BA-03-04

Neutrino Democracy And Other Phenomenology From 5D $SO(10)$

Qaisar Shafi[a 1], Zurab Tavartkiladze[b,c 2]

[a] *Bartol Research Institute, University of Delaware, Newark, DE 19716, USA*
[b] *Institute for Theoretical Physics, Heidelberg University, Philosophenweg 16, D-69120 Heidelberg, Germany*
[c] *Institute of Physics, Georgian Academy of Sciences, Tbilisi 380077, Georgia*

Abstract

We present a five dimensional supersymmetric $SO(10)$ model compactified on an orbifold $S^{(1)}/Z_2 \times Z_2'$. The gauge symmetry $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$, realized on one of the fixed points (branes), is spontaneously broken to the MSSM via the higgs mechanism. Employing a flavor $\mathcal{U}(1)$ symmetry and suitably extending the 'matter' sector enables us to understand large mixings in the neutrino sector via a democratic approach, versus the small CKM mixings. A residual $\mathcal{R}$-symmetry on G_{422} brane helps eliminate the troublesome dimension five nucleon decay, while the $\mathcal{U}(1)$ symmetry plays an essential role in suppressing dimension six decay. For rare leptonic decays we expect BR($\mu \to e\gamma$) $\sim$ BR($\tau \to \mu\gamma$) $\sim$ BR($\tau \to e\gamma$).

[1] E-mail address: shafi@bartol.udel.edu
[2] E-mail address: Z.Tavartkiladze@ThPhys.Uni-Heidelberg.DE

1 Introduction

The recent SuperKamiokande [1], [2] and KamLAND [3] data have provided increasingly strong evidence for oscillations between active neutrino flavors. Namely, the atmospheric neutrino deficit is due to (near) maximal $\nu_\mu \to \nu_\tau$ oscillations with

$$\sin^2 2\theta_{\mu\tau} \simeq 1 \ , \qquad \Delta m^2_{\rm atm} \simeq 3 \cdot 10^{-3} \ {\rm eV}^2 \ , \tag{1}$$

while the preferred solution for the solar neutrino anomaly is the large mixing angle (LMA) solution with $\nu_e \to \nu_{\mu,\tau}$ oscillations, such that

$$\sin^2 2\theta_{e\mu,\tau} \approx 0.8 \ , \qquad \Delta m^2_{\rm sol} \sim 10^{-4} \ {\rm eV}^2 \ . \tag{2}$$

These developments require a physics beyond the Standard Model (SM). The most popular extension is the introduction of right handed (SM singlet) neutrinos with superheavy Majorana masses, which induce suppressed neutrino masses through the see-saw mechanism [4]. A consistent framework for the solution of the gauge hierarchy problem is provided by supersymmetry which leads one to think that a supersymmetric GUT theories such as $SO(10)$ [5] are quite compelling. In building realistic models, in addition to the neutrino data, one should also take into account the charged fermion sector and see how these two sectors blend together. The three quark-lepton families display a clear intergeneration hierarchy. Only the top quark mass [$m_t = (174.3 \pm 5.1)$ GeV] is close to the electro-weak symmetry breaking scale, which means that the top Yukawa coupling $\lambda_t \simeq 1$. Introducing the parameter $\epsilon \simeq 0.2$ (close in value to the Cabibbo angle V_{us}) one can express the hierarchies between the Yukawa couplings as follows:

$$\lambda_t \sim 1 \ , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^8 : \epsilon^4 : 1 \ , \tag{3}$$

$$\lambda_b \sim \lambda_\tau \ , \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^4 : \epsilon^2 : 1 \ , \tag{4}$$

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1 \ . \tag{5}$$

As far as the CKM mixing angles are concerned, they have relatively small values

$$V_{us} \sim \epsilon \ , \quad V_{cb} \sim \epsilon^2 \ , \quad V_{ub} \sim \epsilon^3 \ . \tag{6}$$

It is a challenging task to provide a compelling framework for understanding the origin of the small CKM mixing angles in (6) versus the large neutrino mixings. The hierarchies in (3)-(5) also should be adequately understood. Some flavor symmetry is well motivated in this context, and one simple choice is provided by an abelian $\mathcal{U}(1)$ symmetry, which can be successful in building the desired charged fermion [7], [8] and neutrino [9]-[18] sectors. With a $\mathcal{U}(1)$ symmetry, the variety of textures for neutrino mass matrices can be constructed, giving either maximal or large neutrino mixings. This has been pursued within νMSSM [9], meaning that MSSM is augmented by right handed neutrinos (models such as $SO(10)$ [5], $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ [6] etc. automatically introduce right handed neutrinos) and $\mathcal{U}(1)$ flavor symmetry, and in some cases yields interesting predictions. The same approach has been used in realistic GUTs such as $SU(5)$ [10], $SO(10)$ [11], G_{422} [12], flipped $SU(5)$ [16], pseudo-Goldstone $SU(6)$ [13] and E_6 [14]. A $\mathcal{U}(1)$ symmetry was also used within $SU(6) \times U(1)$ [18], $SU(3)_c \times SU(3)_L \times SU(3)_R$ [19] and $SU(6) \times SU(2)$ [15][3].

An interesting possibility, different from models relying on specific textures to obtain large neutrino mixings is the so-called *democratic* approach [22], [16], [17], in which all flavors of left handed lepton doublets l_α ($\alpha = 1, 2, 3$ is a generation index) transform identically under $\mathcal{U}(1)$. That is, the flavor symmetry does not distinguish neutrino flavors and consequently one naturally expects large mixings between them. At the same time, the $\mathcal{U}(1)$ symmetry is still crucial for understanding hierarchies between charged fermion masses and the CKM mixing angles. It was shown in [16] that within the MSSM, neutrino democracy nicely blends with the charged fermion sector, but within $SU(5)$ one encounters difficulties. Namely, with $\mathcal{U}(1)$ flavor symmetry and neutrino democracy, $SU(5)$ GUT with minimal matter content gives unacceptably small $V_{us}(\sim \epsilon^3 \sim 1/125)$. The root of this problem is in unified $SU(5)$ multiplets where the quark-lepton states come from. Similar difficulty can also be expected in other GUTs with matter states embedded in a single GUT representation. It is possible to avoid this problem either by considering extended GUTs such as flipped $SU(5)$ [16] or constructing $SU(5)$ GUT in five dimensions [17]. With a minimal matter sector these theories typically yield unsatisfactory Yukawa matrices, such as $\hat{M}_d^{(0)} = \hat{M}_e^{(0)}$ in $SU(5)$, and $\hat{M}_u^{(0)} \propto \hat{M}_d^{(0)} = \hat{M}_e^{(0)}$, $\hat{V}_{CKM} = \mathbf{1}$ in $SO(10)$. This indicates the need for an extended matter sector as one possible way to resolve these problems.

We also note that orbifold constructions [23]-[33] turn out to be very efficient and powerful in resolving GUT problems such as doublet-triplet (DT) splitting, nucleon stability, unwanted asymptotic relations, GUT symmetry breaking etc. For earlier attempt within superstring derived models see for instance ref. [34]. In addition, the five dimensional setting offers an economical way to realize neutrino democracy as well as the desired charged fermion pattern [17]. Within the orbifold construction, different ways for realizing large neutrino mixings in the presence of $\mathcal{U}(1)$ symmetry were considered in [32], [33]. The scenarios with democratic approach have some peculiar features which can be tested in future experiments. Namely, models with democracy between l_α states predict that BR$(\mu \to e\gamma) \sim$BR$(\tau \to \mu\gamma) \sim$BR $(\tau \to e\gamma)$. Also, SUSY GUT democratic scenarios yield nucleon decay with emission of charged leptons e, μ with comparable partial lifetimes

[3] These three groups can emerge through E_6 breaking [20], [21] and have interesting phenomenological implications.

$\tau(p \to Ke) \sim \tau(p \to K\mu)$. This would distinguish them from models in which the process $p \to K\mu$ dominates.

