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A LOGICAL ANALYSIS OF SOME VALUE CONCEPTS¹

FREDERIC B. FITCH

The purpose of this paper is to provide a partial logical analysis of a few concepts that may be classified as value concepts or as concepts that are closely related to value concepts. Among the concepts that will be considered are *striving for*, *doing*, *believing*, *knowing*, *desiring*, *ability to do*, *obligation to do*, and *value for*. Familiarity will be assumed with the concepts of logical necessity, logical possibility, and strict implication as formalized in standard systems of modal logic (such as S4), and with the concepts of obligation and permission as formalized in systems of deontic logic.² It will also be assumed that quantifiers over propositions have been included in extensions of these systems.³

There is no intention to provide exhaustive logical analyses, or to provide logical analyses that reflect in detail the usage of so-called ordinary language. This latter task seems impossible anyhow because of the ambiguities of ordinary language and the obvious inconsistencies and irregularities of usage in ordinary language. Furthermore, the term 'ordinary language' is itself rather vague. Whose ordinary language? Should English be preferred to Chinese? Various arguments that invoke English or Latin grammatical usage are seen to be without foundation from the standpoint of Chinese.

Just as the concepts of necessity and possibility used in so-called ordinary language correspond in some degree to the concepts of necessity and possibility used in modal logic, so too it is to be hoped that the ordinary language concepts of striving, doing, believing, desiring and knowing will correspond in some degree to the concepts that we will partially formalize here. Also, just as there are various slightly differing concepts of possibility

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¹ An earlier draft of this paper was presented as a retiring presidential address to the Association for Symbolic Logic; read before the Association at Atlantic City, New Jersey, December 27, 1961.

² For example see A. R. Anderson, *The formal analysis of normative systems*, Technical Report No. 2, Contract No. SAR/Nonr-609(16), Office of Naval Research, Group Psychology Branch, 1956; also, by the same author, *A reduction of deontic logic to alethic modal logic*, *Mind*, n.s. vol. 67 (1958), pp. 100-103.

³ Such quantifiers can be introduced by methods analogous to those used in R. C. Barcan (Marcus), *A functional calculus of first order based on strict implication*, this JOURNAL, vol. 11 (1946), pp. 1-16; *The deduction theorem in a functional calculus of first order based on strict implication*, *ibid.*, pp. 115-118; and F. B. Fitch, *Intuitionistic modal logic with quantifiers*, *Portugaliae mathematica*, vol. 7 (1948), pp. 113-118. See also, R. Carnap, *Modalities and quantification*, this JOURNAL, vol. 11 (1946), pp. 33-64.

and necessity corresponding to differing systems of modal logic, so too there are presumably various slightly differing concepts of striving, doing, believing, and knowing, having differing formalizations.

We begin by assuming that striving, doing, believing, and knowing all have at least some fairly simple properties which will be described in what follows, and we leave open the question as to what further properties they have.

First of all, we assume that striving, doing, believing, and knowing are two-termed relations between an agent and a possible state of affairs. It is convenient to treat these possible states of affairs as propositions, so if I say that *a* strives for *p*, where *p* is a proposition, I mean that *a* strives to bring about or realize the (possible) state of affairs expressed by the proposition *p*. Similarly, if I say that *a* does *p*, where *p* is a proposition, I mean that *a* brings about the (possible) state of affairs expressed by the proposition *p*. We do not even have to restrict ourselves to *possible* states of affairs, because impossible states of affairs can be expressed by propositions just as well as can possible states of affairs. In the case of believing and knowing, there is surely no serious difficulty in regarding propositions as the things believed and known. So we treat all these concepts as two-termed relations between an agent and a proposition. In a similar way, the concept of *proving* could also be regarded as a two-termed relation between an agent and a proposition.

For purposes of simplification, the element of time will be ignored in dealing with these various concepts. A more detailed treatment would require that time be taken seriously. One method would be to treat these concepts as a three-termed relation between an agent, a proposition, and a time. Another method would be to avoid specifying times explicitly, but rather to use a temporal ordering relation between states of affairs. This latter method might be more in keeping with the theory of relativity, in either its special or general form.

