Subcarrier Allocation for Multiuser Two-Way OFDMA Relay Networks with Fairness Constraints

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Abstract—In this paper, we propose a new adaptive subcarrier allocation for a multiuser two-way OFDMA relay network. In the proposed algorithm, subcarriers are allocated to the user-pairs and relays to maximize the achievable sum-rate over all user-pairs while satisfying the minimum rate requirement for each user-pair. Simulation results show that the proposed algorithm improves spectral efficiency and fairness.

Index terms — Subcarrier allocation, two-way relaying, OFDMA, fairness constraints.

I. INTRODUCTION

Diversity techniques are widely employed to mitigate the performance degradation caused by multipath fading in wireless networks. Multiple-input multiple-output (MIMO) in the wireless networks achieves spatial diversity so that the spectral efficiency and reliability of the network are improved [1], [2]. However, deploying multiple antennas at a user may be impractical due to the limitations such as the size, power, and complexity of the user. In a cooperative diversity, one or more users and relays share their resources to form a virtual antenna array so that spatial diversity is obtained without deploying multiple antennas at the ends of users and relays [3]-[5].

Orthogonal frequency division multiple access (OFDMA) is one of efficient techniques to mitigate the problems of frequency selective fading and inter-symbol interferences. In an OFDMA network, a total bandwidth is divided into a number of subcarriers and multiple users transmit their information simultaneously on the different subcarriers. It is known that the OFDMA network provides improved performance by adaptive resource allocation, which implies that the subcarrier, bit, and power are allocated to a user based on a channel coefficient of each subcarrier [6]-[8].

A conventional half-duplex one-way relay network requires additional resources because one symbol is transmitted in two time-slots. Compared with the conventional one-way relay network, a two-way relay network provides improved spectral efficiency by using either superposition coding or network coding [9], [10]. In a two-way OFDM relay network having a single user-pair and a single relay, the sum capacity for both users over all subcarriers is maximized by power allocation and tone permutation [11]. In [12], resource allocation for a multiuser two-way OFDMA relay network is investigated to support two-way communication between the base station and each of multiple users. However, resource allocation for a two-way OFDMA relay network having multiple user-pairs and multiple relays is not investigated yet.

In this paper, we investigate an adaptive subcarrier allocation scheme for a multiuser two-way OFDMA relay network having multiple user-pairs and multiple relays. We formulate an optimization problem to maximize the achievable sum-rate over all user-pairs and propose a new adaptive subcarrier allocation algorithm. The proposed algorithm is compared with the static and greedy algorithms in terms of spectral efficiency, outage probability, and fairness index.

The rest of this paper is organized as follows. Section II describes a system model. In section III, an optimization problem with fairness constraints is formulated and a new adaptive subcarrier allocation algorithm is proposed. Simulation results are presented in section IV. Section V concludes this paper.

II. SYSTEM MODEL

Consider a multiuser two-way OFDMA relay network which consists of $K$ user-pairs and $M$ relays with $N$ subcarriers. Suppose that the two users of the $k$-th user-pair, $A_k$ and $B_k$, $k = 1, ..., K$, exchange information with each other via one or more relays. Assume that there is no direct path between the users. Also assume that each of users and relays is equipped with a single antenna and does not transmit and receive simultaneously.

Assume that the channel has a frequency selective Rayleigh fading. Also assume that the bandwidth of each subcarrier is much smaller than the coherence bandwidth of the channel, and so the channel of each subcarrier has a flat fading.

