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On coincidence and common fixed point theorems of eight self-maps satisfying an F_M-contraction condition*

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Abstract. In this paper, a new type of contraction for several self-mappings of a metric space, called F_M-contraction, is introduced. This extends the one presented for a single map by Wardowski [Fixed points of a new type of contractive mappings in complete metric spaces, *Fixed Point Theory Appl.*, 2012:94, 2012]. Coincidence and common fixed point of eight self mappings satisfying F_M-contraction conditions are established via common limit range property without exploiting the completeness of the space or the continuity of the involved maps. Coincidence and common fixed point of eight self-maps satisfying F_M-contraction conditions via the common property (E.A.) are also studied. Our results generalize, extend and improve the analogous recent results in the literature, and some examples are presented to justify the validity of our main results.

Keywords: coincidence point, common fixed point, common (CLR$_{(AB)(ST)}$) property, common property (E.A.), weakly compatible.

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1 Introduction

Using a function F satisfying three conditions (F1)–(F3), Wardowski [22] introduced a new concept of F-contraction for a single-valued self map of a complete metric space and proved that every F-contraction possesses a unique fixed point. Inspired by Wardowski, many authors extended and improved this concept by generalizing the definition space, relaxing or excluding some of conditions (F1)–(F3) or generalizing the shape of the respective F-contraction.

Thus, Minak et al. [10] and Wardowski and Dung [23] introduced the concept of *Ćirić-type F-contraction* for a single-valued map in metric spaces. Afterward, Cosentino and Vetro [4] introduced the *Hardy–Rogers-type F-contraction*. Meanwhile, there are some other efforts attempted to study the possibility of weakening conditions (F1)–(F3) of Wardowski's F-contraction. We briefly present some existing cases. First, Piri and Kumam [11] replaced condition (F3) by the continuity of F, which is essentially motivated by the fact that most of utilized functions in the existing literature are continuous. Second, Vetro [21] extended the F-contraction by replacing the constant τ with a function. Third, Secelean and Wardowski [16] by weakening condition (F1) and considering the family of certain class of increasing functions ψ, introduced so called *ψF-contraction* and *weak ψF-contraction*. Most recently, Lukács and Kajántó [9] defined a new version of F-contraction by omitting (F2) condition in b-metric spaces.

On the other hand, in 1982, Sessa [17] introduced the concept of *weak commutativity* and proved a common fixed point theorem for weakly commuting maps. Later on, Jungck [6] introduced the notion of *compatible mappings*, which generalizes the concept of weakly commuting pairs of mappings studied by Sessa [17]. In 1996, Jungck and Rhoades [7] defined *weakly compatible mappings*. After that, Aamir and Moutawakil [1] presented the notion of *property* (E.A.), which is a special case of tangential property due to Sastry and Murthy [14]. In 2011, Sintunavarat and Kumam [18] obtained the notions of property (E.A.) always requires the completeness (or closedness) of underlying subspace/space for existence of common fixed point. Hence, they coined the idea of *common limit in range property* (CLR), which relaxes the requirement of completeness (or closedness) of the underlying subspace/space. Clearly, a pair of self-maps satisfying the property (E.A.) along with closedness of subspace/space always enjoys the common limit in range property. Also, the continuity is not necessary for self-maps to satisfy property (CLR).

Motivated by the above, the aim of this paper is to establish the existence and uniqueness of coincidence and common fixed point of eight self-maps in a (noncomplete) metric spaces satisfying a new type contraction condition, called F_M-contraction via common (CLR$_{(AB)(ST)}$) property or common property (E.A.). Our results generalize, extend and improve the results of Wardowski [22], Batra et al. [2], Tomar et al. [19] and others existing in the literature (for instance, Chatterjea [3], Cosentino and Vetro [4], Ćirić [5], Kannan [8], Reich [12], Wardowski and Dung [23], Roldan and Sintunavarat [13] and references therein) without using completeness or closedness of subspace/space or containment requirement of range space of involved maps or continuity of involved maps. Moreover, the new type F_M-contraction defined by us is more comprehensive than the one introduced by Piri and Kumam [11] and Wardowski [22].

2 Preliminaries

We denote as usually the set of all real numbers by $\mathbb{R}$, the set of all positive numbers by $\mathbb{R}^+$ and the set of positive integers by $\mathbb{N}$.

First of all, we recall the concept of F-contraction introduced by Wardowski [22].

Let $\mathcal{F}$ be the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying the following conditions:

(F1) F is strictly increasing, i.e., for all $\alpha, \beta \in (0, \infty)$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$;
(F2) for each sequence $\{\alpha_n\}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;
(F3) there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Definition 1. (See [22].) Let (X, d) be a metric space and $H : X \to X$ be a map. H is said to be an F*-contraction* if $F \in \mathcal{F}$ and there exists $\tau > 0$ such that

$$\forall x, y \in X, \quad d(Tx, Ty) > 0 \quad \Longrightarrow \quad \tau + F\bigl(d(Hx, Hy)\bigr) \leqslant F\bigl(d(x, y)\bigr). \tag{1}$$

From (F1) and (1) it is easy to see that every F-contraction H is contractive, i.e., $d(Hx, Hy) < d(x, y)$ for all $x \neq y \in X$, and hence it is necessary continuous.

Taking different functions F, we obtain a variety of F-contractions, some of them being already known in the literature.

Remark 1. Any Banach contraction of ratio $r \in (0, 1)$ is an F-contraction, where $F : \mathbb{R}^+ \to \mathbb{R}$, $F(t) = \ln t$ and $\tau = -\ln r$. Moreover, there exist F-contractions, which are not Banach contractions (see, e.g., [15, 22]).

