Cooperative Beamforming for OFDM-based amplify-and-forward Relay Networks

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Abstract

In this paper, we investigate the cooperative beamforming (BF) design problem for amplify-and-forward relay networks over frequency-selective channels. Orthogonal frequency division multiplexing (OFDM) transmission is employed to resist the multipath effects. We focus on the time-domain (TD) BF due to the lower implementation complexity and less feedback requirement from the destination to perform BF. Our aim of the BF design is maximizing the minimum signal-to-noise-ratio (SNR) over all subcarriers at the destination, first under the total power constraint (TPC) and then under the per-relay power constraint (PPC). We show that the BF designs lead to non-convex problems generally. Three approaches to approximate these problems to convex problems are proposed. In the first approach, semidefinite relaxation (SDR) techniques with randomization methods is applied. This approach provides an upper bound of the optimum value of the design problem but with higher complexity. In the second approach, the existing iterative method for cooperative beamforming of multi-group multicasting (MGM) relay networks is extended to solve our problem. And the second approach is further improved by appropriately choosing the initial phase rotation in the third approach. Simulation results demonstrate that the third approach always outperforms the other two approaches. Moreover, the second approach performs closely to the first approach when the filter length is relatively small. When longer TD filter is employed, the second approach outperforms the first approach.

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1. Introduction

Recently, cooperative relay networks have been studied widely as they can exploit spatial diversity to improve network performance with the cooperation of multiple single-antenna terminals[1]. Cooperative diversity can be achieved using either amplify-and-forward (AF) or decode-and-forward (DF) protocol. In the AF relaying, the relay nodes simply scale their received signals, while in the DF relaying, the relay nodes have to decode and re-encode the source messages. Thanks to its low implementation complexity, AF relaying is of practical interest.

With partial or full knowledge of channel state information (CSI) in the AF relaying scheme, distributed beamforming can efficiently improve the capacity and reliability of relay networks[2]. Several distributed BF approaches for AF relay networks have been studied in [2, 3] and references therein. However, only flat fading channels were considered in those works. With the demand of higher transmission rate, frequency-selectivity of the channel cannot be ignored. Unfortunately, techniques optimized for flat fading channels cannot be directly extended to the case of frequency-selective channels. Thus, designing distributed BF over frequency-selective channels is well motivated.

Employing OFDM transmission is an effective approach for cooperative systems to compensate for the multi-path effects. BF design for cooperative OFDM networks over frequency-selective channels was considered in [4, 5, 6], but it was limited to the single relay case. Later, [7] proposed frequency-domain (FD) and time domain (TD) BF schemes for the case of multiple relays. In FD-BF scheme, the BF weights are applied in the FD while cyclic BF filters (C-BFFs) are used on the TD signal in TD-BF schemes. Further, [7] optimized the FD-BF weights and the TD cyclic BF filters (C-BFFs) by maximizing the average mutual information (AMI) per sub-carrier. However, the optimization was subject to total power constraint (TPC) and the BF vectors were designed by ignoring the effects of source-to-relays CSI on the power constraint, which may undermine the TPC under some particular CSI conditions. Further, since each node in a network has its own battery generally, the per-relay power constraint (PPC) BF for AF relaying is more practical than TPC BF.

This paper considers the BF design problem for the relay network with one source, one destination and multiple relays under the assumption of
frequency-selective channels between nodes. Since the TD BF scheme requires lower implementation complexity and less feedback from destination, we mainly focus on the TD BF design. Unlike [7] which used the AMI as design criterion, we design the TD C-BFFs weights by maximizing the minimum (max-min) signal-to-noise-ratio (SNR) at the destination over all subcarriers in order to achieve good quality of service (Qos) performance, subject to two different types of power constraints: TPC as well as PPC. Moreover, the instantaneous CSIs between the source and the relays are taken into consideration; this will guarantee the TPC or PPC can hold true for all channel realizations. Both TPC and PPC design problems are non-convex generally. To solve the BF design problem efficiently, we propose two approaches to approximate the original non-convex BF problems to convex problems. The first approach is using the semidefinite relaxation (SDR) technique, which approximately solve the BF design problem by a convex semidefinite programming (SDP) problem. The SDR approach provides an upper bound of the optimum value of the BF design problem but it increases the computational complexity and the randomization operation followed can not guarantee good performance. The second approach is applying the existing iterative method for cooperative beamforming of multi-group multicasting (MGM) relay networks to solve our problem and the performance of this method. The second approach is further improved by choosing the appropriate initial phase rotation in the third approach. Simulation results show that the second approach performs closely to the first approach when the filter length is relatively small. When longer TD filter is employed, the second approach outperforms the first approach. The third approach always performs better than the second approach with SNR improvement of about 0.6 dB.