As a further step of simplification we will often ignore the agent and thus treat each of the concepts under consideration as a *class* of propositions rather than as a two-termed relation. For example, by 'striving' we will mean the class of propositions striven for (that is, striven to be realized), and by 'believing' we will mean the class of propositions believed, relativizing the whole treatment to some unspecified agent. But the agent can always be specified if we wish to do so, and we can replace classes by two-termed relations.

A class of propositions (in particular such classes of propositions as striving, knowing, etc.) will be said to be *closed with respect to conjunction elimination* if (necessarily) whenever the conjunction of two propositions is in the class so are the two propositions themselves. For example, the class of true propositions is closed with respect to conjunction elimination

because (necessarily) if the conjunction of two propositions is true, so are the propositions themselves. If α is a class closed with respect to conjunction elimination, this fact about α can be expressed in logical symbolism by the formula,

$$(\phi)(q)[(\alpha[\phi \ \& \ q]) \rightarrow [(\alpha\phi) \ \& \ (\alpha q)]],$$

where ' \rightarrow ' stands for strict implication.

We assume that the following concepts, viewed as classes of propositions, are closed with respect to conjunction elimination:

striving (for),
doing,
believing,
knowing,
proving.

For example, in the case of believing we assume:

$$(\phi)(q)[(\text{believes } [\phi \ \& \ q]) \rightarrow [(\text{believes } \phi) \ \& \ (\text{believes } q)]].$$

Here are some further concepts which are evidently closed with respect to conjunction elimination:

truth,
causal necessity (in the sense of Burks),⁴
causal possibility (in the sense of Burks),
(logical) necessity,
(logical) possibility,
obligation (deontic necessity),
permission (deontic possibility),
desire for.

A class of propositions will be said to be *closed with respect to conjunction introduction* if (necessarily) whenever two propositions are in the class, so is the conjunction of the two propositions. If α is a class closed with respect to conjunction introduction, this fact about α can be expressed in logical symbolism by the formula,

$$(\phi)(q)[[(\alpha\phi) \ \& \ (\alpha q)] \rightarrow (\alpha[\phi \ \& \ q])].$$

Except for causal, logical, and deontic possibility, all the concepts so far regarded as closed with respect to conjunction elimination could perhaps also be regarded as closed with respect to conjunction introduction, or some varieties of them could. For present purposes, however, we do not need to commit ourselves on this matter except to say that truth and causal, logical, and deontic necessity are all indeed closed with respect to conjunction introduction.

⁴ A. W. Burks, *The logic of causal propositions*, *Mind*, n.s. vol. 60 (1951), pp. 363–382.

A class of propositions will be said to be a *truth class* if (necessarily) every member of it is true. If α is a truth class, this fact about α can be expressed in logical symbolism by the formula, $(p)[(\alpha p) \rightarrow p]$. The concepts truth, causal necessity, and logical necessity are clearly truth classes. It also seems reasonable to assume that doing, knowing, and proving are truth classes, and so we make this assumption. Thus, whatever is true or causally or logically necessary is true; and (as we assume) whatever is done, known, or proved is also true.

The following two theorems about truth classes will be applied to some of the above-mentioned truth classes in subsequent theorems.

THEOREM 1. If α is a truth class which is closed with respect to conjunction elimination, then the proposition, $[p \ \& \ \sim(\alpha p)]$, which asserts that p is true but not a member of α (where p is any proposition), is itself necessarily not a member of α .

