Abstract: Model Predictive Control has been, and continues to be, one of the major success stories of advanced control in industry. This is a result of several factors including its ability to deal with constraints. Early implementations were concentrated in the process industries. These implementations were characterised by several distinctive features including long sample periods and constant set-points. However, new techniques and faster sampling rates have opened the door to new classes of applications where neither of these earlier features apply. In this paper we re-examine the question of reference tracking in MPC and formulate a novel strategy (termed PF-MPC) aimed at problems where the reference is not constant. The strategy combines both preview and feedforward of the reference signal, with which we seek to improve nominal MPC tracking performance. This preliminary study stands as the conceptual basis for a future robust output-feedback MPC strategy focused on improving tracking of time-varying references.

Keywords: Model predictive control; robust control; reference feedforward; preview.

1. INTRODUCTION

Model Predictive Control has become synonymous with advanced control in the context of industrial control. In part this is because it is easily adapted to include non-linear dynamics and because it seamlessly allows hard constraints to be applied Goodwin et al. (2005); Mayne et al. (2000). Model Predictive Control also has a rich supporting theory. For example, procedures are available that allow stability certificates to be constructed for many standard scenarios, e.g. see Mayne et al. (2000) and the references therein. Recent research in this area has focused on issues of robustness. Various treatments are available covering unmodelled dynamics and unmeasured disturbances, see for example Mayne et al. (2006); Løvaas et al. (2010); Langson et al. (2004); Kouvaritakis et al. (2000).

Because of its early concentration on process control, it has been natural to think of the reference signal as being constant. In such cases, the MPC problem can be reformulated as one of regulation about a target for input and states Muske and Rawlings (1993). However, recent advances in computer speed coupled with enhanced understanding of the structure of MPC solutions, has opened the door to new “high speed” applications. These include electromechanical, power electronics and telecommunication problems. In these areas, it is no longer true that the reference signal can be considered constant, or even piecewise constant.

The current paper addresses the problem of MPC-design with non-constant reference signals. In particular, two inter-related issues will be addressed, namely

(i) reference signal feedforward, and
(ii) reference signal preview.

The only previous paper that we are aware of that addresses “feedforward” in MPC is Rossiter and Valencia-Palomo (2009). In the terminology of the current paper, their focus is on preview rather than feedforward although these topics are closely related. Rossiter and Valencia-Palomo conclude that one needs to be careful with preview since the control can “move” too early if later controls are constrained to be constant due to the choice of the control horizon. We do not address this topic here. Instead, we focus on the feedforward design aspect and how it can be used to improve reference tracking in the MPC set up.

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In the current paper we address conceptual and design issues. We deliberately utilise a simplified framework to help with the exposition of the core ideas. In a companion paper Carrasco and Goodwin (2011) we take a first step towards analysing the robustness of the proposed scheme in the presence of output feedback, unmodelled dynamics and unmeasured disturbances.

For clarity of exposition, we restrict attention to the SISO case. However, we expect that most of the ideas will carry over to the MIMO case with appropriate technical modifications.

The layout of the remainder of the current paper is as follows: in Section 2 we briefly introduce the terms “preview”, “feedforward” and “feedback”. We also compare and contrast these ideas. In Section 3 we give brief details of our design strategy in the unconstrained case. In Section 4 we outline the corresponding constrained MPC strategy. Section 5 presents examples. Finally, in Section 6 we draw conclusions.

2. PREVIEW, FEEDFORWARD AND FEEDBACK

In the context of the current paper, we define “preview” as the provision of prior information about future reference changes. For example, one might know that in 10 minutes time, we would like to change from product stream A to product stream B. Such information is clearly useful in control since it allows the controller to prepare for the change. Also, it is known Middleton et al. (2004) that the availability of preview weakens the fundamental limitations that apply to set-point tracking.

By feedforward, we mean that an entirely different control policy is applied to the reference signal as is used for the feedback part of the control action. We can think of this as providing a “second degree of freedom” in the controller. This “second degree of freedom” does not use observations from the actual plant response. Instead, this is left to the “feedback” component which monitors the actual response and makes appropriate adjustments.

