

The Quartic Higgs Coupling at Hadron Colliders

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The quartic Higgs self-coupling is the final measurement in the Higgs potential needed to fully understand electroweak symmetry breaking. None of the present or future colliders are known to be able to determine this parameter. We study the chances of measuring the quartic self-coupling at hadron colliders in general and at the VLHC in particular. We find the prospects challenging.

The LHC and a future linear collider are widely regarded as an ideal combination of experiments to understand electroweak symmetry breaking, *i.e.* study the Higgs boson and measure its couplings to all Standard Model bosons and fermions. According to the electroweak precision data [1] we expect to discover and identify a light Standard Model Higgs boson at the LHC [2, 3, 4]. At the ILC we will be able to measure Higgs couplings to all Standard Model particles with great precision [5].¹ A particularly exciting task is the measurement of the trilinear Higgs self-coupling: for a Standard Model Higgs boson heavier than 150 GeV this coupling can be measured at a luminosity-upgraded LHC (but not at the ILC) [7, 8, 9]. In contrast, for small Higgs masses around 120 GeV it can be measured at the ILC (but possibly not at the LHC) [10, 11, 12].

Higgs potential at Colliders: The measurement of the Higgs self-coupling is crucial to determine the Higgs potential — the way we think the electroweak symmetry is broken. A general parameterization of the Higgs potential with one doublet Φ (as in the Standard Model) is [13]:

$$V(\Phi) = \sum_{n \geq 0} \frac{\lambda_n}{\Lambda^{2n}} \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^{2+n} = \lambda_0 \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 + \mathcal{O} \left(\frac{1}{\Lambda^2} \right), \quad (1)$$

where $v = (\sqrt{2}G_F)^{-1/2}$ is the vacuum expectation value, and G_F is the Fermi constant. In the Standard Model, $\lambda_0 = \lambda_{SM} = m_H^2/(2v^2)$. If we consider the Standard Model an effective theory, λ_0 stands for two otherwise free parameters, namely the trilinear and the quartic scalar self couplings. An upper limit can be determined using unitarity arguments, assuming the model's validity to high energy scales [14]. In the Standard Model, the two self-couplings are linked as the leading terms in eq.(1), namely $\lambda_3/\lambda_4 = v$, and higher dimension operators are not expected to appear much below the Planck scale. If we allow for an intermediate scale $\Lambda \ll M_{\text{Planck}}$ and include the higher dimensional terms $n = 1, 2$ both self couplings receive different corrections: $\lambda_3 \rightarrow \lambda_3 [1 + \lambda_1 v^2/(\lambda_0 \Lambda^2)]$ and $\lambda_4 \rightarrow \lambda_4 [1 + 6\lambda_1 v^2/(\lambda_0 \Lambda^2) + 4\lambda_2 v^4/(\lambda_0 \Lambda^4)]$. In general, it is not even guaranteed that both self-couplings have to be positive, since the stability of the general Higgs potential is guaranteed by the sign of the highest power in the Higgs field alone.

The relation between the Higgs mass and each self-coupling as well as between the two self-couplings can change dramatically when we move to the MSSM with its two Higgs doublets. If we replace the Standard Model Higgs with the light CP-even scalar h^0 the relation between the self coupling becomes $\lambda_{3h}/\lambda_{4h} = v \sin(\beta + \alpha)/\cos 2\alpha$ [15]. As usual, $\tan \beta$ is the ratio of the two vacuum expectation values and α is the mixing angle between the two Higgs scalars. However, if we assume a mass hierarchy between the light Higgs scalar and the remaining Higgs sector the difference to a Standard-Model like Higgs is very small.

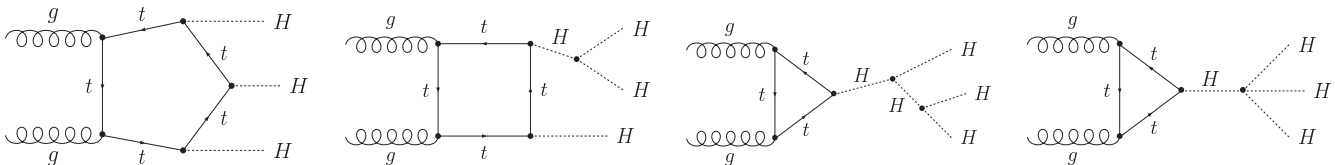


Figure 1: Examples of Feynman diagrams contributing to the process $gg \rightarrow HHH$.

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¹ Recently, it has also been shown that in supersymmetric models the Higgs couplings to neutralinos and charginos can be determined at linear colliders to high precision [6].