Proof. Suppose, on the contrary, that $[p \ \& \ \sim(\alpha p)]$ is a member of α ; that is, suppose $(\alpha[p \ \& \ \sim(\alpha p)])$. Since α is closed with respect to conjunction elimination, the propositions p and $\sim(\alpha p)$ must accordingly both be members of α , so that the propositions (αp) and $(\alpha(\sim(\alpha p)))$ must both be true. But from the fact that α is a truth class and has $\sim(\alpha p)$ as a member, we conclude that $\sim(\alpha p)$ is true, and this contradicts the result that (αp) is true. Thus from the assumption that $[p \ \& \ \sim(\alpha p)]$ is a member of α we have derived contradictory results. Hence that assumption is necessarily false.

THEOREM 2. If α is a truth class which is closed with respect to conjunction elimination, and if p is any true proposition which is not a member of α , then the proposition, $[p \ \& \ \sim(\alpha p)]$, is a true proposition which is necessarily not a member of α .

Proof. The proposition $[p \ \& \ \sim(\alpha p)]$ is clearly true, and by Theorem 1 it is necessarily not a member of α .

THEOREM 3. If an agent is all-powerful in the sense that for each situation that is the case, it is logically possible that that situation was brought about by that agent, then whatever is the case was brought about (done) by that agent.

Proof. Suppose that p is the case but was not brought about by the agent in question. Then, since doing is a truth class closed with respect to conjunction elimination, we conclude from Theorem 2 that there is some actual situation which could not have been brought about by that agent, and hence that the agent is not all-powerful in the sense described.

THEOREM 4. For each agent who is not omniscient, there is a true proposition which that agent cannot know.⁵

⁵ This theorem is essentially due to an anonymous referee of an earlier paper, in 1945, that I did not publish. This earlier paper contained some of the ideas of the present paper.

Proof. Suppose that p is true but not known by the agent. Then, since knowing is a truth class closed with respect to conjunction elimination, we conclude from Theorem 2 that there is some true proposition which cannot be known by the agent.

THEOREM 5. If there is some true proposition which nobody knows (or has known or will know) to be true, then there is a true proposition which nobody can know to be true.

Proof. Similar to proof of Theorem 4.

THEOREM 6. If there is some true proposition about proving that nobody has ever proved or ever will prove, then there is some true proposition about proving that nobody can prove.

Proof. Similar to the proof of Theorem 4, using the fact that if p is a proposition about proving, so is $[p \ \& \ \sim(\alpha p)]$.

This same sort of argument also applies to the class of logically necessary propositions, since this is a truth class closed with respect to conjunction elimination. Thus by Theorem 1 we have the result that every proposition of the form $[p \ \& \ \sim\Box p]$ is necessarily not logically necessary, and hence necessarily possibly false, where ' \Box ' denotes logical necessity. In other words, the proposition $\Box\sim\Box[p \ \& \ \sim\Box p]$ is true for every proposition p .⁶ In particular, if p is a true proposition which is not necessarily true, then $[p \ \& \ \sim\Box p]$ is a true proposition which is necessarily possibly false.

I now wish to describe a relation of *causation*, or more accurately, *partial causation*, which will be used in giving a definition of doing in terms of striving and a definition of knowing in terms of believing, as well as some other definitions.

I will assume that partial causation, expressed by 'C', satisfies the following axiom schemata C1–C4:

- | | | |
|-----|---|-----------------|
| C1. | $[[p \ C \ q] \ \& \ [q \ C \ r]] \ \rightarrow \ [p \ C \ r].$ | (transitivity) |
| C2. | $[p \ \& \ [p \ C \ q]] \ \rightarrow \ q.$ | (detachment) |
| C3. | $[p \ \& \ [[p \ \& \ q] \ C \ r]] \ \rightarrow \ [q \ C \ r].$ | (strengthening) |
| C4. | $[[p \ C \ q] \ \& \ [p \ C \ r]] \ \equiv \ [p \ C \ [q \ \& \ r]].$ | (distribution) |

Here ' $p \ \equiv \ q$ ' is defined as ' $[p \ \rightarrow \ q] \ \& \ [q \ \rightarrow \ p]$ '.