Secelean [15] proved the following lemma.

Lemma 1. (*See* [15, Lem. 3.2].) *Let* $F : \mathbb{R}^+ \to \mathbb{R}$ *be an increasing mapping and* $\{\alpha_n\}$ *be a sequence of positive real numbers. Then the following assertions hold*:

(a) *if* $\lim_{n\to\infty} F(\alpha_n) = -\infty$*, then* $\lim_{n\to\infty} \alpha_n = 0$;
(b) *if* $\inf F = -\infty$ *and* $\lim_{n\to\infty} \alpha_n = 0$*, then* $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

By proving Lemma 1 Secelean showed that condition (F2) in Definition 1 can be replaced by an equivalent but a more simple one:

(F2′) $\inf F = -\infty$;

or, also by

(F2″) there exists a sequence $\{\alpha_n\}$ of positive real numbers such that the limit $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

In [11], Piri and Kumam used the following condition instead of (F3) in Definition 1:

(F3′) F is continuous on $\mathbb{R}^+$.

They denoted the set of all functions satisfying conditions (F1), (F2′) and (F3′) by $\mathfrak{F}$.

Wardowski [22, Thm. 2.1] and Piri and Kumam [11, Thm. 2.1] proved the existence and uniqueness of fixed points of F-contractions, where $F \in \mathcal{F}$ and $F \in \mathfrak{F}$, respectively.

In 2014, Minak, Helvaci and Altum [10] extend the work of Wardowski, Piri and Kumam and introduced the concept of *generalized Ćirić-type F-contractions*, where $F \in \mathcal{F}$ for a self-mapping H of a metric space (X,d) for which there exists $\tau > 0$ such that

$$\tau + F\big(d(Hx,Hy)\big) \leqslant F\bigg(\max\bigg\{d(x,y),\ d(Hx,x),\ d(Hy,y),\ \frac{1}{2}\big(d(Hx,y)+d(Hy,x)\big)\bigg\}\bigg), \tag{2}$$

whenever $x, y \in X$, $Hx \neq Hy$.

Theorem 1. (*See* [10, Thm. 2.2].) *Let (X,d) be a complete metric space and $H: X \to X$ be a Ćirić-type generalized F-contraction. If H or F is continuous, then H has a unique fixed point in X.*

For the sequel, we will denote by $\mathcal{F}_M$ the family of all continuous functions $F: \mathbb{R}^+ \to \mathbb{R}$.

Definition 2. A pair of self-maps S and T of a metric space (X,d) have a *coincidence point* $x \in X$ if $Sx = Tx$. Further, a point $x \in X$ is a *common fixed point of S and T* if $Sx = Tx = x$.

We say that two pairs of self-maps (S,T), (P,Q) of a metric space (X,d) have a *common coincidence point* if there exists $x \in X$ such that $Sx = Tx = Px = Qx$.

Definition 3. (See [18].) A pair (S,T) on a metric space (X,d) is said to be:

(a) *compatible* if $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$;

(b) *weakly compatible* if the pair commutes on the set of their coincidence points, i.e., for $x \in X$, $Sx = Tx$ implies $STx = TSx$.

Definition 4. (See [1].) We say that a pair (S,T) on a metric space (X,d) has:

(a) the *property* (E.A.) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

(b) the *common limit property* with respect to S, denoted by (CLR$_S$), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in S(X)$.

Definition 5. Two pairs (A,S) and (B,T) of self-maps of a metric space (X,d) are said to satisfy:

(a) the *common property* (E.A.) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in X$.

(b) the *common limit range property with respect to S and T*, denoted by (CLR$_{ST}$), if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in S(X) \cap T(X)$.

Definition 6. We say that a pair of self-maps (A,S) of a metric space (X,d) constitutes a *Ćirić-type F_M-contraction* if there exist $F\in\mathcal{F}_M$ and $\tau>0$ such that, for all $x,y\in X$ with $d(Ax,Ay)>0$,

$$\tau+F\bigl(d(Ax,Ay)\bigr)\leqslant F\biggl(\max\biggl\{d(Ax,Sx),\ d(Ay,Sy),\ d(Sx,Sy),\ \frac{d(Ax,Sy)+d(Ay,Sx)}{2}\biggr\}\biggr).$$

Definition 7. Two pairs of self-maps (A,S) and (B,T) of a metric space (X,d) constitute a *Ćirić-type F_M-contraction* if there exist $F\in\mathcal{F}_M$ and $\tau>0$ such that, for all $x,y\in X$ with $d(Ax,By)>0$,

$$\tau+F\bigl(d(Ax,By)\bigr)\leqslant F\biggl(\max\biggl\{d(Ax,Sx),\ d(By,Ty),\ d(Sx,Ty),\ \frac{d(Sx,By)+d(Ax,Ty)}{2}\biggr\}\biggr). \tag{3}$$

Proposition 1. *Let A, B, S, T, L and M be self-maps of a metric space (X,d) satisfying the following conditions*:

(α) $L(X)\subseteq ST(X)$ (*resp.* (α') $M(X)\subseteq AB(X)$);
(β) *the pair (L,AB) satisfies the* (CLR_{AB}) *property* (*resp.* (β') *the pair (M,ST) satisfies the* (CLR_{ST}) property);
(γ) *$ST(X)$ is a closed subset of X* (*resp.* (γ') *$AB(X)$ is a closed subset of X*);
(δ) *there exists $\tau>0$ and $F:\mathbb{R}^+\to\mathbb{R}$ such that, for all $x,y\in X$ with $d(Lx,My)>0$,*

$$\tau+F\bigl(d(Lx,My)\bigr)\leqslant F\biggl(\max\biggl\{d(Lx,ABx),\ d(My,STy),\ d(ABx,STy),\ \frac{d(ABx,My)+d(Lx,STy)}{2}\biggr\}\biggr). \tag{4}$$

Then the pairs (L,AB) and (M,ST) share the ($\mathrm{CLR}_{(AB)(ST)}$) *property.*

Proof. Since the pair (L,AB) satisfies the (CLR_{AB}) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty}Lx_n=\lim_{n\to\infty}ABx_n=z, \tag{5}$$

where $z\in AB(X)$. In view of (α), for $\{x_n\}\subset X$, there exists a sequence $\{y_n\}\subset X$ such that $Lx_n=STy_n$ for all $n\in\mathbb{N}$. Hence, by (γ), $z\in AB(X)\cap ST(X)$.