The rest of this paper is organized as follows: we describe our model for OFDM based cooperative relay network in Section 2. In Section 3, we formulate the BF design problem based on max-min criterion and propose three approaches to solve it efficiently. In Section 4, we present the numerical results to verify our analysis. Section 5 presents the conclusions of this research.

Notations: Superscripts *, $T$ and $H$ denote the complex conjugate, transpose and Hermitian operators respectively. $\odot$ denotes the hadamard product. $\text{diag}(\mathbf{x})$ denotes the diagonal matrix whose diagonal is vector $\mathbf{x}$. $\mathbf{I}_N$ denotes the $(N \times N)$ identity matrix. $\lambda_{\text{max}}\{\mathbf{A}\}$ denotes the maximum eigenvalue of matrix $\mathbf{A}$ and $\mathbf{P}\{\mathbf{A}\}$ denotes the eigenvector corresponding to $\lambda_{\text{max}}\{\mathbf{A}\}$. 

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2. System model

Considering a relay network with a source, a destination and $N_r$ relays as illustrated in Figure 1. We assume that there is no direct link between the source and destination and $N_r$ relays cooperatively establish the communication link between the source and destination. Each node is equipped with a single half-duplex antenna. The channel between nodes are assumed to be frequency-selective and mutually uncorrelated. Let $h_i$ denote the channel impulse response (CIR) from the source to the $i$th relay and $w_i$ represents the CIR from the $i$th relay to the destination. Without loss of generality, we assume that the frequency-selective channels $h_i, i = 0, \cdots, N_r - 1$ are of equal length, denoted by $L_h$. Similarly, $w_i, i = 0, \cdots, N_r - 1$ are of equal length, denoted by $L_w$.

The AF relaying scheme consists of two phases. In the first phase, the source sends data to the relays. In this paper, the signalling is based on OFDM. In the second phase, the received signals at the relays are AF beamformed and then transmitted to the destination.

As shown in Figure 1, the data block $d = [d(0), d(1), \cdots, d(N_c - 1)]^T$, which satisfy $\mathcal{E}\{|d(n)|^2\} = P_s$, are taken to modulated by IDFT at the source. Therefore, the transmitted symbol vector $x = [x(0), x(1), \cdots, x(N_c - 1)]^T$
can be expressed as
\[ x = F^H d \]  
(1)

where \( F = \frac{1}{\sqrt{N_c}} \left[ e^{-j2\pi mn/N_c} \right]_{m,n=0}^{N_c-1} \) is the \((N_c \times N_c)\) DFT matrix.

After removing cyclic prefix (CP) and normalizing the signals by \( \rho = 1/\sqrt{P_s + 1} \), the signal received at the \( k \)th relay is given by
\[ s_k = \rho H_k x + \rho n_k \]  
(2)

where \( H_k \) is an \( N_c \times N_c \) circulant matrix whose first column is \([h_k(0), \cdots, h_k(L_k-1), 0, \cdots, 0]^T\), \( n_k \) is an additive white Gaussian noise (AWGN) vector with correlation matrix \( \mathbf{I}_{N_c} \), where \( \mathbf{I}_{N_c} \) denotes the \( N_c \times N_c \) identity matrix.

For TD-BF systems, the above signal is cyclic filtered at each relay by a filter \( g_k \) of length \( L_g \leq N_c \). The filtered signal can be expressed as
\[ t_k = \rho G_k H_k x + \rho G_k n_k \]  
(3)

where \( G_k \) is an \( N_c \times N_c \) circulant matrix whose first column is \([g_k(0), \cdots, g_k(L_g-1), 0, \cdots, 0]^T\). More details on the C-BFFs implementation can be found in [8].

