Predicting performance measures using linear regression and neural network: A comparison

Anyaeche C. O.* and Ighravwe D. E.

Department of Industrial and Production Engineering, University of Ibadan, Nigeria.

Accepted 25 June, 2013

ABSTRACT

The need to make judicious use of organizational resources has put a lot of pressure on production managers and demand planners; thus, it is necessary to accurately predict what resources will yield what output. Well planned activities result in improved performance of organizational goals among which are productivity, price recovery and profitability. This work uses artificial neural network, Back Propagation Artificial Neural Network (BP-ANN), as an alternative predictive tool to multi-linear regression, for establishing the interrelationships among productivity, price recovery and profitability as performance measures. A 2-20-20-1 back propagation artificial neural network was proposed. Productivity and price recovery served as independent variables while profitability was used as the dependent variable in the BP-ANN architecture. It was observed that BA-ANN model has Mean Square Error (MSE) of 0.02 while Multiple Linear Regression (MLR) has MSE of 0.036. This study concluded that artificial neural network is a more efficient tool for modeling interrelationships among productivity, price recovery and profitability. This approach can be applied in predicting performance measures of firms.

Keywords: Sigmoid function, artificial neural network, multiple linear regressions, profitability.

INTRODUCTION

The quest to make judicious use of organizational resources has put a lot of pressure on production managers and demand planners thus the need to accurately predict what resources will yield what. Well planned activities result in improved performance of organizational goals among which are productivity, price recovery and profitability. In pursuance of these improvements, several manufacturing concepts have been proposed and applied, such as Just In Time (JIT), Total Quality Management, Quality Deployment Function, Lean Manufacturing and Six Sigma. Though they have resulted in high performance when applied, the problem of forecasting how much such application will result to have not been adequately investigated in the performance literature.

Some authors use regression analysis in predicting dependent variables performance. One of such study is that of Phusavat and Aneksitthisin (2000) which applied Multiple Linear Regressions (MLR) for establishing the interrelationship among productivity, price recovery and profitability. Also, Babalola (2001) used quadratic curve estimation, while Oluleye et al. (2012) applied activity based model in their works. This work uses the Artificial Neural Network, but compares the result with predication from Linear Regression and draws conclusion.

The Artificial neural network (ANN) is an intelligent tool developed for prediction, classification, optimization and other purposes. The ANN was developed to solve complex computational problems like natural neurons in living organisms. This is achieved through processing of information from input layer to output layer. Processing of information involves weights manipulation and adjustment has different patterns (that is, inputs with corresponding output are fed into an ANN model one after the other). The purpose of adjusting weights is to ensure that the discrepancy between predicted and actual value is minimal. This is evaluated using statistical measures (Mean Square Error [MSE], Mean Absolute Percentage Error and others), which serve as objective functions for ANN models. Due to the ANN potential of
achieving minimal error when compared with other predictive models, its applications span over several areas of human endeavours.

The ANN has been applied in addressing various managerial problems like sales forecasting, price elasticity modeling, brand analysis, new product acceptance research, and market segmentation and more (Hakimpoor et al., 2011). Shahin et al. (2001) reviewed application of ANN in geotechnical engineering and concluded that ANN has the potential of performing better or as well as other traditional predictive model in the literature. Paulo et al. (2006) also conducted review of the ANN application but along clinical domain (decision making in cancer). They conclude their study by acknowledging the superior quality of ANN results over other nonlinear forecasting models. From the studies of Shahin et al. (2001), Paulo et al. (2006) and Hakimpoor et al. (2011), it is obvious that ANN application in profitability prediction will contribute to the knowledge frontiers.

The idea of extending the successes of the ANN models in other fields to profitability prediction serves as contribution to existing literature on profitability prediction. The ANN has enjoyed wide application in industrial settings; some of these studies are as follows: Saanzogni and Kerr (2001) which applied feed-forward ANN in evaluating milk production and Fast et al. (2010) which investigated the use of the ANN in condition and diagnosis of a combined heat and power plant. It has been applied in controlling inventory in manufacturing systems (Gumus et al., 2000; Lin et al., 2009), as well as in quality control (Pacella et al., 2004). Bisconti and Park (2000) extended the ANN successes to lean production system while Mozer and Wolniewics (2000) examined its potential as a predictive model in the telecommunication industry.

The ANN has the benefit of using multiple independent variables (inputs) in developing nonlinear equations for predicting one or more dependent variables (outputs). This is done by training patterns over several epochs with specified error range and then testing the proposed network, whereas multiple linear regressions (MLR) can only predict one dependent variable at a time as its output. The interrelationships among productivity, price recovery and profitability have been established by Phusavat and Anekstithin (2000) using MLR model as shown in Equation 1. Though applied in the banking sector (Bakar and Tahir 2009), the documentation of ANN as applied to profitability prediction is sparse in the performance literature.