Therefore, it suffices to prove that $\lim_{n\to\infty}My_n=z$, that is, the limit $\lim_n d(Lx_n,My_n)=0$. Indeed, on the contrary, there are $\varepsilon>0$ and an infinite set $I\subset\mathbb{N}$ such that $d(Lx_n,My_n)>\varepsilon$ for every $n\in I$. Then, putting $x=x_n$, $y=y_n$ in (4) and taking into account that $Lx_n = STy_n$, we obtain

$$\tau + F\bigl(d(Lx_n, My_n)\bigr) \leqslant F\left(\max\left\{d(Lx_n, ABx_n),\ d(My_n, STy_n),\right.\right.$$
$$\left.\left. d(ABx_n, STy_n),\ \frac{d(ABx_n, My_n)}{2}\right\}\right). \tag{6}$$

From (5) it follows that one can find $N \in \mathbb{N}$ such that

$$d(Lx_n, ABx_n) < \varepsilon \quad \forall n \geqslant N.$$

Fix $n \in I$, $n \geqslant N$. One has

$$\begin{aligned} d(ABx_n, My_n) &\leqslant d(ABx_n, Lx_n) + d(Lx_n, My_n) \\ &< \varepsilon + d(Lx_n, My_n) \end{aligned}$$

and so, by our assumption,

$$\frac{d(ABx_n, My_n)}{2} < \frac{\varepsilon + d(Lx_n, My_n)}{2} < d(Lx_n, My_n).$$

Consequently

$$\begin{aligned} &\max\left\{d(Lx_n, ABx_n),\ d(My_n, STy_n),\ d(ABx_n, STy_n),\ \frac{d(ABx_n, My_n)}{2}\right\} \\ &\quad = d(Lx_n, My_n). \end{aligned}$$

Now (6) becomes

$$\tau + F\bigl(d(Lx_n, My_n)\bigr) \leqslant F\bigl(d(Lx_n, My_n)\bigr),$$

which is a contradiction.

Hence, $\lim_{n\to\infty} d(Lx_n, My_n) = 0$. Accordingly,

$$\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} STy_n = \lim_{n\to\infty} My_n = z,$$

where $z \in AB(X) \cap ST(X)$, i.e., the pairs (L, AB), (M, ST) share the (CLR$_{(AB)(ST)}$) property.

In the same manner, one can obtain the conclusion using conditions (α'), (β'), (γ'). $\square$

In order to show that the common property (E.A.) of two pairs (L, AB) and (M, ST) can be deduced from containment of $L(X) \subseteq ST(X)$ and property (E.A.) of the pair (L, AB), we can formulate the following result, its proof being analogous to that of Proposition 1.

Proposition 2. *Let A, B, S, T, L and M be self-maps of a metric space (X, d). Suppose that there exists $\tau > 0$ and $F : \mathbb{R}^+ \to \mathbb{R}$ such that inequality* (3) *and the following hypotheses hold*:

(a) $L(X) \subseteq ST(X)$;
(b) $ST(X)$ *is closed and the pair (L, AB) satisfies the property* (E.A.).

Then (L, AB) and (M, ST) satisfy the common property (E.A.).

Remark 2. Proposition 2 assures that the condition of common property (E.A.) of two pairs (L, AB) and (M, ST) is weaker than the containment of $L(X) \subseteq ST(X)$ and property (E.A.) of the pair (L, AB).

3 Main results

We will prove our main results by exploiting Ćirić-type F_M-contraction for eight self-maps via (CLR$_{(AB)(ST)}$) property and common property (E.A.).

Theorem 2. *Let A, B, S, T, L, M, f and g be self-maps of a metric space (X, d). Suppose that the pairs (Lf, AB) and (Mg, ST) satisfy* (CLR$_{(AB)(ST)}$) *property and constitute a Ćirić-type F_M-contraction, that is, there exist $F \in \mathcal{F}_M$ and $\tau > 0$ such that, for all $x, y \in X$ with $d(Lfx, Mgy) > 0$,*

$$\tau + F\bigl(d(Lfx, Mgy)\bigr) \leqslant F\left(\max\left\{d(Lfx, ABx),\ d(Mgy, STy),\ d(ABx, STy),\ \frac{d(ABx, Mgy) + d(Lfx, STy)}{2}\right\}\right). \tag{7}$$

Then there is a common fixed point for both pairs (Lf, AB) and (Mg, ST).

Moreover, if

(i) *both pairs (Lf, AB) and (Mg, ST) are weakly compatible*;
(ii) $AB = BA$, $Lf = fL$, $LfA = ALf$;
(iii) $ST = TS$, $Mg = gM$, $MgS = SMg$;
(iv) $fx = f^2x$, $gx = g^2x$ *for all* $x \in X$;

then A, B, S, T, L, M, f and g have a unique common fixed point in X.