After removing the CP, the signal received at the destination can be written as
\[ r = \rho \sum_{k=1}^{N_r} W_k G_k H_k x + \rho \sum_{k=1}^{N_r} W_k G_k n_k + \eta \]  
(4)

where \( W_k \) is an \( N_c \times N_c \) circulant matrix whose first column is \([w_k(0), \cdots, w_k(L_w-1), 0, \cdots, 0]^T\) and \( \eta \) is an AWGN vector with correlation matrix \( \mathbf{I}_{N_c} \). After the DFT, the signal at the destination can be expressed as
\[ R = \rho \sum_{k=1}^{N_r} W_{k,D} G_{k,D} H_{k,D} d + \rho \sum_{k=1}^{N_r} W_{k,D} G_{k,D} \hat{n}_k + \hat{\eta} \]  
(5)

where \( W_{k,D} = \text{diag}\{W_k\}, \ W_k = [W_k(0), \cdots, W_k(N_c-1)]^T, \ W_k(n) = \sum_{l=0}^{L_w-1} w_k(l)e^{-j2\pi ln/N_c}, \ G_{k,D} = \text{diag}\{G_k\}, \ G_k = [G_k(0), \cdots, G_k(N_c-1)], \ G_k(n) = \sum_{l=0}^{L_w-1} g_k(l)e^{-j2\pi ln/N_c}, \ H_{k,D} = \text{diag}\{H_k\}, \ H_k = [H_k(0), \cdots, H_k(N_c-1)]^T, \ H_k(n) = \sum_{l=0}^{L_h-1} h_k(l)e^{-j2\pi ln/N_c} \) and \( \hat{n}_k = F n_k \), \( \hat{\eta} = F \eta \) are AWGN vectors with correlation matrix \( \mathbf{I}_{N_c} \).

Therefore, the signal received on the \( n \)th subcarrier can now be expressed as
\[ R(n) = \rho G(n) H_{eq}(n)d(n) + \rho G(n) (W(n) \odot \hat{n}(n)) + \hat{\eta}(n) \]  
(6)
where $G(n) = [G_1(n), \cdots, G_{N_c}(n)]$ is the beamforming vector, $H_{eq}(n) = W(n) \odot H(n)$ is the equivalent channel vector, $W(n) = [W_1(n), \cdots, W_{N_c}(n)]^T$, $H(n) = [H_1(n), \cdots, H_{N_c}(n)]^T$ and $\hat{n}(n) = [\hat{n}_1(n), \cdots, \hat{n}_{N_c}(n)]^T$.

For FD-BF systems, the relays apply the BF vector in the FD, which requires additional DFT and IDFT operation at the relays. Moreover, for FD-BF scheme, $N_c N_r$ complex weights are required to be fed back ($N_c$ weights per relay) from the destination to the relays to perform BF, whereas for the TD-BF scheme, the feedback amounts to only $N_r L_g$ complex weights. As pointed out by [7], TD-BF is equivalent to FD-BF when $L_g = N_c$. Due to the advantage of the TD-BF scheme at complexity and feedback requirements, we will focus on the TD-BF design problem in the following.

3. Time-domain beamforming design

In this section, we derive the $N_r$ filters $g_k$ that maximize the minimum SNR over all subcarriers subject to TPC. For notation convenience, we define $g = [g_0^T, \cdots, g_{N_r-1}^T]$.

The max-min SNR problem under TPC can be written as

$$\max \min_n \text{SNR}(n) \quad \text{s.t.} \quad P_{\text{total}} \leq P_{\text{max}} \quad (7)$$

where the SNR at the $n$th subcarrier is defined as $\text{SNR}(n) = \frac{P_{\text{sig}}(n)}{P_{\text{noise}}(n)}$, $P_{\text{sig}}(n)$ and $P_{\text{noise}}(n)$ being respectively the power of the signal and noise, $P_{\text{total}}$ is the total power transmitted from the relays, and $P_{\text{max}}$ is the maximum allowable total power.