This knowledge gap motivated the need for the current investigation which seeks to develop the ANN architecture for establishing interrelationships among productivity, price recovery and profitability. The remaining section of this paper is organized as follows: The research methodology, applicability of the ANN in addressing the objective of this paper and the results, followed by discussion of results; and finally, the conclusion.

METHODOLOGY
Here, we deal with the description of the ANN and MLR as applied to productivity, price recovery and profitability interrelationships. The terms as used are presented as follows.

Nomenclature
The terms used in this study are given as follows:

$$X_1: \text{Productivity}; \quad X_2: \text{Price recovery}; \quad Y: \text{Profitability}; \quad X^*: \text{Normalized value}; \quad Z_{min}: \text{Minimum value of variable}; \quad Z_{max}: \text{Maximum value of variable}; \quad Y^*: \text{Normalized profitability}; \quad S: \text{Hidden layer output}; \quad H^*: \text{Adjacent layer output}; \quad \alpha: \text{Significant level}; \quad \eta: \text{Learning rate}; \quad Pt: \text{Total pattern}; \quad MSE: \text{Mean square error}; \quad MAPE: \text{Mean absolute percentage error}; \quad MAD: \text{Mean absolute deviation}; \quad RMSPE: \text{Root mean square percentage error}; \quad \text{BP-ANN: Back propagation artificial neural network}.$$

Multiple linear regression
This model like other predictive models (exponential smoothing, grey regression, ANN and more) makes use of constant values in determining value of dependent variable through linear combination of explanatory variables with multiplication of their corresponding constants value. Equation 1 presents n-value MLR model.

$$Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \ldots + \beta_nX_n + \epsilon \quad (1)$$

$\beta$ stands for the co-efficient, $n$ represents the explanatory variable, while $Y$ is the dependent variable.

The MLR has been applied extensively as predictive models for engineering and non-engineering domains (Hoyt et al., 2006; Subramanian et al., 2006; Nathans et al., 2012).

Artificial neural network
Since its introduction, several versions of the Artificial Neural Network (ANN) have appeared in the literature (Back-propagation ANN, Feed-forward ANN, Hopfield ANN). The BP-ANN (Rumelhart et al., 1986) has been established as one of the most effective ANN algorithms for supervised learning.

The BP-ANN architecture makes use of three layers (input layer, hidden layer and output layer) like other ANN architectures: the difference between BP-ANN and other ANN algorithms is the way in which weights are adjusted.

$$H = I_1^{-1}W_1 + I_2^{-1}W_2 + I_3^{-1}W_3 \quad (2)$$

Equation 2 shows how an input to a node is converted into output; $H$ (Figure 1) is transformed to actual using transfer function (Sigmoid function). This process is repeated for all preceding nodes in a network till the final layer is reached. After reaching the final layer, the predicted and actual values are compared as shown in Equation 3. The value obtained is then used in adjusting weights that enter the output layers as shown in Equation 4. This procedure of adjusting weights is used in adjusting the remaining weights in the various layers in the architecture as given in Equation 5.

$$E_{ij} = Q_j - F_j \quad (3)$$

$$E_i = W_i + (\eta \ast E_{ij} \ast H) \quad (4)$$
Error for the next back layer is evaluated as follows:

\[ E_i = H_i * (1 - H_i) * (H_i * W_j) \]   \hspace{1cm} (5)

In order to increase the performance of ANN models, the value of inputs and output are normalized, and the normalized value modulates between 0 and 1 or -1 and 1. In this paper, normalized values that modulate between 0 and 1 were used. Equation 6 is used for obtaining these values.

\[ X^* = \frac{X - X_{\text{min}}}{Z_{\text{max}} - Z_{\text{min}}} \]   \hspace{1cm} (6)

Equation 6 was also applied in normalizing productivity and price recovery for simulated data sets. A set of one hundred data was simulated for productivity, price recovery and profitability using Matlab® programming language. 75% of the simulated data was used for training the BP-ANN model while 25% was used for testing the model.

To transform the input from one layer to another, sigmoid transfer function as given in Equation 7 was used.

\[ S = \frac{1}{1 + \left( \exp - \sum_{i=1}^{M} W_i X_i \right)} \]   \hspace{1cm} (7)

**The BP-ANN model**

The proposed BP-ANN model in this work is given in Figure 2. It consists of two inputs (productivity and price recovery), two hidden layers and one output (profitability).

