Development of a User Friendly Toolbox for Advanced Control Education

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Abstract: This paper presents a toolbox developed in Matlab for analysis and design of fractional order control systems that can be used in advanced control courses within the context of master and doctorate education. The toolbox includes a user friendly interface for every function. One can obtain time and frequency analysis tools both for the fractional order control systems and fractional order control systems with parametric uncertainty. On the other hand, the frequency response computation of the uncertain fractional order control systems, which may be required for robust design of the systems, can be easily computed using this toolbox.

Keywords: Advanced control education, User friendly toolbox, Fractional order control, Bode plot, Nyquist plot, Nichols Plot, Bode envelope, Nyquist envelope.

1. INTRODUCTION

Much of the materials in advanced courses on control appear to have changed during the last decade. The concepts of fractional order expressions and fractional order models have strong links to the material thought in recent advanced control courses in master and doctorate education. Fractional order control is an important topic in its own right but its theory and applications are not easy for students. Development of a user friendly toolbox will make the applications of fractional order control systems possible and reinforce the student understanding about this new concept. However, including such analysis and design tools, which are developed for new subjects, to the materials of the control courses will enable the students to be more familiar to new developments in control (Tan et al., 2003). In fact, the fractional calculus has been discussing since 1695. In recent years, it has drawn the attention of many famous mathematicians (Caponetto et al., 2010). During the last two decades, many studies have been done for realization and implementation of the fractional order systems. Consequently many methods have been developed for solution of the fractional order integro-differential equations (Chen et al., 2009). At present, the fractional calculus can easily be used in many areas as an alternative method for solution of control problems. It also finds numerous applications in control engineering (Monje et al., 2008; Malti et al., 2008; Bettou et al., 2008; Hamamcę, 2008).

Recent trends in advanced control education are highly influenced from the development in fractional order calculus. Many control lectures of master and doctorate education have a tendency to include the subjects related with fractional order control systems (FOCS). Development of a toolbox for the analysis and design of FOCS will support the lectures in this direction. There are some powerful graphical tools in classical control, such as Nyquist plot, Bode plots, Nichols plots, and step response analysis which are widely used to evaluate the frequency domain and time domain behaviors of the systems. However, these results are related to the integer order control systems. The extensions of these results to FOCS are important. Some studies in this direction have already been done in (Oustaloup, 1991; Oustaloup et al., 2000; Valerio, 2005; Martin and Milos, 2006; Schegel and Cech, 2004; Yeroglu and Tan, 2009) and the references therein. For example the CRONE (French abbreviation of the ñCommande Robuste dôOrdre Non Entierò) toolbox has been provided for fractional order control systems (Oustaloup, 1991; Oustaloup et al., 2000). Duarte Valerio (2005) proposed the ñToolbox ninteger for Matlab v. 2.3ò (Valerio, 2005). A Java application program working over the internet can be found in (Martin and Milos, 2006; Schegel and Cech, 2004). Some preliminary studies in this direction have been proposed in (Yeroglu and Tan, 2009). But, most of the present studies do not have a user friendly interface for the user and they require sufficient mathematical background of the FOCS which may not be easy to understand for the students.

As usual in control education, the fractional order control subjects include complex and theoretical materials. Some analysis and design tools developed for FOCS will make these subjects easy to analyze (Dzielinski and Sierociuk, 2008). In this paper a user friendly toolbox with an interface is developed to obtain Nyquist plot, Bode plots, Nichols plot and step response for FOCS and fractional order interval control systems (FOICS). The toolbox also provides an option to compute the Bode and Nyquist envelopes for FOICS. The toolbox is named as ñUser Friendly Toolbox for Fractional Order Control Systems - (UFT-FOCS)ò. Usage of the UFT-FOCS is clearly illustrated via examples.

The paper is organized as follows: Section 2 gives some preliminaries of FOCS and FOICS. Section 3 includes brief information about some existing toolboxes. Section 4 introduces ñUFT-FOCSò for FOCS and FOICS. Section 5 provides examples to illustrate the usage of the toolbox. Section 6 includes the concluding remarks.