From eq. (3), the transmitted power of the $k$th relay can be expressed as

$$P_k = \mathcal{E}\{(t_k)^H t_k\} = \rho^2 (P_s G_k H_{k,D} H_{k,D}^H G_k^H + G_k G_k^H) \quad (8)$$

Since $G_k$ can be written as $G_k = g Q_k$, where $Q_k$ is an $L_g N_r \times N_c$ matrix with the $i$th column given by $[0_{1 \times (k-1)L_g}, 1, e^{-j2\pi(i-1)/N_c}, \cdots, e^{-j2\pi(L_g-1)(i-1)/N_c}, 0_{1 \times (N_r-k)L_g}]^T$, eq. (8) can then be rewritten as

$$P_k = g^T T_k g^H \quad (9)$$

where $T_k = \rho^2 (P_s Q_k H_{k,D} H_{k,D}^H Q_k^H + Q_k Q_k^H)$. Thus, the total power transmitted from all relays can be expressed as

$$P_{\text{total}} = \sum_{k=1}^{N_r} P_k = g^T g^H \quad (10)$$
where $T = \sum_{k=0}^{N_r-1} T_k$. It is worth pointing out that $T$ in the above equation was ignored in [7].

From eq. (6), the signal power at the $n$th subcarrier can be written as

$$P_{\text{sig}}(n) = \rho^2 P_s G(n) H_{eq}(n) (H_{eq}(n))^H (G(n))^H$$  \hspace{1cm} (11)$$

and the noise power at the $n$th subcarrier can be obtained as

$$P_{\text{noise}}(n) = \rho^2 G(n) \tilde{W}(n) (\tilde{W}(n))^H (G(n))^H + 1$$  \hspace{1cm} (12)$$

where $\tilde{W}(n) \triangleq \text{diag}\{W(n)\}$. Noting that $G(n) = gK(n)$, where $K(n)$ is an $L_g N_r \times N_r$ matrix with the $i$th column given by $[0_{1 \times (i-1)L_g}, 1, e^{-j2\pi(n-1)/N_c}, \ldots, e^{-j2\pi(L_g-1)(n-1)/N_c}, 0_{1 \times (N_r-i)L_g}]^T$, eq. (11) can be rewritten as

$$P_{\text{sig}}(n) = \rho^2 P_s g \beta(n) (\beta(n))^H g^H$$  \hspace{1cm} (13)$$

and eq. (12) can be rewritten as

$$P_{\text{noise}}(n) = \rho^2 g J(n) g^H + 1$$  \hspace{1cm} (14)$$

where $\beta(n) = K(n) H_{eq}(n)$ and

$$J(n) = K(n) \tilde{W}(n) (\tilde{W}(n))^H (K(n))^H$$

From eq.(13), (14) and eq.(10), the problem (7) can be mathematically formulated as follows:

$$\max_n \min_n \frac{P_s g \beta(n) (\beta(n))^H g^H}{g J(n) g^H + P_s + 1}$$  \hspace{1cm} \text{s.t.} g^H g \leq P_{\text{max}}$$  \hspace{1cm} (15)$$

Letting $g = \tilde{g} T^{-\frac{1}{2}}$, the problem (15) can then be rewritten as

$$\max_n \min_n \frac{P_s \tilde{g} \beta(n) (\beta(n))^H \tilde{g}^H}{\tilde{g} J(n) \tilde{g}^H + P_s + 1}$$  \hspace{1cm} \text{s.t.} \tilde{g} \tilde{g}^H \leq P_{\text{max}}$$  \hspace{1cm} (16)$$

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where \( \tilde{\beta}(n) = T^{-\frac{1}{2}} \beta(n) \) and \( \tilde{J}(n) = T^{-\frac{1}{2}} J(n) T^{-\frac{1}{2}} \). It can be easily proved that the constraint in problem (16) should be met with equality at the optimum solution. Therefore, (16) can be rewritten as

\[
\max \min_n \frac{P_s \tilde{g} \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H \tilde{g}^H}{\tilde{g} \tilde{J}(n) \tilde{g}^H} \quad \text{s.t.} \quad \tilde{g} \tilde{g}^H = P_{\text{max}} \tag{17}
\]

where \( \tilde{J}(n) = \tilde{J}(n) + \left( (P_s + 1)/P_{\text{max}} \right) \mathbf{I}_{L_n N_n} \). Noting that any scaling of \( \tilde{g} \) does not change the value of the objective function in (17), eq. (17) is equivalent to the following unconstrained optimization problem

\[
\max \min_n \frac{P_s \tilde{g} \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H \tilde{g}^H}{\tilde{g} \tilde{J}(n) \tilde{g}^H} \tag{18}
\]

However, the obtained vector from (18) has to be properly scaled to satisfy the power constraint.