In summary we have:

- Preview: The provision of information about future reference signals to the control law with some horizon \( N_p \).
- Feedforward: Control action based entirely on exogenous data (e.g. reference signal) with no correction for the actual observed response.
- Feedback: Corrective action based on the actual observed response.

Obviously these concepts are inter-related. However, the concepts have distinctive meanings. For example, one can use feedforward control whether or not preview or feedback is available. Feedforward introduces an extra degree of freedom in the design. This extra degree of freedom can sometimes be implicit i.e. simply a matter of how one injects the reference signal into the loop. Here we make this extra degree of freedom explicit.

Our ultimate goal in this paper is to place all of these ideas in the context of MPC. However, in an effort to clarify the various terms, we will first examine the ideas in the context of linear unconstrained control.

2.1 Basic Definitions

We consider a linear time-invariant system:

\[
y = G u + d
\]

\[
y_n = y + n
\]

where \( G(q) \) is a discrete time transfer function (the “true plant” transfer function) and where \( y, u, d, n, y_n \) denote unmeasured plant output, plant input, unmeasured disturbance, measurement noise and measured plant output respectively. Also, we write \( G \) as

\[
G = G_o(1 + G_\Delta)
\]

where \( G_o \) is the nominal model and \( G_\Delta \) the multiplicative model error.

Remark 1. Much more general uncertainty models are possible, but the one given above suffices for our current purposes.

The plant dynamics can be modelled as

\[
x^+ = Ax + Bu + J_1(y − C\hat{x} − \hat{d}) \quad (10)
\]

\[
d^+ = \hat{d} + J_2(y − C\hat{x} − \hat{d}) \quad (11)
\]

where

\[
\bar{A} = \begin{bmatrix} A − J_1C & −J_1 \\ −J_2C & 1 − J_2 \end{bmatrix}
\]

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is stable. The observer (11) is what actually provides the
integral action.

We are now in a position to be able to define a (linear, unconstrained) control law, that will serve for the purpose of exposition:

\[ u(k) - u^*(k) = -K(\hat{x}(k) - x^*(k)) + M(q^{-1})y^*(k) \]

where \( u^*(k), x^*(k) \) are regulation set-points

\[ u^*(k) = G_o(1-q^{-1})y^*(k) - \hat{d}(k) \]

and

\[ x^*(k) = (I - A)^{-1}Bu^*(k) \]

and \( M(q^{-1}) \) is a non-proper stable transfer function of the form:

\[ M(q^{-1}) = q^{N_p}M'(q^{-1}) \]

where \( M'(q^{-1}) \) is a proper and stable transfer function and \( q \) is the unit delay operator.

Actually, equation (13) allows us to contrast the three terms “feedback”, “feedforward” and “preview”. The gain \( K \) gives “feedback” action since it determines the part of the input which depends upon \( \hat{x}(k) \) which, in turn, depends (causally) upon the plant observations \( \{y(i)\} \). The transfer function \( M'(q^{-1}) \) gives feedforward action. Note that this part of the input does not take account of the actual response \( \{y(i)\} \). The integer \( N_p \) defines the “preview” horizon i.e. the future interval over which we know the reference signal.

The above formulation has been discussed in the literature relating to unconstrained control. For example Limebeer et al. (1993); Giusto and Paganini (1999); Vilanova et al. (2007); Cerone et al. (2007) discuss the implications of designing \( K \) and \( M \) simultaneously or in a two step procedure in different scenarios. We argue below that, in the case of constrained MPC, one should design the feedforward component first since all injected signals (such as \( M y^* \)) are potentially destabilising and need to be known a priori, before one can design a robust (constrained) feedback control law.

3. UNCONSTRAINED NOMINAL DESIGN

Throughout this section we assume no constraints. Because of these simplifying assumptions, many of the steps could be carried out in different ways. However, we adopt a particular formulation as a prelude to the more difficult robust case (with constraints) to be discussed in the sequel.