2. SOME PRELIMINARIES OF THE FOCS AND FOICS

Fractional calculus can be considered to be generalization of integration and differentiation of the integer order expressions to the non-integer order one. The most frequently used integro-differential definitions are Grünwald-Letnikov, Riemann-Liouville and Caputo expressions (Oldham and Spanier, 2006). Numerical solutions for Grünwald-Letnikov, Riemann-Liouville and Caputo expressions can be found in (Caponetto et al., 2010). On the other hand, the most general formula for the Laplace transformation of the fractional order integro-differential expressions, which are extensively used in control applications, can be given as (Xue et al., 2007),

$$L\left\{\frac{d^m f(t)}{dt^m}\right\} = s^m L\{f(t)\} - \sum_{k=0}^{n-1} s^k \left[\frac{d^{m-1-k} f(t)}{dt^{m-1-k}}\right]_{t=0} \qquad (1)$$

where n is an integer number and m satisfies, $n-1 < m < n$. Generally, dynamic behaviors of the systems can be analyzed using transfer functions of the control systems. Consider a single input single output fractional order control system. Let $y(t)$ be the output and $x(t)$ be the input of the system. The relation between input and output can be defined as

$$\begin{aligned} & a_n \frac{d^{\alpha_n} y(t)}{dt^{\alpha_n}} + a_{n-1} \frac{d^{\alpha_{n-1}} y(t)}{dt^{\alpha_{n-1}}} + \cdots\cdots + a_0 \frac{d^{\alpha_0} y(t)}{dt^{\alpha_0}} \\ & = b_m \frac{d^{\beta_m} x(t)}{dt^{\beta_m}} + b_{m-1} \frac{d^{\beta_{m-1}} x(t)}{dt^{\beta_{m-1}}} + \cdots\cdots + b_0 \frac{d^{\beta_0} x(t)}{dt^{\beta_0}} \end{aligned} \qquad (2)$$

Transfer function of the system can be obtained by taking Laplace transform of Eq. 2 as follows

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0 s^{\alpha_0}} \qquad (3)$$

where, $\alpha_n > .. > \alpha_0 \geq 0$ and $\beta_m > .. > \beta_0 \geq 0$ generally are real numbers, a_k, $(k = 0,1,2...,n)$ and b_l, $(l = 0,1,2...,m)$ are constants (Xue et al., 2007).

Bode, Nyquist and Nichols plots of any control system can be computed using frequency domain behavior of the system. The frequency domain of the control system can be obtained by substituting $s = j\omega$ in the transfer function. Thus, the frequency analysis tools for the FOCS can be obtained by substituting $s = j\omega$ in Eq. 3. On the other hand, simulation of the FOCS can be done using the integer order approximations of the fractional order transfer function. One of the most important approximations for fractional order systems is the Continuous Fractional Expansion method (Caponetto et al., 2010; Xue et al., 2007). Thus, the step response simulation of the fractional order system can be obtained using the Continuous Fractional Expansion method.

Another important topic in control theory is robust control of uncertain systems. Bode and Nyquist envelopes of the fractional order interval transfer function (FOITF) can be used for analysis and design of the FOICS. Bode envelopes of the fractional order interval plant can be obtained using the magnitude and phase extremums of the numerator and denominator of the FOITF. Nyquist envelope is also an important analysis and design tool for FOICS. Suppose that a closed-loop system with an uncertain plant of the following form is given

$$G(s,a,b) = \frac{N(s,b)}{D(s,a)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0 s^{\alpha_0}} \qquad (4)$$