Introducing a new auxiliary variable \( t \), eq. (18) can be equivalently transformed to

\[
\max \quad t \quad \text{s.t.} \quad \frac{P_s \tilde{g} \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H \tilde{g}^H}{\tilde{g} \tilde{J}(n) \tilde{g}^H} \geq t, \quad n = 0, \ldots, N_c - 1 \tag{19}
\]

The problem in (19) is generally non-convex. We will propose two approaches to solve our problem approximately.

3.1. SDR method

Using the definition \( \mathbf{X} \triangleq \tilde{g}^H \tilde{g} \), problem (19) can be equivalently rewritten as

\[
\max \quad t \quad \text{s.t.} \quad \frac{P_s \text{tr} \left( \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H \mathbf{X} \right)}{\text{tr} \left( \tilde{J}(n) \mathbf{X} \right)} \geq t, \quad n = 0, \ldots, N_c - 1 \tag{20}
\]

\[
\mathbf{X} \succeq 0 \quad \text{rank} (\mathbf{X}) = 1
\]
Problem (20) is still non-convex due to the non-convexity of the rank-one constraint. By dropping the rank constraint, we get the semidefinite relaxed problem, which can be expressed as

\[
\begin{align*}
\max & \quad t \\
\text{s.t.} & \quad \frac{P_s \text{tr} \left( \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H X \right)}{\text{tr} \left( J(n)X \right)} \geq t, \quad n = 0, \ldots, N_c - 1 \\
& \quad X \succeq 0
\end{align*}
\]

(21)

Letting \( Q(n) = P_s \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H - tJ(n), \) \( n = 0, \ldots, N_c - 1, \) problem (21) is equivalent to

\[
\begin{align*}
\max & \quad t \\
\text{s.t.} & \quad \text{tr} \left( Q(n)X \right) \geq t, \quad n = 0, \ldots, N_c - 1 \\
& \quad X \succeq 0
\end{align*}
\]

(22)

The problem (22) is quasi-convex, because for any value of \( t \), it turns to the following convex feasibility problem:

\[
\begin{align*}
\text{find} & \quad X \\
\text{s.t.} & \quad \text{tr} \left( Q(n)X \right) \geq t, \quad n = 0, \ldots, N_c - 1 \\
& \quad X \succeq 0
\end{align*}
\]

(23)

In fact, the global optimum value \( t^* \) can be obtained by employing bisection search method, which solves the feasibility problem (23) at each step. Assuming \( t^* \) lies in the interval \([t_l, t_r]\), the bisectio

1. Let \( t = (t_l + t_r)/2 \), solve the feasibility problem (23)
2. If (23) is feasible, then \( t_l = t \), otherwise \( t_r = t \).
3. Repeat step 1 with the new interval until \( (t_r - t_l) < \varepsilon \), where \( \varepsilon \) is the error tolerance value.

Remark1: Note that the feasibility problem (23) is a standard semidefinite programming (SDP) problem, which can be solved using interior point methods [9].

Remark2: It is easy to see from (18) that the maximum SNR achieved at the \( n \)th subcarrier is

\[
\gamma(n) = \mathcal{L}_{\text{max}} \left\{ P_s \left( J(n) \right)^{-1} \tilde{\beta}(n) \left( \tilde{\beta}(n) \right)^H \right\}
\]
where $\mathcal{L}_{\text{max}}\{A\}$ denotes the maximum eigenvalue of matrix $A$ [10]. Thus the initial interval for the bisection search can be selected as $[0, \min_\gamma(n)]$.

The matrix $X^*$, which can achieve the optimum $t^*$ is the global optimum vector for problem (20) but not the optimum vector for the problem (18). Due to the relaxation, the matrix $X^*$ will not be of rank one in general. For the cases where the matrix $X^*$ is rank-one, then $t^*$ is also the global optimum vector for problem (18) and its principal component is the optimal solution to the problem (18). For the cases where the matrix $X^*$ is with rank higher than one, $t^*$ is a upper bound for problem (18) and randomization techniques have to be employed to get a good approximation of the rank-1 solution for problem (18) from $X^*$.

