay transmit power $P_r^*$ is written as:

\[
\tilde{P}_r^* = g_{dr}^2(g_{rs}^2 - g_{ds}^2)P_r < P_r \quad (10)
\]

Note that in $R_1$, the optimal $\beta_r$ is equivalent to $\tilde{P}_r^*$ in (10). For the ranges of source-to-relay link gains within $R_1$, $\tilde{P}_r^*$ is always less than the maximum $P_r$, except at the upper boundary of the regime when $\tilde{P}_r^* = P_r$.

In fast fading it can be impractical for the relay to adapt its transmit power to each channel realization because it is difficult for the relay to obtain instantaneous channel state information (CSI) for the forward link, $g_{dr}$, and the direct link $g_{ds}$. As such, here we analyze the power savings percentage obtained using a suboptimal power $\tilde{P}_r^*$ for IDF relaying in $R_1$, obtained from long-term statistics of channel gains as follows:

\[
\tilde{P}_r^* = \mu_{dr}^{-1}(\mu_{rs} - \mu_{ds})P_r. \quad (11)
\]

In Section VI, we numerically compare the outage performance when using $P_r^*$ and full relay transmit power $P_r$.

**Remark 1.** With perfect instantaneous CSI, relay power savings are realizable for most node distances configurations. Specifically, whenever link condition $\frac{g_{ds}^2}{g_{dr}^2} \leq \frac{g_{ds}^2}{g_{dr}^2} \leq \frac{g_{ds}^2}{g_{dr}^2} + \frac{g_{dr}^2}{g_{ds}^2}$ is satisfied for a specific channel fading realization, link-state regime $R_1$ is applicable and the relay conserves power. However, with long-term CSI, the relay saves power only when the long-term channel satisfies the link conditions for $R_1$, i.e. if $\mu_{ds} < \mu_{rs} \leq \mu_{ds} + \frac{g_{dr}^2}{g_{ds}^2} \mu_{dr}$.

**B. Power Savings Percentage**

In this section, we derive the overall power savings percentage obtained by using only the minimum required relay transmit power in link-state regime $R_1$ with long-term CSI, $P_r^*$. Note that this overall power savings contains both the source and relay transmit power. Using the probability of each link-state regime as derived in (6), the total transmit power when using full relay power ($P_r$) is

\[
P_T = P_s \Pr(\mathcal{R}(0)) + (P_s + P_r)(\Pr(\mathcal{R}(1)) + \Pr(\mathcal{R}(2)))
\]

\[
= P_s + P_r \frac{\mu_{rs}}{\mu_{rs} + \mu_{ds}}, \quad (12)
\]

while the total transmit power when the relay saves power is:

\[
P_M = P_s \Pr(\mathcal{R}(0)) + (P_s + \tilde{P}_r^*)(\Pr(\mathcal{R}(1)) + (P_s + P_r)\Pr(\mathcal{R}(2)).
\]

\[
(13)
\]

Then, the power savings percentage can be derived as in the following corollary.

**Corollary 2.** The power savings percentage ($\Gamma_p$) obtained from utilizing the minimum required relay transmit power is computed as follows

\[
\Gamma_p = \frac{\mu_{rs}(\mu_{dr} + \mu_{ds} - \mu_{rs})}{(\mu_{rs} + \mu_{dr})(\mu_{ds} + \mu_{rs})(P_r^* + \mu_{rs})} \times 100\%
\]

\[
(14)
\]

**Proof.** Obtained from $P_T$ in (12), $P_M$ in (13) and by using $\Gamma_p = (P_T - P_M)/P_T \times 100\%$.

\[ VI. \text{ NUMERICAL RESULTS} \]

In this section, we present numerical results for the outage probabilities of the composite DF scheme. In these simulations, we set the path loss exponent $\gamma = 2.4$, $\tilde{R}$(target) = 5bps/Hz and assume Rayleigh fading such that the average channel gain for each link is $\mu_{ij} = \frac{\alpha_i}{\alpha_j}$. All simulations are obtained using $10^6$ samples for each fading channel. With these settings, we define the average received SNR in dB at the destination for the signal from the source as follows:

\[
\text{SNR} = 10 \log(P/\text{db}_1(a_i)).
\]

(15)

In these simulations, the power allocation parameters are numerically varied to obtain the best outage performance.

Figure 2 shows the outage probability for the composite DF scheme when the relay uses full power and the minimum required power based on the channel statistics as in (11). Results confirm Corollary 1 as the proposed scheme achieves the full diversity order of 2. These results also show that using the minimum required relay power under statistical CSI may or may not degrade the outage performance compared to using full power, depending on the node-distance configuration. (Note there is no outage degradation with perfect CSI.)
In this paper, we analyze the outage performance of a composite independent and coherent DF relaying scheme. The optimal transmission for the considered scheme switches among direct transmission, independent DF relaying, and coherent DF relaying based on the link-state of the system. We analytically derive the outage probability of this composite scheme in a Rayleigh fading environment. Further, we establish that relay power can be conserved in the independent DF regime without degrading outage performance for many inter-node distance configurations under either perfect instantaneous CSI or just long-term statistical CSI at the relay. These characteristics demonstrate the robustness of the composite scheme in fading environments.