Proof. The fact that the pairs (Lf, AB) and (Mg, ST) satisfy the (CLR$_{(AB)(ST)}$) property is equivalent to the existence of two sequences $\{x_n\}$, $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} STy_n = \lim_{n\to\infty} Mgy_n = t, \tag{8}$$

where $t \in AB(X) \cap ST(X)$.

Since $t \in AB(X)$, there exists a point $u \in X$ such that $ABu = t$. Also, since $t \in ST(X)$, there exists a point $v \in X$ such that $STv = t$.

We claim that $d(t, Mgv) = 0$. Suppose, on the contrary, that $d(t, Mgv) = c > 0$. Then there exist $\varepsilon > 0$, $\varepsilon < c$ and $N \in \mathbb{N}$ such that $d(Lfx_n, Mgv) > \varepsilon$ for all $n \geqslant N$. Taking $x = x_n$ and $y = v$ in (7), one obtains

$$\begin{aligned}
&\tau + F\bigl(d(Lfx_n, Mgv)\bigr) \\
&\quad\leqslant F\biggl(\max\biggl\{d(Lfx_n, ABx_n),\ d(STv, Mgv),\ d(ABx_n, STv), \\
&\qquad\qquad \frac{d(ABx_n, Mgv) + d(Lfx_n, STv)}{2}\biggr\}\biggr)
\end{aligned}$$

for all $n \geqslant N$. By passing to the limit in the above inequality, using (8) and the continuity of F at c, we get

$$\tau + F(c) \leqslant F\biggl(\max\biggl\{0, c, 0, \frac{c}{2}\biggr\}\biggr) = F(c),$$

which is a contradiction. Hence, $d(t, Mgv) = 0$, which implies $t = Mgv$.

Therefore, $t = STv = Mgv$, which shows that v is a coincidence point of the pair (Mg, ST).

Similarly, we can also obtain $t = Lfu = ABu$, so u is a coincidence point of the pair (Lf, AB).

Since the pairs (Mg, ST) and (Lf, AB) are weakly compatible and in view of the aforesaid $t = Mgv = STv = Lfu = ABu$ hence $Mgt = STt$, $Lft = ABt$.

In order to show that $Lft = t$, assume that $Lft \neq t$. Using again (7), we have

$$\begin{aligned}
F\bigl(d(Lft, t)\bigr) &= F\bigl(d(Lft, Mgv)\bigr) \\
&\leqslant F\biggl(\max\biggl\{d(Lft, ABt),\ d(Mgv, STv),\ d(ABt, STv), \\
&\qquad \frac{d(ABt, Mgv) + d(Lft, STv)}{2}\biggr\}\biggr) - \tau \\
&= F\bigl(\max\bigl\{0, 0, d(Lft, t), d(Lft, t)\bigr\}\bigr) - \tau \\
&= F\bigl(d(Lft, t)\bigr) - \tau < F\bigl(d(Lft, t)\bigr),
\end{aligned}$$

which is a contradiction. Consequently $Lft = t = ABt$.

Similarly, we also can prove that $Mgt = t = STt$.

Thus, we have $Lft = Mgt = t = ABt = STt$, i.e., t is a common fixed point of Lf, Mg, AB and ST.

Again, taking $x = t, y = St$ in (7) with the assumption $d(Lft, MgSt) \neq 0$, from condition (iii) we have

$$\begin{aligned}
F\bigl(d(Lft, MgSt)\bigr) &= F\bigl(d(t, St)\bigr) \\
&\leqslant F\biggl(\max\biggl\{d(ABt, Lft),\ d(MgSt, STSt),\ d(ABt, STSt), \\
&\qquad \frac{d(ABt, MgSt) + d(Lft, STSt)}{2}\biggr\}\biggr) - \tau
\end{aligned}$$

$$\begin{aligned}
&= F\big(\max\{0, 0, d(t, St), d(t, St)\}\big) - \tau \\
&= F\big(d(t, St)\big) - \tau < F\big(d(t, St)\big),
\end{aligned}$$

which is a contradiction. Hence, $d(Lft, MgSt) = d(t, St) = 0$, i.e., $t = St$. Thus, $t = St = STt = TSt = Tt$.

Similarly, we also can show that $t = At = ABt = BAt = Bt$.

Since $ft = f^2t, gt = g^2t$ and $Lf = fL, Mg = gM$, one has

$$\begin{aligned}
t = Lft = Lfft = fLft = ft \quad &\Longrightarrow \quad Lt = t. \\
t = Mgt = Mggt = gMgt = gt \quad &\Longrightarrow \quad Mt = t.
\end{aligned}$$

Therefore, in view of the aforesaid, we have $t = At = Bt = St = Tt = Lt = Mt = ft = gt$, which shows that A, B, L, M, S, T, f and g have a common fixed point t in X.

Next, we intend to show that this common fixed point is unique. Assume that w is another common fixed point of A, B, L, M, S, T, f and g with $w \neq t$. It follows that $w = Aw = Bw = Lw = Mw = Sw = Tw = fw = gw$. Putting $x = t$, $y = w$ in (7), we have

$$\begin{aligned}
F\big(d(Lft, Mgw)\big) &= F\big(d(t, w)\big) \\
&\leqslant F\bigg(\max\bigg\{d(ABt, Lft),\, d(Mgw, STw),\, d(ABt, STw), \\
&\qquad \frac{d(ABt, Mgw) + d(Lft, STw)}{2}\bigg\}\bigg) - \tau \\
&= F\big(\max\{0, 0, d(t, w), d(t, w)\}\big) - \tau \\
&= F\big(d(t, w)\big) - \tau < F\big(d(t, w)\big),
\end{aligned}$$

which is a contradiction. Hence, $d(t, w) = 0$, i.e., $t = w$.