In the two-way relaying, two users communicate with each other in two phases: the multiple-access (MA) phase and broadcast (BC) phase. In the MA phase, all users transmit their information to relays simultaneously. Let $f_{k,m}^{(n)}$ and $g_{k,m}^{(n)}$ denote the instantaneous channel coefficients of the subcarrier $n$ between the user $A_k$ and relay $R_m$ and between the user $B_k$ and relay $R_m$, respectively. Also, let $x_{A_k}^{(n)}$ and $x_{B_k}^{(n)}$ denote the transmit symbols with unit power, and $p_{A_k}^{(n)}$ and $p_{B_k}^{(n)}$ denote the transmit power of the user $A_k$ and $B_k$ on the subcarrier $n$, respectively. Then the received signal at the relay $R_m$ on the subcarrier $n$ is given by

$$y_{R_m}^{(n)} = \sqrt{p_{A_k}} f_{k,m}^{(n)} x_{A_k}^{(n)} + \sqrt{p_{B_k}} g_{k,m}^{(n)} x_{B_k}^{(n)} + n_{R_m}^{(n)}, \quad (1)$$
where \( n^{(n)}_{R_m} \) is a complex additive white Gaussian noise with zero mean and variance \( \sigma^2 \) at the relay \( R_m \) on the subcarrier \( n \).

In the BC phase, each relay amplifies the received signal and broadcasts it to the users using the same subcarrier. Let \( \rho_{R_m}^{(n)} \) denote the transmit power of the relay \( R_m \) on the subcarrier \( n \), then the amplification factor of the relay \( R_m \) on the subcarrier \( n \) is given by [9]

\[
\beta_{k,m}^{(n)} = \sqrt{\frac{\rho_{R_m}^{(n)}}{p_{A_k}^{(n)} \left| f_{k,m}^{(n)} \right|^2 + p_{B_k}^{(n)} \left| g_{k,m}^{(n)} \right|^2 + \sigma^2}}.
\] (2)

The received signals at the user \( A_k \) and \( B_k \) on the subcarrier \( n \) are given by

\[
y_{A_k}^{(n)} = \beta_{k,m}^{(n)} f_{k,m}^{(n)} y_{R_m}^{(n)} + n_{A_k}^{(n)} \tag{3}
\]

\[
y_{B_k}^{(n)} = \beta_{k,m}^{(n)} g_{k,m}^{(n)} y_{R_m}^{(n)} + n_{B_k}^{(n)}, \tag{4}
\]

respectively, where \( n_{A_k}^{(n)} \) and \( n_{B_k}^{(n)} \) are complex additive white Gaussian noises with zero mean and variance \( \sigma^2 \) at the user \( A_k \) and \( B_k \) on the subcarrier \( n \), respectively.

Assume perfect self-interference cancelation at the users, i.e., the user \( A_k \) can perfectly remove the signal component of \( x_{A_k}^{(n)} \) from \( y_{A_k}^{(n)} \), and similarly for \( B_k \). After perfect self-interference cancelation, the received SNRs at the user \( A_k \) and \( B_k \) on the subcarrier \( n \) are given by

\[
\text{SNR}_{A_k}^{(n)} = \frac{\left| \beta_{k,m}^{(n)} f_{k,m}^{(n)} \right|^2 p_{B_k}^{(n)}}{\left( \left| \beta_{k,m}^{(n)} f_{k,m}^{(n)} \right|^2 + 1 \right) \sigma^2}, \tag{5}
\]

\[
\text{SNR}_{B_k}^{(n)} = \frac{\left| \beta_{k,m}^{(n)} g_{k,m}^{(n)} \right|^2 p_{A_k}^{(n)}}{\left( \left| \beta_{k,m}^{(n)} g_{k,m}^{(n)} \right|^2 + 1 \right) \sigma^2}, \tag{6}
\]

respectively. Then the instantaneous rate of the \( k \)-th user-pair via the \( m \)-th relay on the subcarrier \( n \) is given by [9]

\[
r_{k,m}^{(n)} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{A_k}^{(n)} \right) + \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{B_k}^{(n)} \right). \tag{7}
\]