The numerator and denominator polynomials of the fractional order interval plant are a fractional order interval polynomial. The parameters $a = [a_0, a_1, ..., a_n]$ and $b = [b_0, b_1, ..., b_m]$ might be uncertain parameters of the plant. Exact values of these parameters may not be known. However these parameters can be estimated in certain intervals. Parameters of the plant given in Eq. 4 with parametric uncertainty structure can be defined as, $a_i \in [\underline{a_i}, \overline{a_i}],\ i = 0,1,..,n$ and $b_i \in [\underline{b_i}, \overline{b_i}],\ i = 0,1,..,m$, where, $\underline{a_i}$ and $\underline{b_i}$ are lower limits, $\overline{a_i}$ and $\overline{b_i}$ are upper limits of the parameters respectively. The magnitude and phase extremums of $G(s,a,b)$ can be found from the magnitude and phase extremums of polygons corresponding to $N(s,b)$ and $D(s,a)$. Bode envelopes of the FOICS can be obtained using magnitude and phase extremums of the $N(s,b)$ and $D(s,a)$. Nyquist Envelope of the FOICS can also be obtained using the value set of the system in parameter space. Detailed explanations and related theorems for obtaining Bode and Nyquist envelopes are provided in (Yeroglu et al., 2010a).

3. SOME EXISTING PROGRAMS FOR FRACTIONAL ORDER CONTROL SYSTEMS

Since Matlab is a high-level technical computing language, it deserves to become an indispensible tool for the solution of control problems. Thus, the analysis and design tools can be developed using Matlab. Some analysis and design tools for the FOCS have already been developed in Matlab and other high level languages. This section provides a brief description of some analysis tools developed for FOCS available in the literature.

3.1. CRONE Toolbox

The CRONE toolbox developed using Matlab has been dedicated to engineers and researchers who are interested in automatic control (Oustaloup et al, 2000). Robust analysis of the FOCS can be done using CRONE. The CRONE is made up of three modules that deal with special applications of fractional derivative. A complicated visual sight, the necessity of having the knowledge of complex theoretical materials and many integrated windows of the CRONE are its own disadvantages for students.

3.2. Toolbox "ninteger" for Matlab

Duarte Valerio developed a toolbox in Portugese language in 2000 and published its last version named ñToolbox ninteger for Matlab v. 2.3ò in 2005. It was developed in need of easy analysis of fractional order control systems in Matlab (Valerio, 2005). ñnintegerò is a Matlab based program which is developed for single input single output controllers in time domain and in frequency domain. Itôs suitable for free distribution and usage. The program also includes a simple Simulink library, but there are many *.m files which work on Matlab console and there comes up some difficulties for students. It is necessary to check the userôs manual carefully before using the program.

3.3. PID Control Laboratory 3.0

ñPID Control Laboratory 3.0ò is a Java application program which can be used directly over the internet or can be downloaded to the computer (Martin and Milos, 2006). It can be reached from ñwww.pidlab.comò and on the contrary to the previous referred programs, it is not Matlab based. It can run on every computer which has the Java Runtime Environment set up. ñPID Control Laboratory 3.0ò can be successfully used for FOCS, but the robustness panel of the program is kept complicated that may result some difficulties for students.

4. USER FRIENDLY TOOLBOX FOR FRACTIONAL ORDER CONTROL SYSTEMS (UFT-FOCS)

This section presents a user friendly toolbox with an interface developed in Matlab environment. Some of the program routines for step response, Bode, Nyquist, Nichols plots, Bode envelopes and Nyquist envelopes are benefited from (Yeroglu et al., 2010a; Xue et al., 2007). One can obtain the time and frequency responses of the FOCS and FOICS easily without the necessity of knowledge of the complex mathematical expressions. The main difference between UFT-FOCS and the existing programs in the literature is that the UFT-FOCS provides a user friendly interface, which one can easily enter the parameters of the fractional order plant and fractional order controller. Every function of the UFT-FOCS can be used via a single interface window. Analysis of the FOICS can also be easily done by using this interface. Besides, the interface also provides a section for Bode and Nyquist envelopes of the FOICS. The main window of the UFT-FOCS is presented in Fig. 1.