The steps of BP-ANN algorithm are as presented below:

1. Generate output \( (F) \) for each pattern using Equation 2
2. Calculate error \( (E) \) using Equation 3
3. Adjust weight using Equation 4
4. If \( p < P_t \), go to step 1
5. Calculate MSE
6. If MSE \( \geq \varepsilon \) or \( g < G \), go to step 1
Else, finish.

**MODEL APPLICATION AND RESULTS**

The training data sets (Appendix A) that were generated using Matlab® was used in generating the regression in Equation 8 with the aid of Microsoft excel (2010) by using 75% of that data sets that were generated in this study.

\[ Y = 0.5391 + 0.02623 * X_1 - 0.0591 * X_2 \]   \hspace{1cm} (8)

The mean sum of error (Hagan et al., 1996) was used in training the proposed BP-ANN architecture. After training
the proposed model, the following parameters setting was obtained as shown in Table 1. The results of the comparison of ANN and MLR are as presented in Table 2.

### DISCUSSION

From Table 2, the upper residual errors for MLR and BP-ANN are 0.349 and 0.228 respectively while their lower residual errors are -0.326 and -0.267, respectively. From these residual errors, BP-ANN outperformed MLR. This shows that in all cases, the BP-ANN performed better than MLR. Though these results are necessary conditions to accept that BP-ANN is a superior tool to MLR, we proceed to establish the sufficient condition that the simulated and BP-ANN data sets have the same medians using Wilcoxon rank sum test in Matlab.

The results obtained gave a value of $h = 0$, thus the null hypothesis that both data sets have equal medians at $\alpha = 0.05$ is accepted. From the aforementioned necessary and sufficient conditions, BP-ANN is a more efficient tool for establishing interrelationship among productivity, price recovery and profitability. Our observation is consistent with the study of Bakar and Tahir (2009) as well as Kumar et al. (1995) which applied

---

**Table 1. Proposed BP-ANN parameters setting.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training pattern</td>
<td>75%</td>
</tr>
<tr>
<td>Total epoch</td>
<td>10000</td>
</tr>
<tr>
<td>Testing pattern</td>
<td>25%</td>
</tr>
<tr>
<td>Learning rate</td>
<td>0.3</td>
</tr>
<tr>
<td>Number of hidden layer</td>
<td>2</td>
</tr>
<tr>
<td>Node per hidden layer</td>
<td>20</td>
</tr>
</tbody>
</table>