Let \( \rho_{k,m}^{(n)} \) denote the subcarrier assignment indicator variable. If the subcarrier \( n \) is assigned to the user-pair \( (A_k, B_k) \), and relay \( R_m \), then \( \rho_{k,m}^{(n)} \) is equal to one. Otherwise \( \rho_{k,m}^{(n)} \) is equal to zero. Assume that the subcarrier \( n \) is assigned to only one user-pair and one relay, i.e., if \( \rho_{k,m}^{(n)} \) is equal to one, then \( \rho_{k,m'}^{(n)} \) is equal to zero for all \( k' \neq k \) and \( m' \neq m \), so that there is no interference between users and relays. Then, the achievable rate of the \( k \)-th user-pair is given by

\[
r_k = \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)}. \tag{8}
\]

The achievable sum-rate over all user-pairs is given by

\[
r = \sum_{k=1}^{K} r_k. \tag{9}
\]

III. PROPOSED SUBCARRIER ALLOCATION ALGORITHM

A. Problem Formulation

Let \( P_U \) and \( P_R \) denote the maximum transmit power of each of users and relays, respectively. Let \( r_{\text{min}} \) denote the minimum rate requirement for each user-pair. Then, to maximize the achievable sum-rate over all user-pairs, the optimization problem is formulated as [6]

\[
r^* = \max_{\rho_{k,m}^{(n)}} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)} \tag{10}
\]

subject to:

\[
\rho_{k,m}^{(n)} \in \{0, 1\}, \forall k, m, n \tag{11a}
\]

\[
\sum_{k=1}^{K} \sum_{m=1}^{M} \rho_{k,m}^{(n)} = 1, \forall n \tag{11b}
\]

\[
\sum_{n=1}^{N} p_{A_k}^{(n)} \leq P_U, \forall k \tag{11c}
\]

\[
\sum_{n=1}^{N} p_{B_k}^{(n)} \leq P_U, \forall k \tag{11d}
\]

\[
\sum_{n=1}^{N} p_{R_m}^{(n)} \leq P_R, \forall m \tag{11e}
\]

\[
\sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)} \geq r_{\text{min}}, \forall k \tag{11f}
\]

\[
p_{A_k}^{(n)}, p_{B_k}^{(n)}, p_{R_m}^{(n)} \geq 0, \forall k, m, n. \tag{11g}
\]

The optimization problem in (10) is a combinatorial optimization problem involving both discrete and continuous variables. Due to the high computational complexity of the problem, it is hard to obtain an optimal solution of the problem. To make the problem tractable, we relax the constraint on the subcarrier assignment indicator variable to allow \( \rho_{k,m}^{(n)} \) to be a real value within the interval \([0, 1]\) [6].

Let \( \lambda_n, \zeta_k, \mu_{k,n}, \eta_k, \nu_{k,n}, \kappa_n, \xi_{m,n}, \alpha_k, \) and \( \gamma_{k,m,n} \) be nonnegative Lagrangian multipliers, then the Lagrangian for the relaxed optimization problem is given by [13]

\[
L = -\sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)} + \sum_{n=1}^{N} \lambda_n \left( \sum_{k=1}^{K} \sum_{m=1}^{M} \rho_{k,m}^{(n)} - 1 \right) + \sum_{k=1}^{K} \zeta_k \left( \sum_{n=1}^{N} p_{A_k}^{(n)} - P_U \right) - \sum_{k=1}^{K} \sum_{n=1}^{N} \mu_{k,n} p_{A_k}^{(n)}
\]

\[
+ \sum_{k=1}^{K} \eta_k \left( \sum_{n=1}^{N} p_{B_k}^{(n)} - P_U \right) - \sum_{k=1}^{K} \sum_{n=1}^{N} \nu_{k,n} p_{B_k}^{(n)}
\]

\[
+ \sum_{m=1}^{M} \kappa_n \left( \sum_{n=1}^{N} p_{R_m}^{(n)} - P_R \right) - \sum_{m=1}^{M} \sum_{n=1}^{N} \xi_{m,n} p_{R_m}^{(n)}
\]

\[
+ \sum_{k=1}^{K} \alpha_k \left( r_{\text{min}} - \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)} \right)
\]