In this paper we present a 5D SUSY $SO(10)$ GUT augmented with $\mathcal{U}(1)$ flavor symmetry. By orbifold compactification, on one of the fixed points we realize the gauge symmetry G_{422} [27][4]. To implement neutrino democracy and obtain realistic charged fermion masses, some additional matter states play a crucial role. They can be introduced either in the bulk or directly on the fixed point (so, this model differs from a recently suggested scenario [33], where an abelian flavor symmetry also plays an important role). Once this extension is done, we find that neutrino democracy is an automatic consequence of the model. Due to left-right symmetry at the fixed point, the mass of the lightest right handed neutrino is predicted to be $\sim 6 \cdot 10^{10} \tan^2\beta$ GeV ($\tan\beta$ is a ratio of VEVs of the MSSM higgs doublets). This scale can be interesting for understanding the observed baryon asymmetry either through thermal or inflationary leptogenesis [35], [36], [37]. The question of nucleon decay is also addressed and it is shown that under certain conditions the nucleon can be absolutely stabile. This may be answer to the question of why the search for proton decay has been unsuccessful (at least so far!). In some other schemes that we consider, dimension six proton decay may be accessible in future experiments. We also discuss rare leptonic decays and some other phenomenological consequences.

2 5D SUSY $SO(10)$ On An $S^{(1)}/Z_2 \times Z_2'$ Orbifold

We consider a supersymmetric (SUSY) $SO(10)$ gauge theory in five dimensions (5D) compactified on an $S^{(1)}/Z_2 \times Z_2'$ orbifold. In terms of 4D superfields, 5D $N=1$ SUSY multiplets correspond to 4D $N=2$ supermultiplets. The gauge superfield is $V_{N=2} = (V, \Sigma)$, where V and Σ are vector and chiral superfields respectively, transforming in the adjoint of $SO(10)$. For bulk matter and higgs superfields, each chiral state Φ is accompanied by its mirror $\overline{\Phi}$, so that they constitute an $N=2$ supermultiplet $\mathbf{\Phi}_{N=2} = (\Phi, \overline{\Phi})$. By compactification on $S^{(1)}/Z_2 \times Z_2'$, it is possible to break $SO(10) \to G_{422}$ on one of the fixed points. In terms of G_{422}, an adjoint $\mathbf{45}$ of $SO(10)$ reads

$$\mathbf{45} = L(1,3,1) + R(1,1,3) + C(15,1,1) + B(6,2,2) \ , \tag{7}$$

where the transformation properties under G_{422} are indicated.

The fifth space-like coordinate y of 5D parameterizes a compact circle $S^{(1)}$ with radius R, and the orbifold parities act as follows: $Z_2 : \ y \to -y$, $Z_2' : \ y' \to -y'$ ($y' = y + \pi R/2$). Under $Z_2 \times Z_2'$, the bulk states have definite parities $(P, P') =$

[4]Obtaining the G_{422} model from higher dimensional $SU(4)_c \times SU(4)_{L+R}$ and $SO(12)$ GUTs was discussed in [28], [31]

$(+,+),(+,-),(-,+),(-,-)$. The KK masses of the corresponding states are

$$2n\mu_0\ ,\quad (2n+1)\mu_0\ ,\quad (2n+1)\mu_0\ ,\quad (2n+2)\mu_0\ , \tag{8}$$

where $\mu_0 = 1/R$ and $n = 0, 1, 2, \cdots$ denotes the quantum number of the KK state in mode expansion. From (8) one observes that only states with $(+,+)$ parity contain zero modes, while the remaining states acquire masses $\sim \mu_0$.

The various components from $V(45)$, $\Sigma(45)$ are assigned the following $Z_2 \times Z_2'$ parities:

$$(V_C, V_L, V_R) \sim (+,+)\ ,\qquad V_B \sim (-,+)\ ,$$
$$(\Sigma_C, \Sigma_L, \Sigma_R) \sim (-,-)\ ,\qquad \Sigma_B \sim (+,-)\ . \tag{9}$$

At the fixed point $y = 0$, we have the residual gauge symmetry G_{422}, while at the fixed point $\pi R/2$ we find a $SO(10)$ symmetry. Both at $y = 0$ and $y = \pi R/2$, we also have 4D $N = 1$ SUSY leftover from the original 5D $N = 1$ SUSY.

In order to build the fermion sector at $y = 0$ and arrange for G_{422} breaking down to the MSSM, we should introduce matter and higgs superfields. It turns out that by introduction of appropriate states in the bulk, it is possible to obtain the desired zero mode representations of G_{422} [6]. For this, one needs to introduce additional states [25], the so-called 'copies' (see 1st and 3rd in ref. [26]) in the bulk. Namely, we introduce $\mathbf{16}^F_{N=2} = (16^F, \overline{16}^F)$ and $\mathbf{16}^{F^c}_{N=2} = (16^{F^c}, \overline{16}^{F^c})$ (for each generation) where, under G_{422}

$$16^F = F(4,2,1) + F^{c\,\prime}(\bar{4},1,2)\ ,\quad 16^{F^c} = F^{\prime}(4,2,1) + F^c(\bar{4},1,2)\ , \tag{10}$$

and similarly for $\overline{16}^F$, $\overline{16}^{F^c}$. With the parity prescriptions

$$(F, F^c) \sim (+,+)\ ,\qquad (F^{\prime}, F^{c\,\prime}) \sim (-,+)\ , \tag{11}$$

and opposite parities for the corresponding mirrors, taking into account (9), one can verify that $Z_2 \times Z_2'$ is a symmetry of the whole 5D Lagrangian. According to (11), at $y = 0$ we have the zero modes of F, F^c states, which effectively constitute an anomaly free 16-plet of $SO(10)$. As far as the higgs sector is concerned, the breaking of G_{422} to MSSM proceeds through the VEVs of H^c, $\overline{H}^c$ states, where the transformation H^c under G_{422} is indicated in (10). The zero modes of these states are obtained by introducing the supermultiplets $\mathbf{16}^{H^c}_{N=2} = (16^{H^c}, \overline{16}^{H^c})$, $\mathbf{16}^{\overline{H}^c}_{N=2} = (16^{\overline{H}^c}, \overline{16}^{\overline{H}^c})$, where

$$16^{H^c} = H + H^c\ ,\quad \overline{16}^{H^c} = \overline{H} + \overline{H}^{c\,\prime}\ ,\quad 16^{\overline{H}^c} = H' + H^{c\,\prime}\ ,\quad \overline{16}^{\overline{H}^c} = \overline{H}' + \overline{H}^c\ . \tag{12}$$

With the parity assignments

$$(H^c, \overline{H}^c) \sim (+,+)\ ,\quad (H, \overline{H}') \sim (-,+)\ ,\quad (H', \overline{H}) \sim (+,-)\ ,\quad (H^{c\,\prime}, \overline{H}^{c\,\prime}) \sim (-,-)\ , \tag{13}$$

the H^c, $\overline{H}^c$ will contain zero modes.

As far as the bi-doublet h (containing the MSSM higgs doublets) of G_{422} is concerned, it comes from the 10-plet of $SO(10)$. In 5D we introduce $\mathbf{10}^h_{N=2} = (10^h, \overline{10}^h)$, $\mathbf{10}^D_{N=2} = (10^D, \overline{10}^D)$, where in terms of G_{422},

$$10^h = h(1,2,2) + D'(6,1,1) \ , \quad 10^D = h'(1,2,2) + D(6,1,1) \ , \tag{14}$$

while $\overline{10}^h$, $\overline{10}^D$ have conjugate decompositions. With $Z_2 \times Z_2'$ parity assignments

$$(h, D) \sim (+,+) \ , \quad (h', D') \sim (-,+) \ , \tag{15}$$

and opposite parities for the corresponding mirrors, at the $y = 0$ brane (fixed point) we find non-vanishing zero modes from the h, D states.

It turns out that the G_{422} model with minimal field content encounters difficulties in realizing a realistic fermion sector. To cure this problem, following [12], we introduce in the bulk three families of $\mathbf{10}^f_{N=2} = (10^f, \overline{10}^f)$, $\mathbf{10}^g_{N=2} = (10^g, \overline{10}^g)$ states where, under the G_{422},

$$10^f = f(1,2,2) + g'(6,1,1) \ , \quad 10^g = f'(1,2,2) + g(6,1,1) \ . \tag{16}$$

With parities

$$(f, g) \sim (+,+) \ , \quad (f', g') \sim (-,+) \tag{17}$$

and opposite parities for corresponding mirrors, three families of f and g have zero mode states.