Randomization technique: The basic idea in randomization is to use $X^*$ to generate a set of candidate vectors and then select the best solution $\tilde{g}^*$ among these candidates. Three randomization methods are considered in this paper:

- randA: Calculate the eigen-decomposition of $X^* = V \Lambda V^H$ and choose $\tilde{g}_{A,k} = V \Lambda^{1/2} e_k$, where the elements of $e_k$ are independent random variables, uniformly distributed on the unit circle in the complex plane;

- randB: Choose $\tilde{g}_{B,k} = \text{diag} (\text{diag} (X^*))^{1/2} e_k$;

- randC: Choose $\tilde{g}_{C,k} = V \Lambda^{1/2} e_k$, where the elements of $e_k$ are zero-mean, unit-variance complex circularly symmetric uncorrelated Gaussian random variables;

After obtaining the optimum vector $\tilde{g}^*$ for problem (18), the optimum vector for problem (15) is

$$g^* = \sqrt{P_{\text{max}} \tilde{g}^* T^{-1/2}} / \|\tilde{g}^*\|$$

(24)

3.2. iterative method

In the SDR approach, the number of design variables is increased from $L_g N_r$ to $(L_g N_r)^2$, which increases the computational complexity. Moreover, the randomization methods can not guarantee good solutions. To simplify the optimization problem, we rewrite problem (19) as

$$\max \quad t$$

$$\text{s.t.} \quad \frac{P_s \tilde{g} \tilde{\beta}(n) (\tilde{\beta}(n))^H \tilde{g}^H}{\tilde{g} \tilde{J}(n) \tilde{g}^H} \geq t^2, n = 0, \cdots, N_c - 1$$

(25)
Since $\bar{J}(n)$ is a Hermitian matrix, its singular value decomposition (SVD) can be expressed as $\bar{J}(n) = U(n)D(n)U(n)^H$, where $D(n)$ is an unitary matrix and $V(n)$ is an diagonal matrix.\(^1\) Problem (25) is equivalently to
\[
\max_t \quad t \\
\text{s.t.} \quad \sqrt{P_s}|\tilde{g}\tilde{\beta}(n)| \geq t\|\tilde{g}S(n)\|, \ n = 0, \cdots, N_c - 1
\] (26)

It can be solved through bisection search method by solving the following feasibility problem for some given $t$ at each step:
\[
\text{find } \quad \tilde{g} \\
\text{s.t.} \quad \sqrt{P_s}|\tilde{g}\tilde{\beta}(n)| \geq t\|\tilde{g}S(n)\|, \ n = 0, \cdots, N_c - 1
\] (27)

We notice that similar problem as (27) has been studied in [11] for solving the beamforming design problem for MGM networks cooperative relay networks. Thus the idea proposed in [11] could be extended to solve our problem. Further, we will propose an searching procedure at next section for choosing the appropriate phase rotation at the initial iteration, which can improve the performance of this iterative method.

Introducing the real-valued vector $c = [c_1, \cdots, c_{N_c}]^T$, we can formulate a new feasibility problem
\[
\min_{\tilde{g}, c} \quad 1^T c \\
\text{s.t.} \quad t\|\tilde{g}S(n)\| - \sqrt{P_s}\text{Re}\left(\tilde{g}\tilde{\beta}(n)\right) \leq c_n, \ n = 0, \cdots, N_c - 1
\] (28)

Problem (28) is always feasible and the optimal solution can be denoted by $c_{opt}$ and $\tilde{g}_{opt}$. If $1^Tc_{opt} = 0$, problem (27) is feasible and vice versa. Problem (28) can be solved by Algorithm I proposed by [11].

**Algorithm I:**

**Initialization:** $\beta(n)^1 = \tilde{\beta}(n), \ n = 0, \cdots, N_c - 1$

**for** $i = 1: I$

Solve problem (28) with $\tilde{\beta}(n) = \tilde{\beta}(n)^i$ and the optimum solution is denoted by $\tilde{g}_{opt}$.