I will also employ an identity relation among propositions and will employ the following axiom schemata I1–I9 for this identity relation:⁷

⁶ This result in slightly different form is to be found in the two papers by Anderson cited above. He uses it in constructing a model of deontic logic in alethic modal logic and attributes it to W. T. Parry, *Modalities in the survey system of strict implication*, this JOURNAL, vol. 4 (1939), pp. 137–154, Theorem 22.8.

⁷ It is interesting to observe that I2–I9 may be used to serve as postulates for an algebra like Boolean algebra but somewhat weaker, provided that the identity symbol is regarded as a symbol for equality in such an algebra and that (in place of I1) there are added postulates to the effect that equality is symmetrical and transitive, and that the negates, conjuncts, and disjuncts of equal elements of the algebra are equal.

- I1. $[[p = q] \& (\dots p \dots)] \rightarrow (\dots q \dots)$.
- I2. $p = p$.
- I3. $p = \sim\sim p$.
- I4. $p = [p \& p]$.
- I5. $[p \& q] = [q \& p]$.
- I6. $[p \& [q \& r]] = [[p \& q] \& r]$.
- I7. $[p \& [q \vee r]] = [[p \& q] \vee [p \& r]]$.
- I8. $p = [[p \& q] \vee p]$.
- I9. $[\sim p \& \sim q] = \sim[p \vee q]$.

Notice that we do not have such theorems as $p = [p \& [q \vee \sim q]]$ and $p = [p \vee [q \& \sim q]]$.

Only a few of the axiom schemata listed above will be directly relevant in what follows. The ones most relevant are C2, C4, I1, and I6. The property expressed by C3 reflects the fact that C is only *partial* causation. If C were total causation, then C3 would clearly be unacceptable. It should also be remarked that C need not be regarded as restricted to relating states of affairs that have space-time location, but may relate any state of affairs (e.g., a mathematical truth) to other suitable states of affairs. Otherwise, the sort of knowledge defined below would be knowledge only of states of affairs that have space-time location.

Using the relation C, a definition of doing in terms of striving will now be given. It is perhaps best to regard this definition merely as an axiom schema that provides a necessary and sufficient condition for doing, and similarly in subsequent definitions. As before, reference to the agent and to time are omitted for simplicity.

D1. (does p) $\equiv \exists q[(\text{strives for } [p \& q]) \& [(\text{strives for } [p \& q]) C p]]$.

This means that an agent does p if and only if there is some (possible or impossible) situation q such that the agent strives for p and q , and a result

Also, there should be a postulate to the effect that there are at least two unequal elements of the algebra. Such an algebra provides an algebraic formulation for the Anderson-Belnap system of first degree entailments with quantifiers omitted (A. R. Anderson and N. D. Belnap, Jr., *First degree entailments*, Technical Report No. 10, *ibid.*, 1961, since the assertion that p entails q can be defined as the assertion that p equals the conjunction of p with q , or equivalently as the assertion that q equals the disjunction of q with p . This algebra was suggested to me by a list of theorems on page 21 of my paper, *A system of combinatory logic*, Technical Report No. 9, *ibid.*, 1960, and in part also by some discussions with Anderson. It also bears a close relation to the system of my paper, *The system $C\Delta$ of combinatory logic*, Technical Report No. 13, *ibid.*, 1962 (also forthcoming in this JOURNAL). The system of first degree entailment including quantifiers was also arrived at independently by Miss Patricia A. James and myself as a modified form of the system of my book *Symbolic logic* (New York, 1952) prior to the Anderson-Belnap formulation of that system. This alternative approach to the system of first degree entailment is sketched on p. vii of Miss James's doctoral dissertation, *Decidability in the logic of subordinate proofs* (Yale University, 1962).

of this striving is that p takes place. Using I1, I6, C4, and properties of existence quantification, it is easy to show that this definition gives the result that doing is closed with respect to conjunction elimination.

A definition of knowing in terms of believing is now given:

D2. (knows p) $\equiv \exists q[p \ \& \ q \ \& \ [[p \ \& \ q] \ C \ (\text{believes } [p \ \& \ q])]]$.