In summary, we have shown how zero modes of desired G_{422} representations can be obtained from the bulk supermultiplets. Indeed, one can introduce the needed zero modes directly at the $y = 0$ fixed point. For building the fermion sector and studying its phenomenology the 4D superpotential terms which we consider below are important. The heavy bulk states do not affect our conclusions which are therefore robust. Finally, the G_{422} representations which contain zero modes and with which we will deal below are the matter states:

$$F_\alpha(4,2,1) \ , \quad F^c_\alpha(\bar{4},1,2) \ , \quad f_\alpha(1,2,2) \ , \quad g_\alpha(6,1,1) \ , \tag{18}$$

(α is the family index), and the 'scalar' supermultiplets:

$$H^c(\bar{4},1,2) \ , \quad \overline{H}^c(4,1,\bar{2}) \ , \quad h(1,2,2) \ , \quad D(6,1,1) \ . \tag{19}$$

3 Model At Fixed Point $y = 0$

Five dimensional SUSY does not allow Yukawa and higgs superpotential couplings in the bulk. Because of this, we will construct the 4D theory at the fixed point $y = 0$. We employ the 4D superfield notation after appropriate rescaling from 5D fields.

The breaking of G_{422} to G_{321} can occur through the H^c, $\overline{H}^c$ states of (19). These states contain the MSSM singlets ν^c_H, $\overline{\nu}^c_H$, whose non-zero VEVs break the G_{422} to the G_{321} [39]. In fact, these VEVs break $SU(4)_c \times SU(2)_R$ to $SU(3)_c \times U(1)_Y$, where $SU(3)_c$ is a subgroup of $SU(4)_c$, while $U(1)_Y$ is a superposition of two $U(1)$ factors coming from $SU(4)_c$ and $SU(2)_R$

$$Y = -\sqrt{\frac{2}{5}}Y_{SU(4)_c} + \sqrt{\frac{3}{5}}Y_{SU(2)_R} \ , \tag{20}$$

where $Y_{SU(4)_c}$ and $Y_{SU(2)_R}$ are generators of $SU(4)_c$ and $SU(2)_R$ respectively

$$Y_{SU(4)_c} = \frac{1}{\sqrt{24}}\mathrm{Diag}(1,1,1,-3) \ , \quad Y_{SU(2)_R} = \frac{1}{2}\mathrm{Diag}(1,-1) \ . \tag{21}$$

In (20), the Y hypercharge is given in the 'standard' $SU(5)$ normalization

$$Y = \frac{1}{\sqrt{60}}\mathrm{Diag}(2,2,2,-3,-3) \ . \tag{22}$$

Another superposition, orthogonal to (20), is broken by $\langle \nu^c_H \rangle = \langle \overline{\nu}^c_H \rangle \equiv v$. In this breaking, nine degrees of freedom from the scalars are genuine Goldstone fields that are absorbed by the appropriate gauge fields. These nine degrees come from $\tilde{u}^c$, $\tilde{e}^c$ (tilde indicates scalar components) and $\tilde{\overline{u}}^c$, $\tilde{\overline{e}}^c$ states (from H^c and $\overline{H}^c$ respectively), plus one superposition of singlets $\tilde{\nu}^c_H$, $\tilde{\overline{\nu}}^c_H$. Since these states are complex, there remain nine physical scalars which acquire masses through the D-terms of $SU(4)_c \times SU(2)_R$. Their fermionic superpartners acquire masses through mixing with appropriate gauginos after $G_{422} \to G_{321}$ breaking. In this way, the supersymmetric higgs mechanism is realized within the G_{422} model. As far as the physical colored triplet d^c_H, $\overline{d}^c_H$ states are concerned, through the couplings H^cH^cD, $\overline{H}^c\overline{H}^cD$, taking into account that $D = (d^c_D, \overline{d}^c_D)$, and after substituting the $\langle \nu^c_H \rangle$, $\langle \overline{\nu}^c_H \rangle$ VEVs, they decouple with $\overline{d}^c_D$, d^c_D forming massive states $v d^c_H \overline{d}^c_D$, $v \overline{d}^c_H d^c_D$ [39].

3.1 Charged Fermion Masses And Mixings

The three families of F_α, F^c_α states in (18) constitute the minimal matter content of G_{422}. The Yukawa couplings for the quark and lepton masses are $Y^{\alpha\beta}F_\alpha F^c_\beta h$. Recalling that

$$F = (q,l) \ , \quad F^c = (u^c, d^c, e^c, \nu^c) \ , \quad h = (h_u, h_d) \ , \tag{23}$$

these couplings lead to phenomenologically unacceptable asymptotic relations $\hat{Y}_u^{(0)} = \hat{Y}_d^{(0)} = \hat{Y}_e^{(0)}$, $\hat{V}_{CKM} = 1$. To avoid this, we extend the matter sector of G_{422} by invoking the states $(f+g)_\alpha$ of (18). The content of the latter reads

$$f = (l_f, \overline{l}_f) \ , \quad g = (d^c_g, \overline{d}^c_g) \ . \tag{24}$$

Table 1: $\mathcal{U}(1)$ charges of matter and scalar superfields.

$Q[F_1]$	$Q[F_2]$	$Q[F_3]$	$Q[F_1^c]$	$Q[F_2^c]$	$Q[F_3^c]$
$\frac{2m-p+q}{4}+3$	$\frac{2m-p+q}{4}+2$	$\frac{2m-p+q}{4}$	$\frac{2n-p+q}{4}+5$	$\frac{2n-p+q}{4}+2$	$\frac{2n-p+q}{4}$
$Q[g_1]$	$Q[g_2]=Q[g_3]$	$Q[f_\alpha]=-Q[h]-k$	$Q[H^c]$	$Q[\overline{H}^c]$	$Q[D]$
$\frac{2n-p+q}{2}-k+1$	$\frac{2n-p+q}{2}-k$	$\frac{m+n-p+q}{2}-k$	$\frac{3p-3q-2n}{4}+k$	$\frac{p-q-2n}{4}+k$	$\frac{3q+2n-p}{2}-2k$

This extension is important for building the desired charged fermion sector [12], and is also crucial for the realization of neutrino democracy. We further introduce a $\mathcal{U}(1)$ flavor symmetry for help in realizing the desired hierarchies. To break $\mathcal{U}(1)$ we introduce an $SO(10)$ singlet superfield X with charge $Q_X=-1$. We assume that the scalar component of X develops a VEV

$$\frac{\langle X\rangle}{M_f}\equiv\epsilon\simeq 0.2\ , \tag{25}$$

where M_f is the cut-off scale of the theory.

In Table 1 we present the $\mathcal{U}(1)$ charges for the various states, where m, n, p, q, k are some non negative integers, to be specified below. The charges of F, F^c states are fixed from the values of CKM mixing angles and hierarchies between the up-type quark masses. With these assignments, the couplings responsible for the generation of up-type quark masses are given by

$$\begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} F_1^c & F_2^c & F_3^c \end{array} \\ \left(\begin{array}{ccc} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{array}\right) \end{array} h\ , \tag{26}$$

which, upon diagonalization, yield the desirable hierarchies in (3).

Turning to the down quark and charged lepton sector we will need the g_α and f_α states. From (24), g and f contain fields with the quantum numbers of d^c and l. With the $\mathcal{U}(1)$ charge prescriptions of Table 1, the couplings are

$$\begin{array}{c} \\ F_1^c \\ F_2^c \\ F_3^c \end{array}\!\!\begin{array}{c} \begin{array}{ccc} g_1 & g_2 & g_3 \end{array} \\ \left(\begin{array}{ccc} \epsilon^6 & \epsilon^5 & \epsilon^5 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{array}\right) \end{array} \epsilon^n H^c\ . \tag{27}$$

After substitutution of $\langle H^c \rangle$, this ensures the decoupling of $d^c_{F^c}$ with $\overline{d^c}_g$, so that the light d^c_α states reside in g_α. With the couplings

$$\begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \end{array} \begin{array}{c} \begin{array}{ccc} g_1 & g_2 & g_3 \end{array} \\ \left(\begin{array}{ccc} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{array} \right) \end{array} \frac{\overline{H}^c}{M_f} h \ , \tag{28}$$

the hierarchies in (4) can be realized. Since the left-handed quark doublets q_α reside in F_α, from (26), (28), one can also expect that the desired values in (6) for CKM matrix elements can be realized.