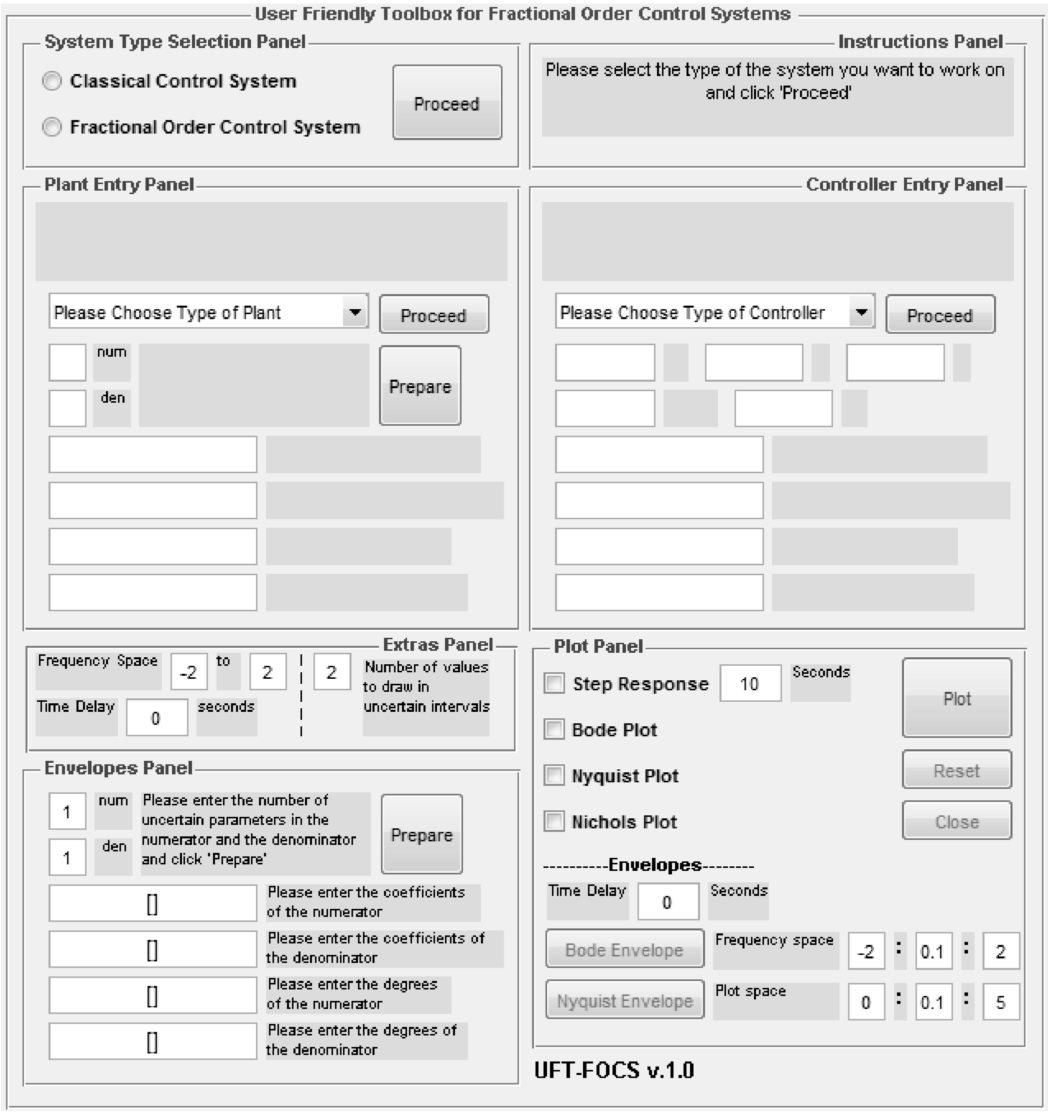

Fig. 1. Main window of the UFT-FOCS

The plot windows in the program appear on the screen independently from each other. The instruction boxes on the main window give short information about usage of the program after each step. If there is a misuse the error dialog boxes show the error message. Some preliminary study of the toolbox has been presented in the conference (ķenol et al., 2011). In this paper, the toolbox is further improved, a user friendly interface is included and the toolbox is extended for computation of the Bode and Nyquist envelopes of the FOICS.