\[
- \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} \gamma_{k,m,n} \rho_{k,m}^{(n)} p_{k,m,n}. \tag{12}
\]
After differentiating $L$ with respect to $\rho_{k,m}^{(n)}$, the following Karush-Kuhn-Tucker (KKT) conditions for the optimal solution are obtained [13]:

$$\frac{\partial L}{\partial \rho_{k,m}^{(n)}} = \lambda_n - (1 + \alpha_k) r_{k,m}^{(n)} - \gamma_{k,m,n} = 0, \forall k, m, n$$  \hspace{1cm} (13)

$$\lambda_n \left( \sum_{k=1}^{K} \sum_{m=1}^{M} \rho_{k,m}^{(n)} - 1 \right) = 0, \forall n$$  \hspace{1cm} (14)

$$\zeta_k \left( \sum_{n=1}^{N} p_{Ak}^{(n)} - P_U \right) = \eta_k \left( \sum_{n=1}^{N} p_{Bk}^{(n)} - P_U \right) = 0, \forall k$$  \hspace{1cm} (15)

$$\kappa_m \left( \sum_{n=1}^{N} p_{Rm}^{(n)} - P_R \right) = 0, \forall m$$  \hspace{1cm} (16)

$$\mu_{k,n} p_{Ak}^{(n)} = \nu_{k,n} p_{Bk}^{(n)} = \xi_{m,n} p_{Rm}^{(n)} = 0, \forall k, m, n$$  \hspace{1cm} (17)

$$\alpha_k \left( r_{k,m}^{(n)} - \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)} \right) = 0, \forall k$$  \hspace{1cm} (18)

$$\gamma_{k,m,n} \rho_{k,m}^{(n)} = 0, \forall k, m, n.$$  \hspace{1cm} (19)

Equation (13) is rewritten as

$$\gamma_{k,m,n} = \lambda_n - (1 + \alpha_k) r_{k,m}^{(n)}.$$  \hspace{1cm} (20)

Because $\gamma_{k,m,n}$ is a nonnegative Lagrangian multiplier, the right-hand side of (20) is also nonnegative, so that $\lambda_n$ becomes

$$\lambda_n \geq (1 + \alpha_k) r_{k,m}^{(n)}.$$  \hspace{1cm} (21)

By substituting (20) into (19), we obtain

$$\left( \lambda_n - (1 + \alpha_k) r_{k,m}^{(n)} \right) \rho_{k,m}^{(n)} = 0.$$  \hspace{1cm} (22)

If and only if $\rho_{k,m}^{(n)}$ has a positive value, the subcarrier $n$ is assigned to the $k$-th user-pair and $m$-th relay. By the complementary slackness condition for $\rho_{k,m}^{(n)}$ [13], if $\rho_{k,m}^{(n)}$ has a positive value, then $\lambda_n - (1 + \alpha_k) r_{k,m}^{(n)}$ is equal to zero and an equality holds in (21). Hence, the subcarrier $n$ is assigned to the $k$-th user-pair and $m$-th relay which maximize $(1 + \alpha_k) r_{k,m}^{(n)}$. Therefore, to maximize the achievable sum-rate over all user-pairs and satisfy the minimum rate requirement for each user-pair, the subcarrier $n$ is allocated to the $k^{*}$-th user-pair and $m^{*}$-th relay such that

$$(k^{*}, m^{*}) = \arg \max_{k,m} (1 + \alpha_k) r_{k,m}^{(n)}.$$  \hspace{1cm} (23)

The exact value of $\alpha_k$ is needed to obtain the optimal solution for the subcarrier allocation and it can be obtained by an iterative searching algorithm [6]. However, the algorithm requires excessive computational complexity. To reduce the complexity, we use an approximated value of $\alpha_k$, instead of its exact value. We obtain the approximated value of $\alpha_k$ from (18). By the complementary slackness condition for $\alpha_k$, if $r_{k,m}^{(n)} - \sum_{m=1}^{M} \sum_{n=1}^{N} \rho_{k,m}^{(n)} r_{k,m}^{(n)}$ has a negative value, which implies that if the achievable rate of the $k$-th user-pair is higher than the required minimum rate, then $\alpha_k$ is equal to zero. Otherwise, $\alpha_k$ has a positive value.