As far as charged leptons are concerned, with the charge assignments in Table 1, the $F_\alpha f_\beta H^c$ type couplings have the structure

$$\begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left(\begin{array}{ccc} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{array} \right) \end{array} \epsilon^m H^c \ . \tag{29}$$

Substituting in (29) the VEV $\langle H^c \rangle$, one can easily verify that the l_F states decouple together with $\overline{l}_f$. Therefore, the light left-handed lepton doublets l_α arise from f_α. Since the f_α have identical transformation properties under $\mathcal{U}(1)$, we see that democracy between the l states is realized! The couplings responsible for charged lepton masses are

$$\begin{array}{c} \\ F^c_1 \\ F^c_2 \\ F^c_3 \end{array} \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left(\begin{array}{ccc} \epsilon^5 & \epsilon^5 & \epsilon^5 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{array} \right) \end{array} \frac{\overline{H}^c}{M_f} h \ . \tag{30}$$

Diagonalization of (30) leads to the required hierarchies in (5).

Note that the up quark mass matrix (26) selects $Q[F^c_1]-Q[F^c_3]=5$, $Q[F^c_2]-Q[F^c_3]=2$, and since hierarchies between the charged lepton masses are $\frac{m_e}{m_\tau} \sim \epsilon^5$, $\frac{m_\mu}{m_\tau} \sim \epsilon^2$, this dictates the democratic selection $Q[f_1]=Q[f_2]=Q[f_3]$. Thus, we have a natural and selfconsistent realization of neutrino democracy.

Summarizing this subsection, with the help of additional $(f+g)_\alpha$ states and $\mathcal{U}(1)$ flavor symmetry, we have obtained the desired charged fermion masses and mixings within 5D SUSY $SO(10)$ model with G_{422} symmetry realized at the fixed point $y=0$. Below we discuss details of the neutrino sector.

3.2 Democracy For Bi-large Neutrino Mixings

The right handed ν^c_α states come from F^c_α with the Dirac couplings given by $\nu^c \hat{Y}^\nu_D l h_u$. The latter arise from (30), which is also responsible for the charged lepton masses. Thus, due to $SU(2)_R$ symmetry which relates ν^c with e^c and h_u with h_d, we have

$$\hat{m}^\nu_D = \hat{m}_e \tan\beta \ . \tag{31}$$

This relation allows us to estimate the Majorana mass of the ν^c_3 state which, in turn, is responsible for the generation of the atmospheric neutrino scale. The ν^c_3 acquires mass through the coupling

$$\epsilon^{2k} \frac{\lambda}{M_f} (F^c_3 \overline{H}^c)^2 \ , \tag{32}$$

which gives

$$M_R^{(3)} = \frac{\lambda \epsilon^{2k}}{M_f} \langle \overline{H}^c \rangle^2 \ . \tag{33}$$

Taking into account (31), we have

$$m_{\nu_3} = \frac{[(\hat{m}^\nu_D)_{33}]^2}{M_R^{(3)}} = \frac{m_\tau^2 \tan^2\beta}{M_R^{(3)}} \ . \tag{34}$$

With hierarchical masses for neutrinos (which indeed must be the case for a democratic scenario), we have $m_{\nu_3} = \sqrt{\Delta m^2_{\rm atm}}$. Thus,

$$M_R^{(3)} = \frac{m_\tau^2 \tan^2\beta}{\sqrt{\Delta m^2_{\rm atm}}} = \begin{cases} 6 \cdot 10^{10}\ \text{GeV}; & \text{for}\ \tan\beta \sim 1 \\ 6 \cdot 10^{12}\ \text{GeV}; & \text{for}\ \tan\beta \sim 10 \\ 2 \cdot 10^{14}\ \text{GeV}; & \text{for}\ \tan\beta \sim 60 \end{cases} \ . \tag{35}$$

For low and intermediate values of $\tan\beta$ the predicted value of $M_R^{(3)}$ is relatively low which opens up an interesting possibility for implementing either inflationary [36] or thermal [37] leptogenesis.

Due to l-democracy, one could naively expect that $m_{\nu_1} \sim m_{\nu_2} \sim m_{\nu_3}$. This scale for ν_2 would be inappropriate for resolving the solar neutrino anomaly. To avoid this, some care should be exercised. One way is to introduce $SO(10)$ singlet states N_1 and N_2 with $\mathcal{U}(1)$ charges $-5-k$ and $-2-k$ respectively. Then, with the couplings $(F^c_1 N_1 + F^c_2 N_2)\overline{H}^c$, after substituting the $\overline{H}^c$ VEV, ν^c_1 and ν^c_2 decouple together with N_1 and N_2. At this stage $m_{\nu_1} = m_{\nu_2} = 0$, since with the introduction of $N_{1,2}$ states, the lepton numbers L_e, L_μ are conserved. To generate a mass for ν_2, we introduce an additional $SO(10)$ singlet $\mathcal{N}$ with charge $Q(\mathcal{N}) = k$. With couplings

$$(\lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3)\mathcal{N} h + M_f \epsilon^{2k} \mathcal{N}^2 \ , \tag{36}$$

integration of the $\mathcal{N}$ state can generate the scale relevant for the solar neutrino anomaly[5]. The additional singlet states are necessary for generating appropriate mass scale for solar neutrino and self consistent bi-large neutrino oscillations.

Let us consider the scheme in some detail. From (31), the $\nu^c - l$ and $e^c - l$ couplings are diagonalized simultaneously. Although the introduction of $N_{1,2}$ makes $\nu^c_{1,2}$ irrelevant, it is still convenient to work with basis in which the matrices in (31) are diagonal. In this case, the lepton mixing matrix will coincide with the unitary matrix which diagonalizes neutrino mass matrix, with non-trivial neutrino mixings coming from the right handed neutrino sector, i.e. from the $\nu_3^c - \mathcal{N}$ mixing term. The $\mathcal{U}(1)$ symmetry allows the coupling $F_3^c \mathcal{N} \overline{H}^c \epsilon^{2k}$, which gives a mixing term $M_{3\mathcal{N}} \nu_3^c \mathcal{N}$, with $M_{3\mathcal{N}} \simeq \langle \overline{H}^c \rangle \epsilon^{2k}$. Taking into account all this and also (33), (36), the relevant matrix is given by

$$\begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_3^c \\ \mathcal{N} \end{array} \begin{array}{c} \begin{array}{ccccc} \nu_1 & \nu_2 & \nu_3 & \nu_3^c & \mathcal{N} \end{array} \\ \left(\begin{array}{ccccc} 0 & 0 & 0 & 0 & \lambda_1 h_u \\ 0 & 0 & 0 & 0 & \lambda_2 h_u \\ 0 & 0 & 0 & \lambda_\tau h_u & \lambda_3 h_u \\ 0 & 0 & \lambda_\tau h_u & M_R^{(3)} & M_{\mathcal{N}} \epsilon' \\ \lambda_1 h_u & \lambda_2 h_u & \lambda_3 h_u & M_{\mathcal{N}} \epsilon' & M_{\mathcal{N}} \end{array} \right) \end{array} , \qquad (37)$$

where we have defined $M_{\mathcal{N}} = M_f \epsilon^{2k}$, $\epsilon' = \frac{M_{3\mathcal{N}}}{M_{\mathcal{N}}} \sim \frac{\langle \overline{H}^c \rangle}{M_f}$.