4.1. A Brief User's Guide for UFT-FOCS Interface

A brief userôs guide is given in this section for the interface of the toolbox. Detailed userôs guide of the UFT-FOCS is provided in (ķenol, 2011).

-*Selecting the System Type*

The first step is to choose the type of the control system desired to work on. Two options are provided in the top left box in Fig. 1. One can make the choice between ñClassical Control Systemò and ñFractional Order Control Systemò.

-*Defining the Plant*

The next step is to define the transfer function of the plant. If one chose ñClassical Control Systemò, the plant has to be entered in integer order representation. Alike this, the plant would be entered in fractional order representation if one chose ñFractional Order Control Systemò.

-*Defining the Controller*

The next step is to enter the controller. While working with classical systems, it is necessary to enter both the plant and the controller with integer order representation. If one desires to study with the fractional order systems, the plant transfer function and the controller can be entered either integer order or fractional order independently from each other.

-*Systems with Parametric Uncertainty*

Plant and controller parameters of the uncertain systems can be entered in similar way with classical and fractional order systems. But the only difference is that, one has to enter the lower and upper limits of the uncertain parameters.

-*Envelopes*

Plant definition in the ñenvelopes panelò is the same with the ñsystems with parametric uncertaintyò section.

-*Plotting*

After entering the parameters of the plant transfer functions and the controller, one can choose which figure to plot. Bottom right box in Fig.1 is the plot section. After the desired figure is selected, the ñPlotò button will provide the figure on a floating window.

5. APPLICATION EXAMPLES

Several applications for FOCS and FOICS, which might be included in advanced courses in control education, can be implemented using UFT-FOCS. Some of the UFT-FOCS applications can be given in the following examples.

Example 1: *Applications of the Bode Nyquist and Nichols plots and step response of the FOCS.*

Podlubny (Podlubny, 1999), presented the PID controller in a generalized form of $PI^{\lambda}D^{\mu}$ where λ is the fractional order of the integrator and μ is the fractional order of the derivative. Let the Fractional order plant and the controller be as follows

$$G_1(s) = \frac{1}{s^{3.16} + 3s^{2.07} + 2s^{1.2}} \tag{5}$$

$$C_1(s) = 3.6 + \frac{1.63}{s^{0.5}} + 4s^{1.2} \tag{6}$$

Parameter entry of $G_1(s)$ and $C_1(s)$ in the user interface of the UFT-FOCS can be easily done as in Fig. 2. In order to define the plant of the system, one can enter the coefficients and the orders of the parameters for the numerator and the denominator. Similarly, one can enter the parameters of the $PI^{\lambda}D^{\mu}$ controller. The next step is to click the "Plot" button in order to obtain the desired plots. Step response plot of the system $C_1(s)G_1(s)$ can be obtained as given in Fig. 3. Bode, Nyquist and Nichols plots of the system can be obtained in similar way.

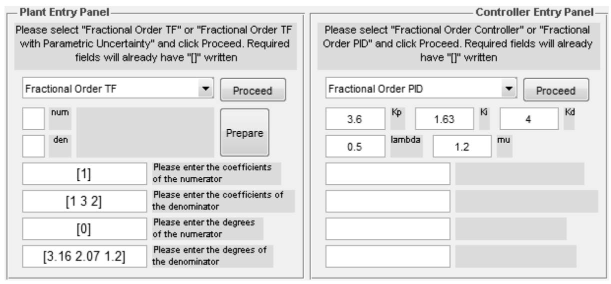

Fig. 2. System Entry Panel for Example 1

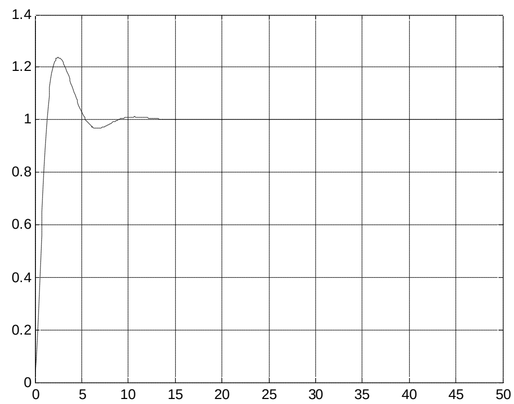

Fig. 3. Step response of the system $C_1(s)G_1(s)$

Example 2: *Applications of Bode Nyquist and Nichols plots and step response for the FOICS.*