Perform the rotation: $\tilde{\beta}(n)^{i+1} = \tilde{\beta}(n)^i \exp(-j\alpha(n)^i) \ n = 0, \cdots, N_c - 1$, where $\alpha(n)^i = \angle(\tilde{g}_{opt}\tilde{\beta}(n))$;

\(^1\) Since $\bar{J}(n)$ is not always positive definite, the Cholesky factorization of $\bar{J}(n)$ may not exist.
Thus the whole bisection search method procedure for solving problem (26) can be summarized as follows:

Assuming $t^*$ lies in the interval $[t_l, t_r]$,
1. Let $t \triangleq (t_l + t_r)/2$, solve the problem (28) by Algorithm I
2. If $1^T c_{\text{opt}} = 0$, then $t_l = t$, otherwise $t_r = t$.
3. Repeat step 1 with the new interval until $(t_r - t_l) < \varepsilon$, where $\varepsilon$ is the error tolerance value.
4. Let $t = t_l$, solve the problem (28) by Algorithm I. Then replace $\tilde{\beta}(n)$ in problem (27) with $\tilde{\beta}(n)^{i+1}$ and solve the new problem (27). Then the obtained $\tilde{g}_{\text{opt}}$ is the optimum solution for problem (26).

Remark 3: Note that the problem (28) and (27) are standard second-order cone programming (SOCP) problems with $N_c$ second-order cone constraint, which can be solved using interior point methods [9].

Remark 4: The initial interval for the bisection search can be selected as $[0, \sqrt{\min_n \gamma(n)}]$.

3.3. initial phase rotation searching algorithm

The iterative algorithm described above is by approximating the modulus of $\tilde{g}\tilde{\beta}(n)$ with its real part initially and improve it successively by applying an appropriate rotation to the individual channel vectors in each iteration. However, the real part of $\tilde{g}\tilde{\beta}(n)$ is not always a good initial approximation of its modulus. In this subsection, we propose a searching procedure to obtain good initial approximation. We introduce the following lemma first:

Lemma: (Linear Approximation of Modulus)[12]: The modulus of a complex number $Z \in \mathbb{C}$ can be linearly approximated with the polyhedral norm given by

$$p_L(Z) = \max_{l \in \mathcal{L}} \left\{ \mathcal{R}e\{Z\} \cos \left( \frac{l\pi}{L} \right) + \mathcal{I}m\{Z\} \sin \left( \frac{l\pi}{L} \right) \right\}$$

(29)

where $\mathcal{L} = \{1, 2, \cdots, 2L\}$, $\mathcal{R}e\{Z\}$ and $\mathcal{I}m\{Z\}$ denote the real and imaginary parts of $Z$ and the polyhedral norm $p_L(Z)$ is bounded by

$$p_L(Z) \leq |Z| \leq p_L(Z) \sec \left( \frac{l\pi}{L} \right)$$

(30)

and $L$ is a positive integer such that $L \geq 2$. □
It can be seen that the modulus of a complex number can be approximated by \( \max_{l \in \mathbb{L}} \Re \{ Z e^{-j \frac{2\pi}{L}} \} \). Thus we can replace \( |\tilde{g}_k^{\beta}(n)| \) with \( \Re \left( \tilde{g}_k^{\beta}(n)e^{-j \frac{2\pi}{L}} \right) \), \( l = 0, \ldots, 2L - 1 \) in problem (26) and choose the best solution \( l_{\text{opt}} \) among these 2L candidate solutions. However, when \( L \) is large, the complexity may be problematic in practical systems. The proposed searching algorithm for the best \( l_{\text{opt}} \) is described as follows:

Algorithm II:

**Initialization:**
Set feasible section = [0, 1, \ldots, 2L], search section = [0, 1, \ldots, 2L – 1].
Set num = 0;

**Iteration \( k + 1 \):**
(a) If length(feasible section) = 1, then terminate; the element of feasible section is the optimal \( l_{\text{opt}} \).
(b) If num > num_{max}, then terminate; randomly choose \( l_{\text{opt}} \) in feasible section.
(c) If length(feasible section) \( \neq 1 \) and num <= num_{max}, then
   - Set \( t \triangleq (t_t + t_r)/2 \);
   - Replace \( \Re \left( \tilde{g}_k^{\beta}(n) \right) \) with \( \Re \left( \tilde{g}_k^{\beta}(n)e^{-j \frac{2\pi}{L}} \right) \), \( l = 0, \ldots, 2L - 1 \) in problem (28) and solve these 2L problems by Algorithm I;
   - If neither of the optimum objective function value of these 2L problems is zero, set \( t_r = t \) and search section = feasible section.
   - If some of the optimum objective function value of these 2L problems are zero, the corresponding values of \( l \) combines feasible section. Set \( t_t = t \) and search section = feasible section.
   - Increase num by 1 and repeat the iteration process.