Thus, A, B, L, M, S, T, f and g have a unique common fixed point t in X. The proof is complete. □

In the next theorem, we will show that one can obtain the results from Theorem 1 without assuming that F satisfies axioms (F2), (F3), respectively (F2), (F3′). We need first the following lemma.

Lemma 2. (*See* [20, Prop. 3].) *Let (X, d) be a metric space, $\{x_n\}$ be a sequence of elements from X, and let Δ be a countable subset of $\mathbb{R}_+$. If $d(x_n, x_{n+1}) \to 0$ and x_n is not Cauchy, then there exist $\eta \in \mathbb{R}^+ \setminus \Delta$, $N \in \mathbb{N}$, and the sequences of positive integers $\{m_k\}$, $\{n_k\}$ such that*

(a) *for all $k \in \mathbb{N}$, $k \leqslant m_k < n_k$, $d(x_{m_k}, x_{n_k}) > \eta$,*
(b) *for all $k \geqslant N$, $n_k - m_k \geqslant 2$, $d(x_{m_k}, x_{n_k - 1}) \leqslant \eta$,*
(c) $d(x_{m_k}, x_{n_k}) \to \eta$, $k \to \infty$,
(d) $d(x_{m_k+1}, x_{n_k+1}) \to \eta$, $k \to \infty$.

Theorem 3. *Let consider a complete metric space (X,d) and H a self-map of X for which there are an increasing function $F:\mathbb{R}^+\to\mathbb{R}$ and $\tau>0$ such that* (2) *holds for all $x,y\in X$, $Hx\neq Hy$. If H or F is continuous, then H has a unique fixed point.*

Proof. Let any $x_0\in X$ and denote $x_n=Hx_{n-1}$ for $n=1,2,\dots$. If there is $n\in\mathbb{N}$ such that $x_n=x_{n-1}$, then x_{n-1} is a fixed point for H. Assume that $d(x_n,x_{n-1})>0$ for all n. Then for each $n\geqslant 1$, we have

$$\begin{aligned}F\bigl(d(x_{n+1},x_n)\bigr)&=F\bigl(d(Hx_n,Hx_{n-1})\bigr)\\&\leqslant F\biggl(\max\biggl\{d(Hx_n,x_n),\ d(Hx_{n-1},\ x_{n-1}),\ d(x_n,x_{n-1}),\\&\qquad\qquad\frac{1}{2}d(Hx_n,x_{n-1})\biggr\}\biggr)-\tau\\&=F\bigl(\max\bigl\{d(x_{n+1},x_n),d(x_n,x_{n-1})\bigr\}\bigr)-\tau\\&=F\bigl(d(x_n,x_{n-1})\bigr)-\tau\end{aligned}\tag{9}$$

because $d(x_{n+1},x_n)\leqslant d(x_n,x_{n-1})$ since, otherwise, we would have

$$F\bigl(d(x_{n+1},x_n)\bigr)\leqslant F\bigl(d(x_{n+1},x_n)\bigr)-\tau,$$

which is a contradiction. By (9) we deduce recursively

$$F\bigl(d(x_{n+1},x_n)\bigr)\leqslant F\bigl(d(x_1,x_0)\bigr)-n\tau\quad\forall n\geqslant 1$$

and so $F(d(x_{n+1},x_n))\to-\infty$. From Lemma 1(a) we deduce that $d(x_{n+1},x_n)\to 0$.

Now, assume that the sequence $\{x_n\}$ is not Cauchy. Since the function F is monotonic, it follows that the set Δ of its discontinuities is at most countable.

According to Lemma 2 there exist $\eta>0$, $\eta\notin\Delta$ and the sequences $\{m_k\}$, $\{n_k\}$ such that

$$d(x_{m_k},x_{n_k})\searrow\eta,\quad d(x_{m_k+1},x_{n_k+1})\to\eta,\quad k\to\infty.$$

By (9), $F(d(x_{m_k+1},x_{n_k+1}))\leqslant F(d(x_{m_k},x_{n_k}))-\tau$ for every $k\in\mathbb{N}$.

Letting $k\to\infty$ and using the continuity of F at η, one obtains

$$F(\eta)\leqslant F(\eta)-\tau,$$

which is a contradiction. Consequently the sequence $\{x_n\}$ is Cauchy so, the space (X,d) being complete, is convergent to some $t\in X$.

If H is continuous then, clearly, $H(t)=t$. The uniqueness of the fixed point results easily from (2).

If F is continuous, the conclusion follows from Theorem 2 taking $H=Lf=Mg$, $AB=ST=I_X$ (the identity map), since, in view of the aforesaid, the pairs (Lf,AB) and (Mg,ST) satisfy (CLR$_{(AB)(ST)}$) property. □

If we take $g = I_X$ (or $f = I_X$) in Theorem 2, then we can obtain the following coincidence and common fixed point result for seven self-maps.

Corollary 1. *Let A, B, S, T, L, M and f be self-maps of a metric space (X, d). Suppose that the pairs (Lf, AB) and (M, ST) satisfy* $(\mathrm{CLR}_{(AB)(ST)})$ *property and are Ćirić F_M-contraction, that is, there exist $F \in \mathcal{F}_M$ and $\tau > 0$ such that, for all $x, y \in X$ with $d(Lfx, My) > 0$,*

$$\tau + F\big(d(Lfx, My)\big) \leqslant F\left(\max\left\{d(Lfx, ABx),\ d(STy, My),\ d(ABx, STy),\ \frac{d(ABx, My) + d(Lfx, STy)}{2}\right\}\right).$$

Then both pairs (L, AB) and (M, ST) have a common fixed point.