In this example, following fractional order plant with parametric uncertainty structure and the fractional order $PI^{\lambda}D^{\mu}$ controller is considered

$$G_2(s) = \frac{[0.8\ 1.2]}{[0.8\ 1.2]s^{3.16} + [2.8\ 3.2]s^{2.07} + [1.8\ 2.2]s^{1.2}} \tag{7}$$

$$C_2(s) = 3.6 + \frac{1.63}{s^{0.5}} + 4s^{1.2} \quad (8)$$

Fig. 4 shows the parameter entry panel for the $C_2(s)G_2(s)$ system. As seen from the plant $G_2(s)$, there is one uncertain parameter in the numerator and there are three uncertain parameters in the denominator. As seen in Fig. 4, the brackets will appear just after one enters the number of uncertain parameters of the numerator and the denominator and clicks the ñPrepareò button . Then, the lower and upper limits of the uncertain parameters can be entered between the brackets.

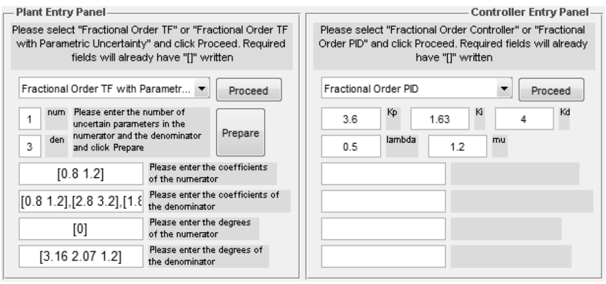

Fig. 4. System Entry Panel for Example 2

As seen from Fig. 5, the Bode plot of the $C_2(s)G_2(s)$ system can be computed for all parameter perturbations of the plant in Eq. 7. In this example, the Bode plots are computed for $4^4 = 256$ different transfer functions of the plant by taking 4 values within the interval of four uncertain parameters. Nyquist, Nichols plots and the step response can be obtained in similar way

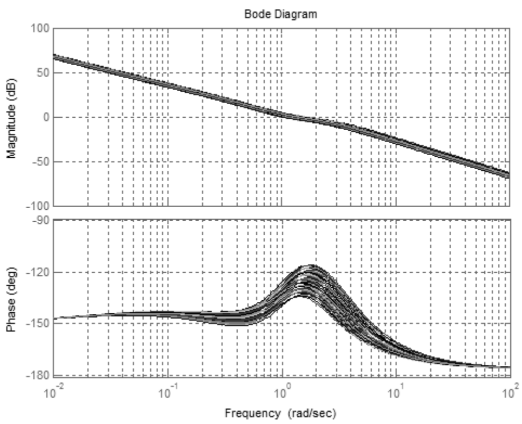

Fig. 5. Bode plot of the system $C_2(s)G_2(s)$

Example 3: *Stability analysis of Nonlinear FOICS using Nyquist envelope.*

First order plus dead time (FOPDT) systems provide simple characterization of a process and gives valuable information about dynamics of many applications in process control industry. Since the plants are commonly modeled with FOPDT transfer functions in the process industry, most of the engineers are familiar with the parameter of FOPDT model (Roy and Ikbal, 2004). A FOPDT system can be represented mathematically as follows,

$$G(s) = \frac{k}{\tau s + a} e^{-Ls} \quad (9)$$

where k is the steady state gain, L represents the process delay time, $\tau > 0$ is the time constant. Sign and magnitude of a determines the open loop stability and steady state gain of the process respectively. Let the plant $G_3(s)$ in Eq. 10 represent a fractional order version of the FOPDT system in Eq. 9.