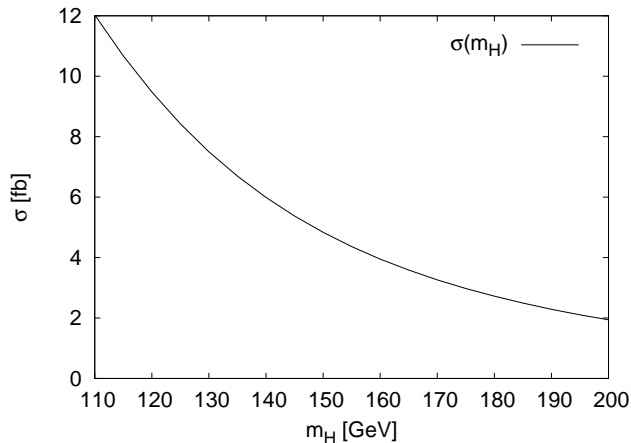


Figure 2: Total cross section for the production process $gg \rightarrow HHH$ at the 200 TeV VLHC in the Standard Model.

The measurement of the trilinear Higgs coupling requires the production of at least two Higgs bosons. The gluon fusion process at the LHC can proceed through a box-shaped top loop or through a triangular top loop and an intermediate Higgs boson. While it is well known that the total rate can be well approximated by the heavy-top approximation [7, 8] (for three Higgs production see Ref. [16]) it has been shown for Higgs pair production that the distributions in particular in the threshold region will come out completely wrong [9]. Precisely this threshold behavior carries the information on the Higgs self-coupling.

In this brief letter we study a possible measurement of the quartic Higgs self-coupling at hadron colliders. In analogy to the measurement of the trilinear coupling we now produce three Higgs bosons in gluon fusion. The four topologies shown in Fig. 1 will appear: (a) continuum production of three Higgs bosons through a pentagon top-loop, (b) the production of two Higgs bosons with a subsequent decay via the trilinear self-coupling, and finally the production of one Higgs boson with a decay through either (c) two three-Higgs vertices or (d) through one quartic self-coupling. From this list it is obvious that it will not be possible to make any statement about the quartic self-coupling without having information on the trilinear coupling and the top Yukawa coupling [17]. Moreover, it is fairly obvious that the LHC even including a major luminosity upgrade will not be able to supply enough three-Higgs events. Instead, we ask the question what a future 200 TeV VLHC [18] could do, keeping in mind that even future high-energy linear colliders like CLIC will not be able to measure this coupling [10, 19]. In other words, we are trying to determine if any future high-energy collider will be able to completely measure the parameters of the Higgs potential (1).

Three-Higgs production: The one-loop diagrams contributing to the process $gg \rightarrow HHH$ are leading order, *i.e.* they are finite for any value of the Higgs self-couplings. We compute the total and differential cross sections using the HadCalc program [20]. The Feynman diagrams are constructed using FeynArts [21], the matrix elements are calculated by FormCalc [22], and the loop integrals are numerically evaluated using LoopTools [23], where we have added the scalar five-point function [24] and modified the general four-point function [25]. For the top mass we use the on-shell value ($m_t = 178$ GeV), because it has been shown to lead to perturbatively stable cross section predictions for the single-Higgs production through a one-loop amplitude [26]. The bottom loops are included in our numerical analysis, but their effect is below one per cent.

We show the total cross section for the production of three Standard Model Higgs bosons at a 200 TeV VLHC in Fig.2. The cross sections are quoted without branching fractions, acceptance cuts, or efficiencies. In Fig. 3 we show the dependence of the LHC and the 200 TeV VLHC cross sections on the trilinear (λ_3) and quartic (λ_4) Higgs self couplings. The central values at the LHC and at the VLHC are $6.25 \cdot 10^{-2}$ fb and 9.45 fb (Standard Model couplings). The fact that λ_3 contributes to many more topologies than λ_4 (with its single diagram) is reflected in the much steeper behavior of the total cross sections as a function of λ_3 than as a function of λ_4 : each of the three topologies (triangle, box, pentagon) alone would yield a rate of $(0.46, 8.20, 17.07) \cdot 10^{-2}$ fb at the LHC. If we compute only the propagator-suppressed triangle contribution and keep either $\lambda_3 \neq 0$ or $\lambda_4 \neq 0$, we are left with $0.17 \cdot 10^{-2}$ fb from the trilinear self-coupling and with $0.08 \cdot 10^{-2}$ fb from the quartic self-coupling, with a constructive interference. This means that the contribution from the quartic self-coupling is suppressed by almost two orders of magnitude.