Moreover, if

(i) *pairs (Lf, AB) and (M, ST) are weakly compatible*;
(ii) $AB = BA$, $Lf = fL$, $LfA = ALf$;
(iii) $ST = TS$, $MgS = SMg$;
(iv) $fx = f^2x$ *for all* $x \in X$;

then A, B, S, T, L, M and f have a unique common fixed point in X.

If we take $f = g = I_X$ in Theorem 2, we can obtain common fixed point result for six self-maps:

Corollary 2. *Let A, B, S, T, L and M be self-maps of a metric space (X, d). Suppose that the pairs (L, AB) and (M, ST) satisfy* $(\mathrm{CLR}_{(AB)(ST)})$ *property and they are Ćirić F_M-contraction, that is, there exist $F \in \mathcal{F}_M$ and $\tau > 0$, such that for all $x, y \in X$ with $d(Lx, My) > 0$,*

$$\tau + F\big(d(Lx, My)\big) \leqslant F\left(\max\left\{d(Lx, ABx),\ d(STy, My),\ d(ABx, STy),\ \frac{d(ABx, My) + d(Lx, STy)}{2}\right\}\right). \tag{10}$$

Then both pairs (L, AB) and (M, ST) have a common fixed point.

Moreover, if

(i) *both pairs (L, AB) and (M, ST) are weakly compatible*;
(ii) $AB = BA$, $LA = AL$;
(iii) $ST = TS$, $MS = SM$;

then A, B, S, T, L and M have a unique common fixed point in X.

If we take $T = I_X$ in Corollary 2, we also can obtain the coincidence and common fixed point result for five self-maps as follows:

Corollary 3. *Let A, B, S, L and M be self-maps of a metric space (X,d). Suppose that the pairs (L,AB) and (M,S) satisfy* (CLR$_{(AB)(S)}$) *property and they are Ćirić F_M-contraction, i.e., there exist $F \in \mathcal{F}_M$ and $\tau > 0$ such that, for all $x,y \in X$ with $d(Lx,My) > 0$,*

$$F\big(d(Lx,My)\big) \leqslant F\left(\max\left\{d(Lx,ABx), d(Sy,My), d(ABx,Sy), \frac{d(ABx,My)+d(Lx,Sy)}{2}\right\}\right) - \tau.$$

Then (L,AB) and (M,S) have a common fixed point.

Moreover, if

(i) *both pairs (L,AB) and (M,S) are weakly compatible*;
(ii) $AB = BA$, $LA = AL$;
(iii) $SM = MS$;

then A, B, S, L and M have a unique common fixed point in X.

Finally, if we take $B = T = I_X$ in Corollary 3, we even obtain the analogous results for four self-maps, which is stated as follows:

Corollary 4. *Let A, S, L and M be self-maps of a metric space (X,d). Suppose that the pairs (L,A) and (M,S) satisfy* (CLR$_{(AS)}$) *property and they are Ćirić F_M-contraction. Then (L,A) and (M,S) have a common fixed point. Moreover, if both pairs (L,A) and (M,S) are weakly compatible, $LA = AL, MS = SM$, then A, S, L and M have a unique common fixed point in X.*

Using now Proposition 1, we have the following result:

Theorem 4. *Let A, S, L and M be self-maps of a metric space (X,d). Suppose that all the conditions* (α)–(δ)*(or* (α')–(δ')) *of Proposition* 1 *hold. Then (L,AB) and (M,ST) have a common fixed point. Moreover, if*

(i) *both pairs (L,AB) and (M,ST) are weakly compatible*;
(ii) $AB = BA$, $LA = AL$;
(iii) $ST = TS$, $MS = SM$;

then A, B, S, L and M have a unique common fixed point in X.

Proof. According to Proposition 1 we deduce that the pairs (L,AB) and (M,ST) have the (CLR$_{(AB)(ST)}$) property. The rest of the proof can be completed along with the routine of the proof of Theorem 2. For the sake of briefness, we omit the tedious presentation. □

In the following, we utilize the common property (E.A.) instead of (CLR$_{(AB)(ST)}$) property of (Lf,AB) and (Mg,ST) in Theorem 2 in order to obtain coincidence and common fixed point results for eight self-maps.

Theorem 5. *Let A, B, S, T, L, M, f and g be self-maps of a metric space (X, d). Suppose that the inequality* (7) *and the following hypotheses hold*:

(a) *the pairs (Lf, AB) and (Mg, ST) have the common property* (E.A.);
(b) *$ST(X)$ and $AB(X)$ are closed subsets of X.*

Then (Lf, AB) and (Mg, ST) have a common fixed point.
Moreover, if

(i) *both pairs (Lf, AB) and (Mg, ST) are weakly compatible*;
(ii) *$AB = BA$, $Lf = fL$, $LfA = ALf$*;
(iii) *$ST = TS$, $Mg = gM$, $MgS = SMg$*;
(iv) *$fx = f^2x$, $gx = g^2x$ for all $x \in X$;*

then A, B, S, T, L, M, f and g have a unique common fixed point in X.

Proof. Since the pairs (Lf, AB) and (Mg, ST) have the common property (E.A.), one can find two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} STy_n = \lim_{n\to\infty} Mgy_n = z,$$

where $z \in X$. Since $ST(X)$ is closed, $\lim_{n\to\infty} STy_n = z = STv$ for some $v \in X$. Also, $AB(X)$ being closed, one has $\lim_{n\to\infty} ABx_n = z = ABu$ for some $u \in X$. The rest of the argument can run along with the lines of Theorem 2. □

Some significant consequences of the previous theorem can be obtained by considering the cases when some of the respective eight functions are I_X.

