Weighted-CS for reconstruction of highly under-sampled dynamic MRI sequences

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Abstract—This paper investigates the potential of the new Weighted-Compressive Sensing approach which overcomes the major limitations of other compressive sensing and outperforms current state-of-the-art methods for low-rate reconstruction of sequences of MRI images. The underlying idea of this approach is to use the image of the previous time instance to extract an estimated probability model for the image of interest, and then use this model to guide the reconstruction process. This is motivated by the observation that MRI images are hugely sparse in Wavelet domain and the sparsity changes slowly over time.

I. INTRODUCTION

Magnetic Resonance Imaging (MRI) is an essential medical imaging tool burdened by its intrinsic slow data acquisition process. Since data acquisition is sequential in MR imaging modalities, the scan time (time to get enough data to accurately reconstruct one frame) is reduced if fewer measurements are needed for reconstruction. Goal of many researchers is therefore to employ a smaller set of samples than normally required to reconstruct the original images. However, when k-space is under sampled, the Nyquist criterion is violated, and conventional Fourier reconstructions exhibit aliasing artifacts.

Compressed sensing (CS) [1]–[4], on the other hand, aims to reconstruct signals and images from significantly fewer measurements than were traditionally thought necessary. CS has already had noteworthy impacts in this field [5], [6] which enables the reconstruction of MRI images from much smaller set of measurements than the number of Nyquist-rate samples. It was shown that MRI has two key features that makes the application of CS successful; first is that MRI images are naturally compressible by sparse coding in an appropriate transform domain (e.g., by wavelet transform [7]); second is that MRI scanners acquire sample images in 2D Fourier transform, which is known to be incoherent w.r.t. sparse domain, rather than acquiring direct pixel samples [8].

Much of the current CS-based works in the literature are, mostly concerned with reconstruction of static MRI images. However, in many important and challenging MRI applications, such as volumetric (3D) MRI or real-time MRI (refers to the continuous monitoring of moving objects in real time), instead of just one image slice, we are dealing with sequences of MRI images which are closely related to each other. Real-time MRI, for example, currently is only possible with low image quality or limited rate of slices per second due to the time-consuming scanning process [9]. Therefore developing CS-based methods, specifically adopted for dealing with dynamic sequences of MRI images, can greatly benefit these applications. To the best of our knowledge, only a few works have developed reconstruction methods for sequences of MRI images (e.g. [8], [9]).

II. THE BACKGROUND

A. Notations

Throughout the paper, vectors are denoted by boldface letters (e.g. f, F) and f_i is the i-th element of the vector f. Scalars are shown by small regular letters (e.g. n, k) and matrices are denoted by bold capital Greek letters (Ψ, Φ). Superscript (t) added to a vector/matrix refers to that of time t. We use the notation Φ|S to denote the sub-matrix containing the columns of Φ with indices belonging to S. For a vector, the notation y|S forms a sub-vector that contains elements with indices in S.

B. Problem Formulation

In its noiseless formulation, the problem can be posed as follows: let f(t) ∈ R^n be the slowly time-varying MRI signal at epoch t, which is sparse in a transform domain (Ψ'(t) = Ψf(t)). S(t) := {k ∈ {1, . . . , n} : F'(t)k ̸= 0} denote the support of the signal in the transform domain. At each slice, the image is measured using a sampling matrix (Φ of size m × n) and y(t) = Φf(t) is the observation vector. The problem, at each time t, is then to recover the original image, f(t), from the corresponding compressive samples (y(t)), assuming that the image of interest is sparse. The most straightforward application of CS for reconstruction of the dynamic signal, would be to use the simple CS at each time frame separately [9]. This means that at each time frame, we search for a signal that satisfies the observations and is sparsest in the transform domain. Equivalently, at each time instant, t, we find a g which is an estimate of f(t), as a solution to the following optimization problem:

f(t) = arg min(||G||_ℓ_1 subject to Φg = y(t)). (1)

where G = Ψg.

In this method, however, the fact that all image slices are closely related is not exploited. The authors of [10], propose a method based on the fact that the MRI images...
Therefore, the conjecture is that we should be able to extract nearby slices are closely related to each other (see figure 3). Based on the observation that in sequences of MRI images, the signal of interest in the transform domain. The idea is of critical a priori knowledge including the estimated support of the signal of interest’s sparsity, where $p_i := P(F_i > 0)$ is the probability of $F_i$ having a non-zero value. It is clear that if $p_i = 0$, then $F_i$ is always zero, while if $p_i = 1$, then it is known beforehand that $F$ has a spike at location $i$ (though its value is unknown). $p_i = 0.5$ basically means that no priori information of $F_i$ sparsity is available and it is as likely as not to have a non-zero value.

In order to incorporate the probability model of the signal into the process of reconstruction, we propose to minimize a weighted $\ell_1$ norm (4), where the weights are adjusted according to the probability of each entry being non-zero:

$$\min_{\Phi g = y} \| W G \|_{\ell_1} \quad \text{subject to} \quad \Phi g = y.$$  

where $W = \text{diag}([w_1, w_2, \ldots, w_n])$. Intuitively, a smaller weight should be given to those entries with higher probability of being non-zero while those elements with small probability should be penalized with larger weights. Naturally, we want to reward and penalize the elements uniformly using a linear function. Thus, the choice of the weight for each element is:

$$w_i = 2(1 - p_i).$$

Figure 1 shows the chosen weight with respect to the value of the probability. It can be seen that as the probability of an element being non-zero, increases, its weight decreases accordingly. Notice that if $p_i = 0.5$ then $w_i = 1$ and $w_i < 1$ when $p_i > 0.5$ (similarly if $p_i < 0.5$ then $w_i > 1$). It should be noted that the weighted $\ell_1$ approach (4), could be seen as a generalized $\ell_1$ minimization, since when no priori information of the sparsity is available (which means for all elements $p_i = 0.5$), it reduces to the conventional $\ell_1$ minimization as all weights would be equal.