The interference between the continuum and the box is indeed destructive (as we would expect from Ref. [9]), which is the primary reason for the steep decline with λ_3 shown in Fig. 3. The interference between the continuum and the triangle diagrams is constructive, but because of the more similar kinematic configuration the destructive interference

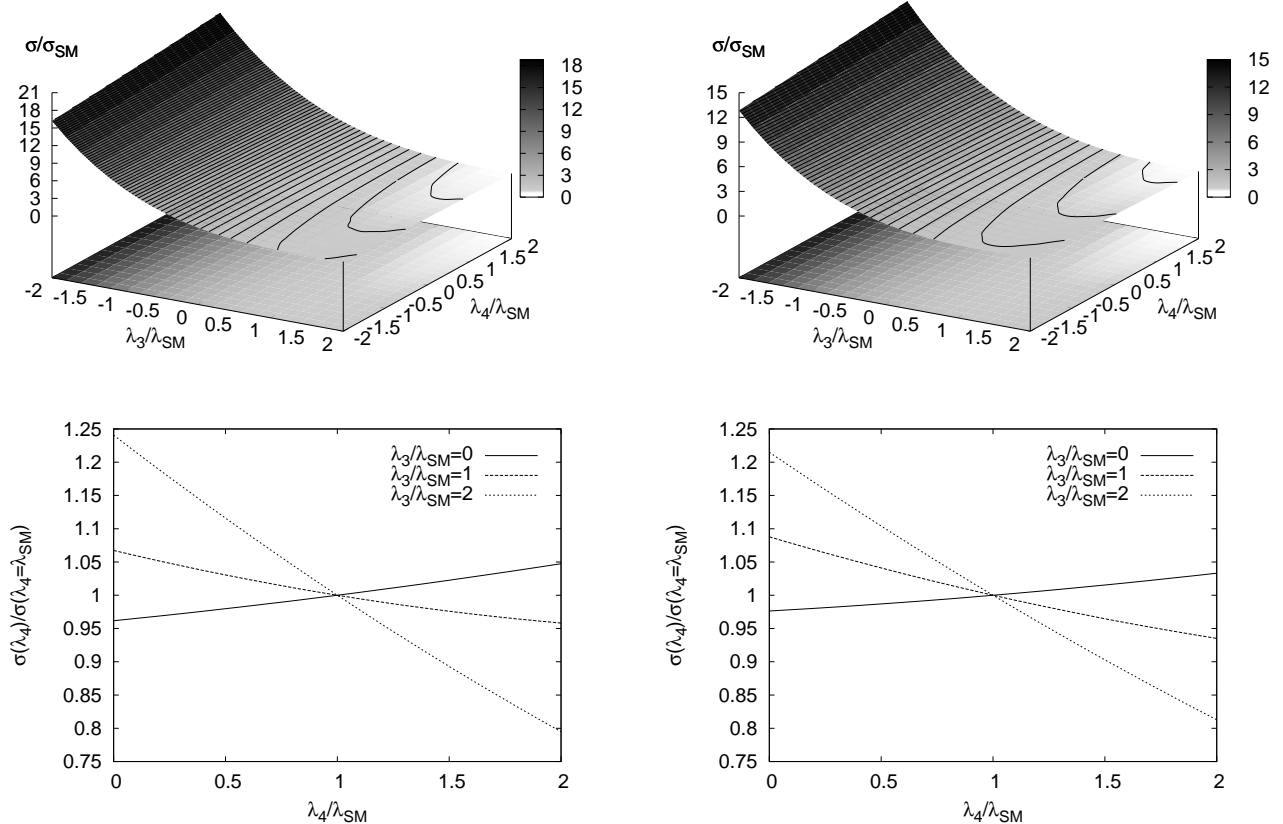


Figure 3: Total cross section ratios normalized to the Standard Model values for the production process $gg \rightarrow HHH$ at the LHC (left) and at the 200 TeV VLHC (right). The Higgs mass is fixed to 120 GeV, the absolute values of triple and quartic self-couplings are varied up to twice the Standard Model values. Contour lines are included every 0.25 steps on the z axis. In the lower set of figures the triple Higgs self-coupling is fixed. The Standard Model values for the cross sections are $6.25 \cdot 10^{-2}$ fb at the LHC and 9.45 fb at the VLHC.

between the box diagrams and the triangle diagrams leads to the slight decrease of the cross section with growing λ_4 . If we switch off the box contributions ($\lambda_3 = 0$) the constructive interference between continuum and triangle topologies switches around the behavior of the total cross section as a function of λ_4 . This behavior can be understood analytically in the limit $m_t \gg m_H$, using the low-energy theorem for the leading form factors in m_H/m_t ($\hat{s} \sim m_H^2$) [27]². These leading form factors for an increasing number of external Higgs scalars can be iteratively derived from the top loop in the gluon self-energy: $F_{(n+1)H} = m_t^2 \partial(F_{nH}/m_t)/\partial m_t$. We obtain $F_{\text{triangle}} = -F_{\text{box}} = F_{\text{pentagon}} = 2/3 + \mathcal{O}(m_H^2/m_t^2)$. This relative sign explains the structure of the constructive and destructive interferences observed above.