From (37) one sees that $\nu_3^c - \mathcal{N}$ mixing plays an essential role. Namely, to have large $\nu_\mu \to \nu_\tau$ oscillations, one should have $\lambda_{2,3} \epsilon' \sim \lambda_\tau$. Having in mind that $\epsilon' \ll 1$, we also have $\lambda_\tau \ll 1$, which means that $\tan\beta$ is not large, but has either a low or an intermediate value. Integrating out $\mathcal{N}$ and ν_3^c in (37), we obtain the light neutrino mass matrix

$$\hat{m}_\nu = \left(\begin{array}{ccc} \lambda_1^2 & \lambda_1 \lambda_2 & \lambda_1 \overline{\lambda}_3 \\ \lambda_1 \lambda_2 & \lambda_2^2 & \lambda_2 \overline{\lambda}_3 \\ \lambda_1 \overline{\lambda}_3 & \lambda_2 \overline{\lambda}_3 & \overline{\lambda}_3^2 \end{array} \right) m + \left(\begin{array}{ccc} \lambda_1^2 & \lambda_1 \lambda_2 & \lambda_1 \lambda_3 \\ \lambda_1 \lambda_2 & \lambda_2^2 & \lambda_2 \lambda_3 \\ \lambda_1 \lambda_3 & \lambda_2 \lambda_3 & \lambda_3^2 \end{array} \right) m' , \qquad (38)$$

where

$$m = \frac{\epsilon'^2 h_u^2}{M_R^{(3)}} , \quad m' = \frac{h_u^2}{M_{\mathcal{N}}} , \quad \overline{\lambda}_3 = \lambda_3 - \lambda_\tau / \epsilon' . \qquad (39)$$

The scales m, m' correspond to atmospheric and solar neutrino anomalies, so $m \gg m'$.

[5]These $SO(10)$ singlet states can be introduced either directly on the brane or in the bulk. In the bulk case, the accompanying mirrors carry opposite orbifold parities and $\mathcal{U}(1)$ charges. The presence of mirrors does not affect results obtained through the brane superpotential couplings.

For analyzing the neutrino mass matrix, it is convenient to rewrite (38),

$$\hat{m} = \begin{pmatrix} \tilde{\lambda}_1^2 & \tilde{\lambda}_1\tilde{\lambda}_2 & \tilde{\lambda}_1\tilde{\lambda}_3 \\ \tilde{\lambda}_1\tilde{\lambda}_2 & \tilde{\lambda}_2^2 & \tilde{\lambda}_2\tilde{\lambda}_3 \\ \tilde{\lambda}_1\tilde{\lambda}_3 & \tilde{\lambda}_2\tilde{\lambda}_3 & \tilde{\lambda}_3^2(1+\delta) \end{pmatrix} m \ , \tag{40}$$

where

$$\tilde{\lambda}_{1,2} = \sqrt{1+m'/m}\lambda_{1,2} \ , \quad \tilde{\lambda}_3 = \frac{1+\lambda_3 m'/(\overline{\lambda}_3 m)}{\sqrt{1+m'/m}}\overline{\lambda}_3 \ , \quad \delta = (1-\frac{\lambda_3}{\overline{\lambda}_3}+\frac{\lambda_3^2}{\overline{\lambda}_3^2})\frac{m'}{m} \ . \tag{41}$$

The value of δ is small and is responsible for the non-zero mass of ν_2. From (40), assuming $\lambda_\alpha \sim \lambda_\tau/\epsilon'$ one obtains

$$m_{\nu_1} = 0 \ , \quad m_{\nu_2} \simeq \frac{(\tilde{\lambda}_1^2+\tilde{\lambda}_2^2)\tilde{\lambda}_3^2}{\tilde{\lambda}_1^2+\tilde{\lambda}_2^2+\tilde{\lambda}_3^2}\delta m \sim \tilde{\lambda}_\alpha^2 m' \ , m_{\nu_3} \simeq (\tilde{\lambda}_1^2+\tilde{\lambda}_2^2+\tilde{\lambda}_3^2)m \ . \tag{42}$$

As expected, the neutrino masses are hierarchical, with $\Delta m^2_{atm} \sim m^2_{\nu_3}$, $\Delta m^2_{sol} \sim m^2_{\nu_2}$. From (39), (42), we see that the relevant scale for the solar neutrino anomaly $m_{\nu_2} \sim 10^{-2}$ eV is generated for $M_{\mathcal{N}} \simeq (3\cdot 10^{11}\tan^2\beta/\epsilon'^2)$ GeV. With $\epsilon' \simeq 0.2$ and $\tan\beta = 1-60$, $M_{\mathcal{N}} \simeq (10^{13}-3\cdot 10^{16})$ GeV. For the atmospheric neutrinos, $m_{\nu_3} \simeq 5\cdot 10^{-2}$ eV is generated with the $M_R^{(3)}$ scales indicated in (35).

As far as the mixing angles are concerned, from the structure of (40), (38), with $\lambda_1 \sim \lambda_2 \sim \lambda_3 \sim \lambda_\tau/\epsilon'$, taking into account (39), (41), one naturally expects large neutrino mixings

$$\sin^2 2\theta_{\mu\tau} \sim 1 \ , \quad \sin^2 2\theta_{e\mu,\tau} \sim 1 \ , \tag{43}$$

as intended.

As was emphasized in [16], [17], within the democratic approach, one also expects a large value for the θ_{13} mixing angle. On the other hand, the CHOOZ data [40] requires $\theta_{13} \lesssim 0.2 (\simeq \epsilon)$, so that some cancellation among the parameters is needed. If future measurements turn out to favor a much smaller ($\ll \epsilon$) θ_{13}, some modification of the demoratic approach would be required.

3.3 Particle Spectra And Gauge Coupling Unification

Below the symmetry breaking scale of G_{422} ($v \simeq M_G$), the massless gauge fields are just those from the MSSM. However, there are some additional vector like supermultiplets with masses below the GUT scale M_G. From (27) and (29), it is easy to see that the masses of decoupled color triplets and $SU(2)_L$ doublet states respectively are

$$m_{t_1} \simeq v\epsilon^{n+6} \ , \quad m_{t_2} \simeq v\epsilon^{n+2} \ , \quad m_{t_3} \simeq v\epsilon^{n} \ ,$$
$$m_{d_1} \simeq v\epsilon^{m+3} \ , \quad m_{d_2} \simeq v\epsilon^{m+2} \ , \quad m_{d_3} \simeq v\epsilon^{m} \ . \tag{44}$$

With the $\mathcal{U}(1)$ charges in Table 1, the superpotential couplings

$$W(H, D) = \epsilon^p H^c H^c D + \epsilon^q \overline{H}^c \overline{H}^c D \ , \tag{45}$$

yield masses

$$m_{T_1} \simeq v\epsilon^p \ , \qquad m_{T_2} \simeq v\epsilon^q \ , \tag{46}$$

for the colored triplet states from 'scalar' supermultiplets.

In the simplest case, one can assume that the compactification scale μ_0 and the G_{422} breaking scale are close to $M_G(\simeq \mu_0 \simeq v)$. On the other hand, from (44), (46) we see that with $m, n, p, q \neq 0$, below the scale v there are additional states which contribute, together with the MSSM states, to the renormalization of the gauge couplings. For the strong coupling constant in particular,

$$\alpha_3(M_Z)^{-1} = [\alpha_3(M_Z)^{-1}]^{\rm min}_{SU(5)} + \frac{9}{14\pi} \ln \frac{M_G^2 m_{d_1} m_{d_2} m_{d_3}}{m_{t_1} m_{t_2} m_{t_3} m_{T_1} m_{T_2}} \ , \tag{47}$$

where the first term on the right hand side corresponds to the value calculated in minimal SUSY $SU(5)$ [MSSU(5)]. In order to preserve the MSSU(5) prediction, one should take $m_{t_1} m_{t_2} m_{t_3} m_{T_1} m_{T_2} \simeq M_G^2 m_{d_1} m_{d_2} m_{d_3}$, which taking into account (44), (46) gives the relation

$$3n + p + q + 3 = 3m \ . \tag{48}$$

If in the right hand side of (48), $3m$ is replaced by $3m + c$ (c=1, 2), one would have the possibility to reduce the somewhat large value of $\alpha_3(M_z)(\simeq 0.126)$ predicted by MSSU5 [41]. Thus, a proper selection of the integers m, n, p, q can yield successful unification with additional matter states playing a crucial role.