3.1 Feedforward Design

As discussed previously, this part of the design makes no use of actual plant measurements. Hence, we use the nominal plant transfer function \( G_o \). We are interested in designing \( u_{ff} = My^* \) so that \( y_{ff} = G_ou_{ff} \) tracks the given reference signal with small error (the subscript “ff” refers to the feedforward model).

Of course, we need a mechanism for restraining the bandwidth to a sensible value (otherwise impractical deadbeat type responses will ensue).

Due to time invariance, we can, without loss of generality, take the current time as 0. Thus we seek to choose \( u_{ff}(0) \). We use a state-space formulation for the nominal model:

\[ x_{ff}^+ = Ax_{ff} + Bu_{ff} \]

\[ y_{ff} = Cx_{ff} \]

where \( x_{ff}(0) \) is assumed to be known. We also assume \( u(-1), u(-2), \ldots \) are given. We seek to minimise a cost function of the form:

\[ J_{ff} = \sum_{k=0}^{\infty} [y_{ff}(k) - \hat{y}^*(k)]^2 + \lambda_{ff} \cdot |u_{ff}(k) - u_{ff}(k-1)|^2 \]

where \( \hat{y}^*(\cdot) \) was defined previously in (9).

Remark 2. A crucial point about (18) is that we use a weighting on the change of \( \{u_{ff}(\cdot)\} \) as a surrogate to limit the bandwidth. By way of contrast, most of the existing MPC literature uses regulation about given values \( x^*, u^* \). Of course, it is clear that these constant regulation set-points cannot be defined when \( \{\hat{y}^*(\cdot)\} \) is not constant (or even piece-wise constant). Nonetheless, there is a simple rule of thumb that links the “bandwidth” achieved via use of (18) and that achieved from the more conventional cost function of the form:

\[ J'_{ff} = \sum_{k=0}^{\infty} [y_{ff}(k) - y^*]^2 + \lambda'_{ff} \cdot |u_{ff}(k) - u^*|^2 \]

where \( y^*, u^* \) are consistent steady state values. Using Parseval’s Theorem, equation (18) can be expressed in the frequency domain as:

\[ J_{ff} = \int_{-\pi}^{\pi} \|Y_{ff} - \hat{Y}^*\|^2 + \lambda_{ff} \cdot \|\delta\|^2 \|U_{ff}\|^2 d\omega \]

where

\[ \delta(e^{j\omega}) = 1 - e^{-j\omega} \]

Also, we can express \( Y_{ff}(e^{j\omega}) \) in the frequency domain as

\[ Y_{ff}(e^{j\omega}) = H(e^{j\omega})x_{ff}(0) + G_o(e^{j\omega})U_{ff}(e^{j\omega}) \]

where

\[ H(e^{j\omega}) = C[e^{j\omega}I - A]^{-1} \]

\[ G_o(e^{j\omega}) = H(e^{j\omega})B \]

Substituting into (20) gives

\[ J_{ff} = \int_{-\pi}^{\pi} \|Hx_{ff}(0) + G_oU_{ff} - \hat{Y}^*\|^2 + \lambda'_{ff} \cdot \|\delta\|^2 \|U_{ff}\|^2 d\omega \]

Heuristically, we see that two frequency regions are of interest

\[ R_1 = \{\omega \in [0, \pi] : |G_o(e^{j\omega})| < \lambda^1_{ff}|\delta(e^{j\omega})|\} \]

\[ R_2 = \{\omega \in [0, \pi] : |G_o(e^{j\omega})| > \lambda^2_{ff}|\delta(e^{j\omega})|\} \]

For \( \omega \in R_1 \), \( Y_{ff} \) is essentially brought to \( \hat{Y}^* \) whereas for \( \omega \in R_2 \), \( U_{ff} \) will be near zero. Thus the bandwidth (BW) can be loosely defined as the value \( \omega = \omega_{BW} \) such that

\[ |G_o(e^{j\omega_{BW}})| = \lambda^1_{ff}|\delta(e^{j\omega_{BW}})| \]