### B. Proposed Subcarrier Allocation Algorithm

Due to the high computational complexity, it is hard to solve the optimization problem in (10). To reduce the complexity, we propose a new suboptimal subcarrier allocation algorithm. Suppose that equal power allocation is adopted in the proposed algorithm.

**Algorithm 1:** Proposed subcarrier allocation algorithm

**Step 1**
Set $K = \{1, 2, \ldots, K\}$, $M = \{1, 2, \ldots, M\}$, $N = \{1, 2, \ldots, N\}$, and $\rho_{k,m}^{(n)} = 0, \forall k, m, n$.

**Step 2**
for $k = 1 : K$
do

if $r_{k,m}^{(n)} < r_{\text{min}}$
do

$$m^{*} = \arg \max_{m} r_{k,m}^{(n)}, m \in M, n \in N;$$

$$\rho_{k,m}^{(n)} = 1, N = N - \{n^{*}\};$$

update $r_{k,m}^{(n)}$;

else

$$n^{*} = \text{rand} \{N\};$$

$$k^{*}, m^{*} = \arg \max_{k,m} r_{k,m}^{(n)}, k \in K, m \in M;$$

$$\rho_{k^{*},m^{*}}^{(n)} = 1, N = N - \{n^{*}\};$$

if $\sum_{n=1}^{N} p_{Ak}^{(n)} = \sum_{n=1}^{N} p_{Bk}^{(n)} > P_U$ then

$$K = K - \{k^{*}\};$$

else

if $\sum_{n=1}^{N} P_{Ak}^{(n)} > P_R$ then

$$M = M - \{m^{*}\};$$

update $r_{k,m}^{(n)}$;

**Step 3**
while $N \neq \emptyset$
do

$k^{*} = \arg \min \{r_{k,m}\}, k \in K;$

if $r_{k^{*}}^{(n)} < r_{\text{min}}$
do

$$m^{*} = \arg \max_{m} r_{k^{*},m}^{(n)}, m \in M, n \in N;$$

$$\rho_{k^{*},m}^{(n)} = 1, N = N - \{n^{*}\};$$

else

$$n^{*} = \text{rand} \{N\};$$

$$k^{*}, m^{*} = \arg \max_{k,m} r_{k,m}^{(n)}, k \in K, m \in M;$$

$$\rho_{k^{*},m^{*}}^{(n)} = 1, N = N - \{n^{*}\};$$

if $\sum_{n=1}^{N} p_{Ak}^{(n)} = \sum_{n=1}^{N} p_{Bk}^{(n)} > P_U$ then

$$K = K - \{k^{*}\};$$

else

if $\sum_{n=1}^{N} P_{Ak}^{(n)} > P_R$ then

$$M = M - \{m^{*}\};$$

update $r_{k,m}^{(n)}$;

end

end

end

end

end

The proposed subcarrier allocation algorithm consists of three steps. In the first step, all sets and subcarrier assignment indicator variables are initialized, where the set of user-pairs, relays, and subcarriers are denoted by $K$, $M$, and $N$, respectively, in the algorithm. In the second step, one relay-subcarrier pair which maximizes the instantaneous rate is allocated to one user-pair for all user-pairs. In the third step, remaining
subcarriers are allocated to the user-pairs and relays under the maximum transmit power constraints of the users and relays. The subcarriers are allocated to the user-pairs in order to maximize the achievable sum-rate over all user-pairs while satisfying the minimum rate requirement for every user-pair. At first, one user-pair with lowest achievable rate is selected. If the selected user-pair does not meet the minimum rate requirement, the relay-subcarrier pair which maximizes the instantaneous rate of the user-pair is allocated to the user-pair. After every user-pair meets the minimum rate requirement, one of the remaining subcarriers is randomly selected and allocated to the user-pair and relay which maximize the achievable sum-rate over all user-pairs. This procedure continues until all subcarriers are allocated.