4 Proton Stability And Automatic Matter Parity

In this section we discuss the issue of nucleon decay within our scheme. Let us start with dimension five left and right handed operators

$$\mathcal{O}_L = qqql \ , \qquad \mathcal{O}_R = u^c u^c d^c e^c \ , \tag{49}$$

whose presence depends on the details of the model. The $\mathcal{O}_L$ can emerge through qqT, $ql\bar{T}$ type couplings, while $\mathcal{O}_R$ can appear through e^ce^cT, $u^cd^c\bar{T}$, once the color triplets T, $\bar{T}$ are integrated out. Consider an $\mathcal{R}$ symmetry under which $W \to e^{2\mathrm{i}\alpha}W$ and $\phi_i \to e^{\mathrm{i}R_i}\phi_i$, where 2α and R_i are the phases of the superpotential W and ϕ_i superfield respectively. It is easy to check that the transformations

$$
\begin{gathered}
(F, F^c, g, f, N_{1,2},\ \mathcal{N}) \to e^{\mathrm{i}\alpha}(F, F^c, g, f, N_{1,2},\ \mathcal{N})\ , \\
(h, H^c, \overline{H}^c, X) \to (h, H^c, \overline{H}^c, X)\ , \quad D \to e^{2\mathrm{i}\alpha}D\ , \quad W \to e^{2\mathrm{i}\alpha}W\ ,
\end{gathered}
\tag{50}
$$

are consistent with the $\mathcal{R}$-symmetry. Note that the direct mass terms $\overline{H}^cH^c$ and D^2 in $W(H,D)$ of (45) are forbidden to all orders. Therefore, the color triplet mass matrix is given by

$$
\hat{M}_T = \begin{array}{c} \\ d^c_H \\ d^c_D \end{array}
\begin{array}{c} \begin{array}{cc} \overline{d}^c_H & \overline{d}^c_D \end{array} \\ \left(\begin{array}{cc} 0 & v\epsilon^p \\ v\epsilon^q & 0 \end{array}\right) \end{array} .
\tag{51}
$$

From (51), the elements of the inverse mass matrix which are relevant for nucleon decay, read

$$
\left(\hat{M}_T^{-1}\right)_{11} = \left(\hat{M}_T^{-1}\right)_{22} = 0\ , \quad \left(\hat{M}_T^{-1}\right)_{12} = \frac{1}{v\epsilon^q}\ , \quad \left(\hat{M}_T^{-1}\right)_{21} = \frac{1}{v\epsilon^p}\ .
\tag{52}
$$

Thus, for nucleon stability it is crucial that the triplets from the D state do not couple with the matter fields. It is easy to check, using (50), that the couplings

$$
FFD\ , \quad FfD\overline{H}^c\ , \quad F^cgD\overline{H}^c\ , \quad F^cF^cD\ ,
\tag{53}
$$

are forbidden. Thus, $d=5$ operators from the color triplet exchange are absent.

As far as the Planck scale $d=5$ operators of (49) are concerned, one can also verify that the couplings

$$
FFFf\overline{H}^c\ , \qquad F^cF^cF^cg\overline{H}^c\ ,
\tag{54}
$$

from which these operators could emerge, are also forbidden.

In principle, in (51) instead of zeros, entries of the order of SUSY breaking scale $m_S \sim 1$ TeV can be expected. The latter can come from the Kähler potential after SUSY is broken. By the same reasoning, operators (53), (54), suppressed by m_S/M_f, may also emerge. However, proton decay is then strongly suppressed and essentially unobservable. We conclude that through a proper $\mathcal{R}$-charge selection, dimension five nucleon decay is eliminated.

Turning to dimension six nucleon decay, with all matter supermultiplets introduced in the bulk, the 5D bulk kinetic terms are irrelevant for nucleon decay [25]. This follows since through the exchange of V_X, V_Y bosons, the quark-lepton states are converted into heavy states with masses of order $1/R$. A different source for $d=6$ nucleon decay can be brane localized operators, which respect G_{422} and the orbifold symmetries. This kind of operator has the form [29]

$$\delta(y)\psi_1^+(\partial_5 e^{2\hat{V}} - \hat{\Sigma}e^{2\hat{V}} - e^{2\hat{V}}\hat{\Sigma})\psi_2 \ , \tag{55}$$

where ψ_1 and ψ_2 denote appropriate matter superfields with zero modes, and $\hat{V}$, $\hat{\Sigma}$ are fragments from the coset $SO(10)/G_{422}$. In order for these operators to be invariant under $\mathcal{U}(1)$, either the multiplier $(X^+)^{Q_1}X^{Q_2}$ or $X^{Q_2-Q_1}$ should be present, where Q_1, Q_2 are the $\mathcal{U}(1)$ charges of ψ_1, ψ_2. If either Q_1, Q_2 or Q_2-Q_1 is not an integer, the corresponding operator is not allowed. The operators $F^+\partial_5 V_B F^c$, $f^+\partial_5 V_B g$ carry $\mathcal{U}(1)$ charge $(n-m)/2$ and will not be allowed for $n-m=2k'+1$ (k' is an integer). Thus, with the help of $\mathcal{U}(1)$ symmetry, dimension six nucleon decay can also be avoided [17] within 5D SUSY $SO(10)$ GUT.

Alternatively, if the selection $n-m=2k'$ is made, the appropriate operators will be allowed with a suppression factor $\epsilon^{k'}$. Together with this, there will appear additional powers of ϵ, coming from the $\mathcal{U}(1)$ charges of the light families. The appropriate amplitude has at least an ϵ^2 suppression. Consequently, the amplitudes are proportional to $\epsilon^{2k'+2}$, which even for $k'=0$, gives an enhancement factor ~ 650 for the lifetime, in comparision to the estimates in [29]. This is well within the current experimental bounds [38]. It is interesting to note that the operator $\epsilon^{2k'+2}q_1 l_\alpha u_1^{c+}d_2^{c+}$, relevant for decay with emmision of charged leptons, would give $\tau(p\to Ke)\sim\tau(p\to K\mu)$. This happens due the fact that the l_α states all have the same $\mathcal{U}(1)$ charge.

To conclude this section, we note that the mass term for the bidoublet h is forbidden by $\mathcal{R}$-symmetry and therefore at this level the μ-term is zero. This gives a good starting point for the resolution of SUSY μ problem, although some mechanism [42], [43] for its generation should be applied. The same $\mathcal{R}$-symmetry also forbids all matter parity violating operators otherwise allowed by the symmetry G_{422}. That is, together with the suppression of $d=5$ baryon number violating operators and μ-term, the $\mathcal{R}$-symmetry also ensures automatic matter parity [44], [12].

5 Rare $l_\alpha \to l_\beta\gamma$ Decays

In our scenario neutrino masses are generated by the see-saw mechanism and within the SUSY framework, it turns out to be also the dominant source for lepton flavor violating (LFV) rare processes [45] such as $l_\alpha\to l_\beta\gamma$. Since the masses of the right handed neutrinos are below the GUT scale, they contribute to renormalization so that universality (assumed to hold at high scales) among the soft slepton masses is violated. For non-universal contributions, the relevant scales are those where the corresponding right handed neutrinos decouple. It is convenient to work with a basis in which the right handed neutrino mass matrix is diagonal. The non-universal contributions to the slepton masses are then

$$(\delta m^2)_{\alpha\beta} \approx -\frac{1}{8\pi^2}(A+3)m_S^2 \sum_k (\tilde{Y}_\nu^T)_{\alpha k} \ln \frac{M_G}{M_R^{(k)}} (\tilde{Y}_\nu)_{\beta k} \ , \tag{56}$$

where we have assumed universality and proportionality at M_G. Also, in (56) SUSY breaking occurs through $N=1$ SUGRA. The $\tilde{Y}_\nu$ comes from the term $N_R' \tilde{Y}_\nu l h_u$, and N_R' denotes the right handed neutrino state in a mass eigenstate basis. From (37),

$$\tilde{Y}_\nu = \begin{pmatrix} -\lambda_1 \epsilon' & -\lambda_2 \epsilon' & \lambda_\tau - \lambda_3 \epsilon' \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} . \tag{57}$$

From (56), (57), taking into account that $\epsilon' \ll 1$, one finds

$$(\delta m^2)_{\alpha\beta} \approx -\tfrac{1}{8\pi^2}(A+3)m_S^2 \begin{pmatrix} \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_1\lambda_2 & \lambda_2^2 & \lambda_2\lambda_3 \\ \lambda_1\lambda_3 & \lambda_2\lambda_3 & \lambda_3^2 \end{pmatrix}_{\alpha\beta} \cdot \ln \tfrac{M_G}{M_{\mathcal{N}}} \ . \tag{58}$$

Thus, the $\mathcal{N}$ state provides the dominant contribution to LFV. Due to democracy, the λ_α couplings all have similar values, and therefore one can expect

$$\mathrm{BR}(\mu \to e\gamma) \sim \mathrm{BR}(\tau \to \mu\gamma) \sim \mathrm{BR}(\tau \to e\gamma) \ . \tag{59}$$

This is a characteristic signature of the democratic scenario.