By a similar argument, the bandwidth for cost function (19) is achieved at \( \omega = \omega'_{BW} \) where

\[ |G_o(e^{j\omega'_{BW}})| = (\lambda'_{ff})^\frac{1}{2} \]

Now, say we want to achieve a bandwidth \( \omega'_{BW} \) satisfying (28) for a given \( \lambda'_{ff} \), then all we have to do (in principle) is to choose \( \lambda_{ff} \) such that
\[ \lambda_{ff} = \frac{\lambda_{ff}^\circ}{|\delta(e^{j\omega c_{BW}})|^2}. \] (29)

### 3.2 Feedback Design

Next we turn to the design of the feedback component. For this purpose, we define the total plant input as
\[ u(k) = u_{fb}(k) + u_{ff}(k) \] (30)

Our goal here is to determine \( u_{fb}(k) \). Again, our main objective is to track \( y^* \), but here we also want to reject the (unmeasured) disturbance \( d \). For simplicity we assume \( d \) is constant, although more general formulations are possible. Now, since we do not measure either \( x \) (the true plant state) or the disturbance \( d \), we use an observer of the form given earlier in (10), (11).

Without loss of generality, we take the current time as \( k = 0 \) and we assume that \( \hat{x}(0), \hat{d}(0), u_{fb}(-1), u_{fb}(-2), \ldots \) are known.

We then can choose a feedback cost function of the form:
\[ J_{fb} = \sum_{k=0}^{\infty} [\hat{y}^p(k) + \hat{d}p(k) - y^*(k)]^2 + \lambda_{fb}[u_{fb}(k) - u_{fb}(k-1)]^2 \] (31)

where \( \{\hat{y}^p(\cdot)\} \) and \( \{\hat{d}p(\cdot)\} \) are predicted quantities satisfying:
\[ \begin{align*}
(\hat{x}p)^+ &= A\hat{x}p + B(u_{fb} + u_{ff}), \quad \hat{x}p(0) = \hat{x}(0) \\
(\hat{d}p)^+ &= \hat{d}p, \quad \hat{d}p(0) = \hat{d}(0) \\
\hat{y}p &= C\hat{x}p
\end{align*} \] (32) (33) (34)

**Remark 3.** Note that the same effect of including \( x^*, u^* \) in (13) is embedded in the cost function (31).

**Remark 4.** An important point is that, generally, one would expect to choose \( \lambda_{fb} > \lambda_{ff} \). The reason being that the feedback control uses observations from the true plant, hence, if the true plant differs from the nominal model, then the feedback signal can be potentially destabilising. (Note that this is never an issue for the feedforward control which does not use plant observations). Therefore, one needs to ensure robust stability for the feedback signal based on the worst possible case of plant uncertainty, whereas the feedforward signal can be designed in a more optimistic fashion. 

**Remark 5.** In the ideal case (no disturbances, no noise, no uncertainty) it holds that \( d(0) = 0, \dot{x} = x_{ff} \). In this case, if \( \lambda_{ff} \) were to be chosen small, then \( y_{ff} \to y^* \). Moreover, from (32), (34), we see that \( u_{fb} = 0 \) suffices for \( \hat{y}p \to y_{ff} \). Hence, we see that \( u_{fb} \simeq 0 \) is a good choice since the feedforward signal has “done all the work” and there is nothing left for the feedback to do. Of course, if there are disturbances, noise, model uncertainty, etc. then the feedback signal may have some “cleaning up” to do.

**Remark 6.** It is important to note that the feedback MPC design also has the ability to do reference preview, as seen from (31). The fact that this controller has to deal with more than just tracking is the reason the feedback design can make an improvement.

### 4. CONstrained MPC DESIGN

The constrained MPC design parallels that given for the nominal case in section 3. We note that now it is essential for the feedforward design to be done first, since the feedback strategy can then stabilise the system with the known input \( u_{ff} \) applied.