After obtain the best \( l_{\text{opt}} \) by Algorithm II, replace \( |\tilde{g}_k^{\beta}(n)| \) with \( \Re \left( \tilde{g}_k^{\beta}(n)e^{-j \frac{2\text{opt}}{L}} \right) \) in problem (26) and then the optimum solution is obtained.

**PPC case:** For the PPC case, the individual power constraint on \( i \)th relay can be written as \( g^T_k g^H_k \leq P_{k,\text{max}} \), where \( P_{k,\text{max}} \) is the maximal transmitted power of the \( i \)th relay. Here these inequality power constraint can not be equivalent to equality constraint. But such individual relay power constraints can be expressed in the semidefinite matrix constraint or second-order constraint form. Thus the optimization problem for the TD-BF scheme subject to PPC can be solved similarly.
4. Simulation Results

In this section, the performance of TD-BF by three approaches are evaluated for a cooperative network with \( N_r = 2 \) relays. The total number of data (subcarriers) of each block is \( N_c = 64 \) and the data symbols are drawn from BPSK constellations. \( L_h \) and \( L_w \) are set to 10 in the simulations. We also assume that the channel taps \( h_i(l), w_i(l) \) are uncorrelated zero-mean complex gaussian random variables with exponential power delay profile \( p_l = \lambda \exp(-0.2l) \), where \( \lambda \) is a scalar that ensures that the total energy of the channel taps is normalized to unity. The channels corresponding to different relays are assumed uncorrelated and known perfectly at the destination. The latter performs BF design and feedback a BF vector to each relay. In this paper we assume that this feedback is error-free. In the first approach, 100 candidates are generated for each randomization technique. The value of \( I \) in Algorithm I and Algorithm II is chosen to be 5. In Algorithm II, \( num \) is set to be 5 and \( L \) is set to be 2.

Figure 2 depicts the achieved minimum SNR under TPC versus total power per data block for TD-BF with \( L_g = 4 \). The label ‘bound’ represents the non rank-one solution from SDR method, which can not be achieved. The label ‘randA’, ‘randB’ and ‘randC’ represent the rank-one solution from randomization method, respectively. The label ‘iterative’ represents the solution from the iterative method proposed in 3.2 and ‘iterative with search’ represents the solution from the method proposed in 3.3. It is shown that the ‘iterative with search’ method outperforms other algorithms, with about 0.6 dB SNR improvement, compared to the ‘iterative’ method. The ‘randC’ method performs closely to the ‘iterative’ method and performs better than other two randomization techniques.

Figure 3 depicts the achieved minimum SNR under TPC versus total power per data block for TD-BF with \( L_g = 6 \). As can be observed, the ‘iterative with search’ method also outperforms other algorithms, with about 0.6 dB SNR improvement, compared to the ‘iterative’ method. The ‘iterative’ method performs better than SDR techniques with randomization methods and ‘randC’ has the best performance among the three randomization techniques.

Figure 4 shows the performance of the BF schemes under TPC versus filter length \( L_g \) with \( P_{\text{max}}/N_r = 21 dB \). From Figure 4, we can find that the minimum SNR over all subcarriers is improved with the increase of the filter length.
Figure 2: SNR versus average relay power, $L_g = 4$

Figure 3: SNR versus average power, $L_g = 6$
5. Conclusion

In this paper, we addressed the problem of BF design for OFDM based cooperative networks over frequency-selective channels. Based on max-min SNR criterion, the TD BF vectors were obtained through three methods. The first method was based on SDR technique with randomization methods. The second approach was an iterative method which was further improved by choosing appropriate initial phase rotation in the third method. It was shown that the third method achieved the best performance.

References