More precisely, taking into account that $\lambda_\alpha \sim \lambda_\tau/\epsilon'$, eq. (58) and the expressions given in [46], the branching ratios are

$$\mathrm{BR}(l_\alpha \to l_\beta\gamma) = \frac{\alpha_{em}^3 [(\delta m^2)_{\alpha\beta}]^2}{G_F^2 m_S^8} \tan^2\beta \sim \frac{\alpha_{em}^3 (A+3)^2}{64\pi^4 G_F^2 m_S^4} \left(\frac{\lambda_\tau}{\epsilon'}\right)^4 \tan^2\beta \ln^2 \frac{M_G}{M_{\mathcal{N}}} \ . \tag{60}$$

In order to satisfy the most stringent bound $\mathrm{BR}(\mu \to e\gamma) \lesssim 10^{-14}$ [47], for $m_S \sim 1$ TeV, [and if there is no cancellation in (60)], one should take $\lambda_\tau/\epsilon' \sim 5 \cdot 10^{-2}$. With $\epsilon' \sim 0.2$ this gives $\lambda_\tau \sim 10^{-2}$, indicating a preference for low $\tan\beta$ regime, which is also suggested by the charged fermion and neutrino sectors.

In conclusion, let us note that in contrast to previously discussed LFV scenarios [48], our scheme predicts 'democracy' in the sector of LFV rare processes which hopefully can be probed in the near future.

6 Conclusions

We have presented a realistic model of charged fermion and neutrino masses and mixings in which a number of key ideas play an essential role. Namely, $SO(10)$ grand unification, an extra dimension, orbifold and higgs breaking and an abelian $\mathcal{U}(1)$ flavor symmetry. Some additional states are introduced in order to realize a democratic approach in the neutrino sector, while yielding the CKM mixings in the quark sector. Note that it is possible to replace $\mathcal{U}(1)$ with a discrete symmetry, or with a vectorlike 5D gauge symmetry [49]. The issue of proton stability is resolved and interesting predictions for rare leptonic decays have been obtained.

Acknowledgments
We acknowledge the support of NATO Grant PST.CLG.977666. This work is partially supported by DOE under contract DE-FG02-91ER40626. Z.T. would like to thank the Bartol Research Institute for warm hospitality during his visit there.

References

[1] S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 88 (2000) 3999; N. Fornengo et al., Nucl. Phys. B 580 (2000) 58.

[2] S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 86 (2001) 5651; J. Bahcall, P. Krastev, A. Smirnov, hep-ph/0006078; M.C. Gonzalez-Garcia et al., Nucl. Phys. B 573 (2000) 3.

[3] K. Eguchi et al., [KamLAND collaboration], Phys. Rev. Lett. 90 (2003) 021802; S. Pakvasa, J. Valle, hep-ph/0301061.

[4] M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity, eds. D. Freedman et al. (North-Holland, Amsterdam, 1980); T. Yanagida, in proc. KEK workshop, 1979; R. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[5] H. Fritzsch, P. Minkowski, Ann. Phys. 93 (1975) 193; H. Georgi, in Particles and Fields - 1974, Proc. of the APS 1974 Meeting, ed. C.A. Carlson, AIP, New York, 1975.

[6] J. Pati, A. Salam, Phys. Rev. D 10 (1974) 275.

[7] C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[8] G. Lazarides, Q. Shafi, Nucl. Phys. B 350 (1991) 179; P. Ramond, R.G. Roberts, G.G. Ross, Nucl. Phys. B 406 (1993) 19; L. Ibañez, G.G. Ross, Phys. Lett. B 332

(1994) 100; E. Dudas, S. Pokorski, C. Savoy, Phys. Lett. B 369 (1995) 255; See Z. Berezhiani, Z. Tavartkiladze in [10], [11]; K. Choi, E.J. Chun, H. Kim, Phys. Lett. B 394 (1997) 89; N. Irges, S. Lavignac, P. Ramond, Phys. Rev. D 58 (1998) 035003; See also references therein.

[9] P. Binetruy et al., Nucl. Phys. B 496 (1997) 3; J. Elwood, N. Irges, P. Ramond, Phys. Rev. Lett. 81 (1998) 5064; M. Gomez, G. Leontaris, S. Lola, J. Vergados, Phys. Rev. D 59 (1999) 116009; F. Vissani, JHEP 9811 (1998) 025; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 448 (1999) 46; Phys. Lett. B 482 (2000) 145; B. Stech, Phys. Lett. B 465 (1999) 219; J. Feng, Y. Nir, Phys. Rev. D 61 (2000) 113005; S. Barr, I. Dorsner, Nucl. Phys. B 585 (2000) 79; D. Suematsu, Phys. Rev. D 64 (2001) 073013; S. King, N. Singh, Nucl. Phys. B 596 (2001) 81; T. Kitabayashi, M. Yasue, Nucl. Phys. B 609 (2001) 61; I. Gogoladze, A. Perez-Lorenzana, Phys. Rev. D 65 (2002) 095011; K.S. Babu, R.N. Mohapatra, Phys.Lett.B 532 (2002) 77; F. Feruglio, A. Strumia, F. Vissani, Nucl. Phys. B 637 (2002) 345; T. Ohlsson, G. Seidl, Nucl. Phys. B 643 (2002) 247; V. Antonelli et al., Phys. Lett. B 549 (2002) 325.

[10] Z. Berezhiani, Z. Tavartkiladze, Phys. Lett. B 396 (1997) 150; J. Sato and T. Yanagida, Nucl. Phys. Proc. Suppl. 77 (1999) 293; G. Altarelli, F. Feruglio, Phys. Lett. B 451 (1999) 388; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 451 (1999) 129; G. Altarelli, F. Feruglio, I. Masina, JHEP 0011 (2000) 040; M. S. Berger, K. Siyeon, Phys. Rev. D 63 (2001) 057302.

[11] See 1st ref. in [8]; Z. Berezhiani, Z. Tavartkiladze, Phys. Lett. B 409 (1997) 220; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 487 (2000) 145; N. Maekawa, Prog. Theor. Phys. 106 (2001) 401; J.C. Pati, hep-ph/0209160.

[12] Q. Shafi, Z. Tavartkiladze, Nucl. Phys. B 549 (1999) 3.

[13] Q. Shafi, Z. Tavartkiladze, Nucl. Phys. B 573 (2000) 40.

[14] M. Bando, T. Kugo, Prog. Theor. Phys. 101 (1999) 1313; M. Bando, N. Maekawa, Prog. Theor.Phys. 106 (2001) 1255.

[15] N. Haba, T. Matsuoka, Prog. Theor. Phys. 99 (1998) 831; M. Matsuda, T. Matsuoka, Phys. Lett. B 499 (2001) 287; Y. Abe et al., Prog. Theor. Phys. 108 (2002) 965.

[16] Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 550 (2002) 172; For earlyer study of flipped $SU(5)$ model with $\mathcal{U}(1)$ flavor symmetry, see Q. Shafi, Z. Tavartkiladze in [9].