APPENDIX A: PROOF OF LEMMA 1

The probabilities for link-state regimes are given as follows:

- \( P_r(R0) = \int_{g_{ds}}^{\infty} \int_{g_{rs}}^{\infty} f_2(g_{ds}, g_{rs}) dg_{rs} dg_{ds} \),
- \( P_r(R1) = \int_{g_{ds}}^{\infty} \int_{g_{rs}}^{\infty} f_1(g_{ds}, g_{dr}, g_{rs}) dg_{rs} dg_{dr} dg_{ds} \),
- \( P_r(R2) = 1 - P_r(R1) - P_r(R0) \).
for the outage probability
\[ P_{\text{outage}} \] at the destination is given as follows.

\[ P_{\text{dest}} = \int_{1}^{\infty} \left( \int_{0}^{1} f_1(g_{ds}, \tilde{g}_{dr}, g_{rs}) \frac{4 g_{ds} g_{rs}}{\mu_{ds} \mu_{rs}} \exp \left[ -\left( \frac{g_{ds}^2}{\mu_{ds}} + \frac{g_{rs}^2}{\mu_{rs}} \right) \right] \right) \frac{1}{2 \pi} ds \]

\[ f_2(g_{ds}, g_{rs}) = \frac{4 g_{ds} g_{rs}}{\mu_{ds} \mu_{rs}} \exp \left[ -\left( \frac{g_{ds}^2}{\mu_{ds}} + \frac{g_{rs}^2}{\mu_{rs}} \right) \right] \] (17)

By evaluating these expressions, we obtain the probability for each link-state regime as in Lemma 1.

**Appendix B: Proof of Lemma 2 and Theorem 2**

**A. Proof of Lemma 2**

In \( \mathcal{R}_0 \), DT is optimal and outage can only occur at the destination, denoted as \( P_{\text{dt}} \). In \( \mathcal{R} \), outage can only occur at the relay because the path is used in this regime. In \( \mathcal{R}_2 \), outage can occur separately at the relay or at the destination. Outage at the destination in \( \mathcal{R}_2 \) is denoted as \( P_{\text{dest}} \). If we combine the outage at the relay in \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \), we obtain \( P_{\text{delay}} \).

For the outage formulation in Lemma 2 to be correct, the following identity should be true.

\[ 1 - P_{\text{outage}} = P[R \leq J_0] P[0] P[R_0] + P[R \leq \min(J_1, J_2)] P[R \leq J_1] P[R \leq J_2] \]

where \( P_{\text{outage}} = P_{\text{dt}} + P_{\text{delay}} + P_{\text{dest}} \) as in (7). Note that all three link-state regimes are disjoint. As such, (18) can be simplified to the identity \( (1 = P[R_0] + P[R_1] + P[R_2]) \).

**B. Proof of Theorem 2**

First, let \( g_{ds} = \sqrt{P_{\text{ds}} g_{dr}} \). Then, the outage probability for each case in (7) can be analyzed as follows,

1) **Outage Probability of Direct Transmission**: The analysis for the outage probability \( P_{\text{dt}} \) in (8) is straightforward by using the exponential distribution of \( g_{ds}^2 \) and \( g_{rs}^2 \) as shown in [8].

2) **Outage Probability at the Relay**: From (7), the outage at the relay is given as follows.

\[ P_{\text{relay}} = P[(R > J_1) \cap (R \leq J_2)], \]

where \( \beta_s = P_s \) in \( \mathcal{R}_1 \). Formula (19) is similar to \( P_{\text{dt}} \) in (7) except replacing \( \beta_s \) with \( P_s \), \( g_{ds} \) and \( g_{rs} \) with \( g_{ds} \) and \( g_{rs} \) with \( g_{ds} \). Hence, \( P_{\text{relay}} \) in (8) is similar to \( P_{\text{ds}} \) except replacing \( \beta_s \) with \( P_s, \lambda_{ds} \) with \( \lambda_{ds} \) and \( \lambda_{ds} \) with \( \lambda_{ds} \).

3) **Outage Probability at the Destination**: From (7), the outage at the destination is given as follows.

\[ P_{\text{dest}} = P[\xi_1 < 2R - 1, g_{rs} > \max(\eta_1, \eta_3)] \]

\[ = P[g_{ds} < \beta_1, g_{dr} > \xi_1, g_{rs} > \eta_1] + P[g_{ds} < \beta_1, g_{dr} > \xi_1, g_{rs} > \eta_3] \]

\[ = P(g_{ds} < \beta_1, g_{dr} < \xi_1, g_{rs} > \eta_1) + P(g_{ds} < \beta_1, g_{dr} < \xi_1, g_{rs} > \eta_3) \]

where \( f_1 \) as in (17), \( \xi_1 = \frac{g_{dr}^2 P_s + g_{ds}^2 P_s + 2 g_{ds} g_{dr} \sqrt{P_s \lambda_{ds}}}{\beta_s} \), \( \eta_1 = \sqrt{\frac{2R - 1}{\beta_s}} \), \( \eta_3 = \sqrt{\frac{2R - 1}{\beta_s}} \) and \( \beta_1 = \sqrt{\frac{2R - 1}{P_s}} \).

By evaluating these expressions, we obtain the probability for each link-state regime as in Lemma 1.

**Acknowledgment**

This work has been supported in part by the Office of Naval Research (ONR, Grant N00014-14-1-0645) and National Science Foundation (NSF, Grant No. DGE-1325256).

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the ONR or NSF.

**References**