Corollary 5. *Let A, B, S, T, L and M be self-maps of a metric space (X, d). Suppose that the inequality* (10) *and the following hypotheses hold*:

(a) *the pairs (L, AB) and (M, ST) have the common property* (E.A.);
(b) *$ST(X)$ and $AB(X)$ are closed subsets of X or*
(b′) *$\overline{L(X)} \subseteq ST(X)$ and $\overline{M(X)} \subseteq AB(X)$, where the bar means the closure, or*
(b″) *$L(X)$ and $M(X)$ are closed subset of X and $L(X) \subseteq ST(X)$ and $M(X) \subseteq AB(X)$.*

Then, (L, AB) and (M, ST) have a common fixed point.
Moreover, if

(i) *both pairs (L, AB) and (M, ST) are weakly compatible*;
(ii) *$AB = BA$, $LA = AL$*;
(iii) *$ST = TS$, $MS = SM$*;

then A, B, S, T, L and M have a unique common fixed point in X.

Proof. The results follow in the same manner as in the proof of Theorem 5 by changing both f and g with I_X. □

In the following, we present some examples to support our main results.

Example 1. Let $X = [1, \infty)$ and d be the Euclidean metric defined by $d(x, y) = |x - y|$ for all $x, y \in X$. Define A, B, S, T, L and $M{:}X \to X$ by

$$Ax = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 6 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 1 & \text{if } x \geqslant 4, \end{cases} \qquad Bx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 5 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 1 & \text{if } x \geqslant 4, \end{cases}$$

$$Sx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 9 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2 & \text{if } x \geqslant 4, \end{cases} \qquad Tx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 4 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2 & \text{if } x \geqslant 4, \end{cases}$$

$$Lx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 6 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2 & \text{if } x \geqslant 4, \end{cases} \qquad Mx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 10 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2, & \text{if } x \geqslant 4, \end{cases}$$

$$fx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 7 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2 & \text{if } x \geqslant 4, \end{cases} \qquad gx = \begin{cases} 3 & \text{if } x = 1, 2, 3, \\ 8 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \\ 2 & \text{if } x \geqslant 4. \end{cases}$$

Hence,

$$Lfx = \begin{cases} 3 & \text{if } x = 1, 2, 3 \text{ and } x \geqslant 4, \\ 2 & \text{if } x \in [1, 4) - \{1, 2, 3\}, \end{cases} \qquad Mgx = \begin{cases} 3 & \text{if } x = 1, 2, 3 \text{ and } x \geqslant 4, \\ 2 & \text{if } x \in [1, 4) - \{1, 2, 3\}. \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X that $x_n = 4 + 1/n$, $y_n = 3$, then

$$\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} Lf\left(4 + \frac{1}{n}\right) = 3,$$
$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} AB\left(4 + \frac{1}{n}\right) = 3,$$

and

$$\lim_{n\to\infty} Mgy_n = \lim_{n\to\infty} Mg(3) = 3,$$
$$\lim_{n\to\infty} STy_n = \lim_{n\to\infty} ST(3) = 3.$$

Hence, $\lim_{n\to\infty} Lfx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Mgy_n = \lim_{n\to\infty} STy_n = 3$, $3 \in AB(X) \cap ST(X)$, i.e., (Lf, AB) and (Mg, ST) share the (CLR$_{(AB)(ST)}$) property.

Also, $Lfx = ABx = 3$, $Mgx = STx = 3$, where $x \in \{1, 2, 3\}$ and $x \geqslant 4$, i.e., the pairs (Lf, AB) and (Mg, ST) have coincidence points in X.

Moreover, $LfABx = ABLfx$, $MgSTx = STMgx$, where $x \in \{1, 2, 3\}$ and $x \geqslant 4$, i.e., (Lf, AB) and (Mg, ST) are weakly compatible. We also can easily check that $AB = BA$, $fL = Lf$, $Mg = gM$,$ST = TS$, $LfA = ALf$, $MgS = SMg$, $f^2 = f$ and $g^2 = g$. Further, A, B, S, T, L, M, f and g satisfy Ćirić-type F_M-contraction assumption (4) for $\tau = \ln 3$ and $F(\alpha) = \ln\alpha$.

Hence, all the conditions of Theorem 2 are satisfied, and $x = 3$ is the unique common fixed point of A, B, S, T, L, M, f and g. Moreover, all self-maps are discontinuous at common fixed point.

Example 2. Let $X = [1,\infty)$ and d be the ordinary metric defined by $d(x,y) = |x - y|$ for all $x, y \in X$. Define A, B, S, T, L and $M{:}X \to X$ by

$$Ax = \begin{cases} 3 & \text{if } x = 1,2,3,\\ 6 & \text{if } x \in [1,4) - \{1,2,3\},\\ 1 & \text{if } x \geqslant 4, \end{cases} \qquad Bx = \begin{cases} 3 & \text{if } x = 1,2,3,\\ 5 & \text{if } x \in [1,4) - \{1,2,3\},\\ 1 & \text{if } x \geqslant 4, \end{cases}$$

$$Sx = \begin{cases} 3 & \text{if } x = 1,2,3,\\ 9 & \text{if } x \in [1,4) - \{1,2,3\},\\ 2 & \text{if } x \geqslant 4, \end{cases} \qquad Tx = \begin{cases} 3 & \text{if } x = 1,2,3,\\ 4 & \text{if } x \in [1,4) - \{1,2,3\},\\ 2 & \text{if } x \geqslant 4, \end{cases}$$

$$Lx = \begin{cases} 3 & \text{if } x = 1,2,3 \text{ and } x \geqslant 4,\\ 4 & \text{if } x \in [1,4) - \{1,2,3\}, \end{cases} \qquad Mx = \begin{cases} 3 & \text{if } x = 1,2,3 \text{ and } x \geqslant 4,\\ 2 & \text{if } x \in [1,4) - \{1,2,3\}. \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X that $x_n = 4 + 1/n$, $y_n = 3$, then

$$\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} L\left(4 + \frac{1}{n}\right) = 3,$$
$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} AB\left(4 + \frac{1}{n}\right) = 3,$$

and

$$\lim_{n\to\infty} My_n = \lim_{n\to\infty} M(3) = 3,$$
$$\lim_{n\to\infty} STy_n = \lim_{n\to\infty} ST(3) = 3.$$

Hence, $\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} My_n = \lim_{n\to\infty} STy_n = 3$, $3 \in AB(X) \cap ST(X)$, i.e., (L, AB) and (M, ST) share the ($\mathrm{CLR}_{(AB)(ST)}$) property.