**B. Estimation of sparsity probability vector**

In this subsection, we discuss estimation of the sparsity probability model for a signal at time $t$, from the reconstructed signal at time $t - 1$. Based on the assumption that sparsity changes smoothly with time, given a spike in the signal at time $t - 1$, there is a good chance that either it remains in the same location, or shifts to some point in the same vicinity in the next time frame ($t$). Similarly, at time $t$ it is expected that zeros appear in the vicinity of zeros at time
More specifically, suppose there is a spike in the $i^{th}$ location of $\mathbf{F}^{(t-1)}$. In the next time instant, there is a very high probability this spike remains in the same location but also some possibility that it moves to some other point in the vicinity. Thus, the probability of the spike appearing at the each location decreases as we get farther from $(i)$. This motivated us to use a Gaussian distribution (figure 2(a)) to provide an estimate of the probability of the progression of a spike in the next time frame.

Thus the probability of the $j^{th}$ element of $\mathbf{F}^{(t)}$ being non-zero ($p_{j}^{(t)}$) is the accumulated probability of the spikes of $\mathbf{F}^{(t-1)}$ moving to location $j$ at time $t$ as follows:

$$p_{j}^{(t)} = \sum_{i \in S^{(t-1)}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(j-i)^2}{2\sigma^2}}$$

(6)

where $S^{(t-1)}$ is the support of $\mathbf{F}^{(t-1)}$ and $\sigma^2$ is the variance of the Gaussian distribution which is set proportional to the signal’s rate of change with time. Figure 2(b) shows a synthetic signal $\mathbf{F}^{(t-1)}$ together with the probability of the elements of $\mathbf{F}^{(t)}$ being non-zero in the next time frame ($p_{j}^{(t)}$) which is the dashed red line. From this figure and equation (6), it can be seen that the maximum value of $p_{j}^{(t)}$ is 1. This coincides with the locations where in the previous time instant, $F_{j}^{(t-1)}$ and all elements in its vicinity ($[j-3\sigma,j+3\sigma]$) are non-zero. Similarly, $p_{j}^{(t)}$ is 0, if there is not any spike in this span.

The above formulation can be extended for 2D images using a 2D Gaussian distribution.

It should be mentioned that if no priori knowledge on the sparsity of the signal of interest is available, including at $t = 1$ where there is no previous time instant to estimate the probability from, we set all $p_{i} = 0.5$ and basically we solve a basic $\ell_1$ minimization.

### C. Reconstruction of the time sequences of sparse signals

In the earlier sub-sections, we explained the methods for estimating the sparsity probability model and its use for guiding the reconstruction. Using these methods, the algorithm for reconstruction of sequences of MRI images is summarized in Algorithm 1 below. It should be noted that at time $t = 1$, if no priori knowledge of the image is available, we solve using $\ell_1$ minimization thus requiring more samples to achieve a perfect reconstruction.

#### Algorithm 1 Reconstruction of a sequences of sparse signals varying with time using Weighted-CS.

**Input:** $\Phi$ and $\mathbf{y}^{(t)}$.

**Output:** $\mathbf{g}^{(t)}$.

1. If $t = 1$ then
   $$\mathbf{g}^{(t)} = \arg \min \{ \| \mathbf{G} \|_{\ell_1} \text{ subject to } \Phi \mathbf{g} = \mathbf{y}^{(1)} \};$$
2. $t = t + 1$;
3. $S^{(t-1)} := \{ k \in \{1,\cdots,n\} : \mathbf{c}_k^{(t-1)} \neq 0 \}$
4. Compute $\mathbf{p}^{(t)} := [p_{1}^{(t)}, \cdots, p_{n}^{(t)}]$ from (6).
5. Compute $\mathbf{W}^{(t)}$ from (5).
6. $\mathbf{g}^{(t)} = \arg \min \{ \| \mathbf{W}^{(t)} \mathbf{G} \|_{\ell_1} \text{ subject to } \Phi \mathbf{g} = \mathbf{y}^{(t)} \}$;
7. Go to step 2

### IV. Experimental Results

We tested our algorithm on 5 sequences of MRI images of the foot, knee, ankle, neck (all of size 512 × 512 × 20) and skull base (512 × 512 × 40), obtained from [14]. Figure 3 shows some of these MRI images.

For the first image in each sequence, $t = 1$, since no priori knowledge is available, 30% of samples are taken along 150 radial lines in the Fourier domain (see sampling mask in figure 4(a)) while in the successive frames, only 10% of samples are taken along 50 radial lines (figure 4(b)). The reconstruction is carried out in the Wavelet domain which is assumed to be the sparse domain.

Figure 5(a)-(e) compares the reconstruction performance of the Weighted-CS with $\sigma$ set to 12 for MRI sequences.
with those obtained using the modified-CS, priori-CS and \( \ell_1 \) minimization (CS). It can be seen that our method outperforms the other methods significantly in terms of the improved PSNR.

V. CONCLUSION

We have employed a Weighted-CS based method to reconstruct sequences of MRI images from a lower that conventionally possible number of samples. Our method uses the image of the previous time instance to efficiently reconstruct the current image of interest using a weighted-\( \ell_1 \) minimization. Our experimental results on MRI datasets, has shown that our algorithm can achieve a significant reduction in the number of samples needed compared to the other state-of-art CS algorithms. One possible extension of our work would be to use different measures of sparsity such as [15], [16] or [17], to further reduce the number of samples needed.

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REFERENCES

Fig. 5. PSNR of the reconstructed images vs. time.