This means that an agent will be said to know p provided that p and some (possibly other) situation q are both true, and provided that the fact that they are both true causes the agent to believe the fact that they are both true. Thus the known fact p must be causally efficacious (as part of the conjunction $[p \ \& \ q]$) in bringing about the agent's belief that $[p \ \& \ q]$ is the case, and hence that p itself is the case, since belief is assumed closed with respect to conjunction elimination. It is easy to show that knowing, as thus defined, is a truth class closed with respect to conjunction elimination.

Ability to do can be defined in the following way:

D3. (can do p) $\equiv \exists q[(\text{strives for } [p \ \& \ q]) \ C \ p]$.

This definition can be shown to give the result that ability to do is closed with respect to conjunction elimination.

Obligation to do can be defined in terms of doing and the concept of obligation as expressed by the operator 'O' of deontic logic, as follows:

D4. (should do p) $\equiv O(\text{does } p)$.

Obligation to do, as thus defined, can be shown to be closed with respect to conjunction elimination and also with respect to conjunction introduction.

I now wish to propose a definition of desire, as follows:

D5. (desires p) $\equiv \exists q[(\text{believes } (\text{can do } [p \ \& \ q])) \ C \ (\text{strives for } [p \ \& \ q])]$.

This means that an agent desires a situation p if his belief that he can achieve the conjunction of p with some (possibly other) situation causes him to strive for that conjunction of situations. Desire as thus defined can be shown to be closed with respect to conjunction elimination.

A concept of *value*, which I now wish to consider, can be defined in the following way:

D6. (value p) $\equiv \exists q \exists r[q \ \& \ ((\text{knows } q) \ C \ (\text{strives for } [p \ \& \ r]))]$.

This means that a situation p is a value for an agent if (and only if) there is an actual situation q and situation r such that if the agent knows q then he will strive for the conjunction of p and r . In knowing q the agent may be supposed to have all the knowable relevant information concerned with the effect of his striving for the conjunction of p and r , and if this knowledge causes him to strive for this conjunction, it must be because this conjunction, and in particular p itself, is of value to him. To see why q may be supposed to contain all the knowable relevant information for the purpose at hand, let us suppose, on the contrary, that q does not contain all such relevant information. Then there might be some additional information s such that knowledge of the conjunction of q and s would cause

the agent not to strive for any conjunction of the form [$p \ \& \ t$]. But in the hypothetical case that the agent knew [$q \ \& \ s$], he would also know q because of the fact that knowing is closed with respect to conjunction elimination, and this knowledge of q , by assumption, would cause him to strive for [$p \ \& \ r$]. Thus he would be caused to strive for [$p \ \& \ r$] and also caused not to strive for [$p \ \& \ r$], and the assumption that he could know such a proposition as [$q \ \& \ s$] leads to an absurdity. Hence q may be regarded as containing all the knowable relevant information. It can be shown easily that value as thus defined is closed with respect to conjunction elimination.

The objection might be raised against the above definition of value that the agent must be assumed to be rational, since otherwise he might have all the relevant knowledge to enable him to make a choice in his own interest, and yet, being irrational, he would be caused by this knowledge to make some other choice and to strive for some outcome that would not be of value to him. One way, and perhaps the only way, to attempt to meet this objection is to maintain that all irrationality is due to lack of sufficient knowledge, so that the having of sufficient relevant knowledge already rules out any relevant amount of irrationality. According to this view, any sort of insanity would be curable simply by giving the patient sufficient knowledge of himself and of the world around him. This view would not deny that in practice there might be insuperable obstacles that prevent the communication of this knowledge to the patient, but the existence of such obstacles would not prove that irrationality was not essentially a lack of knowledge.

This definition of value of course does not guarantee that there are any values in this sense, though it seems to me not unreasonable to assume that there may be values in this sense.

A more difficult problem is the problem of the comparison of values, that is, the problem of greater and less among values. This problem will not be dealt with here.

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