Also, $Lx = ABx = 3$, $Mx = STx = 3$, where $x \in \{1,2,3\}$ and $x \geqslant 4$, i.e., the pairs (L, AB) and (M, ST) have coincidence points in X.

Moreover, $LABx = ABLx$, $MSTx = STMx$, where $x \in \{1,2,3\}$ and $x \geqslant 4$, i.e., (L, AB) and (M, ST) are weakly compatible. We also can easily check that $AB = BA$, $AL = LA$, $MS = SM$ and $ST = TS$. Further, A, B, S, T, L and M satisfy Ćirić-type F_M-contraction assumption (6) for $\tau = \ln 3$ and $F(\alpha) = \ln\alpha$.

Hence, all the conditions of Corollary 2 are satisfied and $x = 3$ is the unique common fixed point of A, B, S, T, L and M. Moreover, all self-maps are discontinuous at common fixed point.

Example 3. Let $X = (2,10)$ and d be the ordinary metric defined by $d(x,y) = |x-y|$ for all $x,y \in X$. Define A, B, S, T, L and M:$X \to X$ by

$$Ax = \begin{cases} 3 & \text{if } x \in (2,4], \\ 7 & \text{if } x \in [4,6), \\ 4 & \text{if } x \in [6,10), \end{cases} \qquad Bx = \begin{cases} 3 & \text{if } x \in (2,4], \\ 8 & \text{if } x \in [4,6), \\ 4 & \text{if } x \in [6,10), \end{cases}$$

$$Sx = \begin{cases} 3 & \text{if } x \in (2,4], \\ 7 & \text{if } x \in [4,6), \\ 4 & \text{if } x \in [6,10), \end{cases} \qquad Tx = \begin{cases} 3 & \text{if } x \in (2,4], \\ 10 & \text{if } x \in [4,6), \\ \frac{3}{2} & \text{if } x \in [6,10). \end{cases}$$

$$Lx = \begin{cases} 3 & \text{if } x \in (2,4] \cup [6,10), \\ 12 & \text{if } x \in (4,6), \end{cases} \qquad Mx = \begin{cases} 3 & \text{if } x \in (2,4] \cup [6,10), \\ 9 & \text{if } x \in (4,6). \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X that $x_n = 6 + 1/n$, $y_n = 4 - 1/n$, then

$$\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} L\left(6 + \frac{1}{n}\right) = 3,$$

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} AB\left(6 + \frac{1}{n}\right) = 3,$$

and

$$\lim_{n\to\infty} My_n = \lim_{n\to\infty} M\left(4 - \frac{1}{n}\right) = 3,$$

$$\lim_{n\to\infty} STy_n = \lim_{n\to\infty} ST\left(4 - \frac{1}{n}\right) = 3.$$

Hence, $\lim_{n\to\infty} Lx_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} My_n = \lim_{n\to\infty} STy_n$, $3 \in (2,10)$, i.e., (L,AB) and (M,ST) satisfy the common property (E.A.).

Also, $Lx = ABx = 3, Mx = STx = 3$, where $x \in (2,4] \cup (4,10)$, i.e., the pairs (L,AB) and (M,ST) have coincidence points in X.

Moreover, $LABx = ABLx, MSTx = STMx$, where $x \in (2,4] \cup (4,10)$, i.e., (L,AB) and (M,ST) are weakly compatible. We also can easily check that $AB = BA$, $AL = LA$, $MS = SM$, $ST = TS$ and $ST(X)$, $AB(X)$ are closed subsets of X. Further, A, B, S, T, L and M satisfy Ćirić-type F_M-contraction assumption (6) for $\tau = \ln 2$ and $F(\alpha) = \ln \alpha$.

Hence, all the conditions of Corollary 5 are satisfied, and $x = 4$ is the unique common fixed point of A, B, S, T, L and M. Moreover, all self-maps are discontinuous at common fixed point.

4 Conclusion

In our main results, we established coincidence and common fixed point theorems for more than six self-maps on metric spaces via common (CLR$_{(AB)(ST)}$) property or common property (E.A.) with neither assuming continuity nor containment of the range space of the involved maps nor completeness of subspace/space, which generalize the result of Tomar et al. [19] from four maps to eight maps, assuming only that F is continuous without imposing conditions (F1)–(F3). Moreover, the maps are discontinuous even at the common fixed point. Whereas, Batra et al. [2] established coincidence point of a pair of self-maps by taking containment of range space of involved maps, completeness of space along with continuity and commutativity of both maps. The weak compatibility used here is indeed weaker than the commutativity of a pair of maps. Since an F-contraction is a proper generalization of a Banach contraction, our results generalize, extend and improve the results of Wardowski [22] and some other ones existing in the literature. Furthermore, the F_M-contraction introduced here is weaker than the two versions of F-contractions presented by Wardowski [22] and Piri et al. [11], respectively.

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