**Table 2. Comparison of ANN and MLR outputs using simulated data.**

<table>
<thead>
<tr>
<th>S/N</th>
<th>Simulated data</th>
<th>MLR results</th>
<th>MLR residual</th>
<th>BA-ANN results</th>
<th>BA-ANN residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.770</td>
<td>0.996</td>
<td>-0.226</td>
<td>0.778</td>
<td>-0.008</td>
</tr>
<tr>
<td>2</td>
<td>1.090</td>
<td>1.006</td>
<td>0.084</td>
<td>0.942</td>
<td>0.148</td>
</tr>
<tr>
<td>3</td>
<td>0.842</td>
<td>1.030</td>
<td>-0.188</td>
<td>0.904</td>
<td>-0.062</td>
</tr>
<tr>
<td>4</td>
<td>0.950</td>
<td>1.014</td>
<td>-0.063</td>
<td>0.976</td>
<td>-0.026</td>
</tr>
<tr>
<td>5</td>
<td>1.086</td>
<td>1.031</td>
<td>0.055</td>
<td>0.825</td>
<td>0.260</td>
</tr>
<tr>
<td>6</td>
<td>0.860</td>
<td>1.039</td>
<td>-0.18</td>
<td>0.979</td>
<td>-0.119</td>
</tr>
<tr>
<td>7</td>
<td>0.886</td>
<td>1.005</td>
<td>-0.119</td>
<td>1.030</td>
<td>-0.144</td>
</tr>
<tr>
<td>8</td>
<td>1.109</td>
<td>1.024</td>
<td>0.085</td>
<td>0.906</td>
<td>0.202</td>
</tr>
<tr>
<td>9</td>
<td>0.869</td>
<td>1.014</td>
<td>-0.145</td>
<td>0.956</td>
<td>-0.087</td>
</tr>
<tr>
<td>10</td>
<td>1.250</td>
<td>1.006</td>
<td>0.244</td>
<td>1.021</td>
<td>0.230</td>
</tr>
<tr>
<td>11</td>
<td>1.358</td>
<td>1.009</td>
<td>0.349</td>
<td>1.130</td>
<td>0.228</td>
</tr>
<tr>
<td>12</td>
<td>1.186</td>
<td>1.012</td>
<td>0.174</td>
<td>1.197</td>
<td>-0.012</td>
</tr>
<tr>
<td>13</td>
<td>0.922</td>
<td>1.007</td>
<td>-0.084</td>
<td>1.189</td>
<td>-0.267</td>
</tr>
<tr>
<td>14</td>
<td>1.086</td>
<td>1.012</td>
<td>0.075</td>
<td>1.105</td>
<td>-0.019</td>
</tr>
<tr>
<td>15</td>
<td>0.761</td>
<td>1.008</td>
<td>-0.247</td>
<td>0.793</td>
<td>-0.032</td>
</tr>
<tr>
<td>16</td>
<td>1.306</td>
<td>1.040</td>
<td>0.266</td>
<td>1.330</td>
<td>-0.024</td>
</tr>
<tr>
<td>17</td>
<td>1.288</td>
<td>1.035</td>
<td>0.253</td>
<td>1.113</td>
<td>0.175</td>
</tr>
<tr>
<td>18</td>
<td>1.246</td>
<td>1.003</td>
<td>0.243</td>
<td>1.081</td>
<td>0.164</td>
</tr>
<tr>
<td>19</td>
<td>0.866</td>
<td>1.037</td>
<td>-0.171</td>
<td>0.837</td>
<td>0.029</td>
</tr>
<tr>
<td>20</td>
<td>1.093</td>
<td>1.018</td>
<td>0.075</td>
<td>1.175</td>
<td>-0.082</td>
</tr>
<tr>
<td>21</td>
<td>0.703</td>
<td>1.029</td>
<td>-0.326</td>
<td>0.764</td>
<td>-0.061</td>
</tr>
<tr>
<td>22</td>
<td>0.978</td>
<td>1.000</td>
<td>-0.022</td>
<td>1.044</td>
<td>-0.066</td>
</tr>
<tr>
<td>23</td>
<td>0.901</td>
<td>1.011</td>
<td>-0.109</td>
<td>1.114</td>
<td>-0.213</td>
</tr>
<tr>
<td>24</td>
<td>0.798</td>
<td>1.025</td>
<td>-0.227</td>
<td>0.948</td>
<td>-0.150</td>
</tr>
<tr>
<td>25</td>
<td>0.810</td>
<td>1.026</td>
<td>-0.216</td>
<td>0.877</td>
<td>-0.067</td>
</tr>
</tbody>
</table>
Table 3. Forecasting accuracy comparison.

<table>
<thead>
<tr>
<th>Method</th>
<th>MSE</th>
<th>MAD</th>
<th>MAPE (%)</th>
<th>RMSPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP-ANN</td>
<td>0.020</td>
<td>0.115</td>
<td>11.34</td>
<td>13.73</td>
</tr>
<tr>
<td>MLR</td>
<td>0.036</td>
<td>0.169</td>
<td>17.60</td>
<td>20.32</td>
</tr>
</tbody>
</table>

real data set in comparing ANN with regression model. However, if real world data sets were used without simulated data sets in training ANN model for profitability prediction, differences may result. One reason for this variation is change in input and output neurons and also the amount of training and testing data sets which depends on the trainer’s choice. This could be investigated in further studies.

Conclusion

In this study, an alternative approach to multiple-linear regression (MLR) as a predictive model for establishing interrelationship among productivity, price recovery and profitability was proposed using back propagation artificial neural network (BP-ANN).

Simulated data set was used in training and testing four layers of BP-ANN, after which results were compared with those from the MLR. It was observed that BP-ANN results outperformed that of MLR. This study therefore concludes that the ANN is a better predictor than MLR and therefore may be applied in establishing the aforementioned relationship.

Although the quantity of outputs from a given set of input resources can be established using the ANN, the Analytical Network Process (ANP) or Analytical Hierarchical Process (AHP) may be investigated if their use can help in reducing the number of input nodes in the ANN architecture. This can be pursued as a further study. Also, an investigation of the ANN in tracking system resource leakages can be studied. Furthermore, extending the findings of this study in predicting other performance measures (e.g., system effectiveness) of other systems by combining some of the variables in this study can be pursued as further study.

Finally, it may be of interest to investigate the effect of using real-world data which may be more complex than the machine generated datasets. These are refereed also for further studies.

REFERENCES

Babalola JB, 2001. University Funding, Responses and Performances under a Declining Economy in Nigeria, Department of Educational Management, University of Ibadan, Ibadan, Nigeria.


Appendix A

Appendix A shows the simulated data.

Appendix A. Simulated data for training pattern.