[17] Q. Shafi, Z. Tavartkiladze, hep-ph/0210181.

[18] Q. Shafi, Z. Tavartkiladze, Nucl. Phys. B 552 (1999) 67.

[19] N. Maekawa, Q. Shafi, Prog. Theor. Phys. 109 (2003) 279.

[20] Y. Achiman, B. Stech, Phys. Lett. B 77 (1978) 389; Q. Shafi, Phys. Lett. B 79 (1978) 301; G. Lazarides, C. Panagiotakopolous, Q. Shafi, Phys. Lett. B 315 (1993) 325; G. Dvali, Q. Shafi, Phys. Lett. B 403 (1997) 65; M. Bando, T. Kugo, Prog. Theor. Phys. 109 (2003) 87.

[21] S. Cecotti et al., Phys Lett. B 156 (1985) 318; G. Lazarides, Q. Shafi, Nucl. Phys. B 329 (1990) 183.

[22] M. Fukugita, et al., hep-ph/9809554; M. Tanimoto, T. Watari, T. Yanagida, hep-ph/9904338; L. Hall, H. Murayama, N. Weiner, Phys. Rev. Lett. 84 (2000) 2572; H. Fritzsch, Z. Xing, Acta Phys. Polon. B 31 (2000) 1349; N. Haba, H. Murayama, hep-ph/0009174; M. Berger, K. Siyeon, hep-ph/0010245; S. Huber, Q. Shafi, hep-ph/0104293.

[23] Y. Kawamura, Prog. Theor. Phys. 105 (2001) 999; *ibid.* 105 (2001) 691.

[24] G. Altarelli, F. Feruglio, Phys. Lett. B 511 (2001) 257.

[25] A. Kobakhidze, Phys. Lett. B 514 (2001) 131.

[26] L. Hall, Y. Nomura, Phys. Rev. D 64 (2001) 055003; M. Kakizaki, M. Yamaguchi, hep-ph/0104103; A. Hebecker, J. March-Russel, Nucl. Phys. B 613 (2001) 3; R. Barbieri, L. Hall, Y. Nomura, hep-th/0107004; C. Csaki, G. Kribs, J. Terning, hep-ph/0107266; T. Li, hep-ph/0108120; hep-th/0110065.

[27] T. Asaka, W. Buchmüller, L. Covi, hep-ph/0108021; R. Dermisek, A. Mafi, Phys. Rev. D 65 (2002) 055002; H.D. Kim, S. Raby, JHEP 0301 (2003) 056.

[28] Q. Shafi, Z. Tavartkiladze, Phys. Rev. D 66 (2002) 115002 (hep-ph/0108247).

[29] A. Hebecker, Nucl. Phys. B 632 (2002) 101; A. Hebecker, J. March-Russel, Phys. Lett. B 539 (2002) 119.

[30] F. Paccetti Correia, M.G. Schmidt, Z. Tavartkiladze, Nucl. Phys. B 649 (2003) 39; Phys. Lett. B 545 (2002) 153; S. Barr, I. Dorsner, Phys. Rev. D 66 (2002) 065013; N. Haba, M. Harada, Y. Hosotani, Y. Kawamura, hep-ph/0212035; B. Kyae, Q. Shafi, hep-ph/0212331; J. Oliver, J. Papavassiliou, A. Santamaria, hep-ph/0302083.

[31] I. Gogoladze, Y. Mimura, S. Nandi, hep-ph/0301014; hep-ph/0302176.

[32] G. Seidl, hep-ph/0301044.

[33] R. Kitano, T. Li, hep-ph/0302073.

[34] P. Candelas at el., Nucl. Phys. B 258 (1985) 46; E. Witten, Nucl. Phys. B 258 (1985) 75; P.S. Aspinwall at el., Nucl. Phys. B 294 (1988) 193; G. Lazarides, Q. Shafi, Phys. Lett. B 206 (1988) 32; G. Lazarides at el., Nucl. Phys. B 323 (1989) 614; G. Lazarides, C. Panagiotakopoulos, Q. Shafi, Phys. Lett. B 225 (1989) 66; R. Arnowitt, P. Nath, Phys. Rev. Lett. 62 (1989) 2225; A. Font, L. Ibanez, F. Quevedo, Phys. Lett. B228 (1989) 79; A. Faraggi, Nucl. Phys. B 428 (1994) 111; hep-ph/0107094.

[35] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.

[36] G. Lazarides, Q. Shafi, Phys. Lett. B 258 (1991) 305.

[37] See for instance, W. Buchmuller, P. Di Bari, M. Plumacher, Nucl. Phys. B 643 (2002) 367; S. Davidson, hep-ph/0302075; and references therein.

[38] Particle Data Group, Phys. Rev. D 66 (2002) 010001.

[39] I. Antoniadis, G. Leontaris, Phys. Lett. B 216 (1989) 333; S.F. King, Q. Shafi, Phys. Lett. B 422 (1998) 135; R. Jeannerot, S. Khalil, G. Lazarides, Q. Shafi, JHEP 0010 (2000) 012; A. Melfo, G. Senjanovic, hep-ph/0302216.

[40] M. Apollonio et al. (CHOOZ Collaboration), Phys. Lett. B 466 (1999) 415.

[41] I. Antoniadis, J. Ellis, S. Kelley, D. Nanopoulos, Phys. Lett. B 272 (1991) 31; P. Langacker, N. Polonsky, Phys. Rev. D 47 (1993) 4028; Phys. Rev. D 52 (1995) 52; N. Polonsky, hep-ph/0108236.

[42] G. Giudice, A. Masiero, Phys. Lett. B 206 (1988) 480.

[43] S. Dimopoulos, G. Dvali, R. Rattazzi, Phys. Lett. B 413 (1997) 336; E.J. Chun, Phys. Rev. D 59 (1999) 015011; P. Langacker, N. Polonsky, J. Wang, Phys. Rev. D 60 (1999) 115005; T. Han, D. Marfatia, R.J. Zhang, Phys. Rev. D 61 (2000) 013007; K. Choi, H.D. Kim, Phys. Rev. D 61 (2000) 015010; Q. Shafi, Z. Tavartkiladze, Nucl. Phys. B 580 (2000) 83.

[44] I. Antoniadis, J. Hagelin, D. Nanopoulos, Phys. Lett. B 194 (1987) 231; G. Lazarides, C. Panagiotakopoulos, Q. Shafi, Phys. Lett. B 315 (1993) 325; G. Dvali, Q. Shafi, Phys. Lett. B 403 (1997) 65; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 459 (1999) 563; K.S. Babu, I. Gogoladze, K. Wang, hep-ph/0212245.

[45] F. Borzumati, A. Masiero, Phys. Rev. Lett. 57 (1986) 961.

[46] J. Hisano et al., Phys. Rev. D 53 (1996) 2442; J. Hisano, D. Nomura, Phys. Rev. D 59 (1999) 116005.

[47] L. Barkov et al., Proposal for experiment at PSI (1999), http://meg.psi.ch/; M. Brooks et al., Phys. Rev. Lett. 83 (1999) 1521.

[48] J. Sato, K. Tobe, T. Yanagida, Phys. Lett. B 498 (2001) 189; J. Casas, A. Ibarra, Nucl. Phys. B 618 (2001) 171; D. Carvalho et al., Phys. Lett. B 515 (2001) 515; S. Baek et al., Phys. Rev. D 64 (2001) 095001; S. Lavignac, I. Masina, C. Savoy, Phys. Lett. B 520 (2001) 269; A. Kageyama, S. Kaneko, N. Shimoyama, M. Tanimoto, Phys. Rev. D 65 (2002) 096010; K.S. Babu, C. Kolda, Phys. Rev. Lett. 89 (2002) 241802; X. Bi, Phys. Rev. D 66 (2002) 076006; A. Masiero, S. Vempati, O. Vives, Nucl. Phys. B 649 (2003) 189; G. Ross, O. Vives, hep-ph/0211279.

[49] C.A. Lee, Q. Shafi, Z. Tavartkiladze, Phys. Rev. D 66 (2002) 055010.