#### 4.1 Feedforward Design

We again use the feedforward model (17). However, we replace the cost function (18) by the following finite horizon cost function:
\[ J_{ff}^N = \sum_{k=0}^{N} [y_{ff}(k) - \hat{y}^*(k)]^2 + \lambda_{ff} \cdot [u_{ff}(k) - u_{ff}(k-1)]^2 \] (35)

In order to make the control sequence \{\( u_{ff}(\cdot) \)\} realistic, it is crucial that we minimise (35) subject to the appropriate constraints, i.e. \( x_{ff}(k) \in X \) and \( u_{ff}(k) \in \mathcal{U} \), where \( X, \mathcal{U} \) are given convex sets which are consistent with the required state and input constraints.

We then implement \( u_{ff}(0) \) as the first element of the feedforward sequence. More importantly, we pass the entire sequence \( u_{ff}(0), \ldots, u_{ff}(N) \) onto the feedback design stage.

#### 4.2 Feedback Design

We are given \( \hat{x}(0), \hat{d}(0), u_{ff}(0), \ldots, u_{ff}(N) \). Then, in the nominal case, we use the prediction model (32), (33), (34). However, we replace (31) by:
\[ J_{fb}^N = \sum_{k=0}^{N} [\hat{y}p(k) + \hat{d}p(k) - y^*(k)]^2 + \lambda_{fb}[u_{fb}(k) - u_{fb}(k-1)]^2 \] (36)

As for the feedforward design, we need to optimise (36) subject to constraints. A simple (but nonetheless important) point is that the input constraints are on \( u(k) = u_{fb}(k) + u_{ff}(k) \). Hence, the appropriate constraints are \( \hat{x}p(k) \in X \) and \( u_{fb}(k) + u_{ff}(k) \in \mathcal{U} \).

**Remark 7.** An important observation is that, if the feedback design does not “like” the provided sequence of \( u_{ff} \), then it can be completely removed. However, in practice, quite the opposite may occur. That is, the feedback design may decide that \( u_{ff}(k), k = 0 \ldots N \) “does the job” and, in this case, \( u_{fb}(k) = 0 \) is a possible outcome.

**Remark 8.** The need for feedback arises due to the presence of disturbances and model uncertainty. This impacts on the choice of bandwidth for the feedback control loop since it now has to be “careful” about what is does to the plant. On the other hand, the plant input may have hard constraints on its change rate, which also limits bandwidth. Our proposed design takes into account these constraints, but there is a subtle difference. If the limitation on the bandwidth is mainly because of a hard constraint on the input, then our strategy will have essentially the same performance as if we had not included the feedforward component, because the total plant input change rate will...
be limited. On the other hand, if the bandwidth limitation is mainly because of robustness to process uncertainties and unknown disturbances, then it will be beneficial to implement our strategy. This is a consequence of the fact that we separate the regulation problem from the tracking problem.

4.3 Robust Stability

The description of the algorithm given above has been deliberately simplified for clarity of exposition. The authors are quite aware that further embellishments and caveats are necessary to imbue the algorithm with a certificate of robust stability. This latter issue will be addressed, for pedagogical reasons, in a companion paper Carrasco and Goodwin (2011).

5. Example

In this section we present an example to illustrate the PF-MPC strategy. Here, we utilise $\lambda_{fb}$ as a mechanism to ensure robustness by imposing bandwidth restrictions on the feedback component.

For all cases presented, the real plant is assumed to be:

$$G(z) = \frac{0.05}{z^5 \cdot (z - 0.95)}$$

while the nominal plant model is

$$G_o(z) = \frac{0.05}{z^2 \cdot (z - 0.95)}$$

Note that there is an unmodelled delay of three samples. The prediction and control horizons are taken as $N_p = N_c = 5$, and the unknown disturbance is chosen as $d = 0.5$. Also $N_p = 4$ and $\lambda_{ff} = 0$. In the figures presented below, the signal names have the following interpretation:

- $y^*(k)$ is the actual reference signal.
- $y(k)$ is the output of the system with input $u(k)$.
- $y_{fbw}(k)$ is the output of the system without implementing the feedforward component (only feedback).