The proposed subcarrier allocation algorithm is compared with the static and greedy subcarrier allocation algorithms in terms of spectral efficiency, outage probability, and fairness index. The outage probability is defined as the probability that a user-pair is not satisfied with the minimum rate requirement. In the static algorithm [6], for comparison, a subcarrier is allocated to the predetermined user-pair and relay regardless of the instantaneous channel condition. In the greedy algorithm [8], a subcarrier is allocated to the user-pair and relay in order to maximize the achievable sum-rate over all user-pairs. However, the greedy algorithm does not consider the fairness among the users, and so there is a significant gap between the achievable rates of the user-pairs in good channel condition and the user pairs in bad channel condition.

C. Fairness Index

Fairness indicates how equally the resources are allocated among the relays. In this paper, a fairness index is defined as [14]

$$F = \left( \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} p_{Rm}^{(n)}}{M \sum_{m=1}^{M} \sum_{n=1}^{N} (p_{Rm}^{(n)})^2} \right)^2$$

which quantifies the degree of similarity among the transmit power of all relays. The fairness index takes a value within the interval $[0, 1]$. If all relays have the same transmit power, the value of fairness index is equal to one and the system is totally fair.

IV. SIMULATION RESULTS

Suppose that the users and relays are distributed in a two-dimensional region of $200m \times 200m$. Suppose that one of user in each user-pair and the other are uniformly distributed inside two different rectangles: a rectangle bounded by the lines of $x = 0, x = 50, y = 0$, and $y = 200$, and another rectangle bounded by the lines of $x = 150, x = 200, y = 0$, and $y = 200$. Also suppose that the number of subcarriers is 128, the total bandwidth is 10 MHz, and the minimum rate requirement for each user-pair is 2.5 Mbps. Assume that every user has the same maximum transmit power and every relay also has the same maximum transmit power. Assume that the path loss exponent is 4.

Fig. 1 shows the spectral efficiency of the proposed subcarrier allocation algorithm versus SNR for $K = 8$. It is shown that the proposed algorithm achieves about 2 bps/Hz higher spectral efficiency than the static algorithm at SNR = 15 dB. It is also shown that the proposed algorithm provides about 1 bps/Hz lower spectral efficiency than the greedy algorithm at SNR = 15 dB.

Fig. 2 shows the outage probability of the proposed subcarrier allocation algorithm versus SNR for $K = 8$. It is shown that the proposed algorithm achieves much lower outage probability than both the static and greedy algorithms. At the
outage probability of $10^{-3}$, the proposed algorithm achieves the SNR gain of 8 dB compared with the static algorithm.

Fig. 3 shows the fairness index of the proposed subcarrier allocation algorithm versus the number of relays at SNR = 15 dB for $K = 8$. It is shown that the value of fairness index of the proposed algorithm remains one regardless of the number of relays, while the value of fairness index of the greedy algorithm decreases as the number of relays increases. It implies that the proposed algorithm achieves maximum fairness.

V. CONCLUSIONS

In this paper, we propose a new adaptive subcarrier allocation algorithm for a multiuser two-way OFDMA relay network. In the proposed algorithm, subcarriers are allocated to the user-pair with lowest achievable rate to satisfy the minimum rate requirement. After every user-pair meets the minimum rate requirement, remaining subcarriers are allocated to the user-pairs and relays to maximize the achievable rate over all user-pairs. Simulation results show that the proposed algorithm achieves higher spectral efficiency than the static algorithm, and achieves much lower outage probability than both of the static and greedy algorithms. It is also shown that proposed algorithm achieves maximum fairness regardless of the number of relays.

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