Because of the fairly strong dependence on λ_3 , where the cross section decreases for increasing λ_3 , we will be able to exclude the hypothesis of $\lambda_3 = 0$ most easily, exactly the same way as in the case of Higgs pair production [9]. The same trick could work for the quartic self-coupling λ_4 as long as $\lambda_3 \gtrsim \lambda_3^{\text{SM}}$. However, the typical variations in the total rate are above 100 % for varying λ_3 and less than 20 % for varying λ_4 . This means that after including the systematic errors on the cross section (like higher-order QCD uncertainties) and the statistical error on the measurement of λ_3 there is little hope to see an effect of λ_4 in the total rate at the LHC or at the VLHC.

Remembering what we know about the measurement of the trilinear self-coupling at the LHC we can compute the normalized distribution of the partonic center-of-mass energy. At the LHC its shape will have maximum discriminating power, because of the interference effects between continuum and intermediate Higgs topologies at threshold [9]. We show these distributions in Fig. 4, again varying λ_3 and λ_4 independently. Indeed, we see that there is a sizeable shift

² We use the conventions as in Ref. [7]. The form factor is basically the matrix element squared without couplings or additional s-channel propagators. The top Yukawa coupling and the top mass in the propagators are both denoted as m_t .

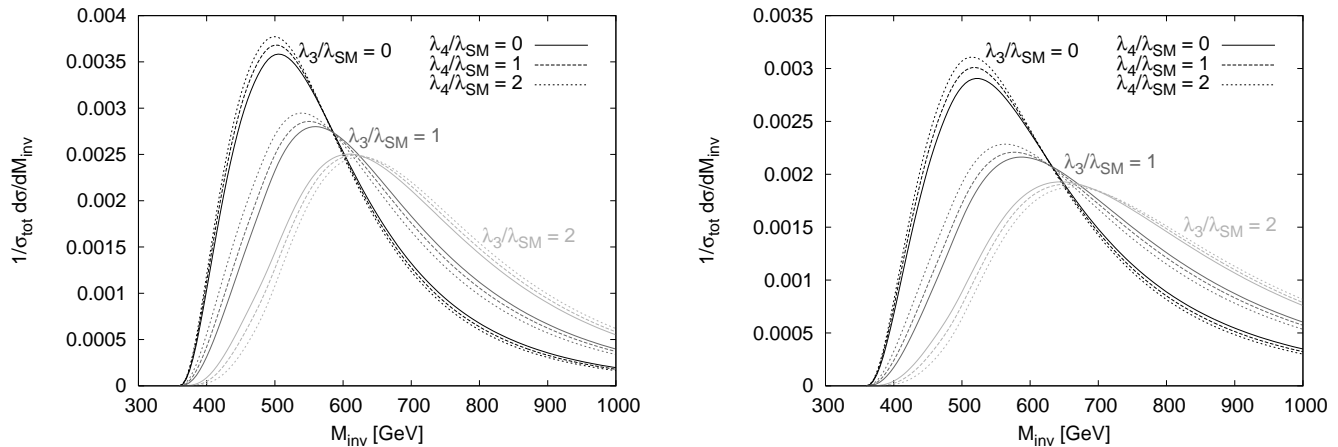


Figure 4: Normalized partonic center-of-mass energy distributions for $gg \rightarrow HHH$ at the LHC (left) and at the 200 TeV VLHC (right). The Higgs mass is fixed to 120 GeV. The trilinear and quartic Higgs self-couplings are varied independently.

when we vary λ_3 while the numerical impact of λ_4 is about an order of magnitude smaller. We also see that for $\lambda_3 = 0$ the order of the three curves for $\lambda_4/\lambda_{\text{SM}} = 0, 1, 2$ turns around together with the sign of the interference. While this distribution will be the key to the LHC measurement of λ_3 , the curves in Fig. 4 look challenging for a measurement of λ_4 , in particular once we again include the error on the measurement of λ_3 .

Summary: To measure the entire set of parameters in the Higgs potential (and completely understand electroweak symmetry breaking) we have to measure the quartic Higgs self-coupling. At the LHC and the ILC the measurement of the trilinear self-coupling will already be a task which requires large luminosities and a very good understanding of the detector [9, 10, 11, 12]. We know that CLIC would not be able to measure the quartic Higgs coupling [19], so what is left is the high-energy mode of the VLHC. To get a rough idea if the quartic coupling might be visible we compute the total cross section as well as the partonic center-of-mass energy distribution for the process $gg \rightarrow HHH$. For a 120 GeV Higgs the cross section at the VLHC is 9.45 fb, so we might even be able to observe triple Higgs production. However, based on this simple study without decays or detector effects we conclude that the measurement of the quartic Higgs self-coupling will be seriously challenging due to the interference structures between the different topologies contributing to the process. Moreover, a measurement of the quartic self-coupling requires a very good knowledge of the value of the trilinear self-coupling, which at the moment is not (yet) established.

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