Remark 9. In summary, the signal $y(k)$ is the result of the implementation of both the feedback and feedforward components described in the previous sections. On the other hand, $y_{fbw}(k)$ is solely the result of the nominal feedback MPC strategy, without ever considering the feedforward signal in the optimisation stage.

For the unconstrained case, Fig. 1 presents all the aforementioned signals for comparison, when $\lambda_{fb} = 0$ and $\lambda_{fb} = 20$ (a value found experimentally that provides acceptable feedback performance in the face of the given mode uncertainty). For the case $\lambda_{fb} = 20$, Fig. 2 shows the same dynamics but now including constraints on the magnitude of the total control signal, namely $|u(k)| < 10$. Two cases are shown: (i) where the constraint is not included in the feedforward optimisation, (ii) where the constraint is taken into account. In both cases, the feedforward optimisation includes knowledge of this constraint.

Fig. 1. Output signal for $\lambda_{fb} = 0$ (top) and $\lambda_{fb} = 20$ (bottom)

Fig. 2. Output signals for $\lambda_{fb} = 20$ with constraints $|u(k)| < 10$, (i) without accounting for it in the feedforward optimisation (top) and (ii) accounting for it (bottom).

5.1 Comments

Fig. 1 clearly shows the success of our proposed MPC framework for dealing with time-varying references. From the case $\lambda_{fb} = 0$, it is clear that the inclusion of our feedforward strategy offers little advantage, since both $y(k)$ and $y_{fbw}(k)$ are essentially the same. Note that the performance of the “closed loop” reference tracking is severely
compromised. This is mainly due to the available bandwidth of the feedback, which dominates over the feedforward, and the huge under-modelling of the time delay of the plant. On the other hand, if we increase the feedback weight to $\lambda_{fb} = 20$, thus restricting the bandwidth of the feedback component, we see that the feedback alone acts very slowly and is not able to track the reference, whereas the feedforward is able to do fast reference tracking as required. The sum of quadratic output errors for the PF-MPC strategy with $\lambda_{fb} = 0$ is $E_{\lambda_{fb}=0} = 143.10$, while for $\lambda_{fb} = 20$ is $E_{\lambda_{fb}=20} = 29.88$. This is a clear demonstration that for certain scenarios PF-MPC vastly improves the performance of time-varying reference tracking.

Fig. 2 shows the results of implementing PF-MPC in a constrained scenario. The results here emphasise the importance of including the constraints, not only in the feedback optimisation stage, but also in the feedforward optimisation. In the top plot, because the constraints are not accounted for in the feedforward optimisation, the optimal value obtained for $u_{ff}$ may not be the one that is applied to the real plant. This means that the feedforward model will be updated with a false control signal, and hence there will be a mismatch in the subsequent control sequences. It is worth noting that what actually tries to correct this mismatch is the feedback signal. In the bottom plot we see that the mismatch is corrected and most of the tracking performance is regained when the constraints are included in the feedforward optimisation stage.

Remark 10. The ideas presented in this paper have a loose connection to the work of Campi and Garatti (2010). Campi and Garatti argue that it may be desirable to replace absolute guarantees of performance (including stability) by high probability guarantees in favor of a possibly significant improvement in the performance. Our work aims at combining an absolute guarantee of robust stability with high bandwidth reference tracking for systems “near” the nominal system. As our presented example shows, “near” can include large unmodelled errors in process delay.

6. CONCLUSIONS

This paper has described a novel strategy for MPC design which incorporates feedback, reference feedforward and preview. Simulation results provided in the paper confirm that the method is capable of yielding significantly better performance than what is achievable by a conventional MPC strategy in the presence of unmodelled dynamics and unmeasured disturbances, especially for non-constant reference signals. In a companion paper, we present preliminary theoretical analysis of the proposed structure.

REFERENCES