$$G_3(s) = \frac{2}{2s^{1.2} + 2} e^{-1.5s} \quad (10)$$

Consider that the nominal values of the parameters of $G_3(s)$ are perturbed in a certain interval. In this case, the Nyquist envelope of this interval plant can be computed and used in several applications. For example, the stability margin of the uncertain parameters of the plant can be calculated for a given describing function using Nyquist envelope of the nonlinear interval control system. Let N be the describing function of the nonlinear system with saturation nonlinearity. Stability margin of the uncertain parameters of the plant can be computed using the negative inverse of the describing function (Yeroglu and Tan, 2010b; Yeroglu and Tan, 2010c). Let $-1/N = -1$ in Fig. 6, which means that the negative inverse of the describing function is defined in [-1, -Ð). One can conclude that the system preserve stability until the Nyquist envelope of the plant touches to the negative inverse of the describing function. Consequently, one can investigate from Fig. 6 that the system in Eq.10 preserve stability for %18 percent perturbations of the parameters k, τ and a. In other words the stability margin of the parameters of the plant is % 18. Let the following equation show %18 perturbations of the parameters k, τ and a of the plant

$$G_{3u}(s) = \frac{[1.64, 2.36]}{[1.64, 2.36]s^{1.2} + [1.64, 2.36]} e^{-1.5s} \quad (11)$$

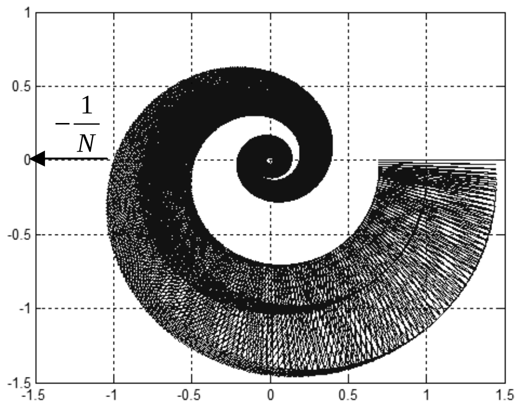

Fig. 6. Plot of $-1/N$ and Nyquist envelope of $G_{3u}(s)$ in Eq. 19 for $0 < \omega < 20$ rad/sec.

Entry of the uncertain parameters of the plant in Eq. 11 can easily be done in similar way of Example 2 as seen in Fig. 7. Nyquist Envelope of the plant in Eq. 11 can easily be computed simply by clicking the ñNyquist Envelopeò button in the user interface of the UFT-FOCS in Fig. 7. Nyquist envelope of the plant $G_{3u}(s)$ and the negative inverse of the describing function can be given as in Fig. 6. One can conclude from the Fig. 6 that the system preserve the stability of the fractional order uncertain plant in Eq. 11 in the presence of saturation nonlinearity.

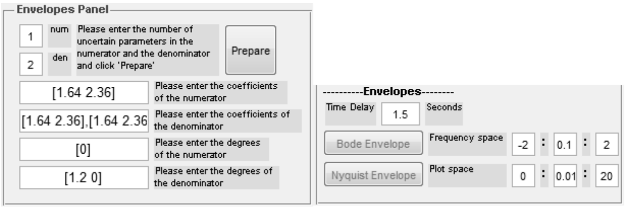

Fig. 7. Parameter Entry Panel for Example 3.

Several applications can also be done using the Bode envelopes but the examples related with Bode envelopes are not included in the paper due to space limitation

5. CONCLUSION

The goal of this paper is to suggest how an additional toolbox would be helpful to the students who study on fractional order control systems, by presenting the toolbox named UFT-FOCS developed for the analysis and design of fractional order control systems. The user friendly toolbox, which one can obtain time and frequency analysis tools for FOCS and FOICS, is presented with all features to researchers and students who may interested in fractional order control systems. The main specialty of the toolbox lies in its easy usage. Consequently, the UFT-FOCS would be useful for applications of the FOCS and FOICS. Bode and Nyquist envelopes of the other types of uncertainties such as affine, multilinear and general uncertainties can be included to the toolbox in the future study.

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